Optimal Investment Allocation with HJB and PINN

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Goal / Problem

- Allocate wealth among stocks, bonds, and cash to **maximize final wealth**.
- Decisions now affect future outcomes: need planning over time.
- Use **dynamic programming** to find optimal strategy.

Model

- Wealth: $W(t)$. Allocation $a(t) = [s, b, c]$ with $s + b + c = 1$.
- Returns: r_s, r_b, r_c for stock, bond, cash.
- Wealth dynamics:

$$\frac{dW}{dt} = W(t)\big(sr_s + br_b + cr_c\big).$$

- Value function $V(t, W)$: best final wealth from (t, W).

Key Equation: HJB

- The value function satisfies:

$$\frac{\partial V}{\partial t} + \max_{s+b+c=1} \left\{ W\left(sr_s + br_b + cr_c\right) \frac{\partial V}{\partial W} \right\} = 0$$

- Terminal condition: $V(T, W) = W$.
- Inner maximization gives optimal allocation $a^*(t, W)$.

Solution Method: PINN

- Approximate $V(t, W)$ by a neural network V_θ.
- Enforce PDE and terminal condition via loss:

$$\mathcal{L}(\theta) = \mathbb{E}\left[\left(\frac{\partial V_\theta}{\partial t} + \max_a \left\{ W(\cdots)\frac{\partial V_\theta}{\partial W} \right\}\right)^2\right] + \lambda\, \mathbb{E}\left[(V_\theta(T, W) - W)^2\right]$$

- Train to minimize $\mathcal{L}$, then extract $a^*(t, W)$.

Data Testing

- Use historical returns to compute covariance and correlation.
- Apply **PCA** to capture main market directions and reduce noise.
- Use **clustering** to define different market scenarios.
- Test learned policy across scenarios and compare with simple baselines.

Summary & Next Steps

- Model: wealth ODE + HJB equation for optimal planning.
- Solver: Physics-Informed Neural Network learns value function.
- Data: PCA and clustering add robustness and insight.
- Next: simulate policy, evaluate performance, and refine scenarios.

Thank you.