Risk Management Project - EVT

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1 Introduction

This report focuses on fitting extreme outcomes of the negative logreturns, per \$100 invested, for two selected stocks: Royal Bank of Canada(RY) and Morgan Stanley(MS), using the Block Maxima Method(GEV) and the General Pareto Distribution(GPD) Method. Weekly returns from the period January 1, 1998 to June 30, 2017 were used for analysis. In this report, individual results of the two stocks are presented first, following which a comparison between the two stocks.

2 Royal Bank of Canada

Royal Bank of Canada stock return extreme value study:

(a) Below is the histogram of observed M_j.

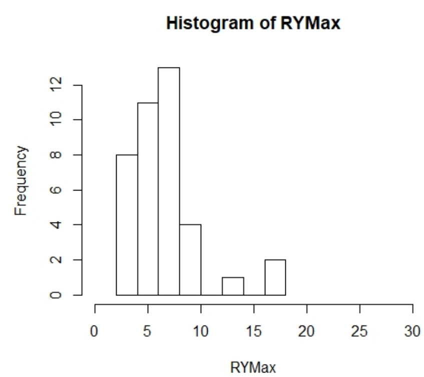

Figure 1: Royal Bank of Canada Stock Return

(b) The fit.GEV function from the R package "QRM" is used. Results for $\hat{\xi}$, $\hat{\mu}$, $\hat{\sigma}$ are summarized in below table:

$\hat{\xi}$	$\hat{\mu}$	$\hat{\sigma}$
0.1122853	4.956501	2.080714

Table 1: Estimated parameters of the GEV model for Royal Bank of Canada

(c) The QQ-plot (Figure 2) indicates an unsatisfactory fit of data with the GEV model. We see that data fitted quite well at the left tail, but poorly at the right end tail, especially for the last 3 data points. No conclusion can be drawn as for now, and further study is required for this model.

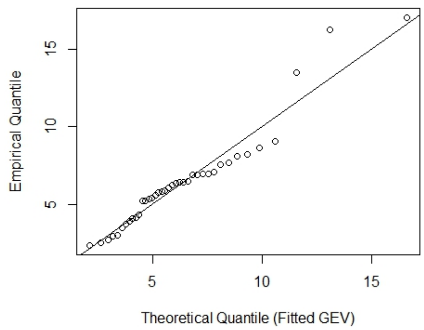

Figure 2: Royal Bank of Canada QQ plot

(d) We now apply likelihood ratio test(LRT) on $H_0 : \xi = 0$ to test if $\xi = 0$. This gives $D = 1.302969$ following the $\chi^2(1)$ distribution, and produces p-value=0.2536716, which is greater than 0.05. Therefore we cannot reject null hypothesis that $\xi = 0$. This is in line with our estimation for $\hat{\xi}$, as 0.1122853 is very close to 0.

(e) We know that the GPD mean excess loss function is a straight line, hence the region where the empirical mean excess loss function becomes approximately linear indicates where the GPD becomes a good approximation to the distribution of excess losses. This means that we need to choose a threshold d such that the plot is approximately linear in the region to the right of d. After plotting Sample Mean-Excess Plot in Figure 3(a) and zooming in the region of interest in Figure 3(b), we select the threshold $d = 6$.

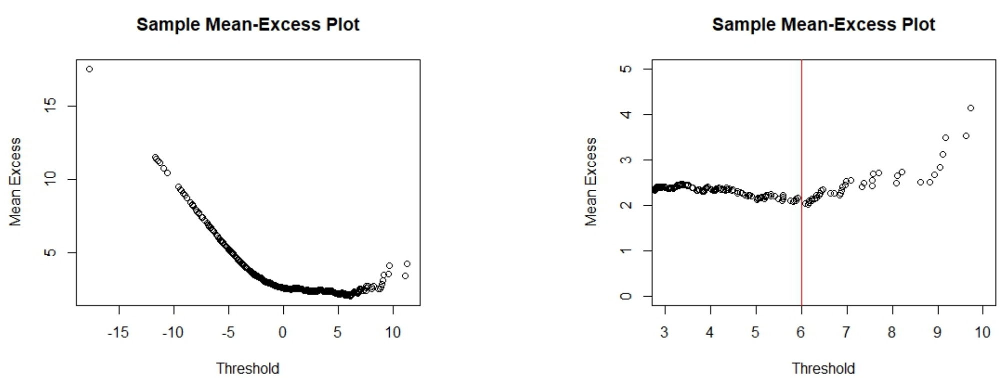

Figure 3: Royal Bank of Canada ME plot

(f) Using the fit.GPD function with threshold $d = 6$, we get:

$\hat{\xi}$	$\hat{\beta}$
0.1905133	1.690758

Table 2: Estimated parameters of the GPD model for Royal Bank of Canada

(g). We now apply likelihood ratio test(LRT) on $H_0 : \xi = 0$ to test if $\xi = 0$. This gives $D = 3.401354$ following $\chi^2(1)$ distribution, and produces p-value$= 0.06514294$, which is greater than 0.05. This shows that we still cannot reject H_0, although the p-value is quite close to 0.05.

(h) $VaR_\alpha(X)$ and $CTE_\alpha(X)$ for $\alpha = 0.99$ and $\alpha = 0.995$ together with empirical VaR and CTE are presented in table below, where $VaR_\alpha = d + \frac{\beta}{\xi}((\frac{S_X(d)}{1-\alpha})^\xi - 1)$ and $CTE_\alpha = \frac{1}{1-\xi}(VaR_\alpha + \beta - \xi d)$.

	VaR	CTE	$\hat{VaR}$	$\hat{CTE}$
$\alpha = 0.99$	8.854395	11.61486	8.912565	11.35086
$\alpha = 0.995$	10.51019	13.66035	9.711564	13.16329

Table 3: VaR, CTE under GPD model and empirical $\hat{VaR}$, $\hat{CTE}$

(i) Our estimate from GPD model is very close to empirical estimate of VaR and CTE, which shows that the this model is fitted fairly well.

3 Morgan Stanley

Morgan Stanley stock return extreme value study:

(a) Below is the histogram of observed M_j.

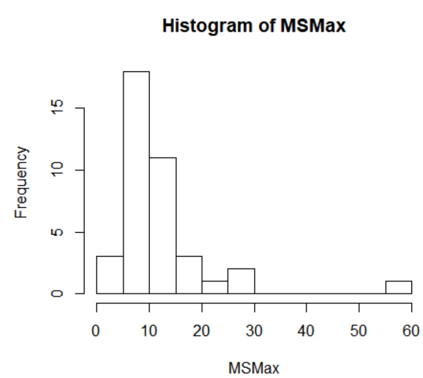

Figure 4: Morgan Stanley Stock Return

(b) We used fit.GEV function and $\hat{\xi}$, $\hat{\mu}$, $\hat{\sigma}$ are summarized in below table:

$\hat{\xi}$	$\hat{\mu}$	$\hat{\sigma}$
0.296817	7.903999	3.927648

Table 4: Estimated parameter of GEV model for Morgan Stanley

(c) A QQ-plot is created in Figure 5 to check if the data fits well with the GEV model. The plot fits quite well with the 45 degree line, except for the last data point.

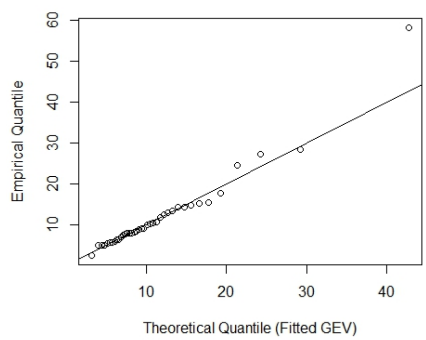

Figure 5: Morgan Stanley QQ plot

(d) We applied likelihood ratio test on $H_0 : \xi = 0$ to test if ratio test will prefer the simpler model. After calculating likelihood ratio from $\xi = 0$ model, we get $D = 13.18707$ following $\chi^2(1)$ distribution, which gives $p = 0.0002818879 < 0.05$, indicating that we should reject null hypothesis. And because estimated ξ is 0.296817, this is expected.

(e) After plotting Sample Mean-Excess Plot in Figure 6(a) and zooming in the region of interest Figure 6(b), we select the threshold $d = 6$.

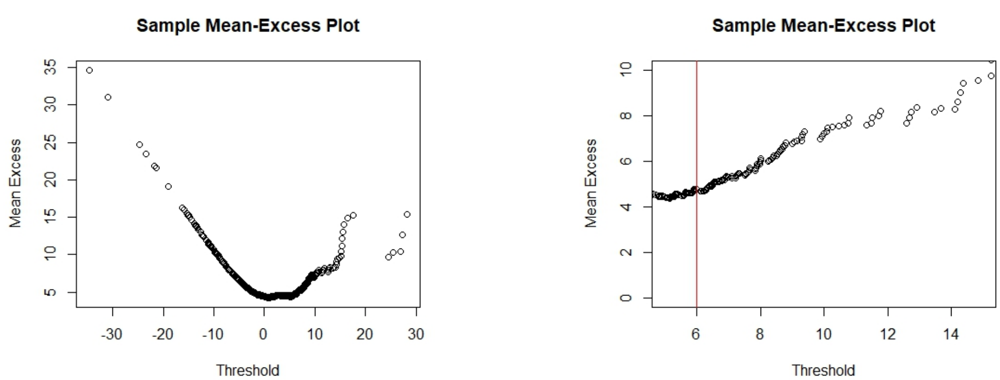

Figure 6: Morgan Stanley ME plot

(f) Using the fit.GPD function with threshold $d = 6$, we get:

$\hat{\xi}$	$\hat{\beta}$
0.3671599	3.041334

Table 5: Estimated parameters of the GPD model for Morgon Stanley

(g) To test $H_0 : \xi = 0$ against $H_1 : \xi \neq 0$ we calculated likelihood ratio.
This gives $D = 44.88074$ following $\chi^2(1)$ distribution, and produces p-value $= 2.09408e{-11}$. This shows that we should reject null hypothesis.

(h) $VaR_\alpha(X)$ and $CTE_\alpha(X)$ for $\alpha = 0.99$ and $\alpha = 0.995$ together with empirical VaR and CTE are presented in table below, where $VaR_\alpha = d + \frac{\beta}{\xi}((\frac{S_X(d)}{1-\alpha})^\xi - 1)$ and $CTE_\alpha = \frac{1}{1-\xi}(VaR_\alpha + \beta - \xi d)$.

	VaR	CTE	$\hat{VaR}$	$\hat{CTE}$
$\alpha = 0.99$	18.08309	29.89928	15.74282	26.84011
$\alpha = 0.995$	23.98553	39.22618	28.51581	35.64741

Table 6: VaR, CTE under GPD model and empirical $\hat{VaR}$, $\hat{CTE}$

(i) From the table above, we observe the estimate from GPD model is not far from the empirical value, but is not as close as what we had for the RBC stock. There are several outliers that are influencing the empirical VaR and CTE so our estimate cannot capture these extremely extreme cases.

4 Comparison

Royal Bank of Canada mainly operates in Canada and UK, where the return on stocks are stable, but tedious; in comparison, Morgan Stanley operates in a riskier environment and is often considered a volatile stock. The volatility of the stocks is demonstrated by the βs listed on the Toronto Stock Exchange - with $\beta_{RY} = 1.073$ and $\beta_{MS} = 1.630$. Hence we'd expect a better estimate on Royal Bank of Canada in comparison to Morgan Stanley.

By comparing the histogram of M_j for both stocks(Figure 7), we find that Morgan Stanley has a more volatile movement than Royal Bank of Canada. Additionally MBMax has a thicker tail, which is in line with $\hat{\xi_{RY}} < \hat{\xi_{MS}}$ calculated using the two methods.

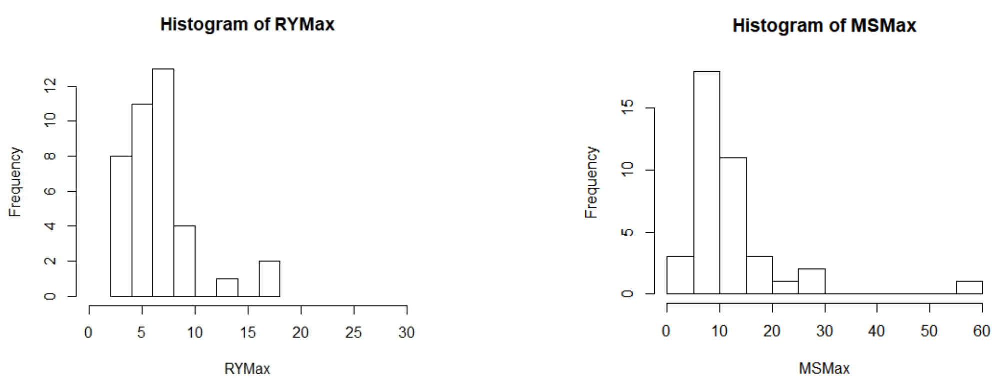

Figure 7: Comparison on Stock Return

QQ plots(Figure 8) show that Morgan Stanley has a better fit with the GEV model comparing to Royal Bank of Canada, as the data points appear to be more in line with the 45 degree line.

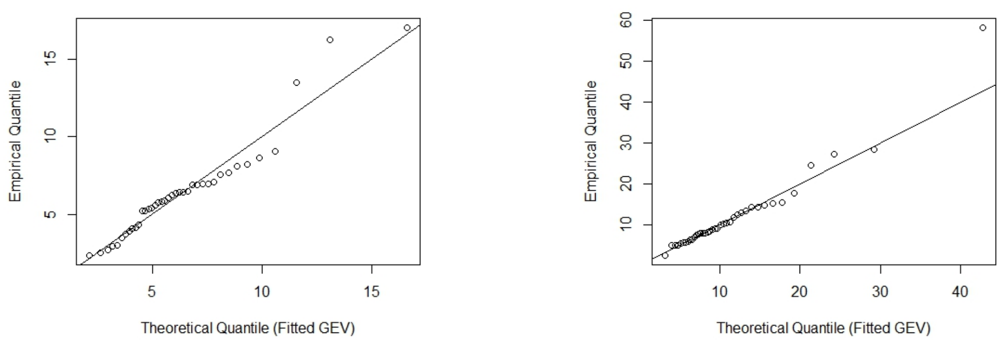

Figure 8: Comparison on QQ plots of fitted GEV model

However, GPD model appears to fit better with Royal Bank of Canada in comparison to Morgan Stanley. This is because the estimated VaR and CTE values using GPD are closer to the empirical estimates. Additionally, we find that the estimated VaR and CTE are higher for Morgan Stanley, which is in line with our expectations that MS is more volatile.

Results of this study show that GPD model is fitted better on Royal Bank of Canada, whereas GEV model is fitted better on Morgan Stanley.