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Evaluation of Competition Indices in Individual Tree Growth Models

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ABSTRACT. In this paper we explore how well distance-independent competition measures explain variation in the height and diameter squared growth of individual conifer trees. We investigated a number of stand-level density indices and individual tree competition indices which incorporate tree sizes, but do not include location. We model growth of individual trees as potential growth reduced by competition. The reduction in mean square error relative to no competition index was used to judge performance of each competition index which varies by species and growth component (height or diameter squared growth). Results are summarized by species and type of competition index. The distance-independent measures are also compared to selected distance-dependent measures shown in recent research work to perform well for conifer species. It was found that a new class of distance-independent indices that includes estimated crown parameters performs on par with the best distance-dependent competition indices when used in conjunction with models of individual tree height and diameter squared growth. FOR. SCI. 41(2):360–377.
ADDITIONAL KEY WORDS. Growth and yield, resource partitioning, growing space, competition index.

COMPETITION AMONG TREES FOR RESOURCES has been studied quantitatively by numerous authors (e.g., Alemdag 1978, Bella 1971, Biging and Dobbertin 1992, Daniels 1976, Daniels et al. 1986, Doyle 1983, Holmes and Reed 1991, Pelz 1978, Pukkala 1989, Pukkala and Kolström 1987, Lorimer 1983, Martin and Ek 1984, and Tomé and Burkhart 1989). There are two major classes of competition measures: those which utilize individual tree location termed *distance-dependent,* and those not using location termed *distance-independent* (Munro 1974). This paper primarily addresses distance-independent indices although it compares them to selected distance-dependent indices shown to perform well in the mixed conifer forests of California (Biging and Dobbertin 1992).

There has been considerable debate in the literature as to whether tree spatial information improves predictions of individual tree growth. From a heuristic viewpoint it seems likely that knowledge of tree locations and their sizes should improve our ability to characterize between-tree competition. However, the literature on this point is mixed. Those finding that distance-dependent competition indices performed well include Doyle (1983) who studied competition in pure and mixed stands of tulip-poplar and white oak. He found that the distance-dependent indices which included some relative measure of subject-competitor size ratio were among the best performing indices. These included Bella's (1971) index, distance-weighted size ratios, and an area polygon model modified so that only one competitor can affect the crown dimensions in any one direction. Noone and Bell (1980) found that Lin's index (Lin 1969), which is based on the ratios of competing tree dbh's to subject tree dbh and competition angle in each of four quadrants, had the highest correlation with measured growth evaluated on a single set of plots for Douglas-fir. Alemdag (1978) found that the distance-dependent indices of Bella (1971), Hegyi (1974), and Alemdag (1978) out-performed other indices tested based on analysis of white spruce plantation data. Daniels et al. (1986) found that for basal area growth of loblolly pine, the weighted area potentially available index (APA) [Moore et al. (1973), Pelz (1978)] had the highest contribution to R^2. Pukkala and Kolström (1987) found that the sum of angles which span competitor dbh's, a measure of vertical angles between the horizontal plane at the top of a tree and the top of competitor trees, and Hegyi's index explained over 50% of the variation in radial-growth in Scots pine stands in Finland. Biging and Dobbertin (1992) found that many of the common distance-dependent measures, which use relative sizes of dbh or total height, did not perform as well as expected for the conifer species they studied. However, they found that a considerable improvement in distance-dependent competition indices was obtained when estimated crown parameters (size, surface area, and cross-sectional area) were included in the indices.

Another portion of the experimental data supports the alternative hypothesis that tree spatial information does not add to our ability to characterize between-tree competition. For example, Lorimer (1983) examined natural even-aged hardwood stands in Wisconsin, but found that intertree distances were of no real value in predicting growth of individual trees. Likewise Martin and Ek (1984) reported improvements to a diameter growth model for red pine, when including a competition index, but obtained only small improvements by using intertree distances. Hatch et al. (1975) working with red pine in Minnesota reported that intertree distances were seldom better than a function of initial dbh in accounting for variability in diameter growth. Daniels et al. (1986) reported similar findings for loblolly pine. In their study they reported that for basal area growth of loblolly pine, weighted APA [Moore et al. (1973), Pelz (1978)] had the highest contribution to R^2, followed closely by tree crown ratio.

From these past studies it is evident that no single index or index type (distance-independent or distance-dependent) has been shown to be a superior competition index, and some indices perform better with specific species in specific situations. It should also be noted that with the exception of work such as Lorimer (1983), Doyle (1983), and Biging and Dobbertin (1992) most studies have been performed on plantation or even-aged stand datasets, and rarely are both classes of indices extensively studied on the same dataset. This fact could be important because in plantations the spacing is relatively controlled and hence knowledge of location is of lesser value.

OBJECTIVES

The objective of this study was to compare the predictive capability of selected distance-independent competition indices against the class of competition indices which uses tree location. The competition indices are used in conjunction with growth models of height and diameter squared for the mixed conifer species of Northern California. Several new distance-independent indices based upon estimated crown geometry are developed and investigated as alternatives to the traditional distance-independent competition indices. Because this study relies on data from mixed species in multiple-aged stands, it is postulated that the range of competitive effects will be greater than in plantations or even-aged stands. If distance-dependent competition indices are superior to distance-independent measures this should become evident in heterogeneous stands, such as those examined in this study, where individual tree competition varies greatly among individuals. Towards this end we compare the performance of the distance-independent indices examined in this paper to several of the distance-dependent indices shown by Biging and Dobbertin (1992) to perform well for height and diameter squared growth in mixed conifer stands in California.

DATA SOURCES

Data for this study were collected in the mixed conifer forest type in cooperation with 12 forest industry contributors of the Northern California Yield Cooperative. The mixed conifer forest type includes ponderosa pine (*Pinus ponderosa* [Laws.]), Douglas-fir (*Pseudotsuga menziesii* [Mirb.] Franco), white fir (*Abies concolor* [Gord. & Glend.] Lindl., red fir (*Abies magnifica* A. Murr.), and sugar pine (*Pinus lambertina* Doubl.). In this report we present results from the two most prevalent species in the dataset—shade tolerant white fir (WF) and intolerant ponderosa pine (PP). The primary modeling data set consisted of stem analysis data described by Biging (1984, 1985) and Biging and Dobbertin (1992). These data consist of measurements on both felled and standing trees. A total of 106 plots [thirty clusters containing three one-fifth ac (0.08 ha) plots each and eight clusters containing two one-tenth ac (0.04 ha) plots each] were established and measured for this stem analysis growth project. Management activities had not taken place on the plots within the last 15 yr, or more, which ensured that the observed growth rates were not affected by recent changes in stand structure.

Every plot was stem mapped and species, dbh, total height, crown length, and 5-yr radial increment were recorded for trees greater than 5.5 in. (14.0 cm) in dbh on the main plot (1/5 or 1/10 ac). Approximately 12 trees per plot were selected for felling, including 4 to 6 dominants to represent the 1 to 2 most prevalent species in the stand, and up to 7 additional trees randomly selected to represent the range of diameter classes present. The plots ranged in stocking from 70 to 305 ft^2 of ba/ac (16 to 70 m^2/ha).

METHODS

All competition indices investigated in this study are evaluated in conjunction with growth models of height and diameter squared for the mixed conifer species of Northern California.

In this work we investigate two types of distance-independent competition measures (with and without estimated tree crown parameters) and a spatial pattern index for their capacity to explain height and diameter squared growth dynamics of individual trees. We compare the performance of these indices with selected distance-dependent indices shown recently to perform well for mixed conifer forests in California.

Conventional Distance-Independent Stand Density Measures

As defined by Munro (1974), distance-independent competition indices are measures of competition which do not utilize spatial information explicitly in their formulation. There are a number of indices that can be researched. In our study we examine some of the most widely used density measures as discussed by Davis and Johnson (1987). Stand density is typically measured by basal area (*BA*), number of trees (*TPA*), crown competition factor (*CCF*), or Reineke's (1933) stand density index (*SDI*). These measures can be improved upon by dividing density into two components. For example, Wykoff et al. (1982) defined a variable formulating basal area in trees larger than the subject tree (*BAL*) for modeling mixed conifer forests in the Intermountain region of the United States. The change from *BA* to *BAL* simply reflects a change in the sample used to calculate the statistic.

Stand density measures reflect the degree of crowding of trees within a unit area (Husch et al. 1982). We rescaled the stand density measures by dividing them by a factor of 500 to reduce the magnitude of the regression coefficients that we were estimating. The negative exponential competition equation used produces a competition factor (CF) on a relative scale between zero (no competition) and one (competition saturation). All distance-independent competition indices examined are listed in Table 1.

Distance-Independent Competition Indices which Utilize Crown Information

The horizontal spacing and density of trees has been used to estimate competition in most competition studies. However, light is a vertically distributed resource whose interception is related to an individual tree's crown area and to the structure of the canopy (Hatch et al. 1975, Doyle 1983). A tree's structural capacity to intercept available light has been shown to be important in the dynamics of mixed-species hardwood stands (Doyle 1983).

In this study we investigate competition indices which incorporate the vertical and horizontal distribution of foliage. Thus, we develop several measures for conifer trees that take into account the vertical position and size of the crown relative to its neighbors. The crown parameters investigated were developed by Biging and Wensel (1990) and also used in the development of distance-dependent competition indices by Biging and Dobbertin (1992). [For a complete formulation of the models see Biging and Dobbertin (1992) or Biging and Wensel (1990)]. In brief, these models predict the total cubic volume (ft^3) occupied by individual tree crowns, the cumulative crown volume (ft^3) from the crown base to any other point within the crown, the crown cross-sectional area (ft^2) at any height within the crown, the total crown surface area (ft^2) and cumulative crown surface area (ft^2) from the crown base to any other point within the crown. The data used to estimate these crown parameters were gathered on a subset of the data used in the present study.

One of the most widely used stand density measures is crown competition factor (*CCF*). *CCF* is defined as the percent of an acre occupied by the crown projection of trees assuming that each individual is open grown. For this research we used the equations of Paine and Hann (1985) to predict open-grown crown radius. *CCF* represents an upper limit on the occupation of trees. This measure has been modified by Ritchie and Hann (1986) to calculate crown competition factor in trees larger than the subject tree (*CCFL*). Throughout this paper *CCFL* is placed into the category of measures which utilizes crowns although it could also be categorized under conventional measures.

In this study we investigate seven crown-related distance-independent measures including *CCFL* (see Table 1). Three other measures which are analogous to *CCFL* in their formulation are examined. They include estimated crown cross-sectional area (ft^2) in trees larger than the subject tree (*CCL*); estimated crown volume (ft^3) in trees larger than the subject tree (*CVL*); and estimated crown surface area (ft^2) in trees larger than the subject tree (*CSAL*). We include crown volume and crown surface indices because other studies have shown that crown volume and especially crown surface area are highly correlated to growth (Assmann 1970, Biging and Dobbertin 1992, Dong and Kramer 1986, Kramer 1988, and Mitchell 1975). Because within a species growth is correlated to a tree's crown foliage area, we hypothesized that crown surface area indices should be superior to either cross-sectional area or crown volume measures.

The other three crown-based indices, evaluated at a percent (p) of the subject tree's height, [crown cross-sectional area (CC_p), crown volume (CV_p), and crown surface area (CSA_p)] are derived from models of crown geometry (Biging and Wensel 1990), but are integrated over the entire plot (or stand) and are not limited to trees larger than the subject tree. Each individual tree's cross-sectional competition index (CC_p) is found by taking a horizontal slice through the canopy at a percent (p) of the ith subject tree's total height (h_i). The crown cross-sectional area at this height is summed and then expanded to a per acre basis. It is then divided by the number of square feet in an acre to produce a competition index. The CC_p index was used by Wensel et al. (1987) for modeling mixed conifer growth dynamics and by Krumland (1982) in modeling growth of redwood and Douglas-fir. This index is graphically displayed in Figure 1 and mathematically expressed in Equation (1) below. In Wensel et al. (1987), the evaluation point of p equal to 66% of the height of each subject tree was selected conceptually without empirical analysis of the mixed conifer data sets they modeled. In another study, Krumland (1982) found that an evaluation point of p equal to 66% of the height of the subject tree produced the greatest reduction in residual sum of squares of his diameter growth model. One purpose of this study was to determine the optimal height from which to evaluate competition for both diameter squared and height growth.

For this study, the heights at which the crown dimensions were calculated are $p = \{25, 33, 50, 66, 75, \text{and } 100\}$ % of the height of the subject tree, and also at the height of the base of the crown. Competition is then computed for each tree ($i = 1, 2, 3, \ldots, n$) in the plot (or stand) as follows:

TABLE 1.

Competition indices examined and their definitions.

Type*	Variable name	Variable definition	Equation
SL/DI	*TPA*	Trees per acre divided by 500	$\frac{\Sigma n_i}{\text{plot size}} \cdot \frac{1}{500}$
SL/DI	*BA*	Basal area (ft²/acre) divided by 500	$\frac{0.005454}{500} \cdot \Sigma DBH_j^2$
SL/DI	*SDI*	Stand density index divided by 500	$SDI = \frac{1}{500} \cdot N \cdot \left[\frac{\overline{D}}{10}\right]^{1.605}$
SL/DI	*BAL*	Basal area in trees larger than the subject tree i (ft²/acre); competitors (j) are trees whose $DBH_j > DBH_i$	$\frac{0.005454}{500}\ \Sigma DBH_j^2$
SL/DI/C	*CCFL*	Crown competition factor in trees larger than the subject tree i, evaluated at crown base of tree j; if $DBH_j > DBH_i$ then j is a competitor of i	$\frac{1}{4 \cdot 43560}\ \Sigma CD_j^2\ (hcb_j) \cdot \pi \cdot 100$
SL/DI/C	*CCL*	Estimated crown cross-sectional area in trees larger than the subject tree i; evaluated at crown base of tree j; if $DBH_j > DBH_i$ then j is a competitor of i	$\frac{1}{43560}\ \Sigma CC_j(hcb_j) \cdot TPA_j$
SL/DI/C	*CVL*	Estimated crown volume in trees larger than the subject tree; evaluated at the crown base of tree j; if $DBH_j > DBH_i$ then j is a competitor of i	$\frac{1}{500000}\ \Sigma CV_j(hcb_j) \cdot TPA_j$
SL/DI/C	*CSAL*	Estimated crown surface area in trees larger than the subject tree i; evaluated at the crown base of tree j; if $DBH_j > DBH_i$ then j is a competitor of i	$\frac{1}{250000}\ \Sigma CSA_j(hcb_j) \cdot TPA_j$
TL/DI/C	CC_p	Crown closure (ft²/acre) evaluated over all competitors j ($j \neq i$) at a percentage p of the ith subject tree's height	$\sum_{j \neq i} \frac{CC_j\ (p \cdot h_i) \cdot TPA_j}{43560\ (\text{ft}^2/\text{ac})}$
TL/DI/C	CV_p	Crown volume (ft³/acre) evaluated over all competitors j ($j \neq i$) at a percentage p of the ith subject tree's height	$\sum_{j \neq i} \frac{CV_j(p \cdot h_i) \cdot TPA_j}{500000\ (\text{ft}^3/\text{ac})}$

TABLE 1. Continued.

Type*	Variable name	Variable definition	Equation
TL/DI/C	CSA_p	Crown surface area (ft^2/acre) evaluated over all competitors j ($j \neq i$) at a percentage p of the ith subject tree's height	$\sum_{j \neq i} \frac{CSA_j(p \cdot h_i) \cdot TPA_j}{250000\ (ft^2/ac)}$
TL/DD/C	H-$H1$	Hegyi's competition index for subject tree i Competitors are chosen with $2m^2$/ha angle gauge	$\sum_{j \neq i} e_{ij} \frac{DBH_j}{DBH_i \cdot (Dist_{ij} + 1)}$
TL/DD/C	ME-$H1$	Martin-Ek competition index for subject tree i Competitors are chosen with $2m^2$/ha angle gauge	$\sum_{j \neq i} e_{ij} \frac{DBH_j}{DBH_i} \exp \left[\frac{16 Dist_{ij}}{DBH_i + DBH_j} \right]$
TL/DD/C	B-$CW2$	Bella's competition index for subject tree i Competitors chosen within a zone 2× the sum of open-grown crown width of subject tree and competitor	$\sum_{j \neq i} e_{ij} \frac{O_{ij} \cdot DBH_j}{Z_i \cdot DBH_i}$
TL/DD/C	CC_{66}-$H2$	Crown cross-sectional competition index evaluated at 66% of subject tree height i for all competitors j ($j \neq i$). Competitors are chosen with a 50° vertical angle gauge from foot of subject tree	$\sum_{j \neq i} e_{ij} \frac{CC_j\ (0.66 \cdot h_i)}{CC_i\ (0.66 \cdot h_i) \cdot (Dist_{ij} + 1)}$
TL/DD/C	CC_{75}-$D2$	Crown cross-sectional competition index evaluated at 75% of subject tree height i for all competitors j ($j \neq i$). Competitors chosen with $2m^2$/ha angle gauge	$\sum_{j \neq i} e_{ij} \frac{CC_j\ (0.75 \cdot h_i)}{CC_i\ (0.75 \cdot h_i) \cdot (Dist_{ij} + 1)}$
TL/DI/C	CV_{HCBU}-$H2$	Crown volume competition index evaluated at the crown base of subject tree i, for all competitors j ($j \neq i$). Competitors are chosen with a 50° vertical angle gauge from foot of subject tree	$\sum_{j \neq i} e_{ij} \frac{CV_j\ (a_{ij})}{CV_i\ (hcb_i)}$
SL/DD/SP	CL-EV	Clark and Evans Index	$2r \cdot \sqrt{P} = 2 \cdot \frac{\Sigma w^2}{n} \sqrt{P}$

* C = utilized estimated crown parameters
DI = distance independent index
DD = distance dependent index
SL = stand level competition index
SP = spatial pattern index

TL	=	tree level competition index
a_{ij}	=	the point where an angle gauge centered on subject tree i intersects the vertical axis of tree j
$CC_i\ (x)$	=	crown cross-sectional area (ft^2) of subject tree i evaluated at a height x
$CC_j\ (x)$	=	crown cross-sectional area (ft^2) of competitor tree j evaluated at a height x
$CD_i\ (x)$	=	open-grown crown diameter (ft) of subject tree i evaluated at a height x
$CD_j\ (x)$	=	open-grown crown diameter (ft) of competitor tree j evaluated at a height x
$CV_i\ (x)$	=	crown volume (ft^3) of subject tree i above a given height x
$CV_j\ (x)$	=	crown volume (ft^3) of competitor tree j above a given height x
$Dist_{ij}$	=	Distance (ft) of subject tree i to competitor j
DBH_i	=	Diameter at breast height (in.) of subject tree i
DBH_j	=	Diameter at breast height (in.) of competitor trees ($j \neq i$)
$\overline{D}$	=	average stand dbh (in.)
e_{ij}	=	linear expansion factor
exp	=	base of the natural logarithm
hcb_i	=	height (ft) of evaluation at the crown base of the subject tree i
hcb_j	=	height (ft) of evaluation at the crown base of the competitor tree j
n	=	number of randomly selected plants
N	=	total number of trees per acre
O_{ij}	=	crown overlap (ft^2) (or influence zone overlap) between subject tree i and competitor j
$p \cdot h_i$	=	height (ft) of evaluation as a percentage (p) of the height of the subject tree i (h_i)
P	=	number of plants per unit area
r	=	average plant-to-nearest-plant distance (ft)
TPA	=	number of trees per acre
w^2	=	squared plant-to-nearest-plant distance (ft)
Z_i	=	crown projection area (ft^2) (or influence zone) of subject tree i

$$CC_p = \sum_j \frac{CC_j(p \cdot h_i)TPA_j}{43{,}560\ (\mathrm{ft}^2/\mathrm{ac})} \tag{1}$$

$$CV_p = \sum_j \frac{CV_j(p \cdot h_i)TPA_j}{500{,}000\ (\mathrm{ft}^3)} \tag{2}$$

$$CSA_p = \sum_j \frac{CSA_j(p \cdot h_i)TPA_j}{250{,}000\ (\mathrm{ft}^2)} \tag{3}$$

where TPA_j is the number of trees per acre represented by tree j, and $\sum_j$ is the sum for all trees on the plot ($j = 1, 2, \ldots, n$). The crown cross-sectional area at the crown base is projected to the ground so that larger trees can have a competitive influence on smaller subject trees (see Figure 1). The notation $CC_j(p \cdot h_i)$ means that the crown cross-sectional area of the jth tree (CC_j) is evaluated at a percentage (p) of the total height (h_i) of the ith subject tree. Dividing the summed crown cross-sectional area by 43,560 converts the absolute area to relative area commonly used for expressing crown closure. This is directly analogous to crown closure percent except that it is evaluated at a given height in the canopy not on the ground. Figure 1 gives an example for the calculation of the cross-sectional area at 66% of the tree height (CC_{66}).

Dividing crown volume, and crown surface area by 500,000 (ft^3) and 250,000 (ft^2), respectively, simply rescaled the values to reduce the magnitude of the regression coefficients that we were estimating. The negative exponential com-

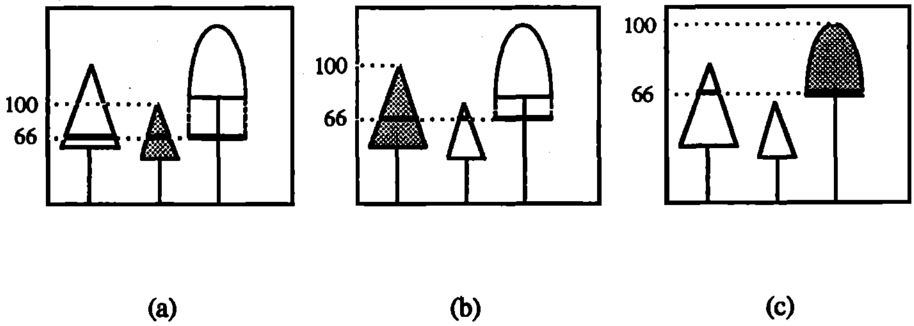

FIGURE 1. Crown cross-sectional area evaluated at 66% of the subject tree's height (CC_{66}). Each competitor and subject tree's contribution to CC_{66} is displayed with a bold line. Subject tree's are labeled a, b, and c.

petition equation used produces a competition factor on a relative scale between zero (no competition) and one (competition saturation). Other empirically determined scaling factors could be used where warranted.

Spatial Pattern Utilized in Competition Indices

The index of Clark and Evans (Pielou 1977), termed *CL-EV*, is a stand-level measure of spatial pattern. It is calculated as $CL\text{-}EV = 2\, r \sqrt{p}$ where r is the average tree-to-nearest-tree distance and p is the number of plants per unit area. This spatial pattern index is, technically, distance-dependent. We place it in a separate subclass of distance-dependent measures (stand-level, distance-dependent, spatial pattern index) because by averaging the information on tree-to-tree distance it yields only a single-value for the stand. For the Clark-Evans index, values greater than one suggest uniformity of spatial pattern, and values less than one suggest clustering. In order to avoid having trees whose nearest neighbors fall outside of the plot dimension for this index only we use an inner plot to define the group of trees for which we calculate the distance from subject tree to nearest neighbor. However, the nearest neighbor can be outside the inner plot, but still inside the outer plot.

Comparison with Distance-Dependent Competition Measures

We compare the best distance-dependent competition indices reported in Biging and Dobbertin's (1992) study against the distance-independent indices examined in this report. The six distance-dependent indices chosen from Biging and Dobbertin's (1992) study were: (a) a distance weighted dbh-ratio index (retaining the notation of Biging and Dobbertin, this is referred to as index *H-H*1) whose competitors were chosen using a height angle of 60° at the stem base of the subject tree; (b) an exponential distance weighted dbh-ratio index whose competitors were chosen using a height angle of 60° at the stem base of the subject tree (*ME-H*1); (c) a crown-overlap index with open-grown crown widths as search radii (*B-CW*2); (d) a size-ratio index using crown cross-sectional area at 75% of the height of the subject tree with competitors chosen by a 2m^2/ha angle gauge (CC_{75}-*D*2); (e) a size-ratio index using crown cross-sectional area at 66% of the height of the subject tree with competitors chosen with a 50° height angle gauge (CC_{66}-*H*2); and (f) a size-ratio index using the crown volume of the competitors evaluated at the crown base of the subject tree when competitors are chosen using a 50° height angle gauge (CV_{HCBU}-$H2$).

Competition Indices Used with Individual-Tree Conifer Growth Models
Wensel et al. (1987) developed growth models based on the exemplar that growth is equal to potential growth, assuming a free-to-grow tree, reduced by competition (Baule 1917). Their equations for potential diameter squared growth (P_D) and height growth (P_H) are of the form:

$$P_D = [(c_0 SI^{c_1} + c_2 DBH^{2\,c_3})^{c_3^{-1}} - DBH^2][1 + e^{(4 - d_2 CR)}]^{-1} \tag{4}$$

$$P_H = [(c_0 SI^{c_1} + c_2 H^{c_3})^{c_3^{-1}} - H]\, d_1\, [1 + e^{(4 - d_2 CR)}]^{-1} \tag{5}$$

Potential growth is then reduced by the competition factor (CF) for diameter or height (CF_D or CF_H, respectively) of the form:

$$CF = e^{-B_1 CI^{B_2}} \tag{6}$$

so that growth is expressed as:

$$\text{Diameter}^2 \text{ growth} = P_D \times CF_D \text{ and}$$
$$\text{Height growth} = P_H \times CF_H$$

where

CI = a competition index for an individual tree

DBH = diameter at breast height (in.)

H = total height (ft)

HCB = height to the base of the live crown (ft)

CR = live crown ratio defined as $CR = \dfrac{(H - HCB)}{H}$

SI = average height of site index trees at breast height age 50 (Biging 1985)

c_0, c_1, c_2, c_3 = species specific coefficients determined via nonlinear regression

b_1, b_2, d_1, d_2 = species specific coefficients determined via nonlinear regression

For this study we estimate the two coefficients of the competition factor (CF) while holding the coefficients of the potential height growth (P_H) and potential diameter squared growth (P_D) models fixed. This allows us to estimate the reduction in growth due to competitive effects.

As an adjunct, a simple one-parameter model was formulated as a control so that the percent of variation explained without a competition index would be known. For comparison to the full model the simple linear adjustment factor B_3 used was:

$$\Delta D^2 = \text{Diameter}^2 \text{ growth} = P_D \times B_3 \text{ and} \tag{7}$$

$$\Delta H = \text{Height growth} = P_H \times B_3 \tag{8}$$

Parameter estimates of B_1, B_2, and B_3 as well as the residual sum of squares

(RSS) and mean square error (MSE) were obtained for all CIs by nonlinear regression analysis using a multivariate secant method to estimate the derivates of the regression function. The sample size for PP and WF was of sufficient size to divide the dataset into a subset used for the regression (75–80% of the trees for the dbh growth, 70% of the trees for the height growth), hereafter termed the *fit dataset,* and the remaining subset of trees were used as the test dataset. The trees for these two subsets were randomly selected.

The test dataset was used to assess how well the models and parameter estimates performed on an "independent" dataset. For the test dataset, residual sums of squares [the sum of (actual values—predicted values)^2] and residual mean sum of squares were calculated using the parameters obtained in the regression of the fit dataset.

We judge how well a competition index performs by observing the reduction of the mean square error ($MSE = \frac{RSS}{n}$; RSS = residual sum of squares) in the diameter squared growth and height growth equations for each species. Thus our comparisons are between potential diameter squared growth reduced by competition [model (4) × model (6)] and model (7) and also between potential height growth reduced by competition [model (5) × model (6)] and model (8). In our opinion, this is a much more meaningful evaluation than comparing the correlations of competition indices to diameter or height growth. Further detail on model evaluation is given in sections below.

RESULTS

Statistics are reported for the fit dataset, the test dataset, and all data combined (i.e., fit plus test data combined by species). Because the results are similar for the fit and the test datasets we will confine our discussion to the case of all data combined (whose values are highlighted in Table 2).

Stand level density measures were calculated either including all trees on the plot (*BA, TPA, SDI,* and *CL-EV*) or only the trees larger than the subject tree (*BAL, CCFL, CCL, CSAL,* and *CVL*). All measures were computed on a per acre basis. Surprisingly, no improvement in MSE was found when any of these nine stand density measures was used as a competition index in the models for intolerant PP, except for *CL-EV* height growth. For the diameter squared and height growth models for WF, basal area (*BA*), basal area in larger trees (*BAL*), number of trees (*TPA*) and crown competition factor in larger trees (*CCFL*) lead to reductions in MSE between 7–24 percentage points. For WF and both dependent variables *BA* was clearly superior to any of the measures calculated for trees larger than the subject tree. *TPA, BAL,* and *CCFL* also showed reductions in MSE for both dependent variables of WF, but were at least 10 percentage points higher in MSE than *BA*. Incorporating estimated crown information into the calculation of distance-independent stand level density measures (*CCL, CSAL,* and *CVL*) did not yield lower mean squared errors. This was the opposite of what was found when crown information was incorporated into distance-dependent indices (Biging and Dobbertin 1992, Table 2).

The distance-independent individual tree competition indices that used estimated crown parameters at fixed percentages (p) of the total height of the subject tree (CC_p, CV_p, CSA_p) substantially reduced the MSEs compared to using no

TABLE 2.

Mean square error for height growth (ΔH) and diameter squared growth (ΔD^2) for competition indices for white fir (*WF*) and ponderosa pine (*PP*) as a percentage of no competition index. Values presented are for fit, test, and all data combined and are rounded to the nearest whole percentage

Competition index	*WF* ΔD^2			*PP* ΔD^2			*WF* ΔH			*PP* ΔH		
	Fit	Test	**All**	Fit	Test	**All**	Fit	Test	**All**	Fit	Test	**All**
No *CI*	224.4	137.8	202.7	160.8	184.0	165.5	6.0	5.3	5.8	3.1	4.0	3.3
No *CI* (100%)	100	100	100	100	100	100	100	100	100	100	100	100
TPA	86	83	**85**	100	99	**100**	89	93	**90**	98	96	**97**
BA	76	77	**76**	99	103	**100**	75	80	**77**	100	105	**102**
SDI	95	94	**95**	100	101	**100**	99	100	**99**	98	93	**97**
CL-EV	99	99	**99**	98	103	**99**	96	94	**96**	90	76	**86**
BAL	90	95	**91**	100	101	**100**	88	96	**90**	100	104	**101**
CCFL	92	95	**92**	100	101	**100**	91	98	**93**	100	103	**101**
CCL	96	99	**97**	100	101	**100**	97	103	**99**	100	104	**101**
CSAL	98	99	**99**	100	101	**100**	98	105	**100**	100	104	**101**
CVL	99	99	**99**	100	101	**100**	97	106	**100**	100	105	**101**
CC_{25}	73	74	**73**	100	103	**101**	77	90	**81**	100	104	**101**
CC_{33}	70	73	**70**	99	109	**101**	74	89	**79**	100	103	**101**
CC_{50}	62	68	**64**	91	99	**93**	67	87	**73**	98	92	**96**
CC_{66}	60	69	**63**	84	87	**85**	62	84	**68**	90	77	**86**
CC_{75}	63	73	**65**	82	83	**82**	62	84	**69**	87	73	**83**
CC_{100}	78	93	**82**	84	82	**84**	73	96	**80**	85	84	**85**
CC_{HCB}	81	81	**81**	98	105	**99**	77	97	**83**	100	102	**101**
CV_{25}	84	91	**86**	100	102	**100**	77	92	**82**	100	103	**101**
CV_{33}	77	88	**80**	98	103	**99**	72	89	**77**	100	99	**100**
CV_{50}	65	74	**67**	88	93	**89**	63	85	**69**	96	84	**92**
CV_{66}	65	72	**66**	83	81	**83**	60	86	**68**	89	73	**84**
CV_{75}	68	77	**70**	83	80	**82**	64	89	**71**	87	74	**83**
CV_{100}	82	96	**86**	86	85	**85**	78	103	**85**	90	88	**89**
CV_{HCB}	96	97	**96**	98	111	**101**	83	102	**89**	100	100	**100**
CSA_{25}	78	91	**81**	99	104	**100**	78	90	**82**	100	104	**101**
CSA_{33}	72	86	**75**	97	104	**99**	73	88	**78**	100	101	**100**
CSA_{50}	63	72	**65**	88	94	**89**	64	83	**70**	95	83	**91**
CSA_{66}	62	71	**64**	82	82	**82**	60	83	**67**	88	72	**83**
CSA_{75}	65	75	**68**	82	80	**81**	61	86	**69**	86	72	**81**
CSA_{100}	80	93	**83**	85	84	**85**	75	100	**82**	90	85	**88**
CSA_{HCB}	92	96	**93**	97	104	**99**	83	100	**88**	100	100	**100**
*H-H*1	68	77	**71**	95	98	**96**	68	75	**70**	95	97	**96**
*ME-H*1	70	75	**71**	97	99	**98**	69	76	**71**	98	94	**97**
*B-CW*2	72	85	**75**	88	85	**88**	78	74	**77**	94	91	**93**
CC_{75}-*D*2	69	77	**71**	85	84	**85**	75	84	**78**	85	81	**84**
CC_{66}-*H*2	67	74	**69**	87	87	**87**	67	78	**71**	84	84	**84**
CV_{HCBU}-*H*2	65	77	**68**	90	89	**90**	61	74	**65**	83	80	**82**
n	685	229	**914**	422	109	**531**	195	84	**279**	104	48	**152**

competition index when the optimal relative tree heights were used. For both species and both dependent variables the CC_p, CV_p, CSA_p indices also significantly outperformed all of the stand level competition indices. Table 2 and Figures 2 and 3 show the mean squared error reductions at the evaluated percentages (p)

of total tree height for CC_p, CV_p, and CSA_p. MSE values for the diameter squared and height growth models were always substantially lower for WF than for PP (Table 2, Figures 2 and 3). The MSEs for the different percentages of the subject trees' heights follow curves whose minima occur either at 66% or at 75% of the subject tree's height. At low percentages of the subject tree's height (25% or 33% as well as the crown base) for both dependent variables for PP, little or no improvement was found by using this type of index compared to having no competition index. The lowest MSEs for WF for diameter squared and height growth were generally found at 50 and 66% of the subject tree's height (Figures 2 and 3). For PP the lowest MSEs were generally found at 66 and 75% tree height for both growth models (Figures 2 and 3). For both species and both dependent variables when evaluated at the 100% of subject tree height level, there was an improvement over the no competition index model. For PP this height yielded MSEs that were close to optimal for both dependent variables. However, for WF there were substantial increases in MSE for both dependent variables when evaluated at 100% of subject tree height compared to lower, more optimal, evaluation points.

The differences among reductions in MSE for CC_p, CV_p, and CSA_p indices are only minor (Figures 2 and 3) compared to the effect of the selection of the height of evaluation (p). For PP the crown surface area competition index (CSA_p) led to slightly lower MSE values than the use of crown volume (CV_p) or crown cross-sectional areas (CC_p) assessed over the range of heights of evaluation (see Figures 2 and 3). This was true for both the diameter squared and the height growth model. The crown cross-sectional area index seemed to be best for the diameter squared growth model for WF (Figure 2), but the crown surface area (CSA_p)

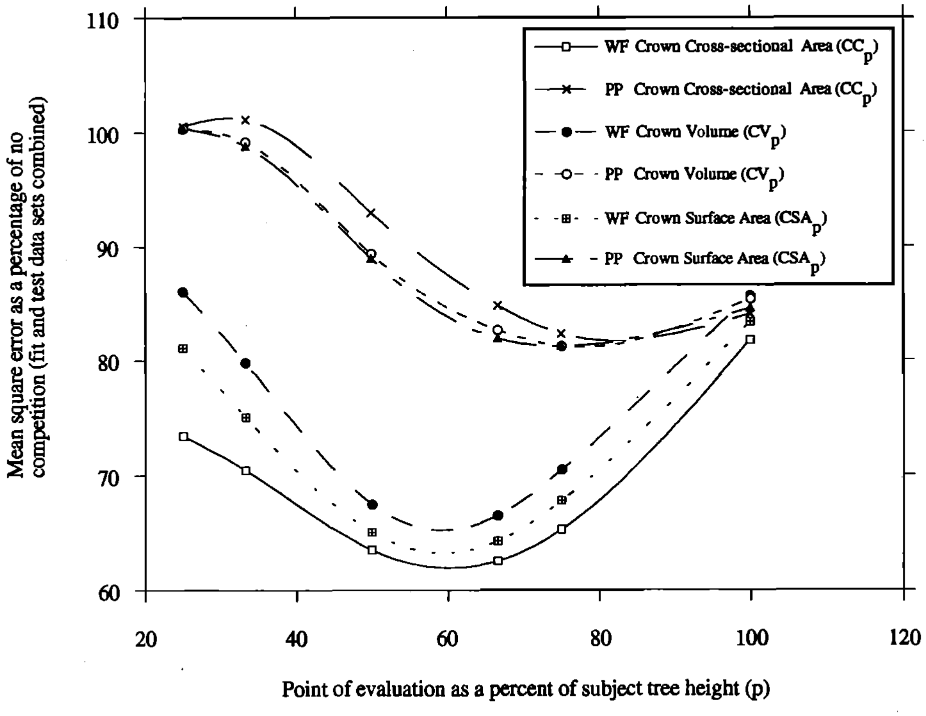

FIGURE 2. Reduction in Mean Square Error (MSE) for three competition indices for diameter^2 growth of white fir and ponderosa pine evaluated at varying percentages of subject tree height.

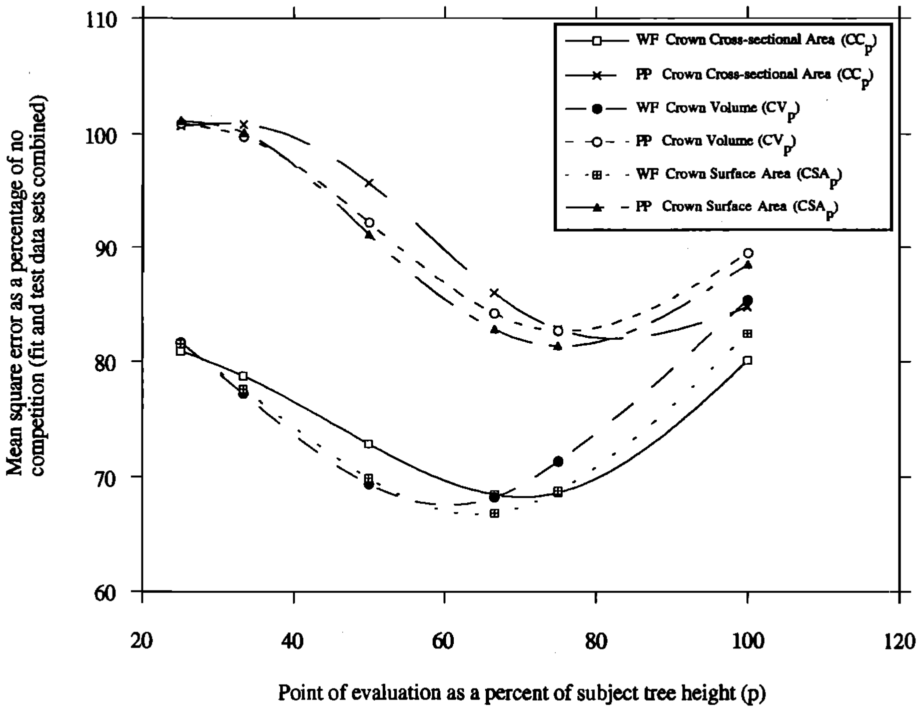

FIGURE 3. Reduction in Mean Square Error (MSE) for three competition indices for height growth of white fir and ponderosa pine evaluated at varying percentages of subject tree height.

index was marginally superior for the height growth model. Thus, the CSA_p index appears to have a slight predictive edge over the other indices. The overall best competition indices for both species in terms of low MSE values were CC_{66}, CC_{75}, CSA_{66}, and CSA_{75} for diameter squared growth (Table 2). The same results held for height growth. The CC_p, CV_p and CSA_p indices evaluated at p = 50, 66, or 75% of the subject tree's height all have lower MSEs than any stand level density index examined. The only stand level index that performed well was *BA* for WF height and diameter squared growth, but it was approximately 10 percentage points higher in MSE than well chosen CC_p, CV_p, and CSA_p indices.

The best distance-dependent indices examined by Biging and Dobbertin (1992) were compared to the best CC_p, CV_p, and CSA_p distance-independent indices. For the diameter squared growth model, and all data combined by species, Biging and Dobbertin's distance-dependent indices CC_{75}-$D2$ and CV_{HCBU}-$H2$ performed a few percentage points worse than the distance-independent indices CSA_{75} and CSA_{66} investigated in this study (Table 2). For WF height growth, the distance-dependent index CV_{HCBU}-$H2$ performed two percentage points better than distance-independent index CSA_{66} and for PP height growth the distance-dependent index CV_{HCBU}-$H2$ performed slightly worse than the distance-independent index CSA_{75}.

CONCLUSIONS

The distance-independent relative size competition indices CC_p, CV_p, and CSA_p performed as well as or slightly better than the best distance-dependent competition indices based on their reduction in MSE. This result is counterintuitive. We had expected that explicit knowledge of tree spatial location would prove superior to other indices which do not take location into account. It is not possible with the data at hand to explore the exact causes of this result. Certainly it is possible that the zone in which competition takes place is greater than was examined in this and other research work on competition. It is difficult, if not impossible, to define an exact zone of influence for use in a competition index (*CI*) for individual trees that includes all competitors and sources of competition for scarce resources. For example, Stiell (1970) found that aboveground competition is concentrated within the area occupied by individual crowns, but that root competition is diffuse and is unpredictable for a given tree. We can only speculate that the distance-independent measures (CC_p, CV_p and CSA_p) outperform their distance-dependent counterparts because the former measures are applied over an entire plot (or stand), whereas the distance-dependent measures are calculated within a zone whose radius is usually less than the plot size. If this is true, then the exact spatial location of trees is less important than expanding the neighborhood considered for calculating competition.

Burger observed in Norway spruce plantations that the crown width of trees is widest at two-thirds of the crown length from the tree top (Burger 1939a,b). This upper crown part, often called the "light crown," receives most of the light and is therefore the most important part of the crown for growth measures. For crown ratios of 0.5, which is the most common crown ratio in the database, this point coincides with the 66% of the tree height. These facts might provide a possible explanation for the much better performance of crown measures at 66 and 75% of the tree height than at lower or higher percentages of total height.

The differences in MSE reduction between CC_p, CV_p and CSA_p are low. The lowest MSEs are usually reached at 66% and 75% of the subject tree height. We expected that *CSA* would be the most significant index for explaining diameter squared and height growth (Assmann 1970, Dong and Kramer 1986), and our data seemed to support this assertion although the differences among these three measures were relatively minor. Part of the reason why CSA_p may not have been a demonstrably superior measure is because it was a derived, not a measured (observed) variate in this analysis. This is because Biging and Wensel (1990) measured crown radii on felled trees to estimate geometric crown volume, but inferred crown surface area and crown cross-sectional area using methods of calculus. Hence *CSA* and *CV* are derived variates. The correlation between crown volume, crown surface area, and crown cross-sectional area exceeds 0.9. If these were observed variables, rather than derived variables, we would expect their intercorrelation to be substantially less than this value. We postulate that if it were measured, crown surface area would be more highly correlated with growth than the other variates.

Traditional stand density measures and spatial pattern variables alone did not reduce the MSE beyond that obtained by using potential growth alone adjusted by a single parameter (observed growth = potential growth $\times$ B_3). For shade tolerant WF only basal area (*BA*) and trees per acre (*TPA*) showed relatively good results.

The use of a spatial pattern index in the growth models did not improve the growth prediction because it was not correlated to actual growth. This is not surprising for the pattern does not give any information about plot density, tree size, or age. Surprisingly, the pattern gave no additional information when used in combination with stand density variables such as *SDI* and *BA*. It could be that the inner plot used to define the group of trees for which we calculated the distances from subject trees to nearest neighbors occurring anywhere on the full plot did not contain enough tree records to provide accurate information about spatial distribution. It should also be pointed out that the distance-independent measures were estimated using ⅕-ac (0.08 ha) plots. Changing the plot size for distance-independent measures could also affect the performance of these estimators. In general, larger plots are preferred because of the reduction in border effects and because competition can be judged over a larger zone.

This work has demonstrated that this new class of distance-independent index formulated to include estimated crown parameters (CC_p, CV_p and CSA_p) performs as good or better than the best distance-dependent competition indices at least for the complex mixed conifer forest type studied and for the competition model forms utilized. Using this construct, the distance-independent simulation models seem to hold a slight predictive advantage over their distance-dependent counterparts. Although running counter to our expectations, it appears that a judicious choice of distance-independent competition index can yield a model fit as good as that obtainable with distance-dependent models. Of course, these results need to be tested in other regions and in other forest types before there is enough experimental evidence to confirm the generalizability of our findings.

This work also points to the need for additional basic research on tree competition. With a better understanding of above- and below-ground competition dynamics we can revisit our formulation of competition indices and search for more physiologically related distance-dependent and independent competition measures.

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