CPSC 121 TUTORIAL 9 - DIRECT PROOFS

Problem 1. Rewrite each of the following theorems using quantifiers and predicates. Then provide a formal proof of the validity of each theorem.

(1) **Theorem:** The fourth power of a positive odd integer can be written in the form $8m + 1$, where m is a non-negative integer.
Note: a positive odd integer can be written as $2i + 1$, where i is a non-negative integer.

(2) **Definition.** The *floor function* assigns to the real number x the largest integer that is less than or equal to x. In other words, the floor function rounds a real number down to the nearest integer. The value of the floor function is denoted by $\lfloor x \rfloor$.

Definition. The *ceiling function* assigns to the real number x the smallest integer that is greater than or equal to x. In other words, the ceiling function rounds a real number up to the nearest integer. The value of the ceiling function is denoted by $\lceil x \rceil$.

Example. $\lfloor \pi \rfloor = 3$. $\lceil \pi \rceil = 4$. $\lfloor -1/2 \rfloor = -1$. $\lceil -1/2 \rceil = 0$. $\lfloor \sqrt{2} \rfloor = 1$. $\lceil \sqrt{2} \rceil = 2$. $\lfloor -5 \rfloor = -5$. $\lceil -5 \rceil = -5$.

Theorem: For any positive integer x, if x is one more than a multiple of 3, then the sum: $2 \cdot \lfloor \frac{x}{3} \rfloor + \lceil \frac{x}{3} \rceil = x$
Note: If x is one more than a multiple of three, then it can be written as $3k + 1$ for some integer k.

(3) **Theorem:** For any integer m, if m is a perfect square, then $m + 2$ is not a perfect square.
Note 1: If m is a perfect square, then it can be written as a^2 for some integer a.
Note 2: To prove that a number is not a perfect square, it is equivalent to prove that it is strictly between two consecutive perfect squares a^2 and $(a + 1)^2$.