Learning to Schedule in Multiclass Many Server Queues

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July 21-22, 2022

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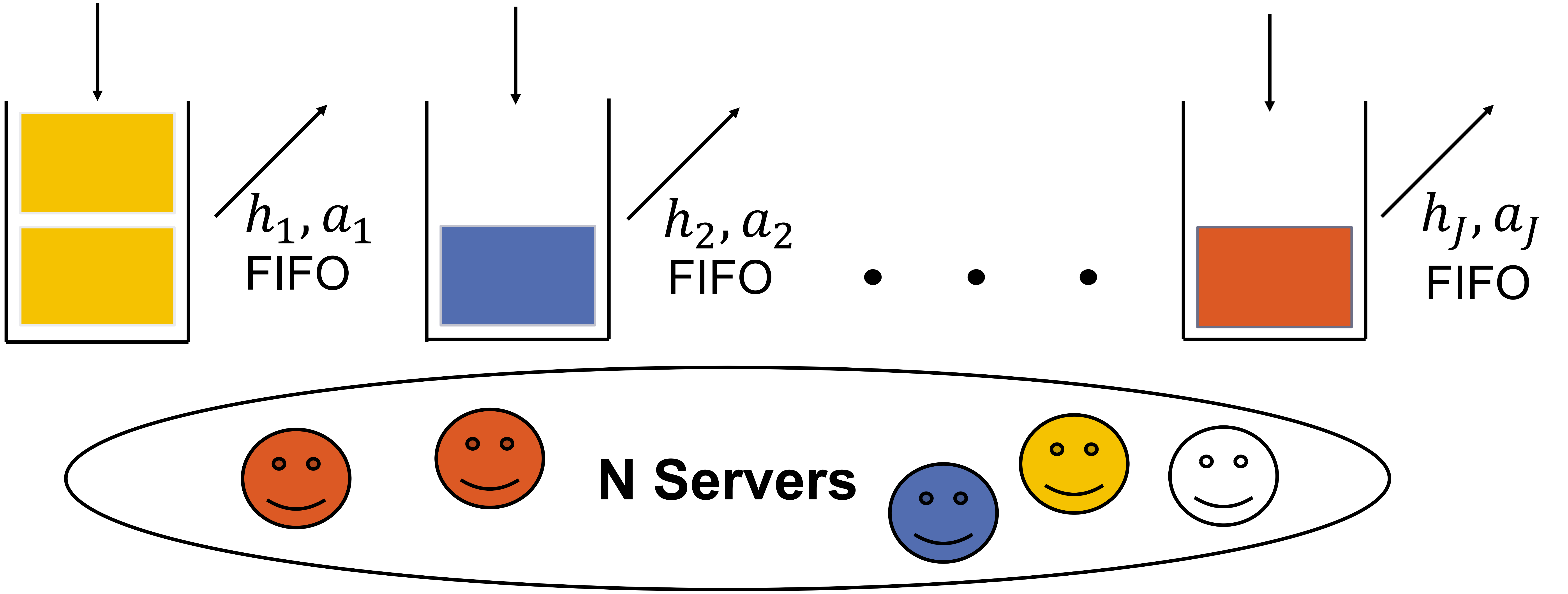
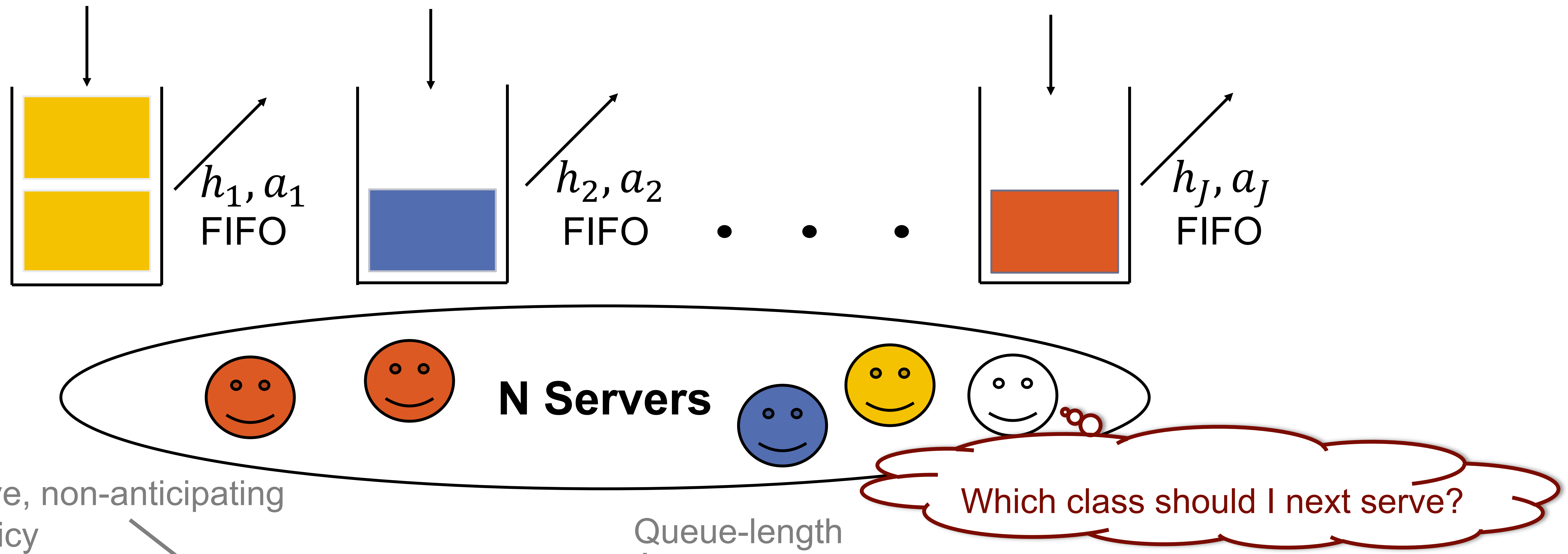
A Canonical Model For Service Systems

Service systems with different classes of customers who have limited patience:

- Multiclass many server queue with abandonment



A Canonical Model For Service Systems



Non-preemptive, non-anticipating scheduling policy

Queue-length

Finite Horizon Cost: $C_T(\pi) \coloneqq \sum_{j=1}^{J} a_j R_j(T,\pi) + \int_0^T h_j Q_j(t,\pi)dt$

Cumulative reneging

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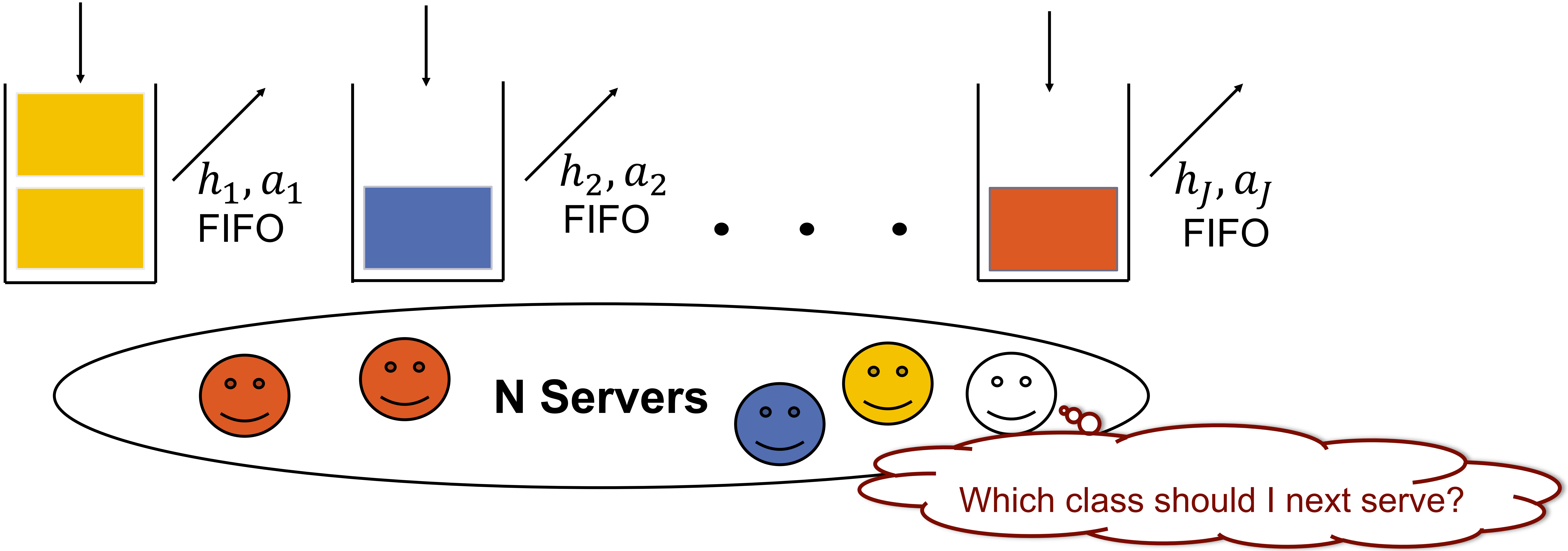

Finite Horizon Cost: $C_T(\pi) \coloneqq \sum_{j=1}^{J} a_j R_j(T,\pi) + \int_0^T h_j Q_j(t,\pi)dt$

Long-Run Average Cost: $C(\pi) \coloneqq \limsup_{T\to\infty} \mathbb{E}[C_T(\pi)]/T$

Known: a_j, h_j
Unknown: $\lambda_j(t)$, μ_j, θ_j

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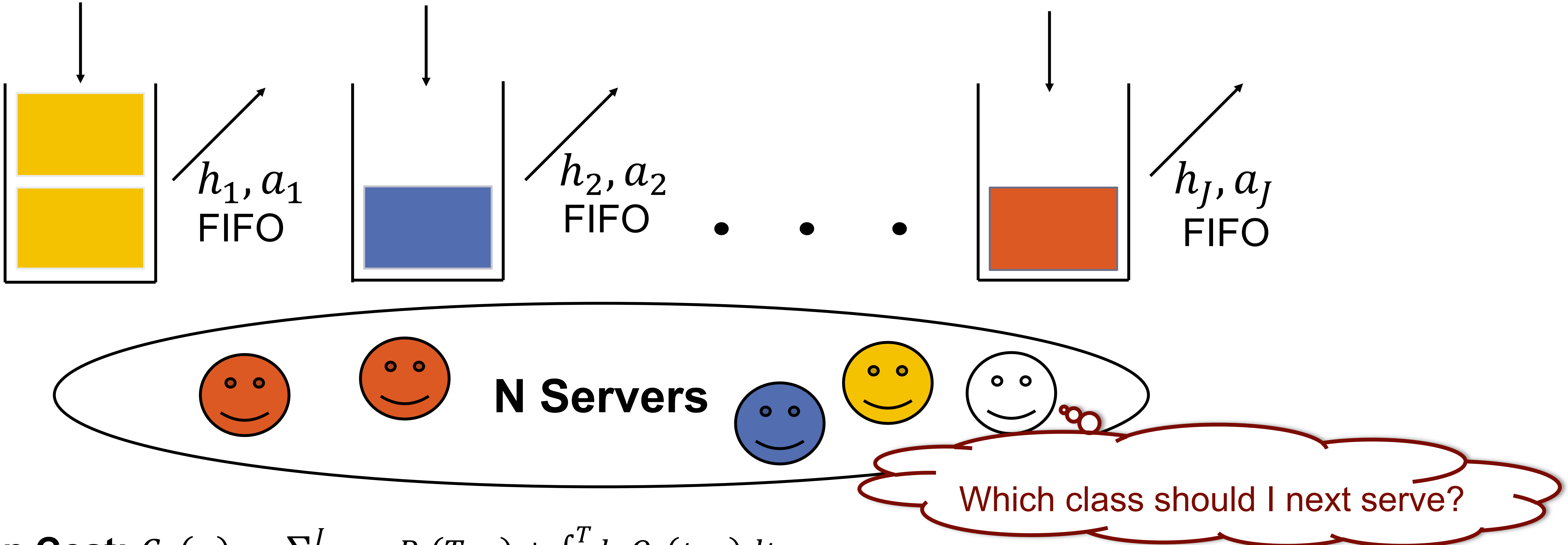

Finite Horizon Cost: $C_T(\pi) := \sum_{j=1}^{J} a_j R_j(T, \pi) + \int_0^T h_j Q_j(t, \pi) dt$

Long-Run Average Cost: $C(\pi) := \limsup_{T\to\infty} \mathbb{E}[C_T(\pi)]/T$

an optimal policy

Objective is to Minimize Regret, $\mathcal{R}(T, \pi) := \mathbb{E}[C_T(\pi)] - \mathbb{E}[C_T(\pi^\star)]$

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Selected Literature Review

Combining statistical learning and optimal control

- Inventory control

  [Kunnumkal and Topaloglu, 2008]

  [Huh and Rusmevichientong, 2014]
- Assortment optimization

  [Saure and Zeevi, 2013]
- Revenue management

  [Besbes and Zeevi, 2012]

  [den Boer and Zwart, 2015]

Not adapted to queueing settings

Learning the scheduling in queueing systems

[Krishnasamy et al., 2018]

[Krishnasamy et al., 2021]

[Stahlbuhk et al., 2021]

[Choudhury et al., 2021]

[Lee and Vojnovic, 2021]

Discrete time,
No abandonment

MAB

[Gittins, 1979]

[Lai and Robbins, 1985]

Parameter estimation in queues

[Walton and Xu, 2021] (survey)

[Asanjarina et al., 2021] (survey)

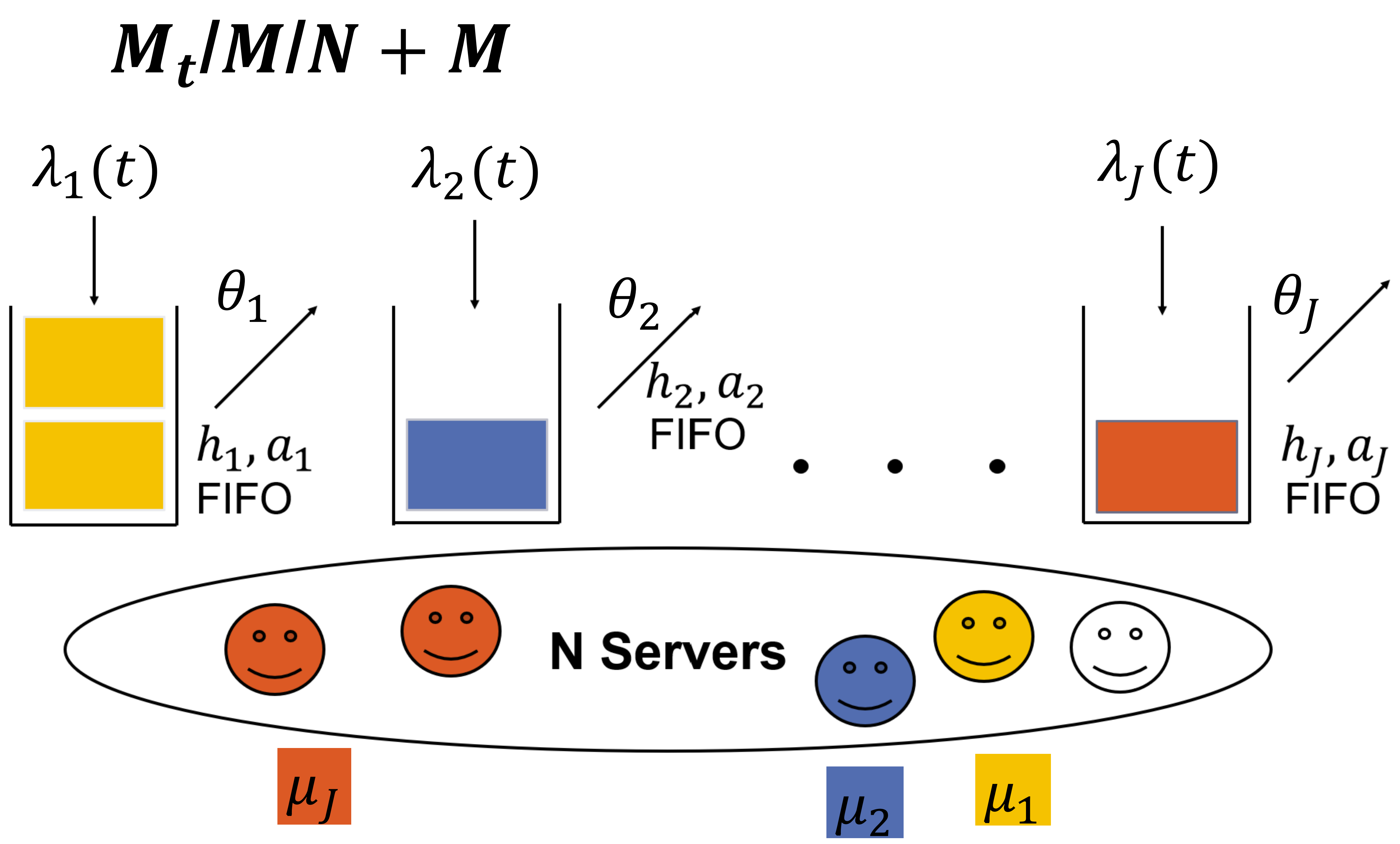
Time-varying queues

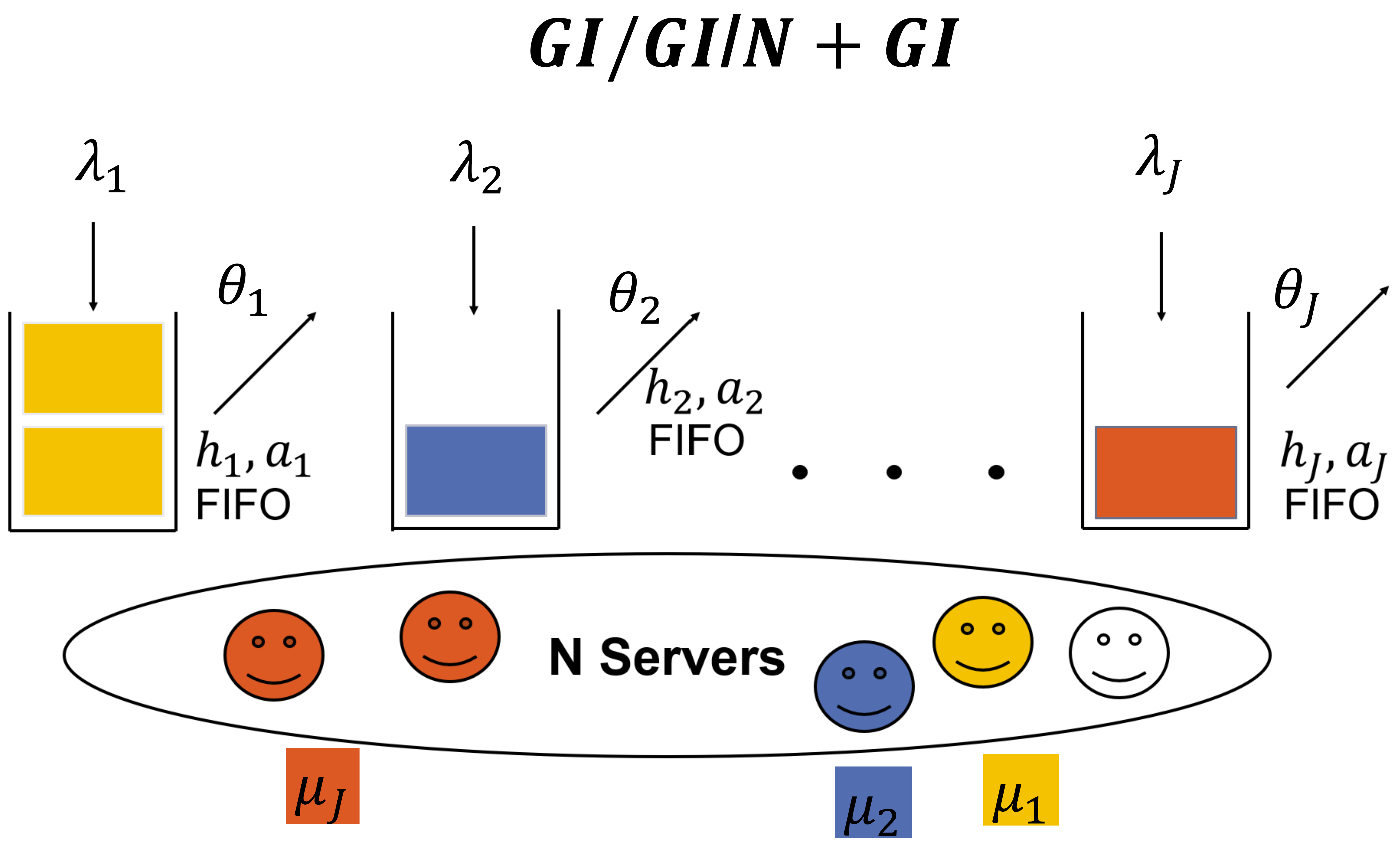
[Mandelbaum et al., 2002]

[Liu and Whitt, 2012]

[Zeifman, 1995]

An Optimal **Policy** π^*





Objective is to Minimize Regret, $\mathcal{R}(T,\pi) := \mathbb{E}[C_T(\pi)] - \mathbb{E}[C_T(\pi^\star)]$

$\pi^\star =$????????

Outline

- Regret Benchmark
- Algorithm & Regret Bounds
- Empirical Performance
- Current Work & Concluding Remarks

An Asymptotically Optimal Policy π^*

Simple and easy to learn policies!!!!

The Static Scheduling Problem:

Long-run average class j arrival rate

Decision variables:

$$\mathbb{B} := \left\{ b \in \mathbb{R}_+^J : b_j \le \frac{\Lambda_j}{\mu_j} \ \forall j \in [J], \sum_{j=1}^J b_j \le 1 \right\}$$

Long-run average fraction of server capacity devoted to class j

Objective ($GI/GI/N+GI$):
[Puha and Ward, 2019]

$$\inf_{b \in \mathbb{B}} \sum_{j=1}^J a_j(\lambda_j - b_j\mu_j) + h_j q_j(b_j) = \sum_{j=1}^J a_j\lambda_j - \sup_{b \in \mathbb{B}} \sum_{j=1}^J a_j \mu_j b_j$$

$= \frac{\lambda_j - b_j\mu_j}{\theta_j}$, if exponential abandonment

Objective ($M_t/M/N+M$):

$$\inf_{b \in \mathbb{B}} \sum_{j=1}^J a_j(\Lambda_j - b_j\mu_j) + h_j \frac{\Lambda_j - b_j\mu_j}{\theta_j} = \sum_{j=1}^J \Lambda_j \left(\frac{a_j\theta_j + h_j}{\theta_j} \right) - \sup_{b \in \mathbb{B}} \sum_{j=1}^J \frac{a_j\theta_j + h_j}{\theta_j} b_j \mu_j$$

Solution ~~($M_t/M/N+M$):~~

($GI/GI/N+GI$):

Assume $h_j = 0$.

$c\mu/\theta$ static priority policy $\pi_{c\mu/\theta}$ (non-idling):

$$(a_1\theta_1 + h_1)\frac{\mu_1}{\theta_1} > (a_2\theta_2 + h_2)\frac{\mu_2}{\theta_2} > \cdots > (a_J\theta_J + h_J)\frac{\mu_J}{\theta_J}$$

Becomes $a_1\mu_1 > a_2\mu_2 > \cdots > a_J\mu_J$.

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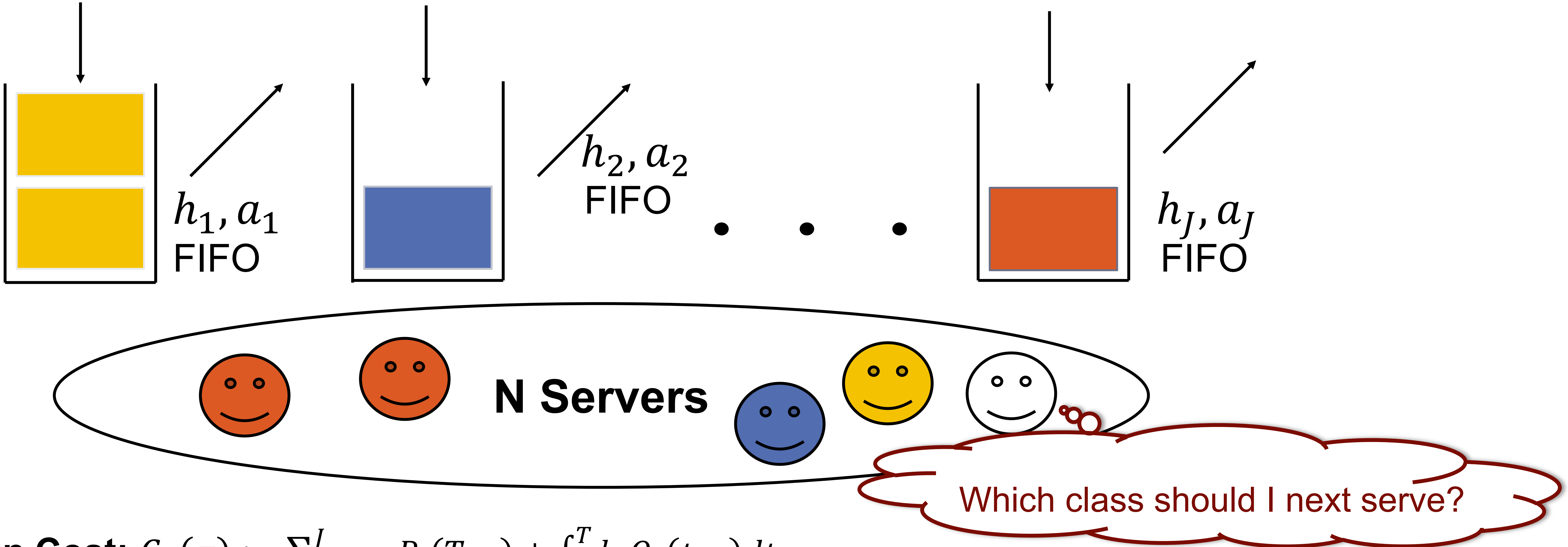
A Canonical Model For Service Systems



Finite Horizon Cost: $C_T(\pi) \coloneqq \sum_{j=1}^{J} a_j R_j(T,\pi) + \int_0^T h_j Q_j(t,\pi)dt$

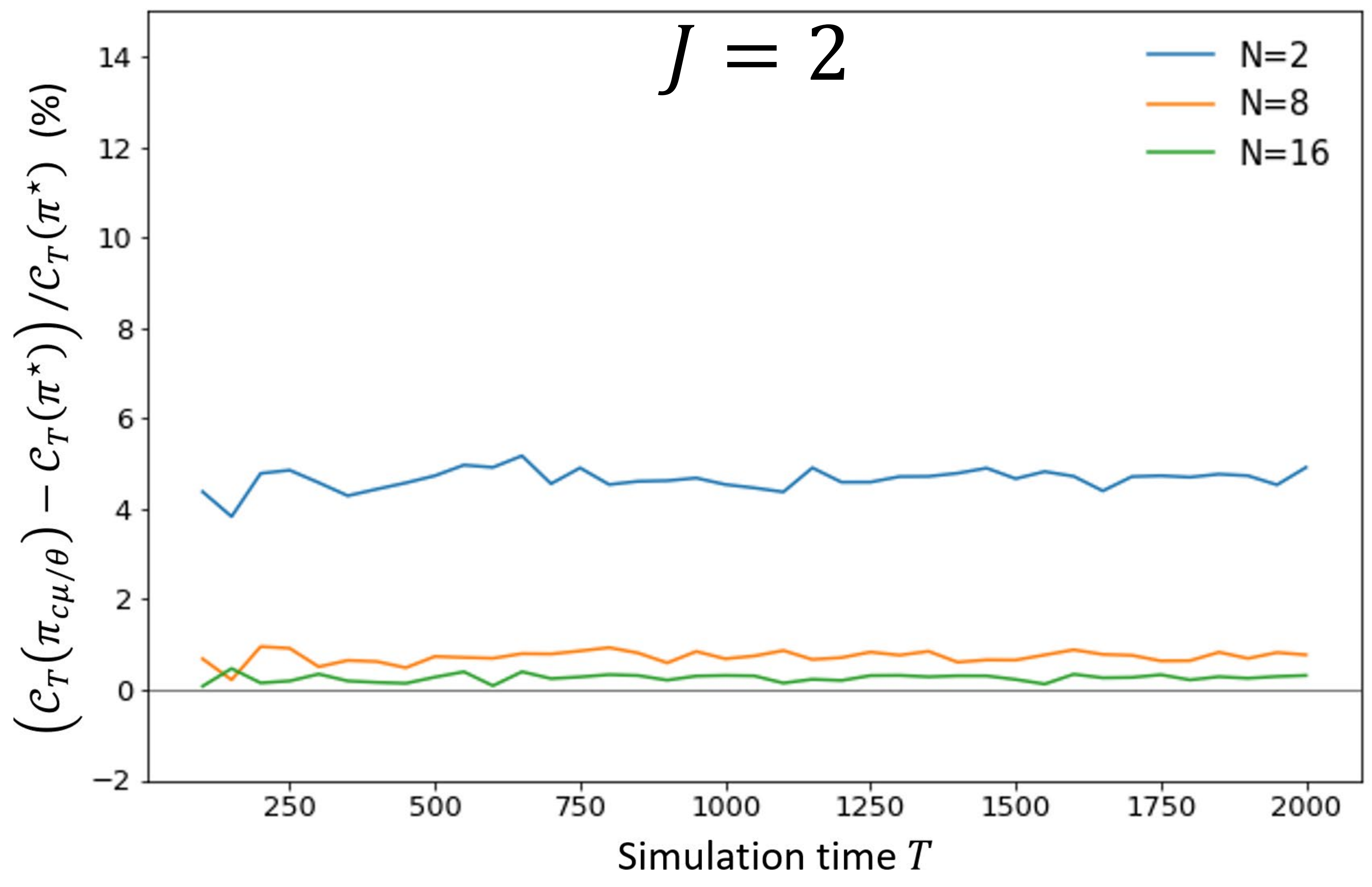
Long-Run Average Cost: $C(\pi) \coloneqq \limsup_{T\to\infty} \mathbb{E}[C_T(\pi)]/T$

~~an optimal policy~~

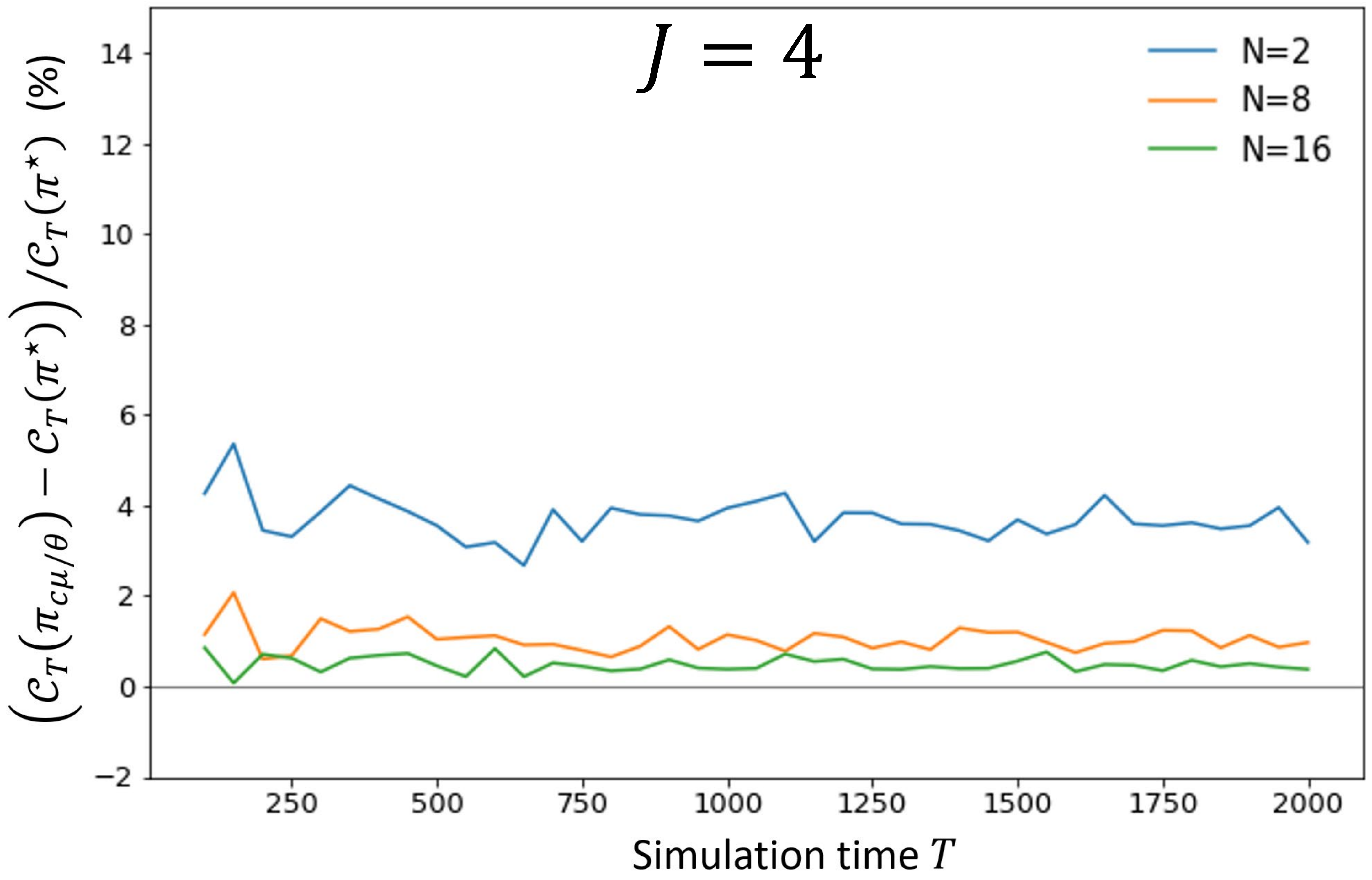
$\pi_{c\mu/\theta}$ the optimal $c\mu/\theta$ policy

Objective is to Minimize Regret, $\mathcal{R}(T,\pi) \coloneqq \mathbb{E}[C_T(\pi)] - \mathbb{E}[C_T\cancel{(\pi^\star)}]$

Performance of $c\mu/\theta$ in a Multiclass *M/M/N+M* Queue



$\lambda_1 = 2N, \lambda_2 = 4N, \mu_1 = \mu_2 = 4, \theta_1 = 1, \theta_2 = 4,$
$h_1 = 1, h_2 = 0.4, a_1 = 1, a_2 = 6$



$\lambda_1 = \lambda_3 = 2N, \lambda_2 = \lambda_4 = 4N, \mu_1 = \mu_2 = \mu_3 = \mu_4 = 4,$
$\theta_1 = \theta_3 = 1, \theta_2 = \theta_4 = 4,$
$h_1 = h_3 = 1, h_2 = h_4 = 0.4, a_1 = a_3 = 1, a_2 = a_4 = 6$

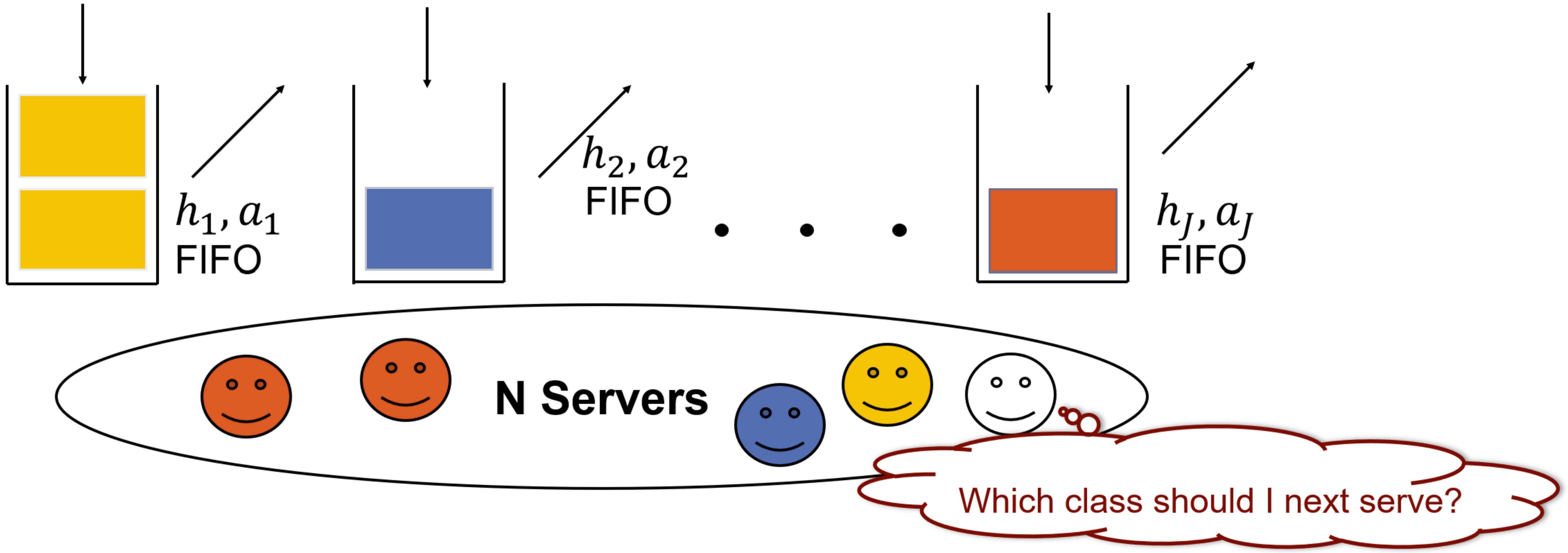
Proposition 3: For the $M_t/M/N + M$ queue, the $c\mu/\theta$ rule is asymptotically optimal, as $N \to \infty$, and then $t \to \infty$.

Analysis Assumptions

Assumption 1 (Arrival Rates):

- $\lambda_j(t)$ is locally integrable on $(0, \infty)$
- $0 < \lambda_j^L \leq \lambda_j(t) \leq \lambda_j^U$,

Finite constants



Assumption 2 (Index Separation):

For any $i \neq j$,

$$\left| \frac{(a_i\theta_i + h_i)\mu_i}{\theta_i} - \frac{(a_j\theta_j + h_j)\mu_j}{\theta_j} \right| \geq \delta$$

Finite positive constant

Objective is to Minimize Regret

$$\mathcal{R}(T, \pi) := \mathbb{E}[C_T(\pi)] - \mathbb{E}[C_T(\pi_{c\mu/\theta})]$$

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Regret Lower Bound

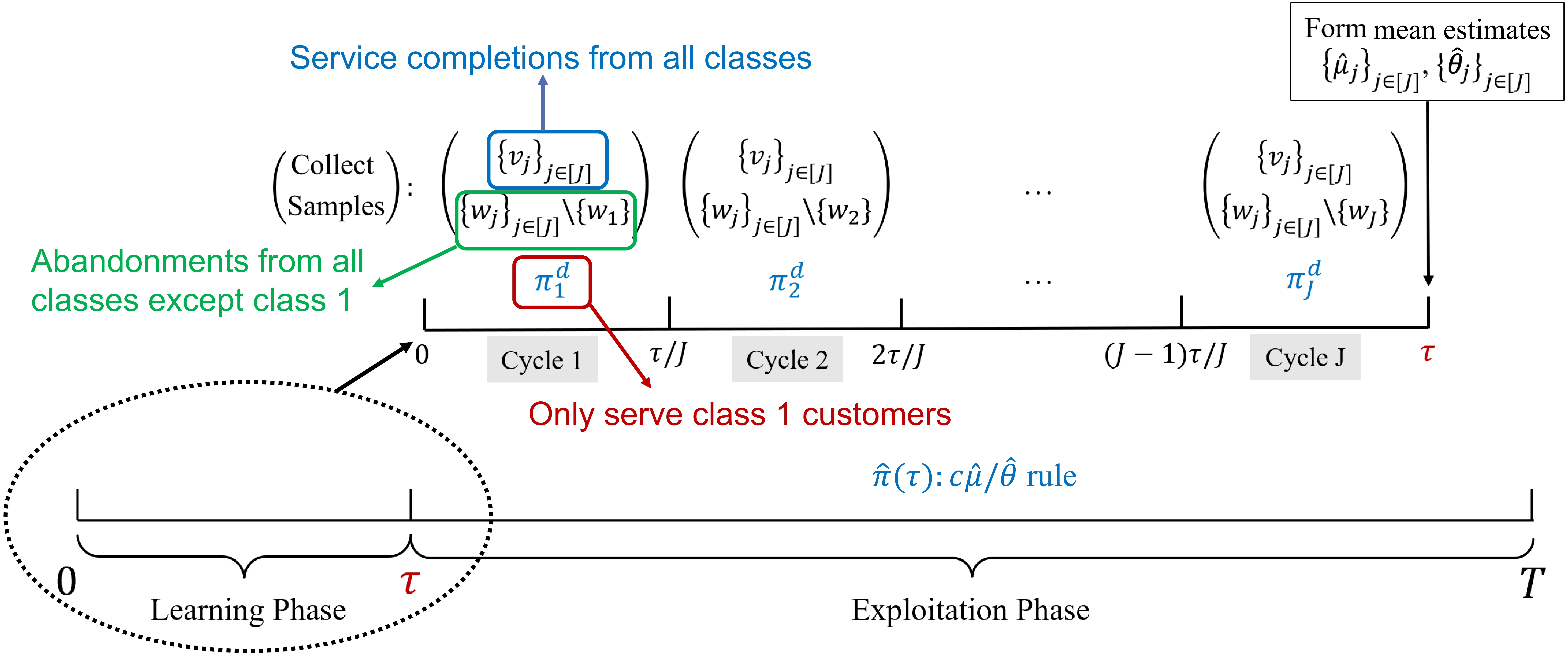
Theorem (Lower Bound on Regret): For any *consistent* non-anticipating and non-preemptive (and possibly randomized and idling) priority scheduling policy φ, there exists a finite constant $c > 0$ such that

$$\mathcal{R}(T, \varphi) \geq c \cdot \log T$$

- Consistency assumption: any "sub-optimal" scheduling policy will be applied for no more than $\mathcal{O}(n^a)$ cumulative fraction of time by the n-th arrival, so the "optimal" scheduling policy (i.e., the benchmark policy) can be eventually identified.

- Multi-Armed Bandit: Lai and Robbins (1985)

Algorithm: Learn-Then-Schedule $\pi_{LTS}(\tau)$



Algorithm Performance

Theorem (Upper Bound on Regret): When $\tau = \mathcal{O}(\log T)$, there exists a finite constant $C > 0$ such that

$$\mathcal{R}(T, \pi_{LTS}(\tau)) \leq C \cdot \log T$$

Theorem (Lower Bound on Regret): $\mathcal{R}(T, \varphi) \geq c \cdot \log T$

Corollary: When $\tau = \mathcal{O}(\log T)$, $\mathcal{R}(T, \pi_{LTS}(\tau)) = \Theta(\log T)$

Regret Analysis Framework

Regret incurred between time τ and time T.

$$\mathcal{R}(T, \pi_{LTS}(\tau)) = \mathcal{R}^{Learning}(T, \pi_{LTS}(\tau)) + \mathcal{R}^{Exploitation}(T, \pi_{LTS}(\tau))$$

Regret incurred between time 0 and τ.

Proposition 1: $\mathcal{R}^{Learning}(T, \pi_{LTS}(\tau)) \leq \left(\sum_{j=1}^{J} \left(\frac{h_j}{\theta_j} + a_j \right) \lambda_j^U \right) \cdot \tau$

Proposition 2: There exist $C_1, C_2, l_1 > 0$ such that

$$\mathcal{R}^{Exploitation}(T, \pi_{LTS}(\tau)) \leq C_1 + C_2(T - \tau)e^{-\ell_1 \tau}.$$

Regret Analysis Framework

$$\mathcal{R}^{Exploitation}(T, \pi_{LTS}(\tau))$$

$$= \mathbb{P}\{\hat{\pi}(\tau) = \pi_{c\mu/\theta}\} \cdot \mathcal{R}^{Exploitation}\left(T, \pi_{LTS}(\tau) | \hat{\pi}(\tau) = \pi_{c\mu/\theta}\right)$$

$$+ \mathbb{P}\{\hat{\pi}(\tau) \neq \pi_{c\mu/\theta}\} \cdot \mathcal{R}^{Exploitation}\left(T, \pi_{LTS}(\tau) | \hat{\pi}(\tau) = \pi_{c\mu/\theta}\right)$$

$$\leq \boxed{\mathcal{R}^{Exploitation}\left(T, \pi_{LTS}(\tau) | \hat{\pi}(\tau) = \pi_{c\mu/\theta}\right)} + \boxed{\mathbb{P}\{\hat{\pi}(\tau) \neq \pi_{c\mu/\theta}\}} \cdot \boxed{\mathcal{R}^{Exploitation}\left(T, \pi_{LTS}(\tau) | \hat{\pi}(\tau) \neq \pi_{c\mu/\theta}\right)}$$

$$\mathbb{E}\left[\left|Q_j^1(t) - Q_j^2(t)\right|^m \mid \left(Q_j^1(0), Q_j^2(0)\right)\right]$$

$$\leq \Psi \cdot e^{-d_j \cdot t} \cdot \left|Q_j^1(0) - Q_j^2(0)\right|^m$$

Exponential loss of memory

(Weakly ergodicity of $M_t/M/N{+}M$)

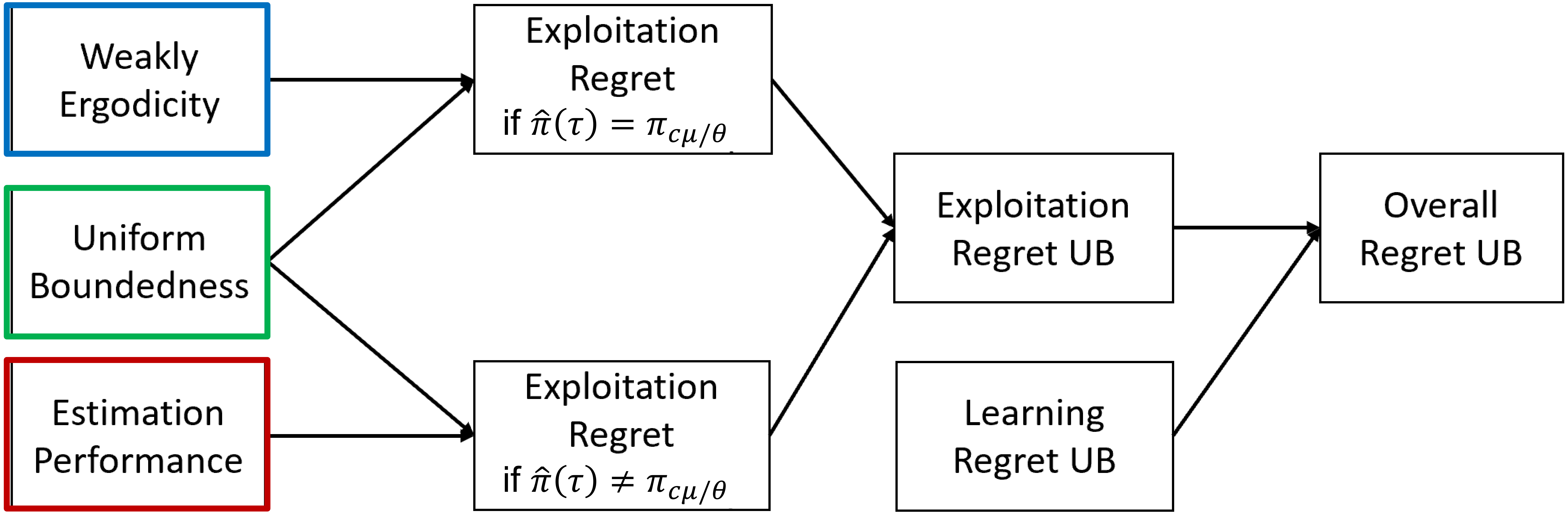
$$\mathbb{P}\{\hat{\pi}(\tau) \neq \pi_{c\mu/\theta}\} \leq 6J \cdot e^{-\ell_1 \cdot \tau}$$

Estimation performance

$$\mathbb{E}[Q_j(t, \pi)] \leq \frac{\lambda_j^U}{\mu_j} + \mathbb{E}[Q_j(0)]$$

Uniform boundedness

Regret Analysis Framework



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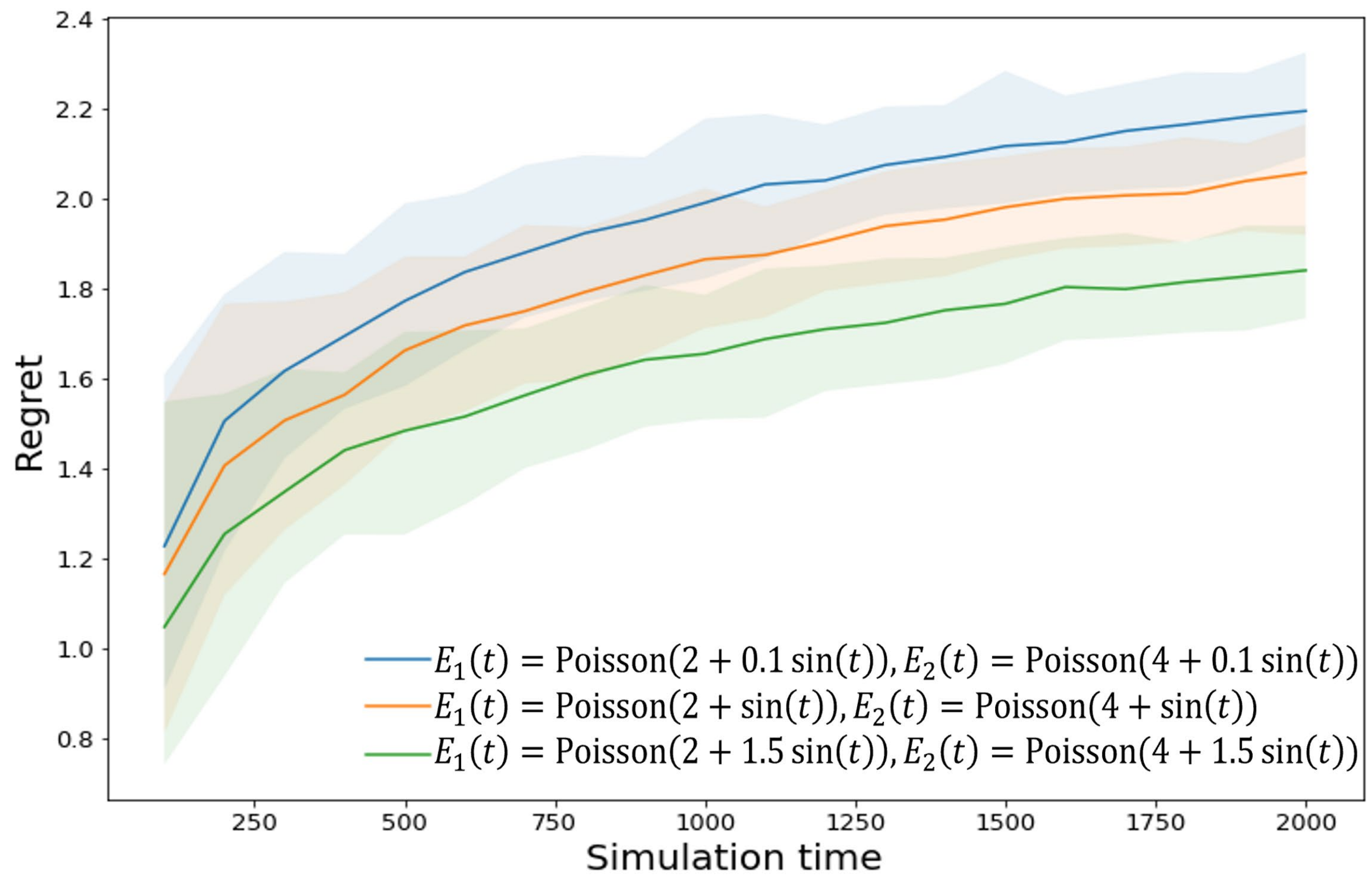
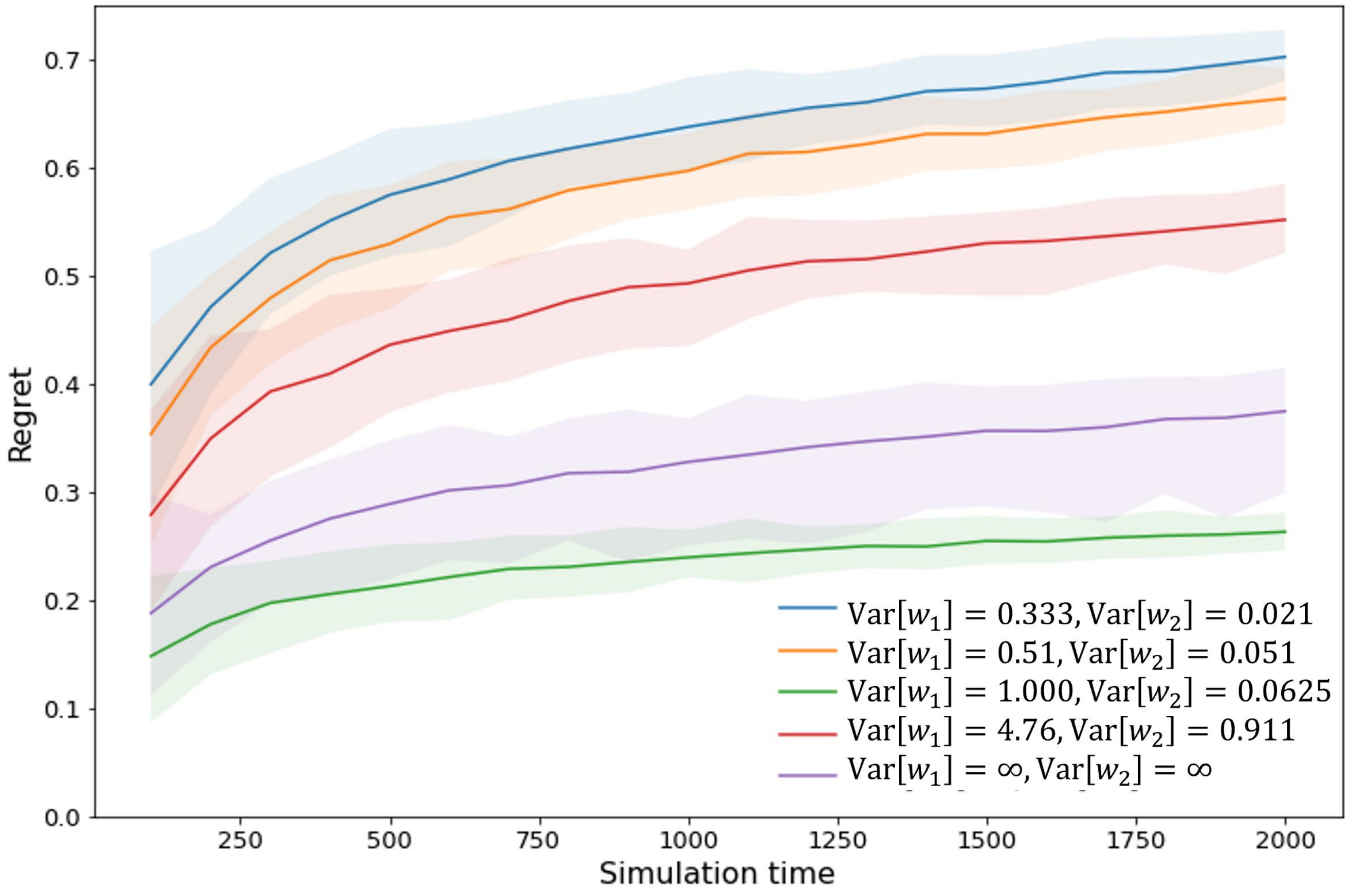
Numerical Study: Two Classes ($N = 8$)

Class	Inter-arrival time	Service time	Patience time	Holding cost h_j	Abandon cost a_j	Index $\left(a_j\theta_j + h_j\right)\frac{\mu_j}{\theta_j}$
1	~~Poisson(2)~~	Exp(4)	~~Exp(1)~~	1	1	8.0
2	~~Poisson(4)~~	Exp(4)	~~Exp(4)~~	0.4	6	7.6





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Current Work: Instance Independent Regret

- Minimax regret:

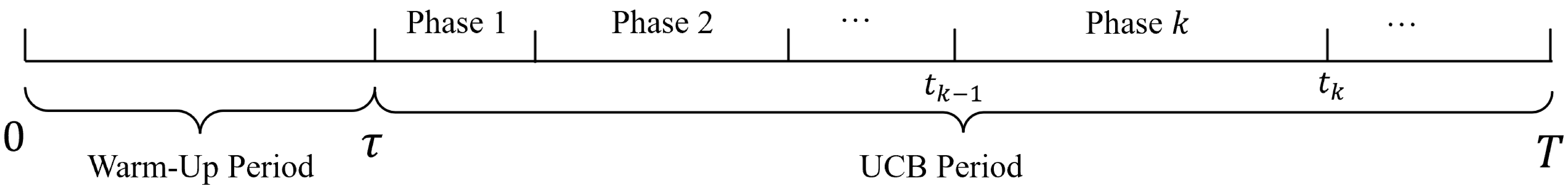
$$Reg(T,\pi) \coloneqq \sup_{\eta\in H} \mathcal{R}(T,\pi;\eta)$$

Parameter space

Theorem (LTS is Not Enough): $Reg(T,\pi_{LTS}(\tau)) = \Theta(T^{2/3})$ when $\tau = \mathcal{O}(T^{2/3})$.

Theorem (Lower Bound): $Reg(T,\pi) \geq \Omega(\sqrt{T})$.

Proposed algorithm: Phased-UCB with forced exploration $\pi_{PUCBFE}(\tau)$



Theorem (Upper Bound): $Reg\left(T,\pi_{PUCBFE}(\tau)\right) \leq \mathcal{O}(\sqrt{T})$, when $\tau = \mathcal{O}(\log T)$.

Concluding Remarks

Summary

- We solve a learning variant of a canonical scheduling problem in a multiclass many server queue with abandonment (specifically, the $M_t/M/N+M$ and the $GI/GI/N+GI$ systems), when model parameters are a priori unknown.
- We propose online algorithms to achieve optimal instance-dependent and minimax regrets.

Follow-On Work

- Extend the results to $G/GI/N+GI$ queue with holding costs and $G_t/GI/N+GI$ queue.
- Bound the suboptimality gap of $\pi_{c\mu/\theta}$ from the true optimal
- Multiple classes of servers

Vision for Learning Problems in Stochastic Systems

- *Exploit asymptotic analysis to define an easier learning problem*

Paper on SSRN: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4090021

Thanks!
Q&A

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