Behavior-Aware Queueing:
When Strategic Customers Meet Strategic Servers

Yueyang Zhong
The University of Chicago Booth School of Business

Raga Gopalakrishnan
Smith School of Business at Queen's University

Amy Ward
The University of Chicago Booth School of Business

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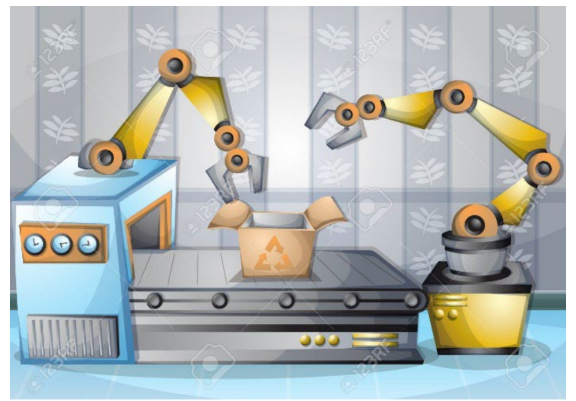
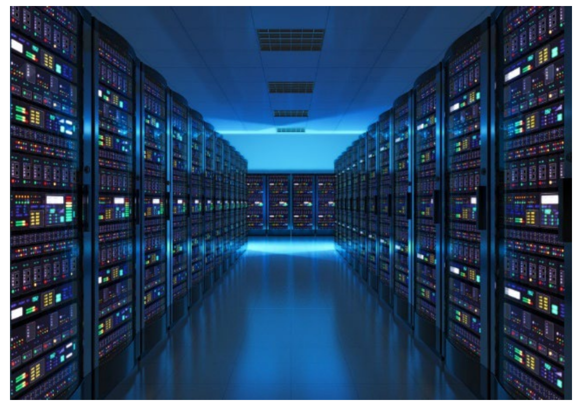
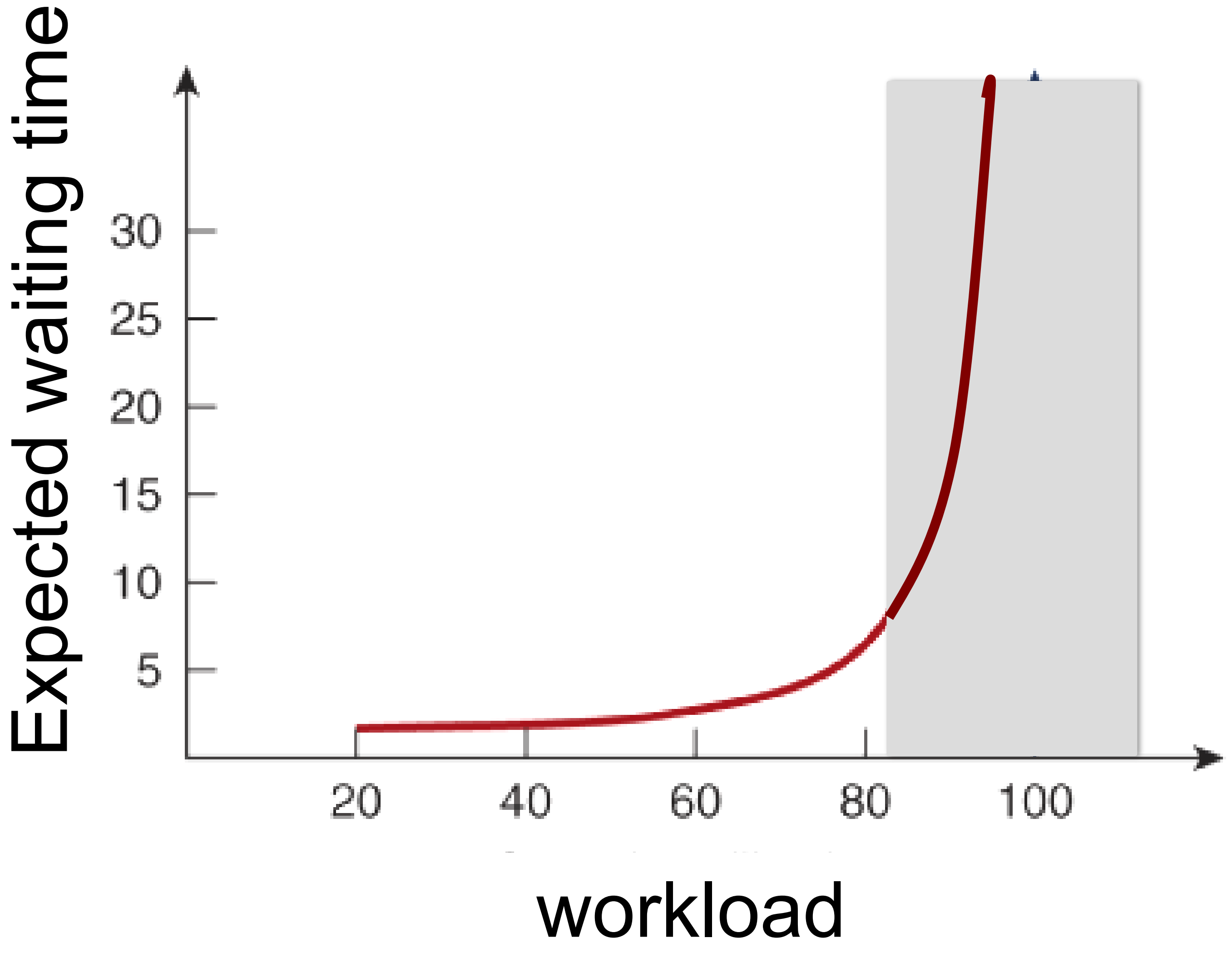
Motivation





	Traditional Queueing (Manufacture/Computer Science)	Behavior-Aware Queueing (Service Operations)
Arrival/ Service processes	Exogeneous	Endogenous
Demand side	Inanimate jobs	Human customers



Inanimate jobs wait forever; Customers may not.

Foundational Econometrica papers
Naor (1969), Knudsen (1972)

⇨ **Finite buffer queue**

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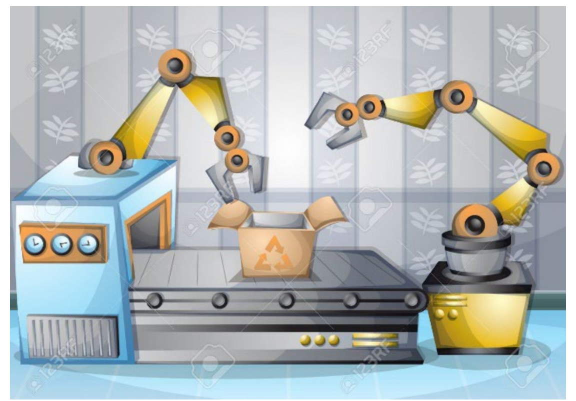
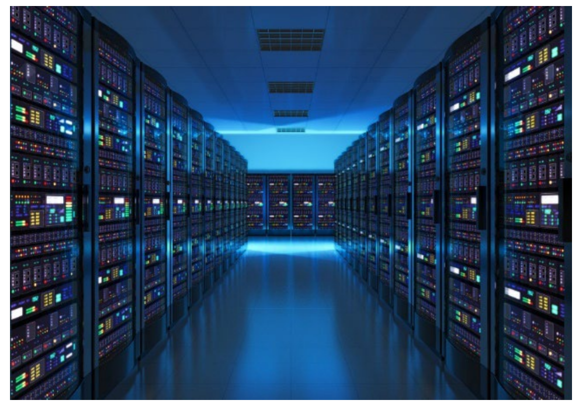
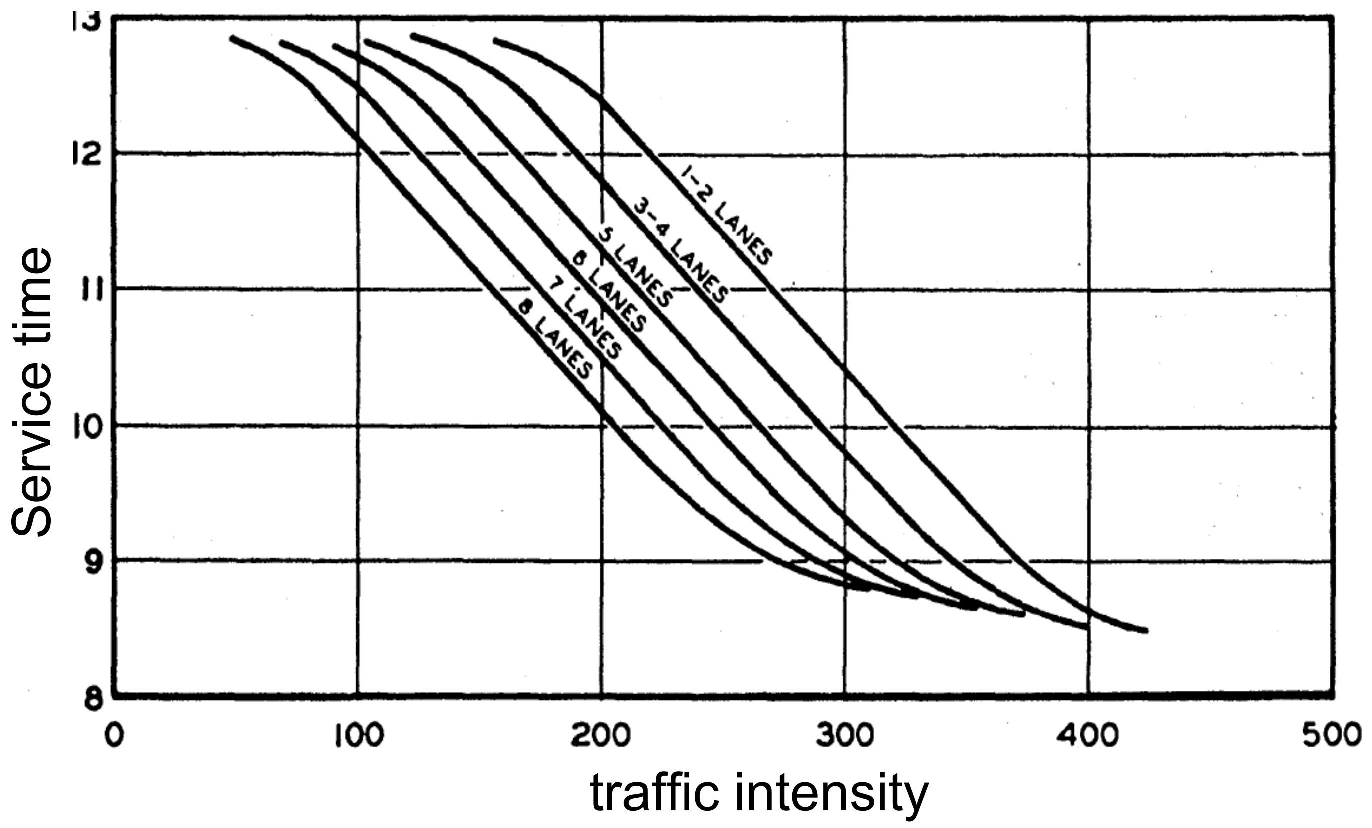



	Traditional Queueing (Manufacture/Computer Science)	Behavior-Aware Queueing (Service Operations)
Arrival/ Service processes	Exogeneous	Endogenous
Demand side	Inanimate jobs	Human customers
Supply side	Inanimate machines	Human workers



Edie (1954), Fig 7: Average booth holding time per vehicle at George Washington Bridge

Selected Literature Review

Human Customers

[Hassin and Haviv, 2003] (survey book)

[Hassin, 2016] (survey book)

[Donahue, Ozer, Zheng, 2020] (survey paper)

[Allon and Kremer, 2018] (survey chapter)

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Human Servers

Empirical:

[Delasay et al., 2019] (survey)

[Kc & Terwiesch, 2009]

[Staats & Gino, 2012]

[Mas & Morretti, 2009]

Large System Asymptotics:

[Ibrahim, 2018]

[Dong and Ibrahim, 2020]

[Zhan and Ward, 2019]

[Gopalakrishnan et al., 2016]

[Zhong et al, 2022]

Session MD51
Service Science Best Student Paper

This talk

Human Customers and Servers

Chung, Ahn, and Righter (2020): N=1 setting

Research Questions

What do we know?

The Customer Side:

Selfish individual | Social optimum | Revenue maximization

Theorem (Naor, 1969; Knudsen, 1972): $k_e^* \geq k_o^* \geq k_r^*$.

Takeaway: Individual selfish customers tend to over-join the queue.

What do we want to know?

Q1 What outcome (equilibrium) do individual ***selfish*** servers induce?

Q2 How does the ***selfish*** server equilibrium compare to social welfare and revenue maximization?

Q3 What happens when ***selfish*** customers and ***selfish*** servers are combined?

Talk Outline

* Subscript e: selfish individual equilibrium,
* Subscript o: social welfare optimization,
* Subscript r: revenue maximization.

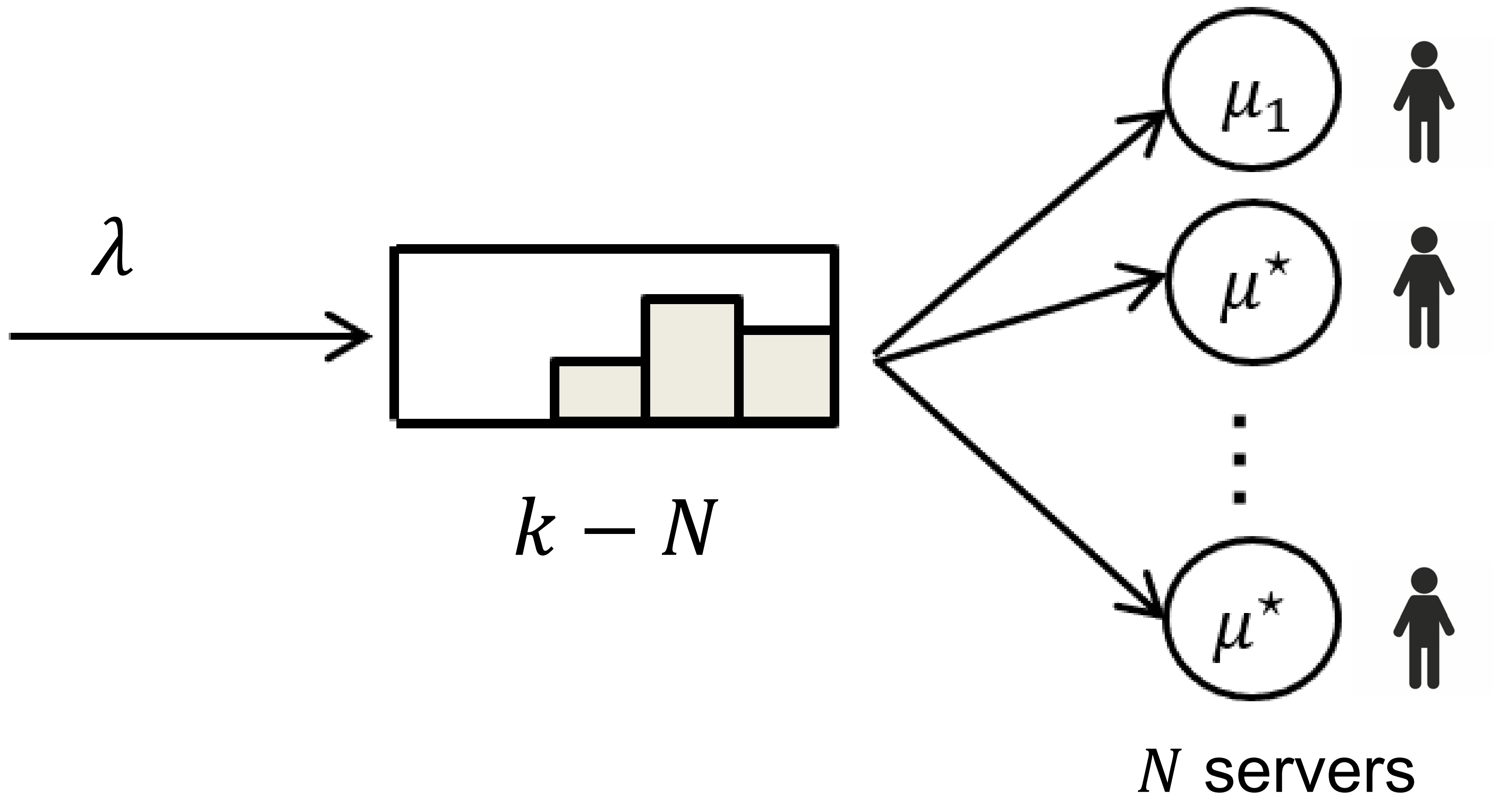
Model: Strategic Servers

The Server Side: $M/M/N/k$ Queue

Definition: Nash Equilibrium Service Rate

The servers want to choose rates $\vec{\mu} = (\mu_1, \mu_2, \dots, \mu_N)$ that satisfy

$$U_i^s(\vec{\mu}) = \underbrace{\overset{\substack{\text{piecerate payment}\\(p^s \geq 0)}}{\boxed{p^s}}\ \mu_i\ \overset{\text{Busyness}}{\boxed{B_i(\vec{\mu})}}}_{\text{Payment}} + v\ \overset{\text{Idleness}}{\boxed{I_i(\vec{\mu})}} - \overset{\text{Effort Cost}}{\boxed{c(\mu_i),}}$$



Definition: Symmetric Equilibrium

$\mu^\star \in \operatorname{argmax}_{\mu_1 \geq 0} U_1^s(\mu_1, \mu^\star)$ where

$$U_1^s(\mu_1, \mu) = p^s \mu_1 B_1(\mu_1, \mu) + v I_1(\mu_1, \mu) - c(\mu_1).$$

Research Q1: **Exact** Equilibrium Analysis

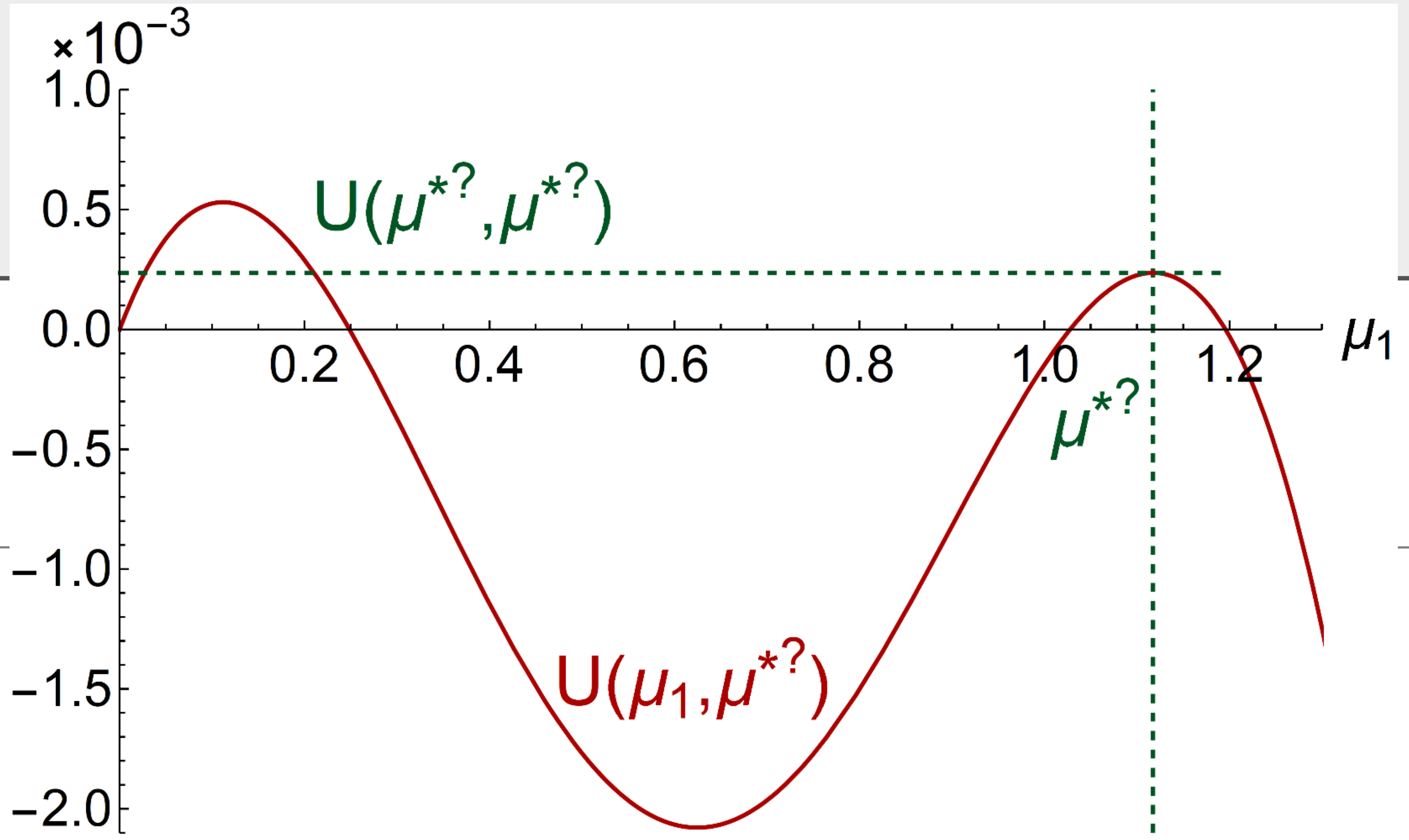
Equilibrium Analysis Steps: (1) Satisfy first-order condition (FOC); (2) Global maximum.

$$\left.\frac{\partial U^s(\mu_1,\mu)}{\partial \mu_1}\right|_{\mu_1=\mu} = 0 \quad \Leftrightarrow \quad p^s\big(1 - I(\mu,\mu)\big) + (v - p^s\mu)\left.\frac{\partial I}{\partial \mu_1}\right|_{\mu_1=\mu} = c'(\mu).$$

Lemma: In an $M/M/N/k$ system with $N-1$ servers operating at rate $\mu > 0$, and a tagged server with rate $\mu_1 > 0$.

$$I(\mu_1,\mu) = \left(1 + \rho\frac{\mu}{\mu_1}\left(\frac{1-C}{N-\rho} + \left(1 - \left(\frac{\rho}{N - \left(1 - \frac{\mu_1}{\mu}\right)}\right)^{k-N}\right)\frac{C}{(N-\rho) - \left(1 - \frac{\mu_1}{\mu}\right)}\right)\right)^{-1},$$

where $\rho = \frac{\lambda}{\mu}$ and $C \coloneqq ErlC(N,\rho) = \frac{\frac{\rho^N}{N!}\cdot\frac{N}{N-\rho}}{\sum_{j=0}^{N-1}\frac{\rho^j}{j!} + \frac{\rho^N}{N!}\cdot\frac{N}{N-\rho}}$.



Research Q1: **Asymptotic** Equilibrium Analysis

Our **asymptotic regime:** a sequence of $M/M/N^{\lambda}/k^{\lambda}$ systems, and let λ become large:

- N^{λ}: the staffing level
- $k^{\lambda} \geq N^{\lambda}$: the system size
- $\mu^{\star,\lambda}$: prelimit equilibrium

Equilibrium Analysis Steps: (1) Satisfy first-order condition (FOC); (2) Global maximum.

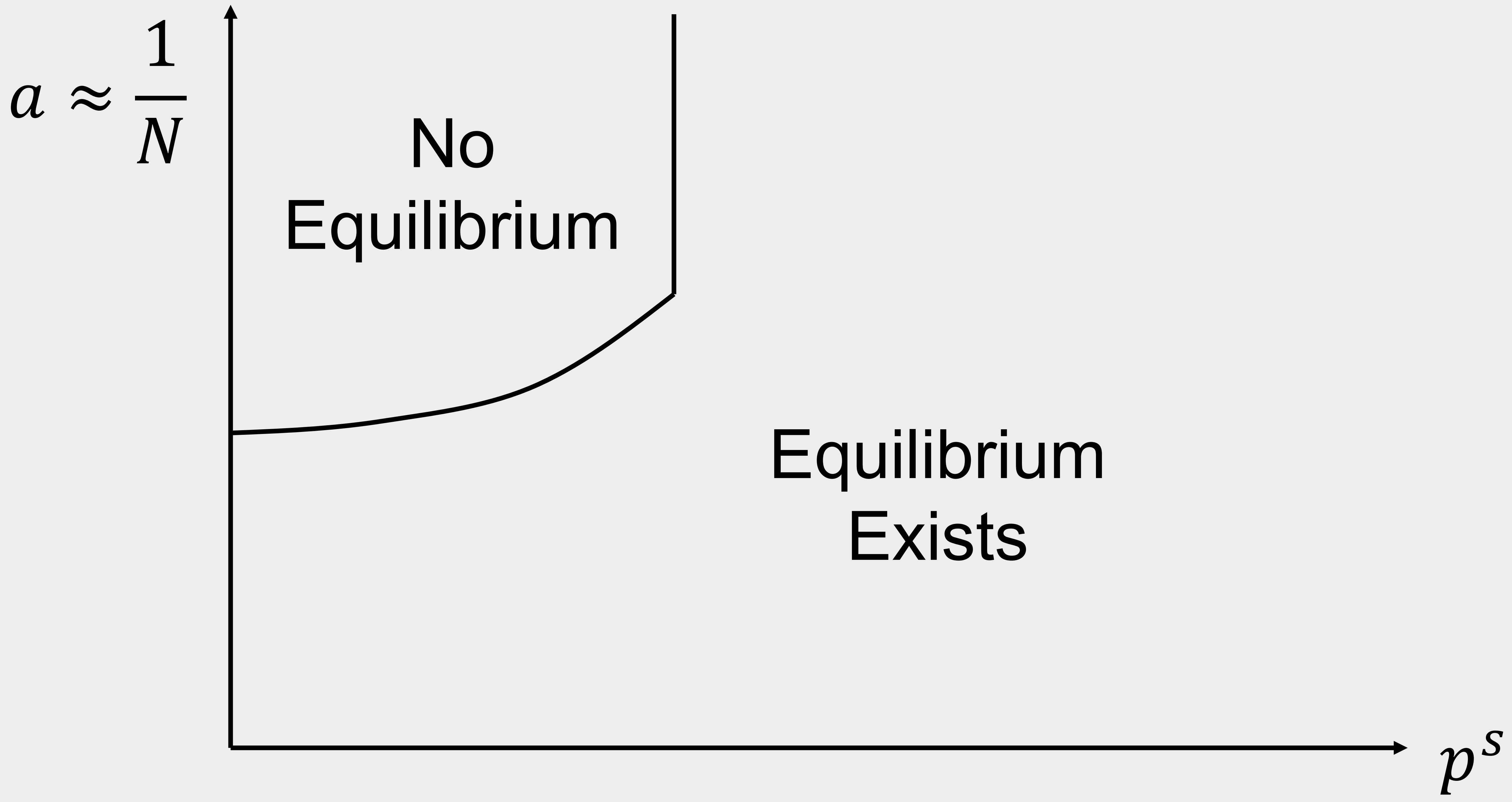
(FOC): $p^s\left(1 - I^{\lambda}(\mu,\mu)\right) + (v - p^s\mu)\left.\frac{\partial I^{\lambda}(\mu_1,\mu)}{\partial \mu_1}\right|_{\mu_1=\mu} = c'(\mu).$

Proposition (Limiting FOC): $p^s\left(1 - \left[1 - \frac{a}{\mu}\right]^+\right) + (v - p^s\mu)\frac{a[\mu-a]^+}{\mu^3} = c'(\mu)$ for $N^{\lambda} = \frac{1}{a}\lambda + o(\lambda)$

Proposition: For large enough λ, $\frac{\partial^2 (U^s)^{\lambda}(\mu_1,\mu)}{\partial \mu_1^2} < 0$ for all $\mu_1 > 0$ and $\mu > 0$.

Research Q1: Equilibrium Existence

Theorem:

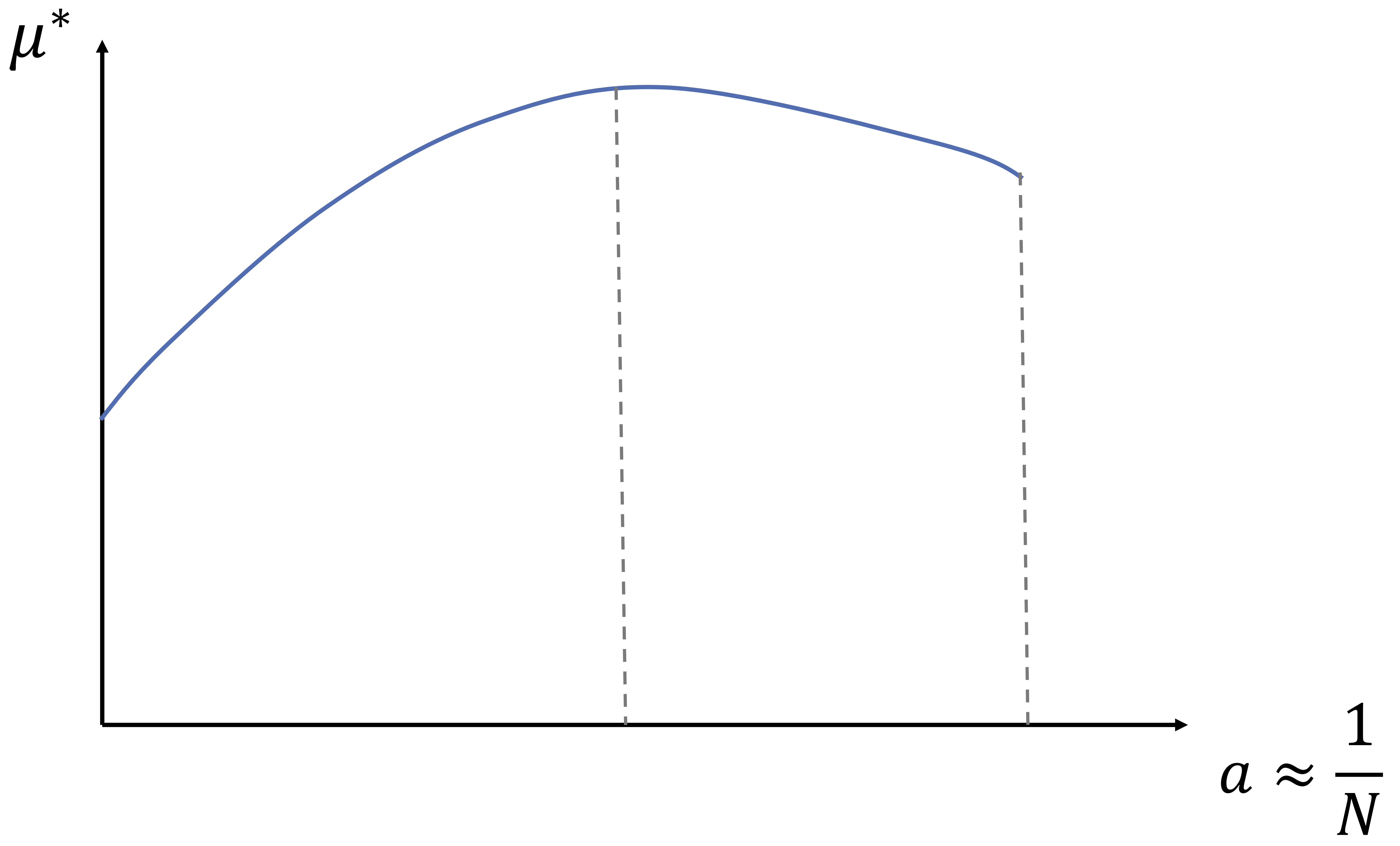


Takeaway: Either staff enough servers or pay them enough to ensure equilibrium existence.

Research Q1: Equilibrium (Non)-Monotonicity

Proposition:

μ^* is strictly increasing in a, and then strictly decreasing in a.



Research Q2: Impact of Selfish Server Behavior

	Definition	Equilibrium
Selfish Utility Maximization [Finite System]	$\mu^* \in \arg\max_{\mu_i \geq 0} U_i(\mu_i, \mu^*; (p^s)_{min}, \lambda, k, N)$	**Selfish Equilibrium** $\mu_e^* = \mu^*(N, (p^s)_{min})$
Social Welfare Maximization [Finite System]	$\max_{p^s} \ N\left(v\, I\big(\mu^*(N, p^s)\big) - c\big(\mu^*(N, p^s)\big)\right)$	**Social Optimum** $\mu_o^* = \mu^*\big(N, {(p^s)_o}^*\big)$
Revenue Maximization [Finite System]	$\min_{p^s} \ p^s \cdot N\mu^*(N, p^s)\left(1 - I\big(\mu^*(N, p^s)\big)\right)$	**Monopolist Optimum** $\mu_r^* = \mu^*\big(N, {(p^s)_r}^*\big)$

Theorem (Limiting System): $\mu_o^* \geq \mu_e^* = \mu_r^*$ (equality only when understaffed or low idleness value).

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* Subscript e: selfish individual equilibrium,
* Subscript o: social welfare optimization,
* Subscript r: revenue maximization.

Research Q3: Combining Selfish Customers and Selfish Servers

Selfish Customers: What should be the buffer size?

Knudsen (1972)

- A customer who arrives to find i customers in the system joins if and only if

$$U^c(i) = \underbrace{\boxed{R}}_{\text{Value for service}} - \underbrace{\boxed{p^c}}_{\text{Service fee}} - \underbrace{\boxed{C}}_{\text{Cost of waiting}} \underbrace{\left(\frac{(i-N+1)^+}{N\mu} + \frac{1}{\mu}\right)}_{\text{Expected time in system}} \geq 0.$$

- Customer equilibrium strategy is to join if and only if there are no more than $k^* - 1$ customers in the system, where

$$k^*(\mu) = \left\lfloor \frac{(R-p^c)N\mu}{C} \right\rfloor.$$

Selfish Servers: What should be the service rate? $\mu^*(k) \in \arg\max_{\mu_i \geq 0} U_i(\mu_i, \mu^*(k); p^s, \lambda, k, N)$

Joint equilibrium: (k^*, μ^*) is a Nash equilibrium if and only if

- $k^* = \left\lfloor \frac{(R-p^c)N\mu^*}{C} \right\rfloor \geq N$, and
- $\mu^* \in \arg\max_{\mu_i \geq 0} U_i(\mu_i, \mu^*; p^s, \lambda, k^*, N)$.

* Subscript e: selfish individual equilibrium,
* Subscript o: social welfare optimization,
* Subscript r: revenue maximization.

Research Q3: Insights from Joint Selfish Behavior

	Definition	Equilibrium
Selfish Utility Maximization [Finite System]	• $k^* = \left\lceil \frac{(R-(p^C)_{min})N\mu^*}{C} \right\rceil \geq N$, and • $\mu^* \in \arg\max_{\mu_i \geq 0} U_i(\mu_i, \mu^*; (p^S)_{min}, \lambda, k^*, N)$	**Selfish Equilibrium** $(k_e^*, \mu_e^*) = \left(k^*((p^C)_{min}), \mu^*(N, (p^S)_{min})\right)$
Social Welfare Maximization [Finite System]	$\max_{p^S, p^C} \left\{\lambda\left(1-\pi_{k^*(p^C)}(\mu^*(N,p^S))\right)\left(R-\frac{C}{\mu^*(N,p^S)}\right) - C \cdot E[Q(\mu^*(N,p^S))]\right\} + \left\{N\left(v\, I(\mu^*(N,p^S)) - c(\mu^*(N,p^S))\right)\right\}$	**Social Optimum** $(k_o^*, \mu_o^*) = \left(k^*((p^C)_o{}^*), \mu^*(N, (p^S)_o{}^*)\right)$
Revenue Maximization [Finite System]	$\max_{p^S, p^C} \left\{p^C \cdot \lambda\left(1-\pi_{k^*(p^C)}(\mu^*(N,p^S))\right)\right\} - \left\{p^S \cdot N\mu^*(N,p^S)\left(1-I(\mu^*(N,p^S))\right)\right\}$	**Monopolist Optimum** $(k_r^*, \mu_r^*) = \left(k^*((p^C)_r{}^*), \mu^*(N, (p^S)_r{}^*)\right)$

* Subscript e: selfish individual equilibrium,
* Subscript o: social welfare optimization,
* Subscript r: revenue maximization.

Research Q3: Insights from Joint Selfish Behavior

	Equilibrium
Selfish Utility Maximization [Finite System]	$(k_e^*, \mu_e^*) = \left(k^*((p^c)_{min}), \mu^*(N, (p^s)_{min})\right)$
Social Welfare Maximization [Finite System]	$(k_o^*, \mu_o^*) = \left(k^*((p^c)_o{}^*), \mu^*(N, (p^s)_o{}^*)\right)$
Revenue Maximization [Finite System]	$(k_r^*, \mu_r^*) = \left(k^*((p^c)_r{}^*), \mu^*(N, (p^s)_r{}^*)\right)$

Recall:

Only selfish customers: $\boxed{k_e^* \geq k_o^*} \geq k_r^*$
Only selfish servers: $\mu_o^* \geq \boxed{\mu_e^* = \mu_r^*}$

Define $b^\lambda := k^\lambda / N^\lambda$

Theorem [Limiting System]: $\boxed{b_o^* \geq b_e^*}$ and $b_r^* \geq b_e^*$; $\mu_o^* \geq \mu_e^*$ and $\boxed{\mu_r^* \geq \mu_e^*}$.

* Subscript e: selfish individual equilibrium,
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Current and Future Research

- Ongoing Work: Optimal Design and Control (e.g., staffing and pricing problem)

- Ongoing Work: Online experiment to test hypotheses

- Future Work: Unknown statistical information

Session TB69 - Learning Algorithms to Manage Service Systems

"Learning the Scheduling Policy in a Multiclass Many Server Queue with Abandonment"
(**Yueyang Zhong**, John R. Birge, Amy R. Ward)

Thanks!
Q&A

Yueyang Zhong: yzhong0@chicagobooth.edu