Visualising the Saddlepoint Approximation

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Cumulant Generating Function

Are there other ways to characterize a distribution other than the probability mass/density function?

$$K(s) = \ln\left(\mathbb{E}\left(e^{sx}\right)\right)$$

The cumulant generating function is uniquely defined by its probability function and vice versa

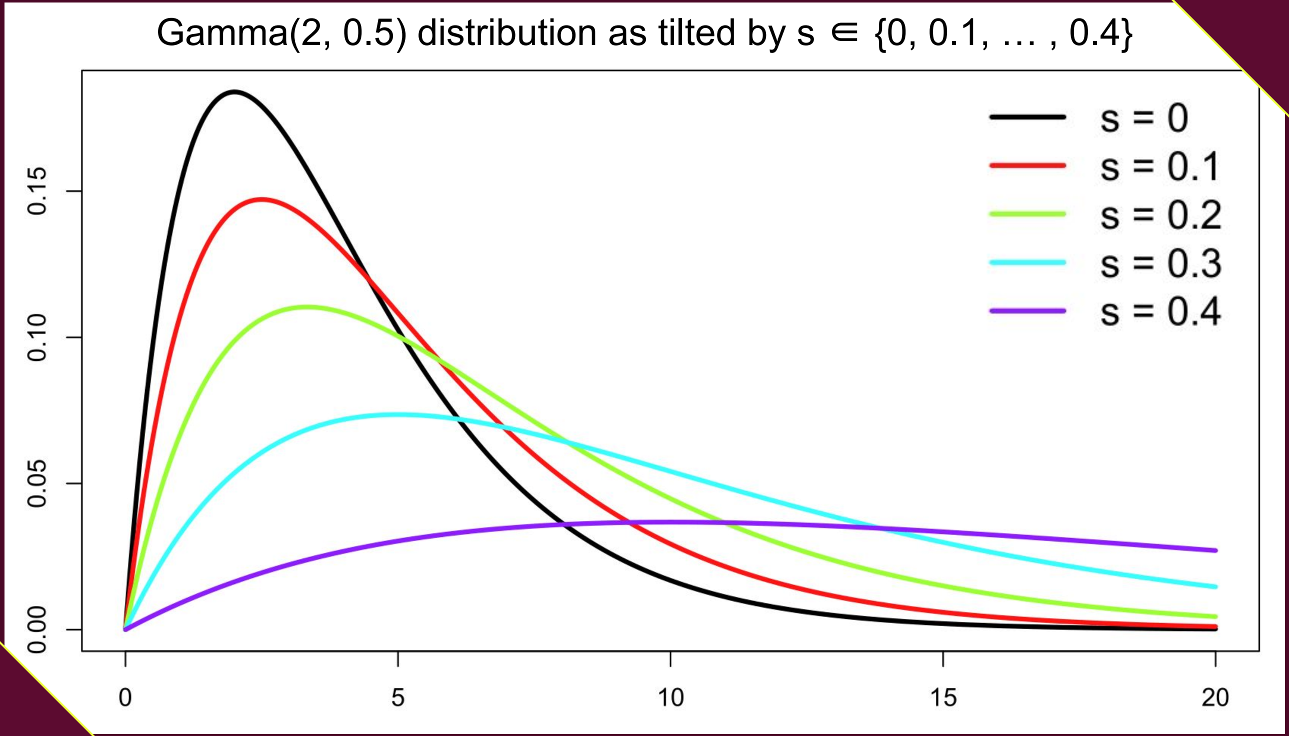
Exponential Tilting

We can rearrange the CGF formula to get:

$$\int_{-\infty}^{\infty} e^{sx - K(s)} f(x)\, dx = 1$$

We can consider the integrand to be a probability density function!

Exponential Tilting



The Saddlepoint Approximation

$$\hat{f}(x) = \frac{1}{\sqrt{2\pi K''(\hat{s})}} \cdot \exp\{K(\hat{s}) - \hat{s}x\}$$

Here, $\hat{s}$ is defined as being the solution to the equation $K'(\hat{s}) = x$

Finding the Saddlepoint Approximation at x

1. Consider the tilted density function as tilted by some s such that the mean of the tilted distribution is x
2. Approximate this tilted density by the normal distribution around x
3. Put back the tilting factor $e^{K(s)-sx}$ to get an approximation for $f(x)$

The Applications

Random variables with common distributions

The sum of two random variables

Multivariate random variables

Saddlepoint Approximation at UoA

- Rachel Fewster has utilised saddlepoint approximations in her work on capture-recapture models for wildlife abundance approximation
- Godrick Oketch is writing code to generate saddlepoint approximations to maximum likelihood estimates and approximate their errors
- Rishika Chopara has work in progress with deviances of the saddlepoint approximation
- Jesse Goodman has been researching its asymptotic accuracy

Thank you!