w6 p.1 | Векторное пр-е, его св-ва, выражение в правом ОНБ. Критерии колл. и компл. в-ров. Двойное в-ное пр-е.

V_i с фикс ориент.

Def. Векторным пр-ем $[\bar{a}, \bar{b}]$ наз-ся вектор, определяемый условиями:

1. $[\bar{a}, \bar{b}] \perp \bar{a}, \bar{b}$
2. $|[\bar{a}, \bar{b}]| = S_{\underset{\bar{a}}{\square}} = |\bar{a}||\bar{b}| \cdot \sin\alpha$
3. $(\bar{a}, \bar{b}, [\bar{a}, \bar{b}])$ — правая тр-ка

Утв. $\bar{a} \parallel \bar{b} \iff [\bar{a}, \bar{b}] = \bar{0}$

Д-во.

$\Rightarrow$ $\bar{a} \parallel \bar{b} \Rightarrow S(\bar{a}, \bar{b}) = 0 \Rightarrow |[\bar{a}, \bar{b}]| = 0 \Rightarrow [\bar{a}, \bar{b}] = \bar{0}$

$\Leftarrow$ $[\bar{a}, \bar{b}] = 0 \Rightarrow S(a, b) = 0 \Rightarrow \bar{a} \parallel b$ ■

Th (о св-х в-го пр-я)

1. Кососим. $[\bar{a}, \bar{b}] = -[\bar{b}, \bar{a}]$
2. Аддитивность по II-му ар-ту $[\bar{a}, \bar{b}_1 + \bar{b}_2] = [\bar{a}, \bar{b}_1] + [\bar{a}, \bar{b}_2]$
3. Однородность по II-му ар-ту $[\bar{a}, \lambda\bar{b}] = \lambda[\bar{a}, \bar{b}]$

Д-во.

1. Пусть $(\bar{a}, \bar{b}, [\bar{a}, \bar{b}])$ — правая тр-ка
$(\bar{b}, \bar{a}, [\bar{a}, \bar{b}])$ — левая
$(\bar{b}, \bar{a}, -[\bar{a}, \bar{b}])$ — правая

2. $\forall c \in V_3$

$$([a, b_1 + b_2], \bar{c}) \overset{*}{=} (\bar{a}, b_1 + \bar{b}_2, \bar{c}) = (\bar{a}, \bar{b}_1, \bar{c}) + (\bar{a}, \bar{b}_2, \bar{c}) =$$
$$= ([\bar{a}, \bar{b}_1], c) + ([\bar{a}, \bar{b}_2], c) = ([\bar{a}, \bar{b}_1] + [\bar{a}, \bar{b}_2], \bar{c})$$ ■

3. Аналогично ■

Note, $*$ — Th $v(\bar{a}, \bar{b}, \bar{c}) = ([\bar{a}, \bar{b}], \bar{c}) = (S(\bar{a}, \bar{b}) \cdot \bar{n}, \bar{c}) = S(a, b)(\bar{n}, \bar{c}) = v(\bar{a}, \bar{b}, \bar{c})$

Утв. Пусть e — базис в V_3

$\bar{a} \underset{e}{\longleftrightarrow} \alpha \quad \bar{b} \underset{e}{\longleftrightarrow} \beta$

$$[\bar{a}, \bar{b}] = \begin{vmatrix} [e_2, e_3] & [e_3, e_1] & [e_1, e_2] \\ \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \end{vmatrix}$$

Д-во.

$$[\bar a, \bar b] = [\sum_i \alpha_i e_i, \sum_j \beta_j e_j] = \sum_i \sum_j \alpha_i \beta_j [e_i e_j] =$$

$$= (\alpha_1\beta_2 - \alpha_2\beta_1)[\bar e_1, \bar e_2] + (\alpha_2\beta_3 - \alpha_3\beta_2)[\bar e_2 \bar e_3] +$$

$$+ (\alpha_3\beta_1 - \alpha_1\beta_3)[\bar e_3, \bar e_1] \blacksquare$$

Сл-е: В ОНБ выр-е для в-го пр-я

$$[\bar a, \bar b] = \begin{vmatrix} \bar e_1 & \bar e_2 & \bar e_3 \\ \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \end{vmatrix}$$

Д-во.

$$[\bar e_1, \bar e_2] = \bar e_3 \quad [\bar e_2 \bar e_3] = \bar e_1 \quad [\bar e_3 \bar e_1] = \bar e_2 \ \blacksquare$$

Т2

$$[\bar a, [\bar b, \bar c]] = \bar b(\bar a, \bar c) - \bar c(\bar a, \bar b)$$

Д-во.

Введем базис (правый ОНБ)

$$\bar a = \alpha_1 \bar e_1 + \alpha_2 \bar e_2 + \alpha_3 \bar e_3$$

$$\bar b = \beta_1 \bar e_1 + \beta_2 \bar e_2$$

$$\bar c = \gamma_1 \bar e_1$$

$$[\bar b, \bar c] = [\beta_1 \bar e_1 + \beta_2 \bar e_2, \gamma_1 \bar e_1] = \beta_2 \gamma_1 [\bar e_2 \bar e_1] =$$

$$= -\beta_2\gamma_1 \bar e_3 = (0, 0, -\beta_2\gamma_1)$$

$$[\bar a, [\bar b, \bar c]] = [\alpha_1 \bar e_1 + \alpha_2 \bar e_2 + \alpha_3 \bar e_3, -\beta_2\gamma_1 \bar e_3] =$$

$$= \alpha_1\beta_1\gamma_1 \bar e_2 - \alpha_2\beta_2\gamma_1 \bar e_1 = (-\alpha_2\beta_2\gamma_1, \alpha_1\beta_1\gamma_1, 0)$$

$$\bar b(\bar a, \bar c) - \bar c(\bar a, \bar b) = \bar b \cdot \alpha_1\gamma_1 - \bar c(\alpha_1\beta_1 + \alpha_2\beta_2) =$$

$$= (-\alpha_2\beta_2\gamma_1, \alpha_1\beta_1\gamma_1, 0) \ \blacksquare$$