## Grafopoli

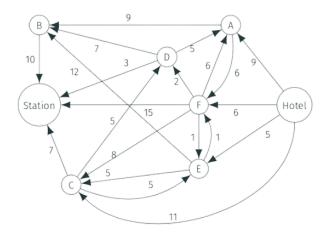
Difficulty level: intermediate

## Keywords

- Graph theory
- Directed graphs
- Dijkstra's algorithm
- Heuristic algorithms
- Integer Linear Programming
- Excel Solver
- Python+PuLP

## Problem description

Remo has to leave the city of Grafopoli and go back to his home. The return train will leave the Central Station at 11:03 but Remo, as always late, only manages to leave the hotel at 10:50. The graph below represents the city network, where each arc is associated with its travel time (in minutes), which is considered reliable.



## **Tasks**

- 1. What is a path that would allow Remo to save the most time possible? Is there only one path or more than one? Why?
- 2. Would Remo have a chance to get to the station in time to catch the 11:03 train? If so, how many minutes early could he arrive? If not, how many minutes late?
- 3. Now suppose that Remo, discouraged by the delay, decides to spend a few more hours visiting the city of Grafopoli. In particular, each node of the graph represents a point of interest to visit (e.g., a museum, a square, a monument, etc.). The table below shows the estimated visiting times for each attraction and the closing times (if there is a "-", it means that the attraction is always open).

|               | A     | В | $^{\mathrm{C}}$ | D     | $\mathbf{E}$ | $\mathbf{F}$ |
|---------------|-------|---|-----------------|-------|--------------|--------------|
| Visiting time |       | 5 | 30              | 60    | 10           | 20           |
| Closing time  | 12:00 | _ | 13:00           | 13:30 | _            | _            |

Assuming that Remo leaves the hotel at 11:00, what path can he take to visit as many attractions as possible and not arrive at the station after 14:00?

- (a) Formulate and describe step by step (i.e., input, output, and all the steps) an algorithm that solves the proposed problem.
- (b) Find a possible solution for the problem by applying the developed algorithm.
- 4. Now formulate the starting problem as an Integer Linear Programming problem, and solve it with the help of Excel Solver.
- 5. Implement the mathematical model in Python and solve it by exploiting the PuLP library.