Graphs, graphs, and more graphs

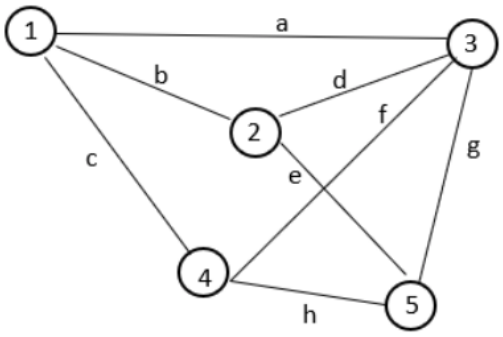
Difficulty level: beginner

Keywords

- Graph theory
- Undirected graphs
- Directed graphs
- Paths

Problem description

Look at the following picture and answer the questions below.

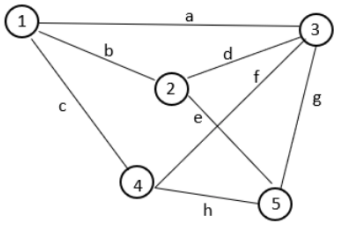


Tasks

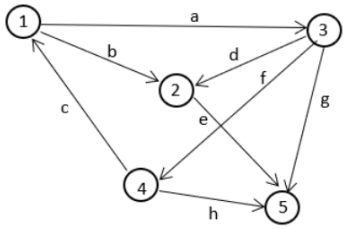
1. What does the picture represent?
2. What is "3"?
3. What does "c" indicate?
4. $\{1, 2, 3, 4, 5\}$ is the set of:
   - ☐ arcs
   - ☐ nodes
   - ☐ edges
   - ☐ graphs
   - ☐ vertices

5. $\{a, b, c, d, e, f, g, h\}$ is the set of:
   - ☐ arcs
   - ☐ nodes
   - ☐ edges
   - ☐ graphs
   - ☐ vertices
6. How many vertices are there in the graph? And how many edges?
7. Write down the degree of each vertex.
8. Draw a graph with 4 vertices, such that one vertex has a degree of 1, two vertices have degree 2, and a vertex has degree equal to 3.
9. Is the graph you have just drawn connected?
10. Draw a connected graph and a disconnected graph.
11. Look at the two following graphs: what can you notice?

Graph A:



Graph B:



12. For both of the two graphs, write the paths linking node 1 to node 5.
13. Looking at Graph A, what is the degree of separation between 1 and 5?
14. Looking at Graph B, what are a and b with respect to 1? And what about c? Similarly, what are b and d to 2?
15. Draw a graph with a disconnected node.

16. Draw a subgraph of the graph at the previous request.

17. Looking at Graph B, motivate your answers to the following questions.

    (a) Does $(1, 3, 2, 4)$ represent a directed path?

    (b) Does $(1, 3, 5, 4)$ represent a directed path?

    (c) $(1, 3, 2, 5)$ represent a directed path?

    (d) Write down another directed path in Graph B.

18. At school there was a bridge tournament in which 8 teams participate: A, B, C, D, E, F, G and H. Team A played with teams B, D and E. Team B with A, E and G. Team D also competed with F and G, instead C with H. In addition, teams G and E also clashed. Draw the relationships between the teams and describe their main characteristics.

19. In May, we are going to visit the village of *Grafopoli*, where the main monuments are connected with each other through some streets (often with only one direction of travel). The main **Place** is connected, by two-way streets, with both the Hamiltonian **M**useum and the **D**ome. Furthermore, from the main **P**lace you can reach the icosahedral **F**ountain and the characteristic **V**ia Leonhard Euler. From the Hamiltonian **M**useum, passing through the particular geometric **Q**uarter, you can arrive at the famous **S**anctuary, from which you can return to the Hamiltonian **M**useum or the main **P**lace, or you can reach the ancient **T**ower of Dantzig. Both from the icosahedral **F**ountain and from the ancient **T**ower of Dantzig you can go towards the characteristic **V**ia Leonhard Euler, from which you can then move towards the **S**anctuary. In *Grafopoli* it is also interesting to visit the **C**athedral of the Quaternions, which can only be directly reached from the **D**ome and from which, however, you can reach the icosahedral **F**ountain and the military **A**rchitecture. From the latter, you can reach the main **P**lace. Finally, you can get to the **D**ome starting directly from the icosahedral **F**ountain. To avoid getting lost in the alleys of *Grafopoli*, draw the map.