SuperAmazingMarket – A chain of stores to be optimized

Difficulty level: advanced

Keywords

- Mathematical modelling
- Linear Programming
- Integer Programming
- Mixed Integer Linear Programming
- Excel Solver

Introduction

Assume you are all members of the Operations Research area of SuperAmazingMarket, a supermarket chain in the province of Brescia. In particular, you are divided into five groups. Every group will focus on solving a different problem related to the company activities. These problems are currently not being managed optimally, or still being solved manually. In order to reach your goals, you are allowed to use the Internet and all the digital technologies you prefer. The results will have to be presented to the executive directors in the form of a presentation.

Every presentation will have to include:

- an analysis of the problem description;
- a manual resolution to compute a feasible solution;
- the model formulation of the problem by applying Linear, Integer, or Mixed Integer Linear Programming;
- an implementation and resolution of the problem performed with Excel Solver;
- the textual description of an analogous problem that arises in a completely different context from the SuperAmazingMarket company.

A) Staff shifts

The SuperAmazingMarket scheduling officer has to reorganize the shifts of its cashier staff in the store located in Rovato, based on the minimum needs expressed in Table 1 for each time slot of the opening hours of the store.

| Time slot | 3–7 | 7-11 | 11–13 | 13-15 | 15-17 | 17-19 | 19-23 | 23-3 |
|-----------------------------|-----|------|-------|-------|-------|-------|-------|------|
| Duration (in hours) | 8 | 8 | 4 | 4 | 4 | 6 | 8 | 8 |
| Minimum number of employees | 7 | 15 | 12 | 20 | 15 | 20 | 10 | 5 |

Table 1: Problem A – Minimum number of employees required in each time slot.

Every employee begins their work shift at the beginning of a time slot. The duration of the shift depends on the slot in which he or she starts working. Those who start working in the night period (i.e., 19–7) have a shift of eight consecutive hours. Those who start working in the evening (i.e., 17–19) have a shift of six consecutive hours. Finally, those who start working during the day (i.e., 11–15) have a shift of four consecutive hours. We want to determine the minimum number of employees such that the needs of the supermarket are met in each time slot.

B) To open or not to open some stores

The SuperAmazingMarket company owns six stores, distributed in the province of Brescia and listed in Table 2, together with their relative distances (in kilometers):

| | Chiari | Coccaglio | Rovato | Iseo | Brescia 1 | Brescia 2 |
|-----------|--------|-----------|--------|------|-----------|-----------|
| Chiari | - | 4 | 5 | 21 | 25 | 27 |
| Coccaglio | 4 | - | 1 | 17 | 22 | 24 |
| Rovato | 5 | 1 | - | 14 | 20 | 22 |
| Iseo | 21 | 17 | 14 | - | 27 | 29 |
| Brescia 1 | 25 | 22 | 20 | 27 | - | 2 |
| Brescia 2 | 27 | 24 | 22 | 29 | 2 | - |

Table 2: Problem B – The SuperAmazingMarket stores and their relative distances (in km).

For example, the store located in Chiari is 5 km far from the one in Rovato, while the supermarket in Iseo is 27 km far from the one named Brescia 1. The SuperAmazingMarket managers wants to reorganize the Sunday opening hours, deciding for each supermarket whether to open it in the morning and/or in the afternoon, such that there is at least one store open within a radius of 20 kilometers.

| | Morning | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$ | Afternoon | Afternoon opening $\mathbf{cost} \ (\mathbb{C})$ | |
|-----------|---------|--|-----------|--|--|
| Chiari | Yes | 20,000 | Yes | 25,000 | |
| Coccaglio | Yes | 15,000 | No | - | |
| Rovato | Yes | 23,000 | Yes | 23,000 | |
| Iseo | No | - | Yes | 17,000 | |
| Brescia 1 | Yes | 30,000 | Yes | 35,000 | |
| Brescia 2 | Yes | 35,000 | Yes | 45,000 | |

Table 3: Problem B – Stores opening possibilities and related opening costs.

Not all stores can always be opened: Table 3 shows the opening possibilities and the related costs. For example, the Iseo store cannot be open in the morning but only in the afternoon, that would cost the company 17,000 euros. Determine which stores to activate according to the possibilities and the constraints, such that the total activation costs are minimal. Are there any superfluous stores?

C) Numismatic policies

A SuperAmazingMarket policy instructs cashiers to try to keep most 1-euro coins at the end of the working day. Geronimo, a cashier working in the store in Coccaglio, really likes this policy because it passionate about numismatics. At the end of his shift, he always checks all the 1-euro coins left in the cash fund, looking for a particularly rare one to add to his collection. One evening, the cash fund is made up of 22 5-euro banknotes, 38 2-euro coins, 15 1-euro coins, 39 50-cent coins, and 34 20-cent coins. Geronimo wants to make sure that, after the last four customers, he has as many 1-euro coins as possible. To do this, he asks the four customers for an estimate of their spending, and the payment method they will use. The four customers reply that the estimated cost of their expense is \mathfrak{C} 47.50, \mathfrak{C} 27.80, \mathfrak{C} 18.50, and \mathfrak{C} 33.60, while the payment methods will be respectively a 50-euro banknote, a 20-euro banknote and a 10-euro banknote, a 20-euro banknote, and finally two 20-euro banknotes. How does Geronimo have to give the change to the four customers, in order to maximize the number of 1-euro coins remaining in the cash fund?

D) Between shelves and warehouses

Tea is a saleswoman working in the Brescia 2 store of the *SuperAmazingMarket* company. Her main task is to arrange several types of products on shelves, by moving items from the warehouse when needed. One day, the trekking shelf contains the products listed in Table 4, together with their quantities on the shelf itself and their stocks in the warehouse.

| | Lighter | Bottle | Pocket knife | Toothpaste | Rubber band | Whistle | Gloves | Helios shoes |
|-----------|---------|--------|--------------|------------|-------------|---------|--------|--------------|
| Shelf | 0 | 2 | 2 | 1 | 1 | 0 | 2 | 0 |
| Warehouse | 4 | 1 | 2 | 3 | 5 | 6 | 4 | 6 |

Table 4: Problem D – Quantities on the shelf and stocks in the warehouse of the products.

The Brescia 2 store director has determined that on the trekking shelf there must be at least three lighters, a bottle, three small pocket knives, three tooth-paste, five rubber bands, a whistle, four gloves, and three Helios shoes. Moreover, from a market survey, it has been found that for each toothpaste on the shelf there must be at least two whistles, and that for each pocket knife there must be at most three bottles.

| | Lighter | Bottle | Pocket Knife | Toothpaste | Rubber band | Whistle | Gloves | Helios shoes |
|---------------------------|---------|--------|--------------|------------|-------------|---------|--------|--------------|
| Price (€) | 12.3 | 7.99 | 4.2 | 5.65 | 9.99 | 7.35 | 10 | 8.7 |
| Volume (cm ³) | 834 | 5100 | 1360 | 570 | 3980 | 4700 | 6000 | 3350 |

Table 5: Problem D – Selling price (in €) and volume (in $\rm cm^3$) of trekking products.

How many quantities of each product must Tea insert or remove from the shelf, such that the company can have the best profit, while taking into account that each product has a certain cost and volume indicated in Table 5, and that the maximum volume of the trekking shelf is 100 dm³?

E) Easter gift baskets

It's Easter and the SuperAmazingMarket company wants to compose some gift baskets to sell to its customers. The several products that can be placed in the baskets are listed in Table 6, together with their prices and weights. Notice that each product can be inserted only once in all the gift baskets.

| Product | Price (€) | Weight (kg) |
|------------------|-----------|-------------|
| Boardgame | 8.7 | 1.4 |
| Chocolate lamb | 2.1 | 0.2 |
| Doll | 6.99 | 0.5 |
| Easter Dove cake | 7.2 | 1 |
| Granola | 5.25 | 0.4 |
| Lamp | 3,99 | 0.7 |
| Notebook | 9,4 | 1 |
| Radio | 4.8 | 0.6 |
| Stamps | 5.75 | 0.9 |
| Tickets | 3.35 | 0.3 |

Table 6: Problem E – List of products that can be inserted into the Easter gift baskets, each item described with its price (in \mathfrak{C}) and weight (in kg).

Five empty baskets are available: one can hold a maximum of 1 kg of products; the second one can hold a maximum of 3 kg; the third and the fourth baskets can both hold a maximum of 2 kg; finally, the last basket can hold 2.5 kg. The SuperAmazingMarket marketing manager wants to determine which baskets to use among those available and which products to insert in each basket, in order to maximize the total profits.