Operations Research at high school: its impact on modelling skills and beyond

Eugenia Taranto[1], Gabriella Colajanni[1], Alessandro Gobbi[2], Marinella Picchi[3], Alice Raffaele[4]

[1]*Department of Mathematics and Computer Science, University of Catania, Italy*
[2]*Department of Mechanical and Industrial Engineering, University of Brescia, Italy*
[3]*IIS Antonietti, Iseo (Brescia), Italy*
[4]*Department of Computer Science, University of Verona, Italy*

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What is the talk about?

- Introduction
- Theoretical background
- Research method
- Results
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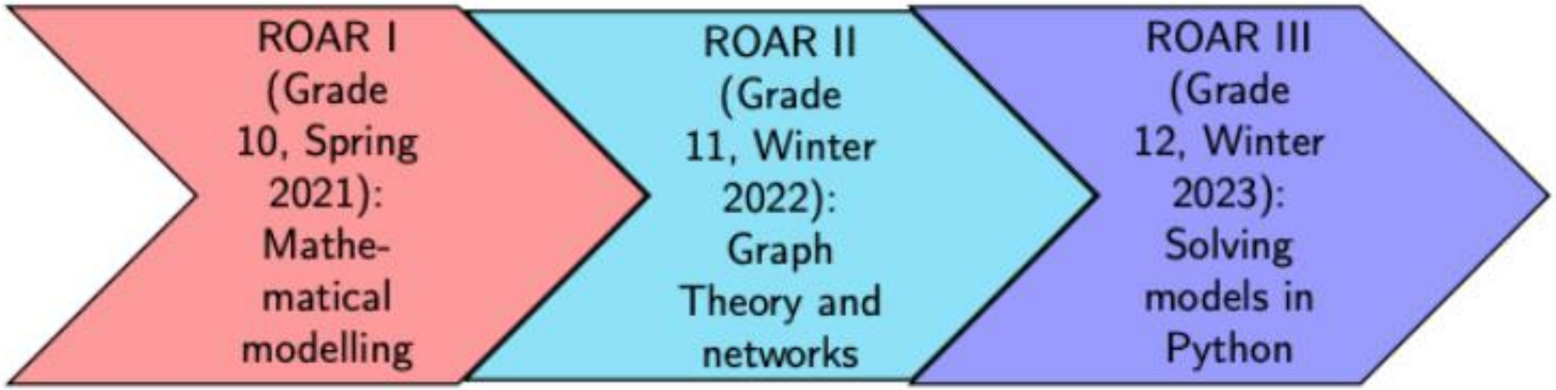
“Model-making is a profound and instinctual human response to understanding the world” (Judson, 1980)

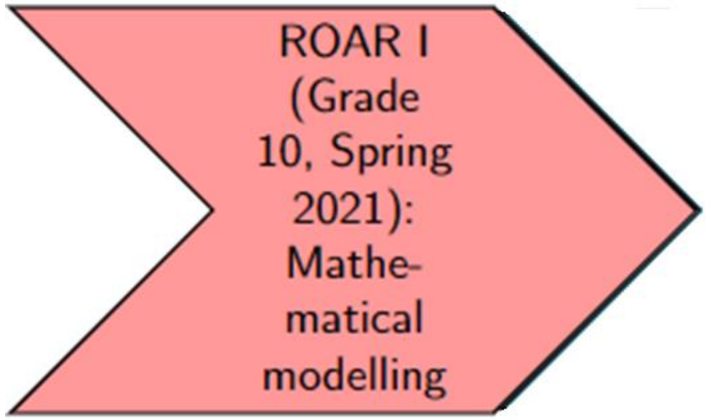
The main object of Operations Research (OR) is the construction of formal models and it could be suitable to stimulate learners’ interest in mathematics and to help them develop problem-solving and modelling skills.

Nevertheless, OR is not typically included in most curricula of higher secondary schools. OR is usually presented mainly at university level.
During the last years, a few initiatives have been developed to introduce OR to students before university level. Raffaele & Gobbi (2021) collected, compared, and classified the OR-based activities addressed to Grades 9–12.

In 2019, ROAR (Ricerca Operativa Applicazioni Reali – Real Applications of Operations Research) educational project was born from an idea of Raffaele and Gobbi, two OR experts, in collaboration with Colajanni e Taranto, respectively OR and mathematics education experts.

ROAR aims to introduce the foundations of OR, by making students work in teams, collaborating with each other most of the time, in studying and analyzing of authentic problems.





tested in a Grade-10 class
at the scientific high school IIS Antonietti in Iseo (Brescia)
between March and May 2021

RESEARCH OBJECTIVES:

- Understanding whether ***it is appropriate to include OR and its typology of problems in regular mathematics lectures***.
- Investigating whether ***modelling and problem-solving skills***, developed with OR, can be ***fostered by implementing a collaborative way of working***, also by making ***use of digital technologies***.
- Knowing what ***impact*** such activities have ***on students' appreciation of OR***.

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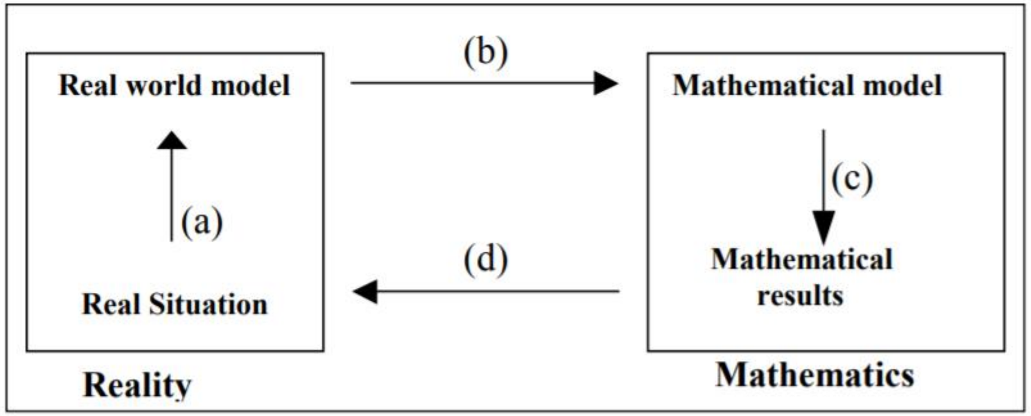
MODELLING

A classification for modelling approaches emerged from an extensive International Commission on Mathematical Instruction (ICMI) study (see Blum et al., 2007).

In ROAR the **realist or applied-modelling perspective** approach: promotion of modelling skills requires students to make their own experiences with authentic modelling problems (Kaiser et al., 2013).

Authentic problems are defined by Niss (1992) as problems that are only slightly simplified and that one might encounter in their everyday work.



Modelling cycle
(from Kaiser et al., 2013, pag. 288).

COLLABORATIVE LEARNING

Many researchers stress the importance of employing the use of models in practical teamwork.

According to Senge (1990), modelling in teams allows for a better understanding of what a model is and how to use it effectively.

Kaiser (2005) states that teamwork allows students to better cope with the uncertainty that is characteristic of modelling activities.

DIGITAL TECHNOLOGIES

In understanding the effectiveness of solving and expressing problems with technology, "[…] *techno-mathematical fluency holds up the entanglement between mathematical and technological knowledge and skills necessary for an efficient activity of problem-solving with technologies*" (Jacinto and Carreira, 2017, p. 1117).

Digital technologies are used to communicate, produce, or represent mathematical ideas, influencing the developed mathematical thinking.

Moreover, the efficient use of a technology is based on an adequate recognition of its affordances, i.e., the set of features attributed to a certain tool or object that invites the subject to undertake an action (Gibson, 1977).

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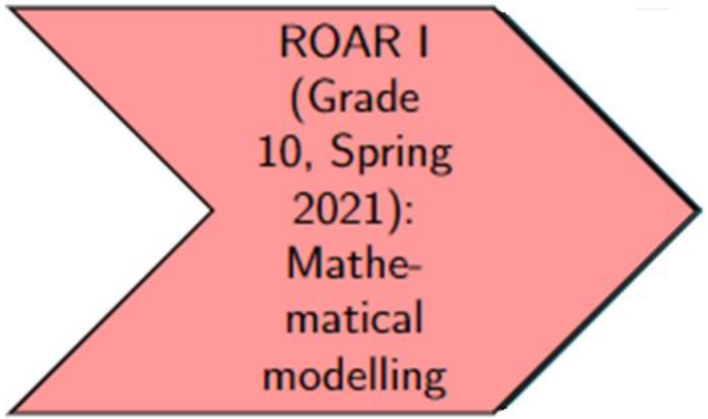

tested in a Grade-10 class
at the scientific high school IIS Antonietti in Iseo (Brescia)
between March and May 2021
by starting a PCTO between the IIS Antonietti and
University of Brescia

- The class* was composed of 16 males and 9 females.
- Classroom teacher: Marinella Picchi
- Two experimenters: Alice Raffaele and Alessandro Gobbi
- Two observers: Gabriella Colajanni and Eugenia Taranto.

Prerequisites:
Mathematics: linear equations, linear inequalities, and some notions of analytic geometry, such as lines and families of straight lines, or plotting a function in a Cartesian coordinate system.
Digital technologies: at least familiar with Microsoft Excel and GeoGebra.

*The students had never dealt with OR-related topics and had never attended supplementary activities on applications or modelling.

6 lectures

1, 2 → MT
3, 4, 5 → 50% MT
6 → on site

Introduction
Homework correction and live polls to activate students' attention

Frontal lecture
Example problem and explanation

Group work #1
Assignment of exercises to students divided in groups and split into breakout rooms

Comparison
Correction and further explanation

Group work #2
...

Conclusion
Homework assignment and live polls to discuss the lecture

	Name	Level
Group 1	Antonella	intermediate
	Barbara	advanced
	Elena	advanced
	Giacomo	advanced
	Marta	advanced
Group 3	Angelo	intermediate
	Bruno	basic
	Carlo	advanced
	Tommaso	advanced
	Pietro	basic

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Sweet Easter

Problem:

The Turin pastry shop "A Little of Cocoa" produces Easter doves and chocolate lambs.
Due to production needs, it has to satisfy the following constraints:

- *the triple of the number of doves plus the number of chocolate lambs must not be less the 750;*
- *the double of the number of doves plus the triple of the number of lambs must not be less than 850;*
- *the difference between the number of doves and the number of chocolate lambs must not exceed 500;*
- *the number of chocolate lambs must not exceed 900.*

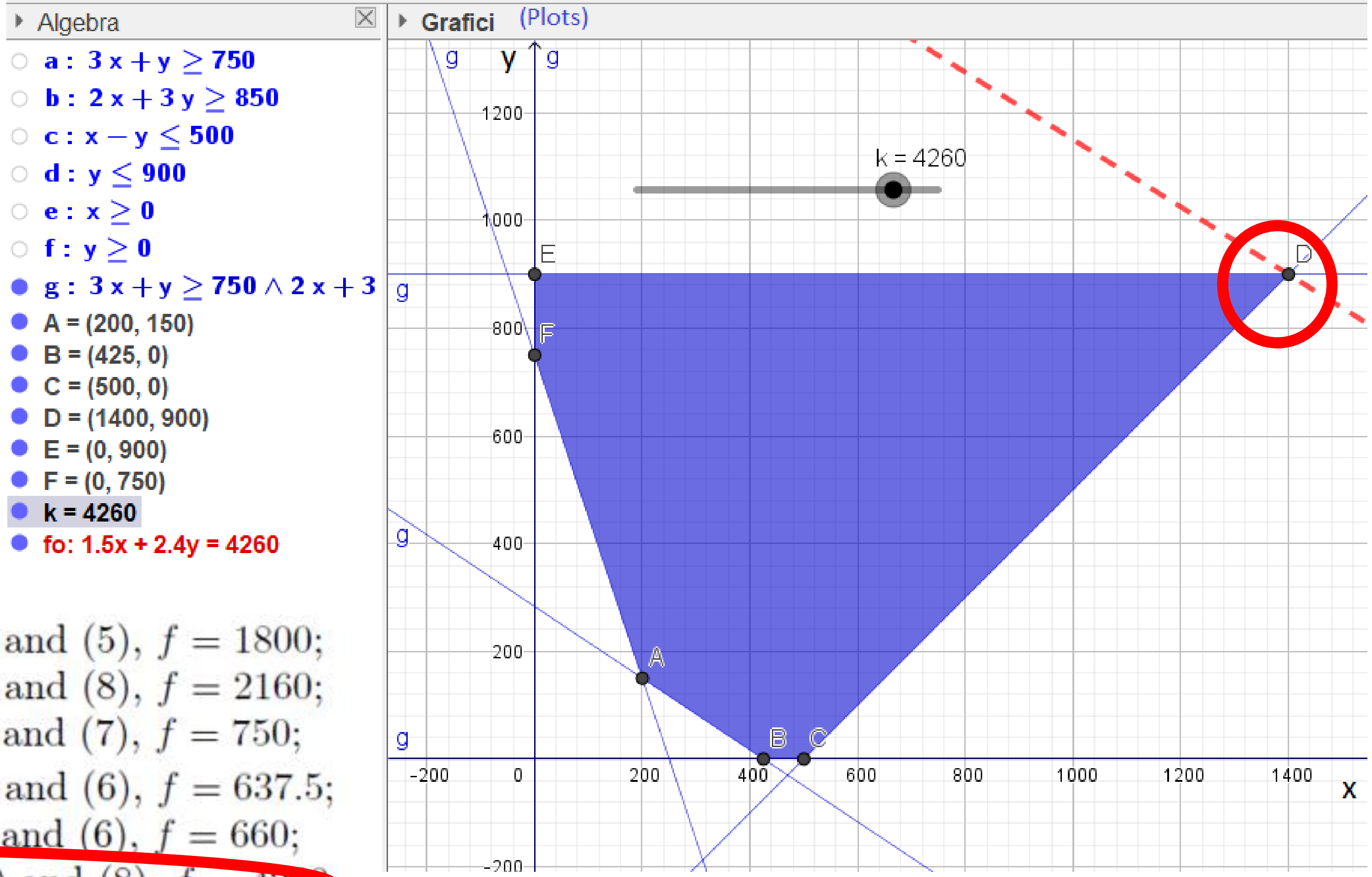
Knowing that the unit selling costs for a dove and a chocolate lamb are 1.50€ and 2,40€ respectively, how many pastries of each kind has the pastry shop to produce, in order to maximize its revenue?

Tasks:

1. *Formulate a Linear Programming model for the problem.*
2. *Compute an optimal solution for the model obtained in Task 1 by using the graphical method.*

x and y indicate the number of Easter doves and the number of chocolate lambs to be produced, respectively

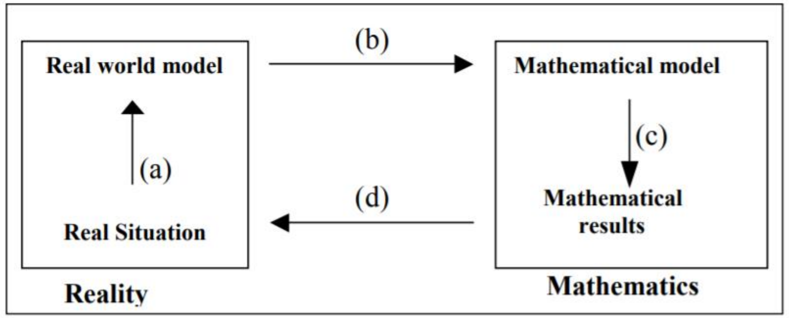
$$
\begin{aligned}
\max \quad & 1.5x + 2.4y && (4)\\
& 3x + y \geq 750 && (5)\\
& 2x + 3y \geq 850 && (6)\\
& x - y \leq 500 && (7)\\
& y \leq 900 && (8)\\
& x, y \geq 0. && (9)
\end{aligned}
$$



- in $(0, 750)$, obtained by the intersection of $x = 0$ and (5), $f = 1800$;
- in $(0, 900)$, obtained by the intersection of $x = 0$ and (8), $f = 2160$;
- in $(500, 0)$, obtained by the intersection of $y = 0$ and (7), $f = 750$;
- in $(425, 0)$, obtained by the intersection of $y = 0$ and (6), $f = 637.5$;
- in $(200, 150)$, obtained by the intersection of (5) and (6), $f = 660$;
- in $(1400, 900)$, obtained by the intersection of (7) and (8), $f = 4260$;

MODELLING

	Name	Level
Group 1	Antonella	intermediate
	Barbara	advanced
	Elena	advanced
	Giacomo	advanced
	Marta	advanced



	Name	Level
Group 3	Angelo	intermediate
	Bruno	basic
	Carlo	advanced
	Tommaso	advanced
	Pietro	basic

both groups conclude the modelling cycle,
satisfying the tasks of the *Sweet Easter* problem

They started modelling the problem by assigning the variables, specifying that the domain was composed of positive numbers.
They all interacted and, in particular, they discussed writing constraints first and then objective function.

MODELLING

	Name	Level
Group 1	Antonella	intermediate
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	Giacomo	advanced
	Marta	advanced

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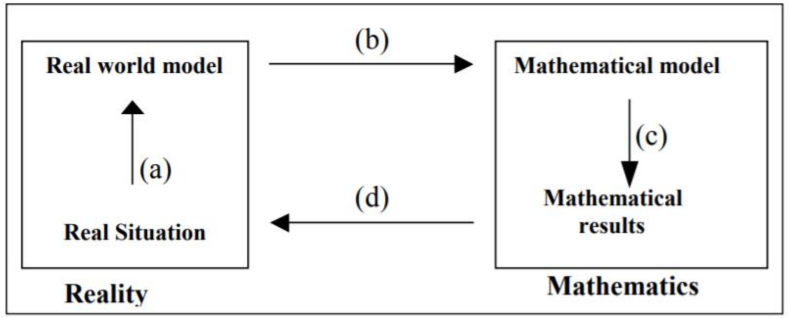

both groups conclude the modelling cycle,
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In phases (a) and (b) of the modelling cycle, **Marta** said that for the objective function it was necessary to maximize x + y, but **Antonella** corrected her immediately, by specifying that it was necessary to integrate coefficients. **Elena** read the correct objective function.

$$\max \quad 1.5x + 2.4y \tag{4}$$
$$3x + y \geq 750 \tag{5}$$
$$2x + 3y \geq 850 \tag{6}$$
$$x - y \leq 500 \tag{7}$$
$$y \leq 900 \tag{8}$$
$$x, y \geq 0. \tag{9}$$

MODELLING

	Name	Level
Group 1	Antonella	intermediate
	Barbara	advanced
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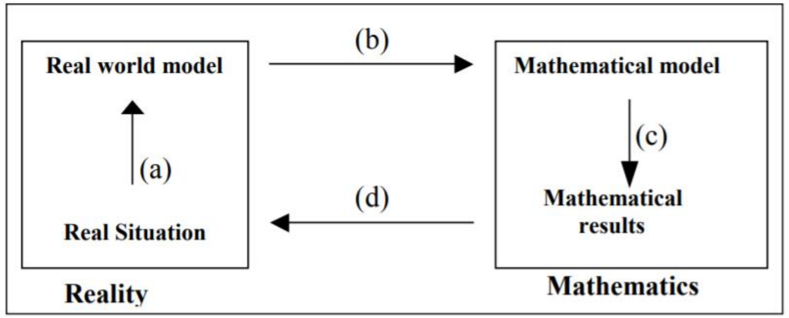
	Marta	advanced



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Moving to phase (c) of the modelling cycle, they decided to work directly with GeoGebra without doing the graphical representation by hand first, except **Barbara**.
Subsequently, the students compared the solutions they have obtained by sharing screens.

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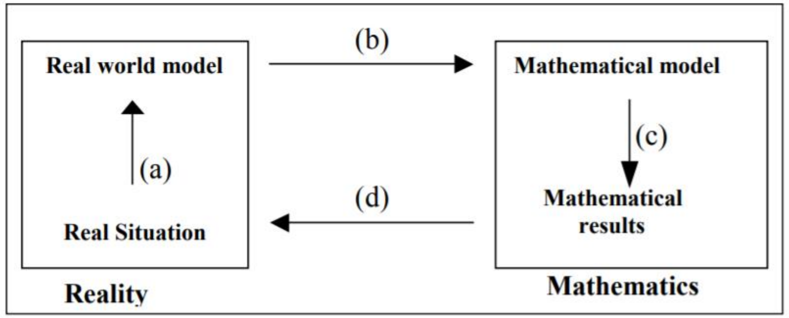

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Meanwhile, **Elena** said she had done the calculations: she told the others that the optimal solution was 1400 doves and 900 lambs.
In the meantime, **Antonella** did the calculations again and confirmed that she had obtained the same result.

$$\max \quad 1.5x + 2.4y \tag{4}$$
$$3x + y \geq 750 \tag{5}$$
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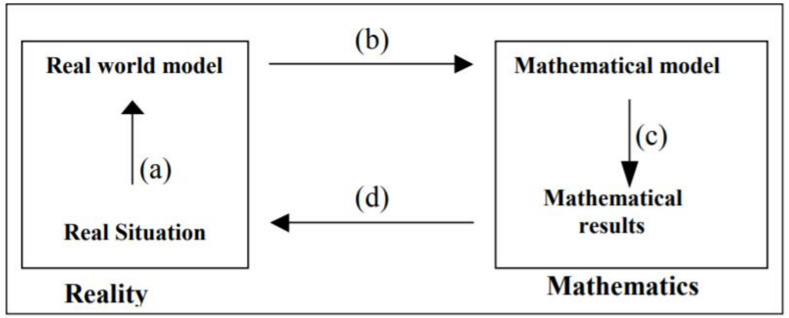

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They moved rather quickly from phase (a) to phase (b) of the modelling cycle. **Angelo** read the text and immediately identified the variables. **Carlo** was uncertain about the sign to be inserted in one of the constraints, but **Angelo** and **Bruno**, re-reading the text, pointed out to him that it should be greater equal.

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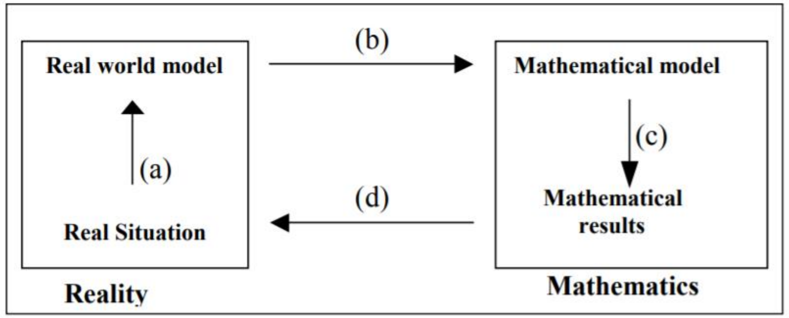

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Students were also confident in moving to phase (c) of the modelling cycle.
They knew that, in order to determine the vertices, they had to consider the associated equations of the constraints. Thus, they wrote them down in explicit form. Then, they switched to GeoGebra and input the constraints as inequalities.

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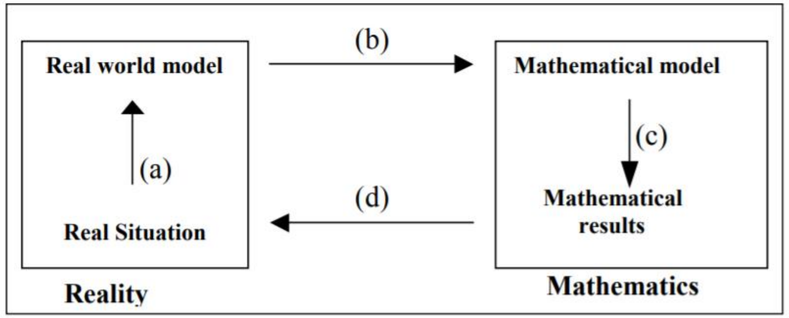

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\end{aligned}
$$

They identified the feasible region, but they all had difficulty marking the vertices of the feasible region on GeoGebra.
They gave up and searched for the intersection points on paper.
By comparing with each other, they found all the six points.

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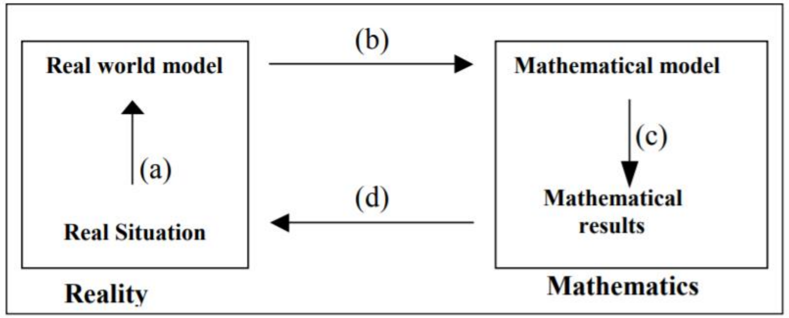

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Subsequently:

Angelo: “*Once we have found the points of intersection, we need to calculate the maximum. The answer is (1400; 900), surely... Because we have to maximize.*”

Tommaso: “*Then the maximum is 1,5 * 1400 + 2,4 * 900 = 4260.*”

That was enough for them to move on to phase (d) of the modelling cycle, thus validating the obtained answer and closing the modelling cycle.

DIGITAL TECHNOLOGIES

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GeoGebra

not all members of the groups
had equal techno-mathematical fluency

Barbara proceeded to work by hand, but she punctually made comparisons with her classmates. **Marta** also failed to fully exploit the affordances of the software, as she could not generate the slider that would have allowed her to represent the family of parallel straight lines. **Giacomo**, also gave up using GeoGebra, but he relied on another software, Desmos.

All the students started working on GeoGebra, but nobody managed to exploit the affordance that would have allowed them to obtain the vertices of the feasible region on the screen. They were forced to switch to paper, outlining little techno-mathematical fluency.

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COLLABORATIVE LEARNING

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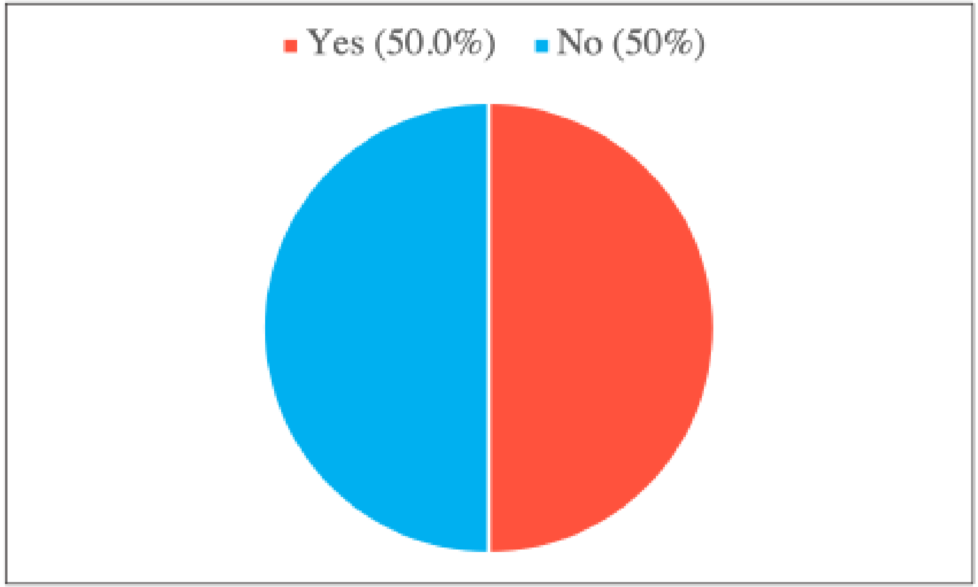
characteristic that emerged
in both groups

The Group 1 members showed themselves capable of a fruitful collaboration that generated formally correct arguments, support among the various members of the group (even if **Barbara** worked manually, she still had confirmation and support from her classmates during the solving process), and co-construction of knowledge.

The Group 3 members showed cohesiveness. In addition, a greater mastery of the mathematical knowledge involved also emerged. They produced rapid and correct modelling of the problem. Only in the validation phase (d, in the cycle) they became less rigorous, letting themselves be guided by intuition.

From the final questionnaire …



"Has ROAR changed or influenced the idea you had about mathematics and its applications in the real world?"

Taranto, E., Colajanni, G., Gobbi, A. Picchi, M. & Raffaele, A. (in press). Fostering students' modelling and problem-solving skills through Operations Research, digital technologies, and collaborative learning. *International Journal of Mathematical Education in Science and Technology* (iJMEST). DOI: https://doi.org/10.1080/0020739X.2022.2115421

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The main goal of ROAR is to engage students
in stimulating modelling and problem-solving activities.

The **understanding of the problem and the writing of its model** constitute
the most important and ambitious part of the modelling process,
an aspect that is mostly neglected in regular mathematics classes
(Kaiser et al., 2013).

The **ROAR problems** are **open**:
there is no one-size-fits-all method to solve them,
but it is up to the student to identify the optimal solving strategy.
In fact, the teaching and learning process is characterised
by autonomous and self-controlled learning.

Care was taken to enable students to be able to work **collaboratively**,
in open-ended problem situations,
while also experiencing feelings of uncertainty and insecurity
that are characteristic for real-life applications
of mathematics in everyday life and science
(Blomhøj and Jensen, 2003).

Students have experienced good learning outcomes
that reflect all of the goals associated with modelling,
ranging from psychological goals such as motivation
to meta-aspects such as fostering working attitudes to pedagogical goals,
i.e., improving the understanding of the world around us.

From the case studies and the quantitative analyses conducted,
it emerges that
the implementation of a modelling pathway such as ROAR
can be successfully tackled by ordinary higher secondary students.

All this supports our position that
it is appropriate to include OR and its type of problems
in regular mathematics lectures,
clearly not every day, but on a regular basis.

Our current and future research perspectives
certainly include the intention to continue with the ROAR project!

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Thank you

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