Graphs, graphs, and more graphs
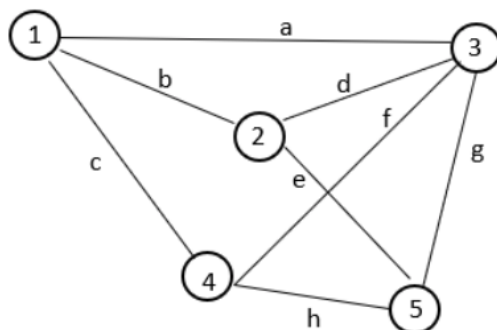
Difficulty level: beginner

Keywords

- Graph theory
- Undirected graphs
- Directed graphs
- Paths

Problem description

Look at the following picture and answer the questions below.


Tasks

1. What does the picture represent?

2. What is "3"?

3. What does "c" indicate?

4. $\{1, 2, 3, 4, 5\}$ is the set of:

   - arcs
   - nodes
   - edges
   - graphs
   - vertices

5. $\{a, b, c, d, e, f, g, h\}$ is the set of:

   - arcs
   - nodes
   - edges
   - graphs
   - vertices

6. How many vertices are there in the graph? And how many edges?

7. Write down the degree of each vertex.

8. Draw a graph with 4 vertices, such that one vertex has a degree of 1, two vertices have degree 2, and a vertex has degree equal to 3.

9. Is the graph you have just drawn connected?

10. Draw a connected graph and a disconnected graph.

11. Look at the two following graphs: what can you notice?

Graph A:
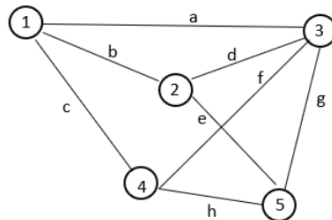
Graph B:
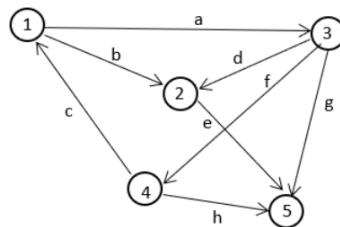
12. For both of the two graphs, write the paths linking node 1 to node 5.

13. Looking at Graph A, what is the degree of separation between 1 and 5?

14. Looking at Graph B, what are a and b with respect to 1? And what about c? Similarly, what are b and d to 2?

15. Draw a graph with a disconnected node.

16. Draw a subgraph of the graph at the previous request.

17. Looking at Graph B, motivate your answers to the following questions.

    (a) Does $(1, 3, 2, 4)$ represent a directed path?
    (b) Does $(1, 3, 5, 4)$ represent a directed path?
    (c) $(1, 3, 2, 5)$ represent a directed path?
    (d) Write down another directed path in Graph B.

18. At school there was a bridge tournament in which 8 teams participate: A, B, C, D, E, F, G and H. Team A played with teams B, D and E. Team B with A, E and G. Team D also competed with F and G, instead C with H. In addition, teams G and E also clashed. Draw the relationships between the teams and describe their main characteristics.

19. In May, we are going to visit the village of *Grafopoli*, where the main monuments are connected with each other through some streets (often with only one direction of travel). The main **Place** is connected, by two-way streets, with both the Hamiltonian **Museum** and the **Dome**. Furthermore, from the main **Place** you can reach the icosahedral **Fountain** and the characteristic **Via** Leonhard Euler. From the Hamiltonian **Museum**, passing through the particular geometric **Quarter**, you can arrive at the famous **Sanctuary**, from which you can return to the Hamiltonian **Museum** or the main **Place**, or you can reach the ancient **Tower** of Dantzig. Both from the icosahedral **Fountain** and from the ancient **Tower** of Dantzig you can go towards the characteristic **Via** Leonhard Euler, from which you can then move towards the **Sanctuary**. In *Grafopoli* it is also interesting to visit the **Cathedral** of the Quaternions, which can only be directly reached from the **Dome** and from which, however, you can reach the icosahedral **Fountain** and the military **Architecture**. From the latter, you can reach the main **Place**. Finally, you can get to the **Dome** starting directly from the icosahedral **Fountain**. To avoid getting lost in the alleys of *Grafopoli*, draw the map.