

Eigenfaces & PCA

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Executive Summary

1. PCA
2. Intro to Facial Recognition
3. Eigenfaces
4. Results
5. Other Methods & Applications
6. Summary



1

PCA



PCA Review

What is PCA?

- ◉ Dimensionality reduction tool
- ◉ Good for clustering & predictive analysis
- ◉ Similar to Factor Analysis
- ◉ Invented by Karl Pearson in 1901

How does PCA work?

- ◉ Performed on a square symmetric matrix such as a covariance matrix
- ◉ Based on orthogonal projections
- ◉ Uses eigenvalue decomposition
- ◉ Each subsequent principal component maximizes the proportion of remaining variance explained



Connecting PCA and SVD

The Connection:

- PCA is equivalent to finding eigenvalues of a covariance matrix
- $\text{Covariance}(A) = A^T A = \Sigma$
- SVD of $A = U \Sigma V^T$
- Then U is an orthogonal matrix, known as the left singular values.
- U will be our eigenfaces (coming soon!)
- We have now avoided the need to calculate the covariance matrix



SVD MAKES PCA FASTER

(SOMETIMES)



2

**Intro to Facial
Recognition**

“ This recognition problem is made difficult by the great variability in head rotation and tilt, lighting intensity and angle, facial expression, aging, etc.

— Woody Bledsoe, Father of Facial Recognition, 1966



Applications of Facial Recognition



iPhone

The latest iPhone X, released in 2017, uses facial recognition “Face ID” that allows your face to be your password.



Android

Android phones introduced the “Trusted Face” feature in 2014 with the release of Android Lollipop.



Snapchat

Snapchat uses facial recognition to allow its users to have cool filters on their face (dog ears filter).



Surveillance

Private intelligence agencies were using facial recognition in their surveillance as early as 1964.



Instagram

Copying Snapchat, Instagram started using facial recognition to also allow their users to have cool filters on their face.



Digital Cameras

Many digital cameras can recognize human faces that allow for clearer and better portrait photos.



3

Eigenfaces



Background Information

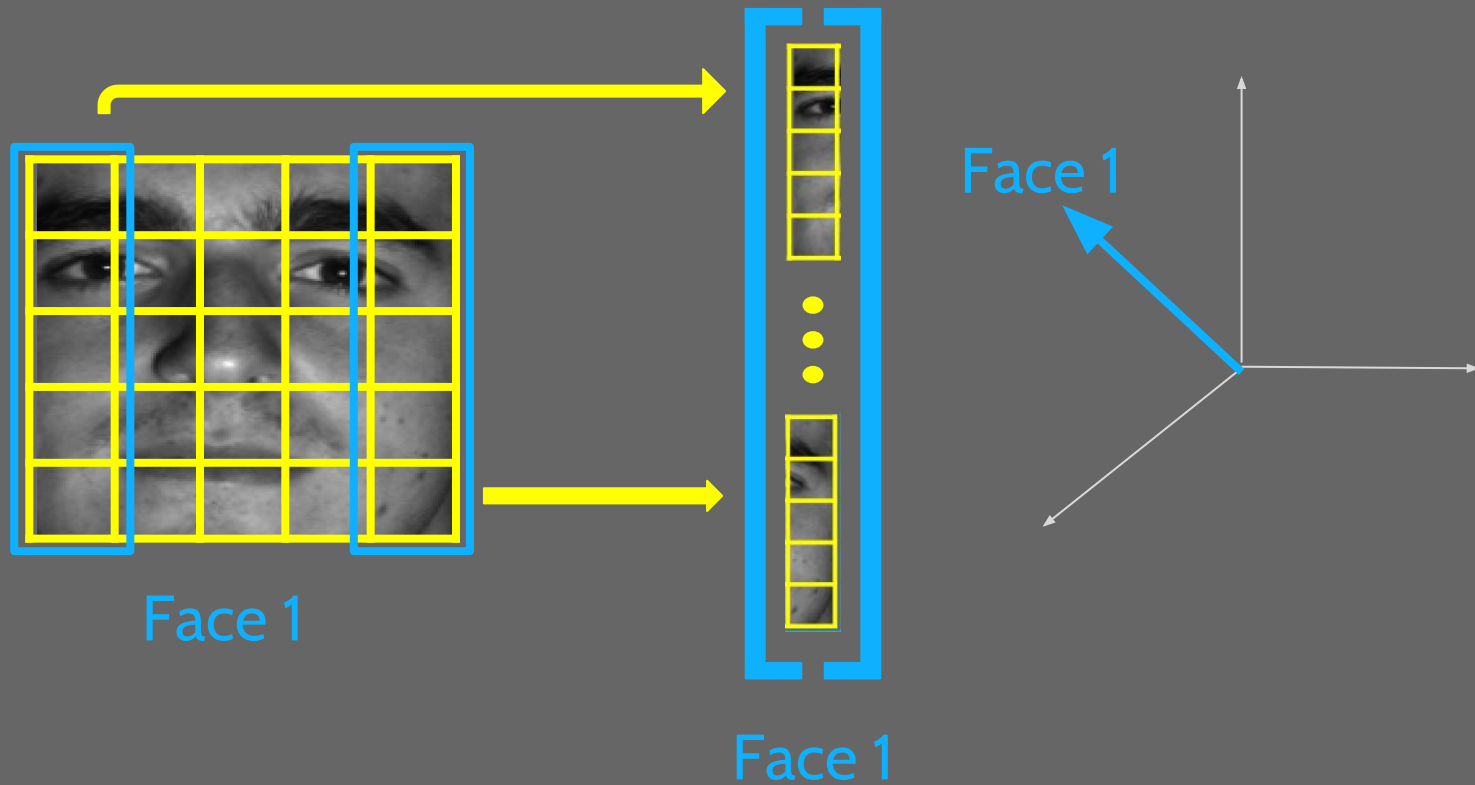
We used Extended Yale Face Database B

- ⦿ 32x32 Data file
- ⦿ This contains faces and their labels
- ⦿ 38 individuals
- ⦿ 9 poses
- ⦿ 64 different illuminations per individual.
- ⦿ The eigenfaces are the PCs of this database

<http://vision.ucsd.edu/~leekc/ExtYaleDatabase/ExtYaleB.html>



Face Vectorization



$$\text{Any Face in Training Set (vector)} = \text{Eigenface 0 (vector)} + C_1 \text{Eigenface 1 (vector)} + C_2 \text{Eigenface 2 (vector)} + \dots$$

Any Face in Training Set
(vector)

Eigenface 0
(vector)

Eigenface 1
(vector)

Eigenface 2
(vector)

Where C_1, C_2, \dots, C_n are constants



Eigenfaces

What

Eigenfaces are eigenvectors used to help computers perform facial recognition.

Eigenfaces form a basis for the set of all training images.

Why

There is a need for low dimensional representation for faces. Eigenfaces decrease required computation time for facial recognition.

How

Eigenvectors are derived from the covariance matrix of the probability distribution over the high-dimensional vector space of face images.



Steps to find the Eigenfaces

Step 1: To create a set of eigenfaces one must first prepare a training set of face images.

Step 2: Standardize the grayscale in each face by subtracting the mean shade

Step 3: Calculate the eigenvectors and eigenvalues of the covariance matrix.

Step 4: Choose the principal components. Sort the eigenvalues in descending order and arrange eigenvectors accordingly. The number of principal components k is determined arbitrarily by setting a threshold ϵ on the total variance.

Step 5: Find k such that k is the smallest number satisfies a certain ϵ tolerance.



4

Results



Our process

Capture Image

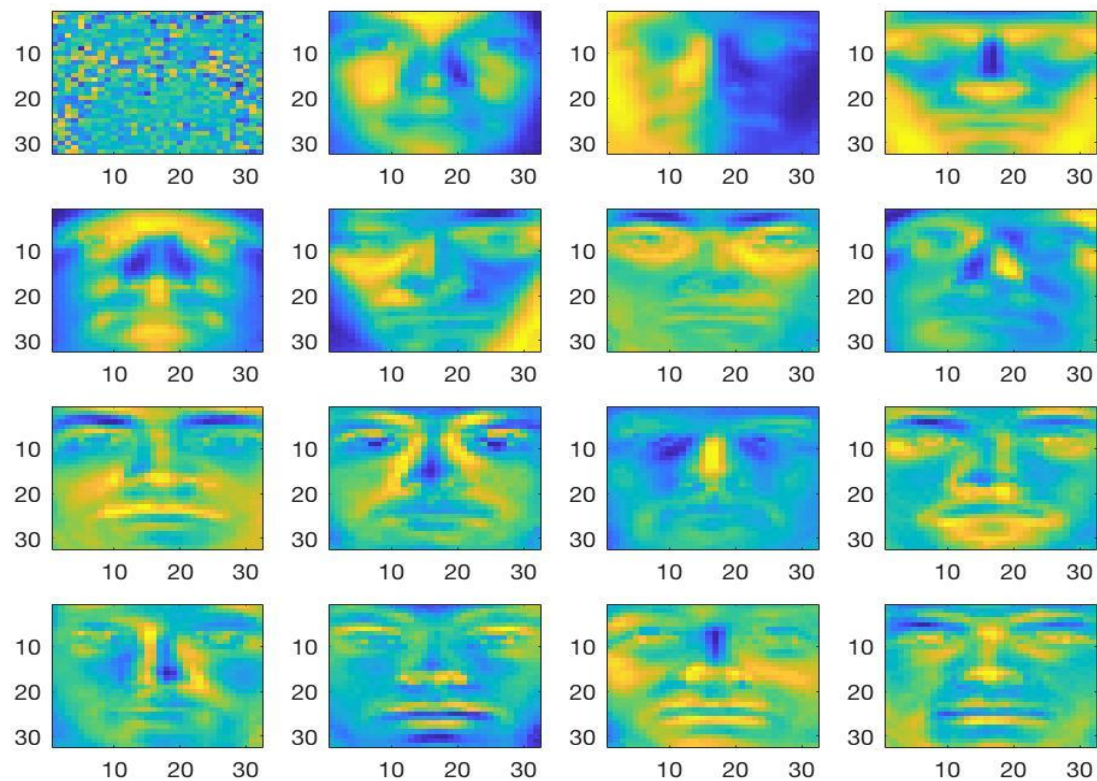
Greyscale,
Align, and
Crop
Image

Run
MATLAB
code

Output
closest
face in
database

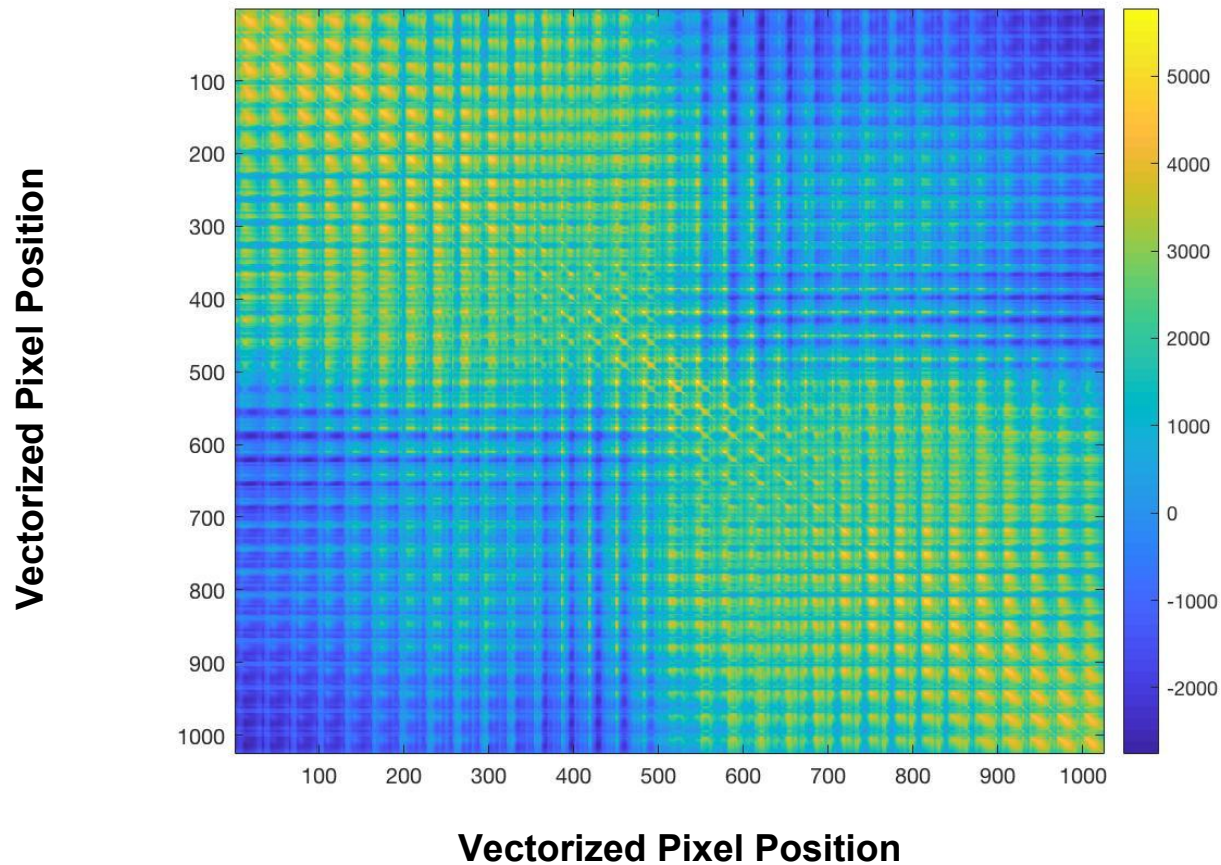
Top 15 Eigenfaces

Y Pixel Coordinate



X Pixel Coordinate

Covariance Matrix for Extended Yale Database B



1.33 Seconds

for computing eigenfaces with a
covariance matrix

1.86 Seconds

for computing eigenfaces with
singular value decomposition

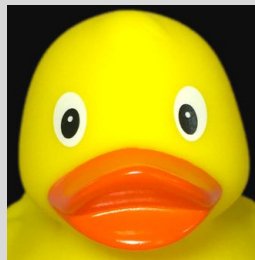
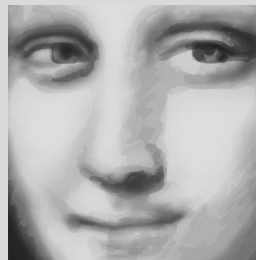
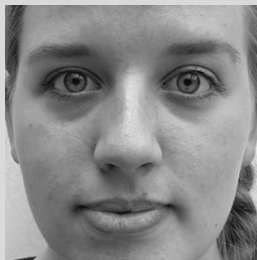
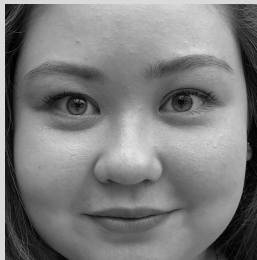


Error Calculation

- We can define weights
- \mathbf{M} is the mean vector
- Let \mathbf{C} be the vector of weights that represent a face image as a linear combination of eigenfaces.
- $\mathbf{C} = \text{transpose}(\mathbf{V})(\mathbf{U} - \mathbf{M})$
- The distance (error) between two faces is the euclidean norm of the difference between the two weight vectors.



Guess Who?



Error: 1364



Error: 1041



Error: 809



Error: 1101

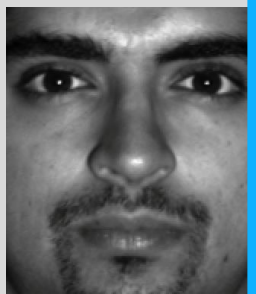
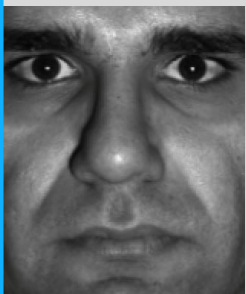
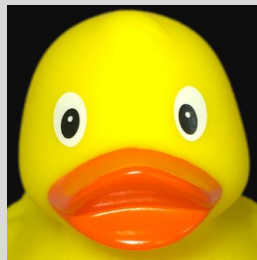
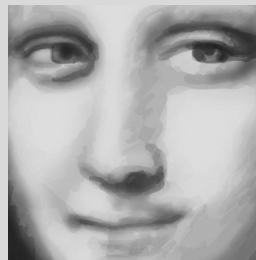
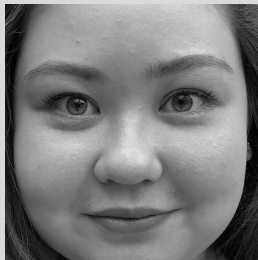


Error: 1874



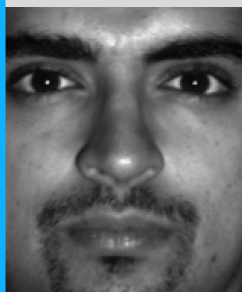
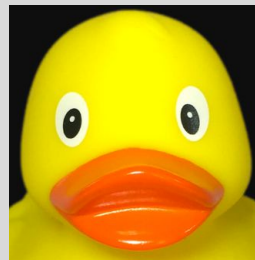
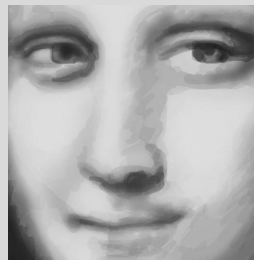
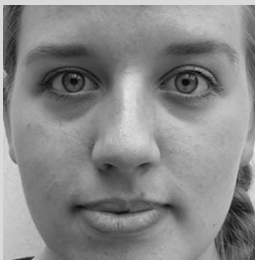
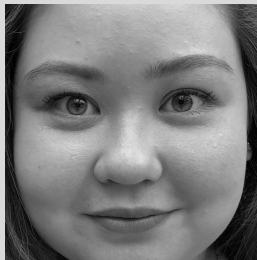


Guess Who?





Guess Who?





Limitations

- ⦿ Faces must be very well centered and adjusted for accurate results
- ⦿ Small training set
- ⦿ Need more data to set error bounds e_1 , and e_2
- ⦿ Not much racial diversity in the training set

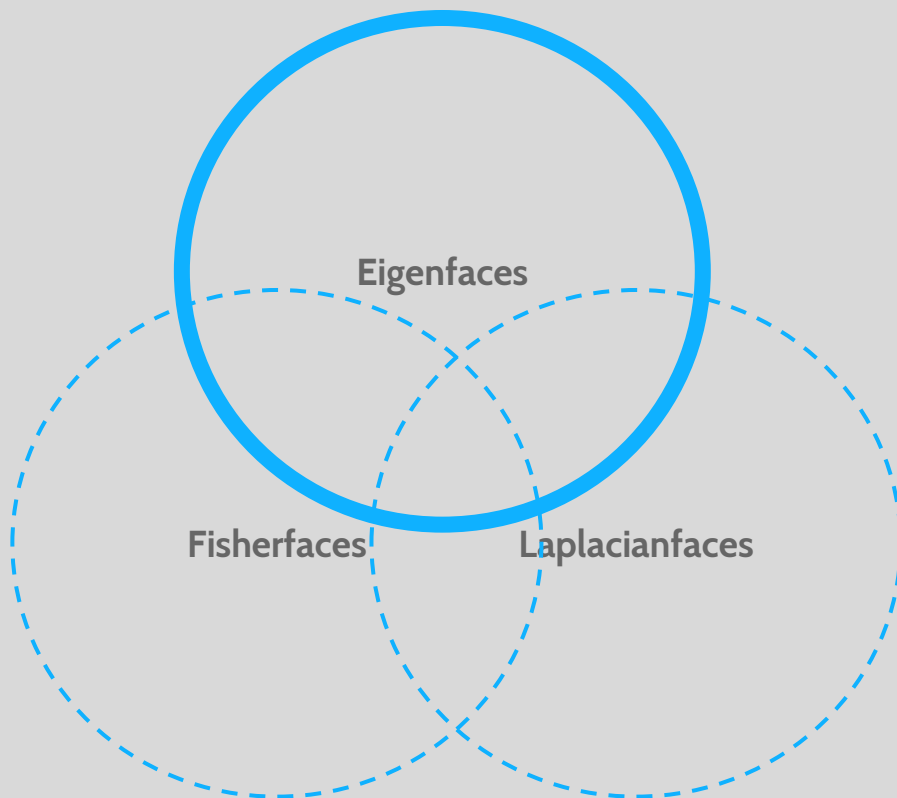


5

**Other Methods
& Applications**



Methods for Facial Recognition





6

Summary



Final Summary

- SVD improves PCA
- Facial Recognition has many applications
- Eigenfaces form a basis for a set of faces
- Eigenface algorithm can find closest face
- Eigenfaces, Fisherfaces, and Laplacianfaces method all do facial recognition
- There are many other facial recognition techniques

“

One is never alone with a
rubber duck.

- Douglas Adams



Bibliography

<https://en.wikipedia.org/wiki/Eigenface>

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Images

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<https://github.com/blackducksoftware>

<https://ducksinthefwindow.com/yellow>

<https://www.queeky.com/gallery/image/mona-lisa-close-up/>

Slide Template

<https://www.slidescarnival.com/>



Eigenface
Fisherface
Laplacianfaces

Fisherface

- ⦿ Fisherface uses Linear Discriminant Analysis (LDA).
- ⦿ Used when the goal is classification rather than representation.
- ⦿ To compute these, we assume the data in each class is Normally distributed with a mean and covariance matrix and probability density function
- ⦿ Problem with fisherfaces is we need to compute the inverse of the within-class scatter matrix. If the sample feature vectors are defined in a p -dimensional space and $p \gg n$ then S is singular and problems arise.
- ⦿ Generalization with large training sample sets



Eigenface
Fisherface
Laplacianfaces

Laplacianfaces

- ⦿ Laplacian faces are an appearance-based approach to human face representation and recognition.
- ⦿ Uses Locality Preserving Projection (LPP) which seeks to capture the details in the geometry of the data as well as the local structure
- ⦿ Extract the low-dimensional manifold structure.
- ⦿ Truncates the unnecessary information