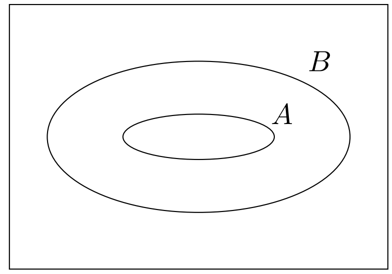
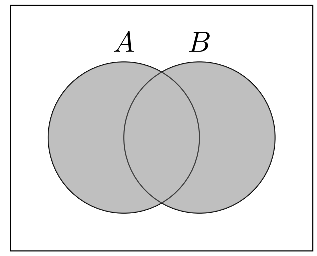
Subset
$A \in B$

if $x \in A$ then $x \in B$



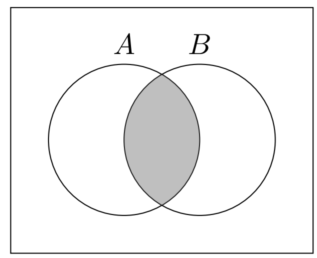
Union
$A \cup B$

$\{x : x \in A \text{ or } x \in B\}$



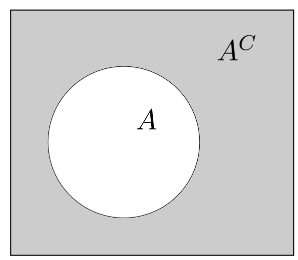
Intersection
$A \cap B$

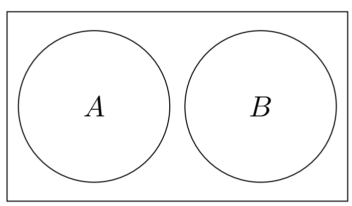
$\{x : x \in A \text{ and } x \in B\}$



Complment
A^C

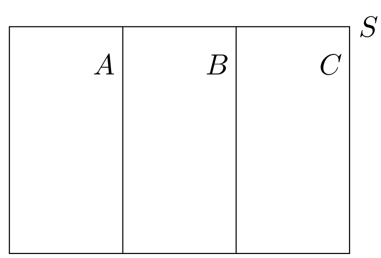
$\{x : x \notin A\}$





Partition

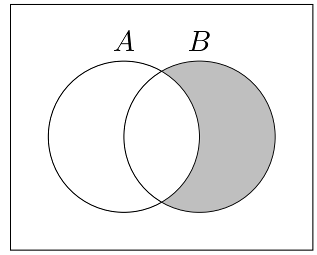
If $A_1, A_2, \ldots$ are pairwise disjoint and $\cup_{i=1}^{\infty} A_i = S$, then the collection $A_1, A_2, \ldots$ forms a partition of S.



Theorems

(a) $P(B \cap A^C) = P(B) - P(A \cap B)$

Proof.



$$
\begin{aligned}
B &= (B \cap A^C) \cup (B \cap A) \\
P(B) &= P(B \cap A^C) \cup P(B \cap A) \\
\Rightarrow P(B \cap A^C) &= P(B) - P(B \cap A)
\end{aligned}
$$

■

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

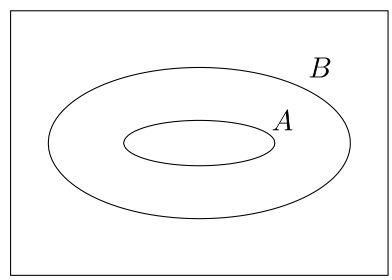
Proof.

$$
\begin{aligned}
A \cup B &= A \cup (B \cap A^C) \\
P(A \cup B) &= P(A) + P(B \cap A^C) \\
&= P(A) + P(B) - P(B \cap A)
\end{aligned}
$$

■

(c) $A \in B \Rightarrow P(A) \leq P(B)$

Proof.



$$
\begin{aligned}
A &= A \cap B \\
P(A) &= P(A \cap B) \\
P(B) &= P(B \cap A^C) + P(B \cap A) \\
&= P(B \cap A^C) + P(A) \\
P(B) &\geq P(A)
\end{aligned}
$$

■