COMPETING RISKS

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ESTIMATING THE CIF

Cumulative Incidence Function

- The **cumulative incidence function** (CIF) is the *unconditional* probability that event type *k* occurs *by* time *t*:

$$F_k(t) = P(T \leq t, K = k)$$

- The probability of **any** event by time *t* is just the sum of the individual CIF's:

$$F(t) = \sum_k F_k(t)$$

Estimating the CIF

- We can estimate the CIF's nonparametrically using the nonparametric estimates of the survival and hazard functions:

$$\hat{F}_k(t) = \sum_{t_m \leq t} \hat{h}_k(t_m)\hat{S}(t_{m-1})$$

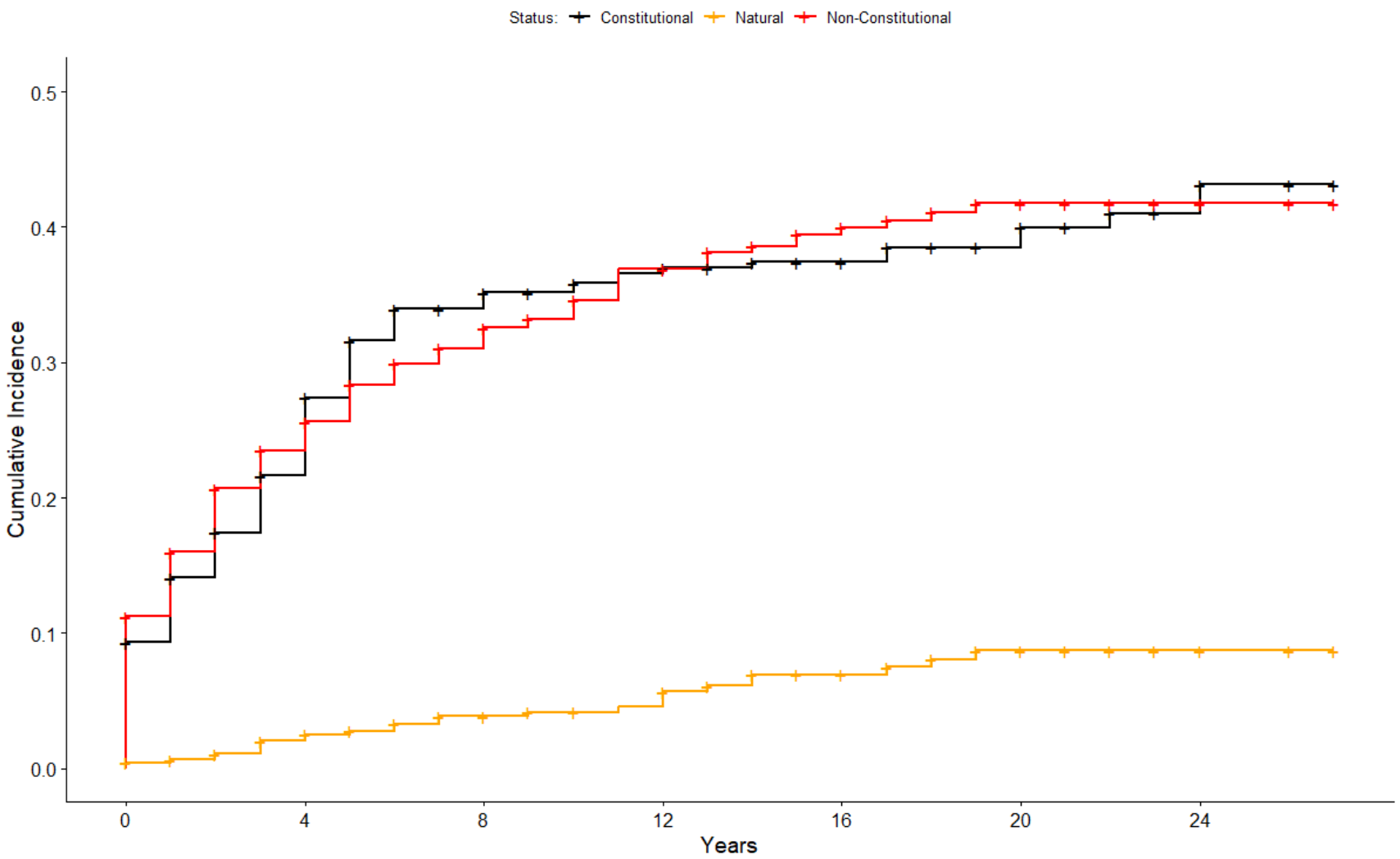
Estimating the CIF's – R

```
lcon_data <- finegray(Surv(years, lost) ~ .,
                      data = leaders, etype = "Constitutional")
lnat_data <- finegray(Surv(years, lost) ~ .,
                      data = leaders, etype = "Natural")
lnon_data <- finegray(Surv(years, lost) ~ .,
                      data = leaders, etype = "Non-Constitutional")

lcon <- survfit(Surv(fgstart, fgstop, fgstatus) ~ 1,
                data = lcon_data, weight = fgwt)
lnat <- survfit(Surv(fgstart, fgstop, fgstatus) ~ 1,
                data = lnat_data, weight = fgwt)
lnon <- survfit(Surv(fgstart, fgstop, fgstatus) ~ 1,
                data = lnon_data, weight = fgwt)
```

Estimating the CIF's – R

```
leadlist <- list(constitutional = lcon, natural = lnat,
                 nonconstitutional = lnon)

ggsurvplot(leadlist, combine = TRUE, fun = "event",
           conf.int = FALSE, break.x.by = 4, ylim = c(0, 0.5),
           legend.title = "Status:",
           legend.labs = c("Constitutional", "Natural",
                           "Non-Constitutional"),
           break.y.by = 0.1, xlab = "Years",
           ylab = "Cumulative Incidence",
           palette = c("black", "orange", "red"))
```

Estimating the CIF's – R



CAUSE-SPECIFIC HAZARD MODEL

Cox Regression Competing Risks

- Typical modeling approach for competing risks is to use separate Cox regression models for **each** cause, treating all other events as censored.
- Essentially, modeling the effects of predictors on the cause-specific hazard:

$$\log h_k(t) = \log h_{0,k}(t) + \beta_1 x_{i,1} + \cdots + \beta_k x_{i,k}$$

Cox Competing Risks – R

```
cox_nat <- coxph(Surv(years, lost == "Natural") ~ manner + start
                 + military + age + conflict + loginc + growth
                 + pop + land + literacy + factor(region),
                 data = leaders)
summary(cox_nat)
```

Cox Competing Risks – R

```
## Call:
## coxph(formula = Surv(years, lost == "Natural") ~ manner + start +
##     military + age + conflict + loginc + growth + pop + land +
##     literacy + factor(region), data = leaders)
##
##   n= 438, number of events= 27
##    (34 observations deleted due to missingness)
##
##                        coef  exp(coef)   se(coef)      z Pr(>|z|)
## manner              3.747e-01  1.455e+00  6.633e-01  0.565    0.572
## start              -5.403e-02  9.474e-01  3.386e-02 -1.596    0.111
## military           -3.646e-01  6.945e-01  7.409e-01 -0.492    0.623
## age                 7.386e-02  1.077e+00  1.840e-02  4.015 5.95e-05 ***
## conflict           -2.609e-01  7.704e-01  4.720e-01 -0.553    0.580
## loginc              3.285e-01  1.389e+00  2.673e-01  1.229    0.219
## growth              8.817e-02  1.092e+00  8.518e-02  1.035    0.301
## pop                 1.991e-03  1.002e+00  2.138e-03  0.931    0.352
## land               -3.969e-05  1.000e+00  1.781e-04 -0.223    0.824
## literacy           -8.796e-03  9.912e-01  1.260e-02 -0.698    0.485
## factor(region)1 -6.427e-01  5.259e-01  8.360e-01 -0.769    0.442
## factor(region)2 -7.776e-01  4.595e-01  9.031e-01 -0.861    0.389
## factor(region)3  6.591e-01  1.933e+00  7.852e-01  0.839    0.401
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Cox Competing Risks – R

```
##                  exp(coef) exp(-coef) lower .95 upper .95
## manner              1.4546     0.6875   0.39644     5.337
## start               0.9474     1.0555   0.88657     1.012
## military            0.6945     1.4400   0.16255     2.967
## age                 1.0767     0.9288   1.03853     1.116
## conflict            0.7704     1.2980   0.30548     1.943
## loginc              1.3889     0.7200   0.82251     2.345
## growth              1.0922     0.9156   0.92423     1.291
## pop                 1.0020     0.9980   0.99780     1.006
## land                1.0000     1.0000   0.99961     1.000
## literacy            0.9912     1.0088   0.96707     1.016
## factor(region)1     0.5259     1.9015   0.10217     2.707
## factor(region)2     0.4595     2.1763   0.07827     2.698
## factor(region)3     1.9330     0.5173   0.41484     9.007
##
## Concordance= 0.819  (se = 0.046 )
## Likelihood ratio test= 32.42  on 13 df,   p=0.002
## Wald test            = 29.47  on 13 df,   p=0.006
## Score (logrank) test = 33.21  on 13 df,   p=0.002
```

AFT Models with Competing Risks

- Accelerated Failure Time models have a similar structure to Cox regression models when dealing with competing risks.
- With AFT Models, distributions need to be evaluated for all types of failure!