ARIMAX and Transfer Functions

PART 1

ARIMAX and Transfer functions

We have modeled a [univariate] time series by using ARIMA (using past lags of Y_t and/or lags of error terms to model future values of Y_t)

We have incorporated “external information” through regression (time for trend, dummy variables for seasonality, sines and cosines for seasonality, etc)

We have incorporated “jumps” or changes in Y_t through Intervention variables

We are now going to look into “multivariate” time series (i.e. more than one time series)

- This can be viewed as transfer functions (series of X_t’s that influence or predict Y_t (and NOT the other way around)
- All of the series are viewed as Y_t’s and influence each other (this gets into VARMA which we will not be covering)

A cautionary note about confidence intervals for prediction: these intervals include variation about the Y_t values, but not X_t values (so they are usually too narrow)

GOAL: Use information from other series (X_t) to predict the series of interest (Y_t)

General Transfer Functions

- Simple transfer functions are time series functions with covariates other than lags of *Y* in the model.

$$Y_t = \beta_0 + \beta X_t + Z_t$$

Covariates used to predict response

- These covariates do NOT have to be at same time point as Y.
- They can be lags of X as well.

General Transfer Functions

- Simple transfer functions are time series functions with covariates other than lags of Y in the model.

$$Y_t = \beta_0 + \beta X_t + Z_t$$

Errors follow an ARIMA model

Milestones are still the same

Stationarity of Residuals

White Noise of Residuals

Good Forecasts

Model signal in Y_t series

Use one or more X_t series

Use time, dummy variables, trig functions or differences

Are residuals stationary?

YES

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NO

Model correlation structure (AR and MA terms)

White Noise?

YES

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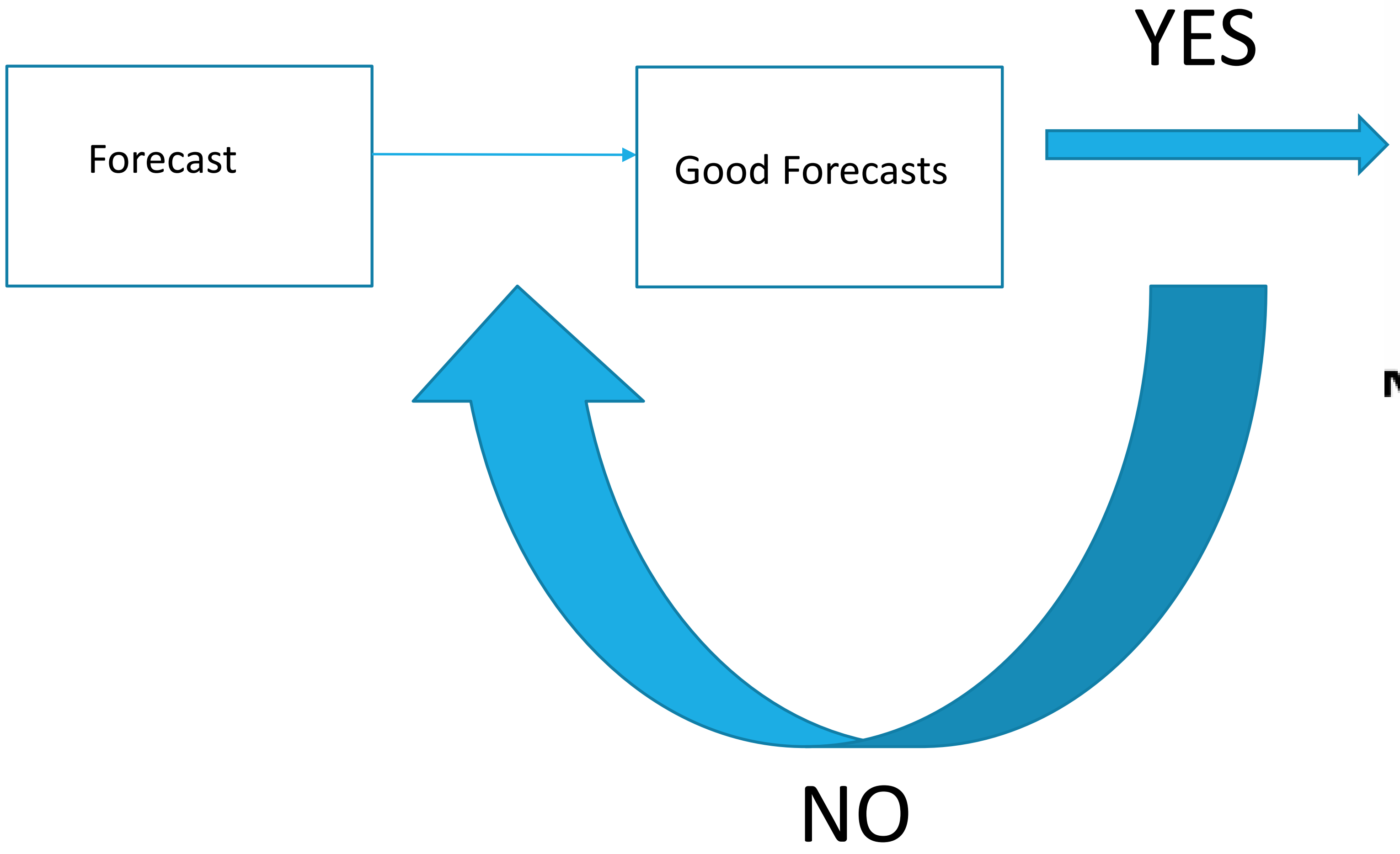
NO



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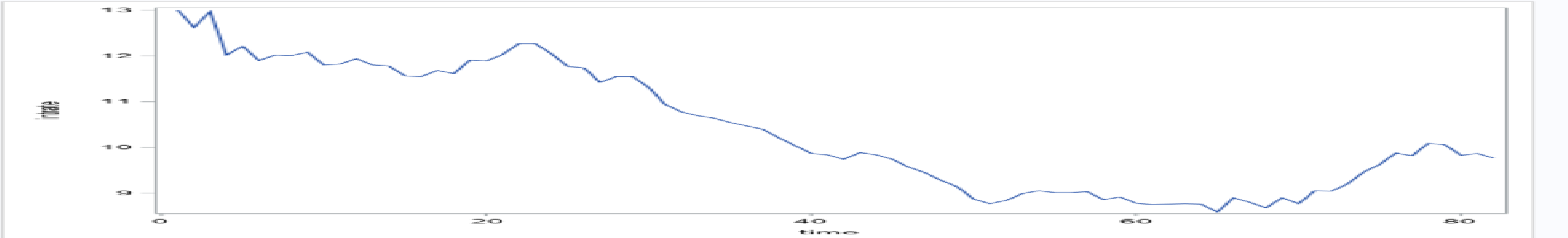
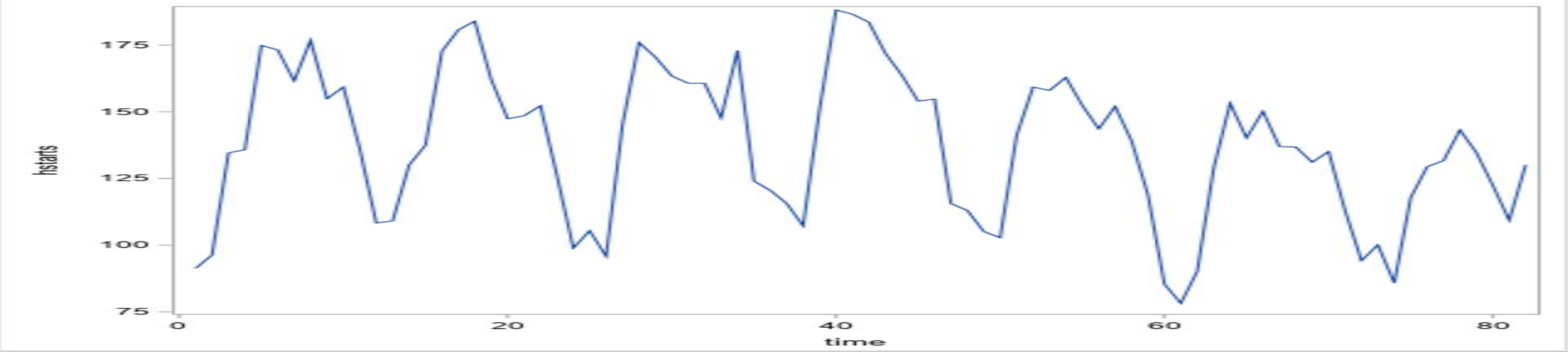
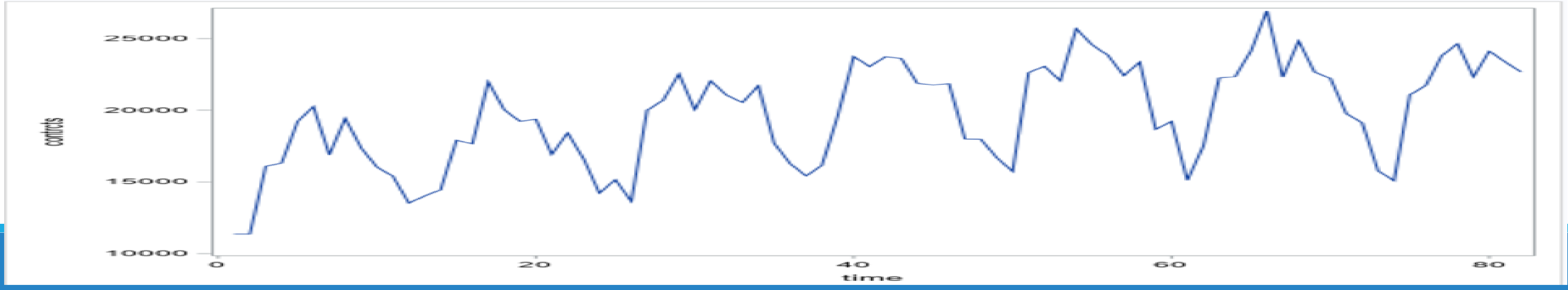
Example

The “hstarts” data set

Response variable is hstarts (# housing starts). We are going to use information from # of construction contractions (contrcts) and mortgage interest rates (intrate) to see if these two variables can provide information on predicting housing starts.


intrate
13
12
11
10
9
0
20
40
60
80
time


hstarts
175
150
125
100
75
0
20
40
60
80
time


contrcts
25000
20000
15000
10000
0
20
40
60
80
time

General Transfer Function Model

- You can also use transfer functions with numerator and denominator factors – similar to intervention variables.
- The denominator factors for a transfer function for an input series are like the AR part of the ARMA model for the noise series.
  - Denominator factors introduce exponentially weighted, infinite distributed lags into the transfer function.
- The numerator factors for a transfer function for an input series are like the MA part of the ARMA model for the noise series.

General Transfer Function Model

- Here is the general transfer function model:

$$Y_t = \omega_0 + \frac{\omega(\mathrm{B})}{\delta(\mathrm{B})}\mathrm{X_t} + \mathrm{Z_t}$$

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$$Y_t = \omega_0 + \frac{\omega(\mathrm{B})}{\delta(\mathrm{B})}\mathrm{X}_\mathrm{t} + \mathrm{Z}_\mathrm{t}$$

Lag structure applied to covariate