UCM in time series

UNOBSERVED COMPONENTS MODELS

UCM

Unobserved Components Models (Structural Models, State Space Models) model the different components in time series data

Components modeled:

- Level
- Trend
- Season
- Cycle

UCM's can incorporate ARMA errors

UCM's can incorporate transfer functions and/or intervention variables

Background on UCM

Each of the components being modeled can be modeled as either stochastic or deterministic

- Deterministic indicates that the “signal” due to that component is consistent across the series (the trend is the same across the series)
- Stochastic indicates that the “signal” from the component changes across the series (for example, the trend is allowed to vary across time...allows more flexibility…time-varying components…this indicates that they can handle random walks and do NOT need to take differences)
- The UCM procedure allows you to test to see if the component is stochastic or deterministic (can also test to see if component is significant in the model)
  - HOWEVER, need white noise for the p-values to be accurate (so again….can be very circular)

We will start with discussing how to model the components and testing whether or not they are deterministic or stochastic

Level and Trend Model

The following model represents the level and trend in a UCM

$$Y_t = \mu_t + \varepsilon_t$$

Where

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \text{ with } \eta_t \sim N(0, \sigma_\eta^2)$$

$$\beta_t = \beta_{t-1} + \xi_t \text{ with } \xi_t \sim N(0, \sigma_\xi^2)$$

$$\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

Level and Trend Model

The following model represents the level and trend in a UCM

$$Y_t = \mu_t + \varepsilon_t$$

Where

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \text{ with } \eta_t \sim N(0, \sigma_\eta^2)$$

$$\beta_t = \beta_{t-1} + \xi_t \text{ with } \xi_t \sim N(0, \sigma_\xi^2)$$

$$\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

This is the noise of the series (often referred to as the irregular component…this will stay in the series)

Level and Trend Model

The following model represents the level and trend in a UCM

$$Y_t = \mu_t + \varepsilon_t$$

Where

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \text{ with } \eta_t \sim N(0, \sigma_\eta^2)$$

$$\beta_t = \beta_{t-1} + \xi_t \text{ with } \xi_t \sim N(0, \sigma_\xi^2)$$

If this variance is 0,
then slope is
deterministic

Level and Trend Model

The following model represents the level and trend in a UCM

$$Y_t = \mu_t + \varepsilon_t$$

Where

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \quad \text{with } \eta_t \sim N(0, \sigma_\eta^2)$$

$$\beta_t = \beta_{t-1} + \xi_t \quad \text{with } \xi_t \sim N(0, \sigma_\xi^2)$$

Test H_0: $\sigma_\xi^2 = 0$

Level and Trend Model

The following model represents the level and trend in a UCM

$$Y_t = \mu_t + \varepsilon_t$$

Where

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \text{ with } \eta_t \sim N(0, \sigma_\eta^2)$$

$$\beta_t = \beta_{t-1} + \xi_t \text{ with } \xi_t \sim N(0, \sigma_\xi^2)$$

If this variance is 0, then level is deterministic

Level and Trend Model

The following model represents the level and trend in a UCM

$$Y_t = \mu_t + \varepsilon_t$$

Where

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \text{ with } \eta_t \sim N(0, \sigma_\eta^2)$$

$$\beta_t = \beta_{t-1} + \xi_t \text{ with } \xi_t \sim N(0, \sigma_\xi^2)$$

Test H_0: $\sigma_\eta^2 = 0$

Level, Trend, Season, Cycle

We can expand this model to incorporate not only level and trend, but season and cycle

$$Y_t = \mu_t + \psi_t + \delta_t + \varepsilon_t$$

Ψ_t is the cycle component (can be deterministic or stochastic…based on variance of that component)

δ_t is the seasonal component (can be deterministic or stochastic based on variance of that component)

PROC UCM

PROC UCM < options > ;

CYCLE < options > ;

ESTIMATE < options > ;

FORECAST < options > ;

IRREGULAR < options > ;

LEVEL < options > ;

MODEL dependent variable < = regressors > ;

SEASON options ;

SLOPE < options > ;

PROC UCM

PROC UCM < options > ;

CYCLE < options > ; ← Specifies a cycle component

ESTIMATE < options > ;

FORECAST < options > ;

IRREGULAR < options > ;

LEVEL < options > ;

MODEL dependent variable < = regressors > ;

SEASON options ;

SLOPE < options > ;

PROC UCM

PROC UCM < options > ;

CYCLE < options > ;

ESTIMATE < options > ; Request ACF and PACF plots

FORECAST < options > ;

IRREGULAR < options > ;

LEVEL < options > ;

MODEL dependent variable < = regressors > ;

SEASON options ;

SLOPE < options > ;

PROC UCM

PROC UCM < options > ;

CYCLE < options > ;

ESTIMATE < options > ;

FORECAST < options > ;

Similar to this function in ARIMA (produces forecast)

IRREGULAR < options > ;

LEVEL < options > ;

MODEL dependent variable < = regressors > ;

SEASON options ;

SLOPE < options > ;

PROC UCM

```
PROC UCM < options > ;
CYCLE < options > ;
ESTIMATE < options > ;
FORECAST < options > ;
IRREGULAR < options > ;
LEVEL < options > ;
MODEL dependent variable < = regressors > ;
SEASON options ;
SLOPE < options > ;
```

Specifies the error of the series..this is where you can put in AR and MA terms

PROC UCM

PROC UCM < options > ;

CYCLE < options > ;

ESTIMATE < options > ;

FORECAST < options > ;

IRREGULAR < options > ;

LEVEL < options > ; Specifies the level component

MODEL dependent variable < = regressors > ;

SEASON options ;

SLOPE < options > ;

PROC UCM

PROC UCM < options > ;

CYCLE < options > ;

ESTIMATE < options > ;

FORECAST < options > ;

IRREGULAR < options > ;

LEVEL < options > ;

MODEL dependent variable < = regressors > ;

This is where you can specify regressors (transfer functions and/or intervention variables)

SEASON options ;

SLOPE < options > ;

PROC UCM

PROC UCM < options > ;

CYCLE < options > ;

ESTIMATE < options > ;

FORECAST < options > ;

IRREGULAR < options > ;

LEVEL < options > ;

MODEL dependent variable < = regressors > ;

SEASON options ;

This is where you specify a season component

SLOPE < options > ;

PROC UCM

PROC UCM < options > ;

CYCLE < options > ;

ESTIMATE < options > ;

FORECAST < options > ;

IRREGULAR < options > ;

LEVEL < options > ;

MODEL dependent variable < = regressors > ;

SEASON options ;

Need to specify the length of the season and how to model the season….dummy variables or trig functions

SLOPE < options > ;

PROC UCM

PROC UCM < options > ;

CYCLE < options > ;

ESTIMATE < options > ;

FORECAST < options > ;

IRREGULAR < options > ;

LEVEL < options > ;

MODEL dependent variable < = regressors > ;

SEASON options ;

SLOPE < options > ;

This is where you specify a slope component

R

There is an R package called rucm (modeled after PROC UCM)

Likelihood Optimization Algorithm Converged in 17 Iterations.

Final Estimates of the Free Parameters					
Component	Parameter	Estimate	Approx Std Error	t Value	Approx Pr > |t|
Irregular	Error Variance	0.00023436	0.0001079	2.17	0.0298
Level	Error Variance	0.00029828	0.0001057	2.82	0.0048
Slope	Error Variance	8.47911E-13	6.2271E-10	0.00	0.9989
Season	Error Variance	0.00000356	1.32347E-6	2.69	0.0072

First look to see if components are random

```
proc ucm data=time.airline;
level;
slope;
season length=12 type=trig;
irregular;
model logpsngr;
run;
```

Significance Analysis of Components (Based on the Final State)			
Component	DF	Chi-Square	Pr > ChiSq
Irregular	1	0.08	0.7747
Level	1	117867	<.0001
Slope	1	43.78	<.0001
Season	11	507.75	<.0001

Likelihood Optimization Algorithm Converged in 17 Iterations.

Final Estimates of the Free Parameters					
Component	Parameter	Estimate	Approx Std Error	t Value	Approx Pr > |t|
Irregular	Error Variance	0.00023436	0.0001079	2.17	0.0298
Level	Error Variance	0.00029828	0.0001057	2.82	0.0048
Slope	Error Variance	8.47911E-13	6.2271E-10	0.00	0.9989
Season	Error Variance	0.00000356	1.32347E-6	2.69	0.0072

ONLY look here IF a component is NOT random!!!! If a component is NOT random AND NOT significant then you can remove it from the model

```
proc ucm data=time.airline;
level;
slope;
season length=12 type=trig;
irregular;
model logpsngr;
run;
```

Significance Analysis of Components (Based on the Final State)			
Component	DF	Chi-Square	Pr > ChiSq
Irregular	1	0.08	0.7747
Level	1	117867	<.0001
Slope	1	43.78	<.0001
Season	11	507.75	<.0001

```
/* Make slope deterministic */
proc ucm data=time.airline;
level;
slope variance = 0 noest;
season length=12 type=trig;
irregular;
model logpsngr;
run;
```

Fit Statistics Based on Residuals	
Mean Squared Error	0.00147
Root Mean Squared Error	0.03830
Mean Absolute Percentage Error	0.54132
Maximum Percent Error	2.19097
R-Square	0.99061

Likelihood Optimization Algorithm Converged in 6 Iterations.

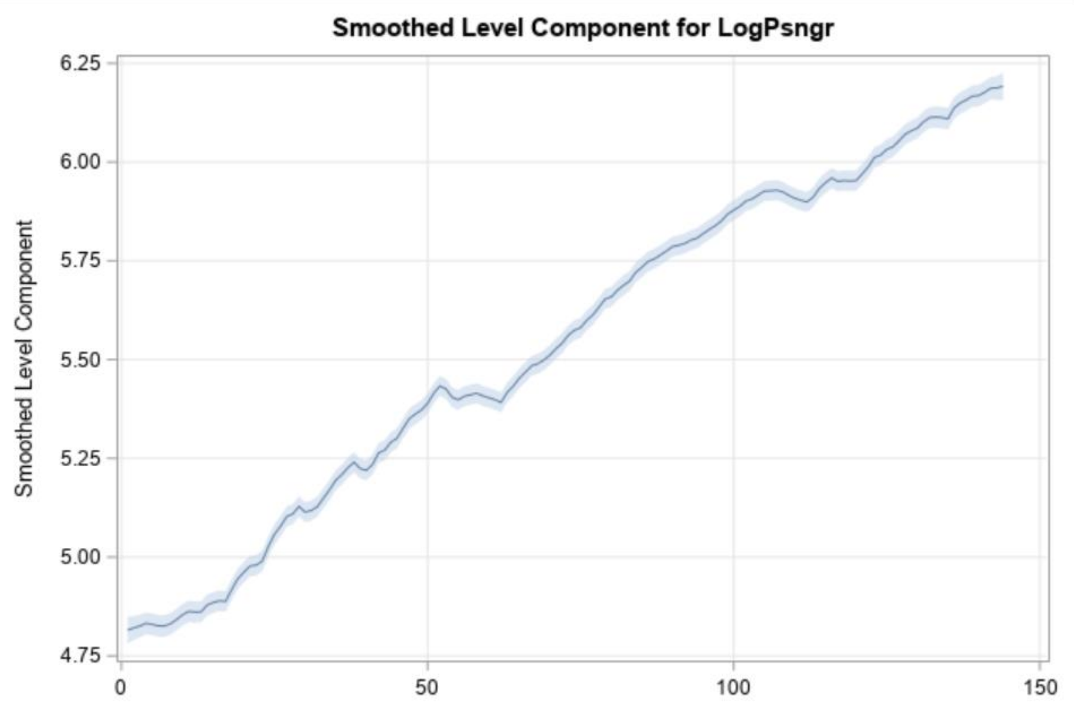
Final Estimates of the Free Parameters

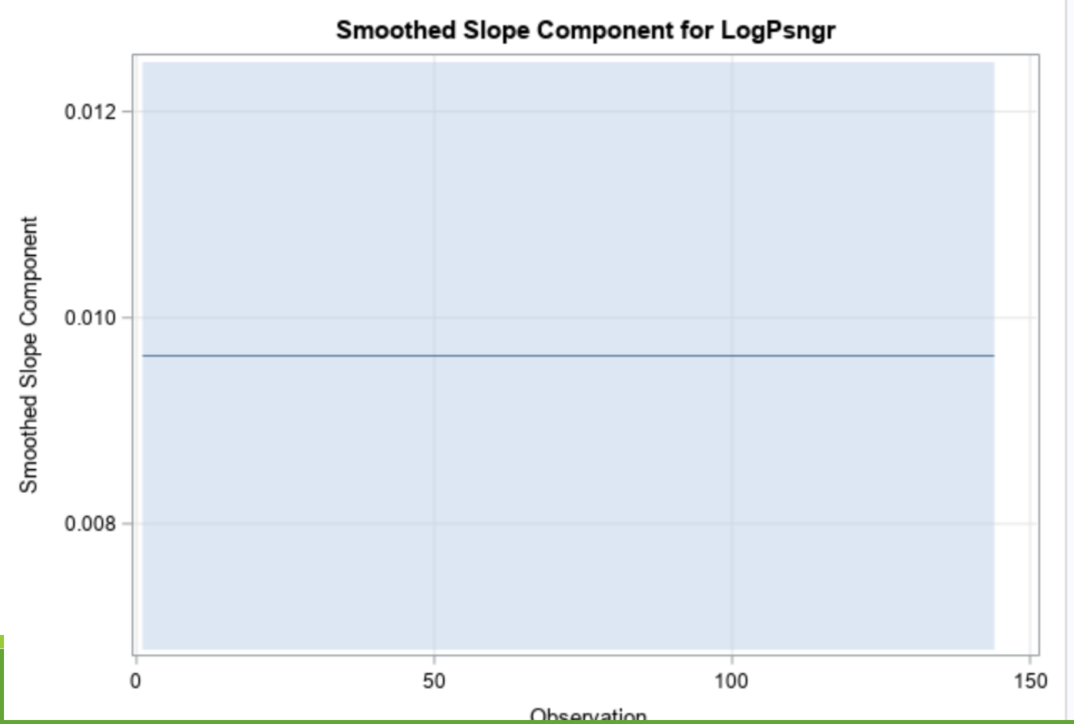
Component	Parameter	Estimate	Approx Std Error	t Value	Approx Pr > |t|
Irregular	Error Variance	0.00023436	0.0001079	2.17	0.0298
Level	Error Variance	0.00029828	0.0001057	2.82	0.0048
Season	Error Variance	0.00000356	1.32347E-6	2.69	0.0072

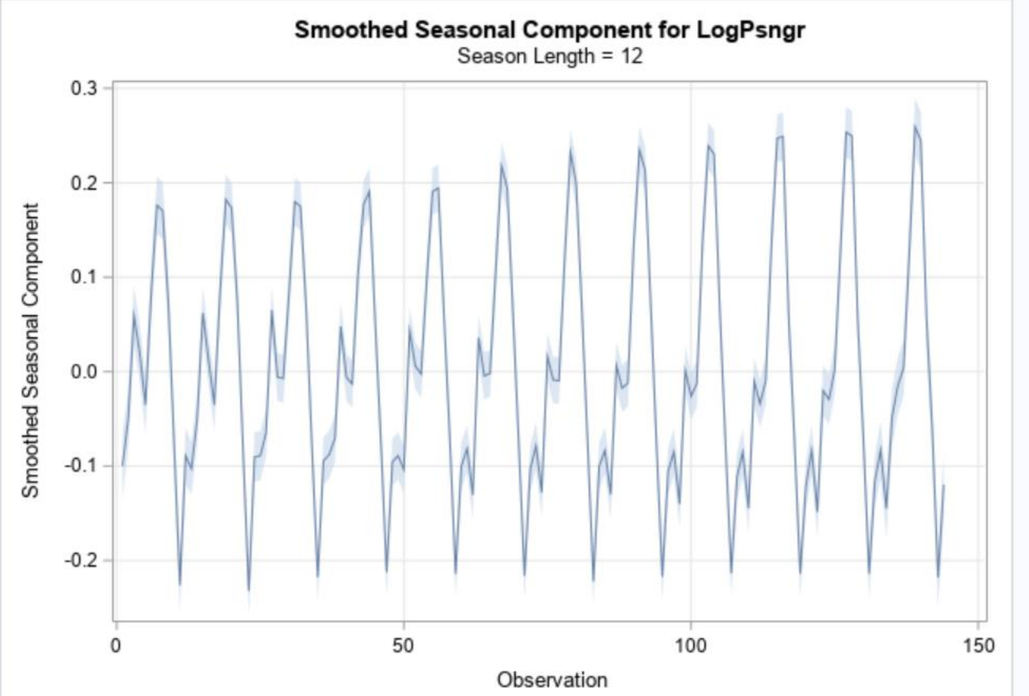
Significance Analysis of Components (Based on the Final State)

Component	DF	Chi-Square	Pr > ChiSq
Irregular	1	0.08	0.7747
Level	1	117867	<.0001
Slope	1	43.78	<.0001
Season	11	507.75	<.0001

```
/* print smoothed plots of components */
proc ucm data=time.airline;
level plot=smooth;
slope variance = 0 noest plot=smooth;
season length=12 type=trig plot=smooth;
irregular;
model logpsngr;
run;
```

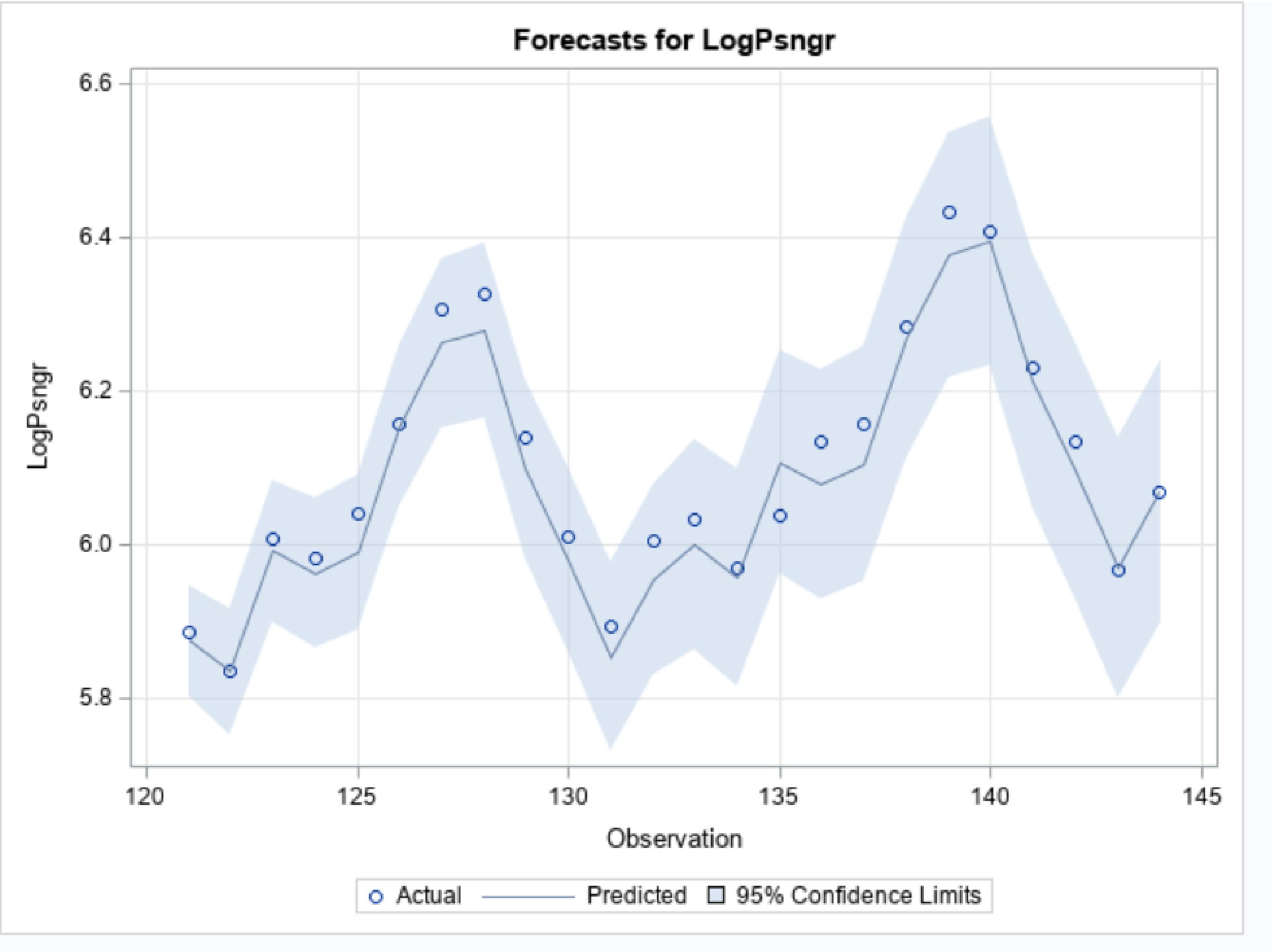






```
/* Forecast */
proc ucm data=time.airline ;
level ;
slope variance = 0 noest;
season length=12 type=trig ;
irregular Q=2;
estimate ;
model logpsngr;
forecast back=24 lead=24;
run;
```

If you want to specify regressors (i.e. other 'X' variables) you can put it here…logpsngr= x1 x2….;

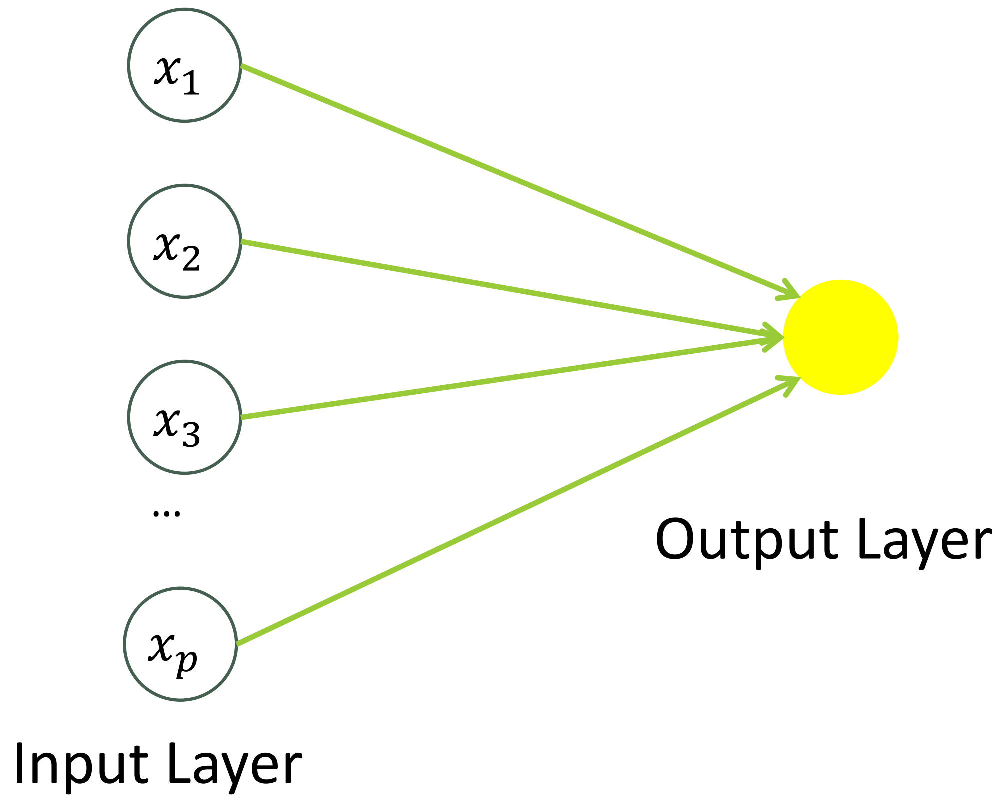


Neural Network Models

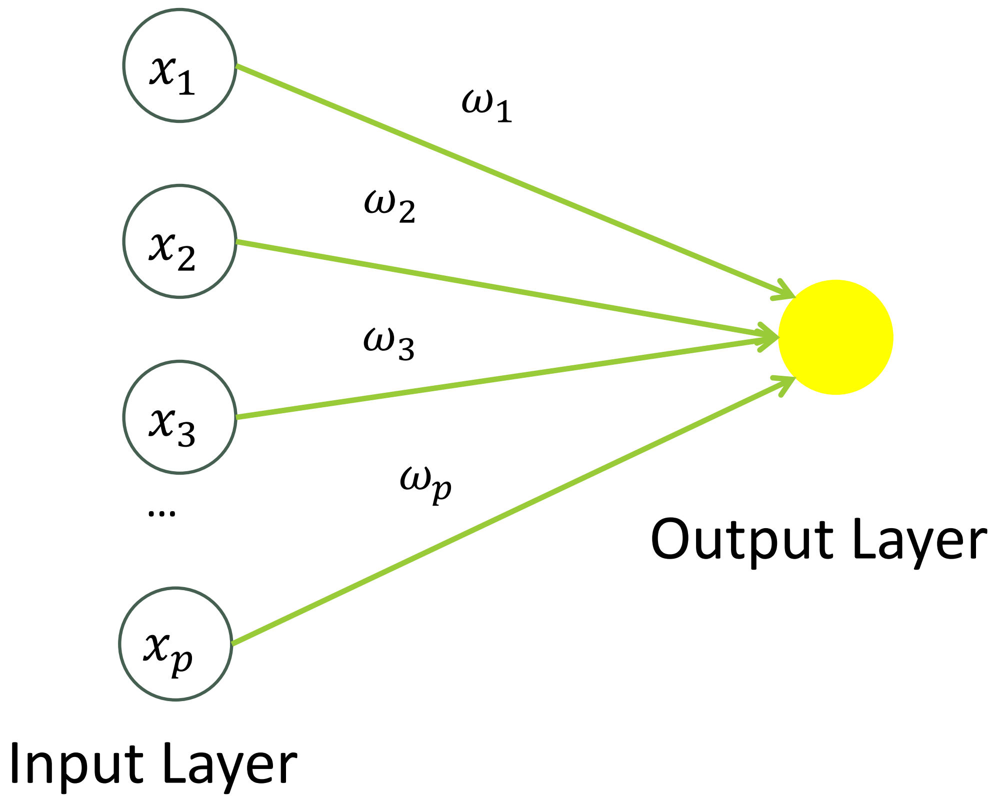
Neural Networks

- Neural network models are models based on mathematical models of how the brain functions.
- They are organized in a network of **neurons** through **layers**.
- The input variables are considered the neurons on the **bottom layer**.
- The output variable is considered the neuron on the **top layer**.
- The layers in between, called **hidden layers**, transform the input variables through non-linear methods to try and best model the output variable.

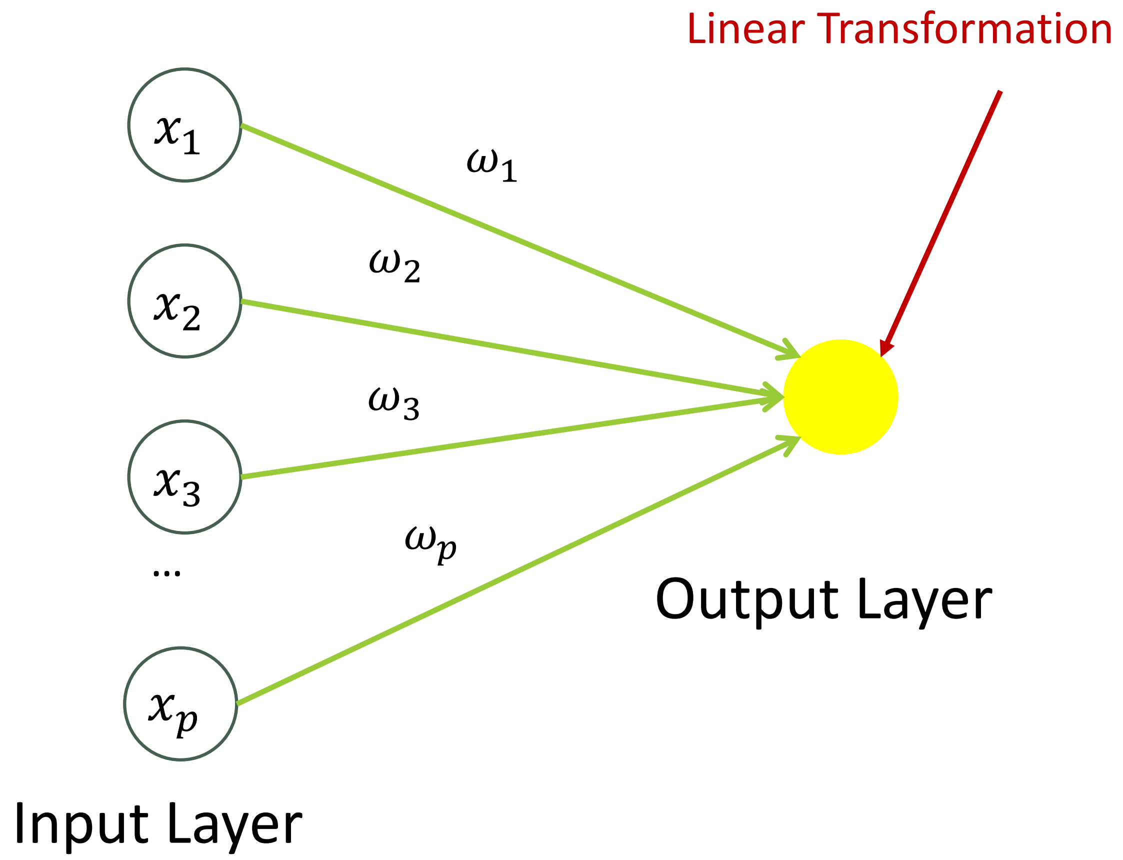
Neural Network Structure



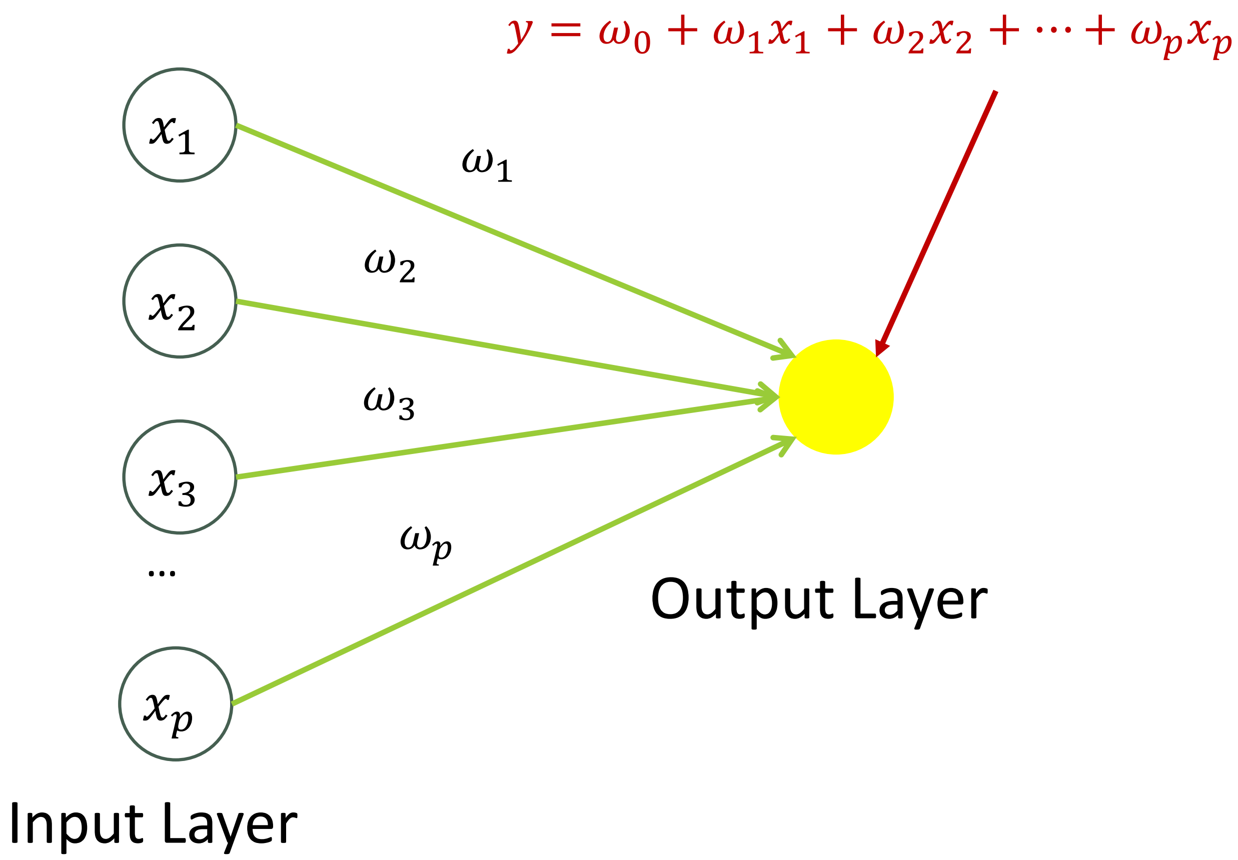
Neural Network Structure



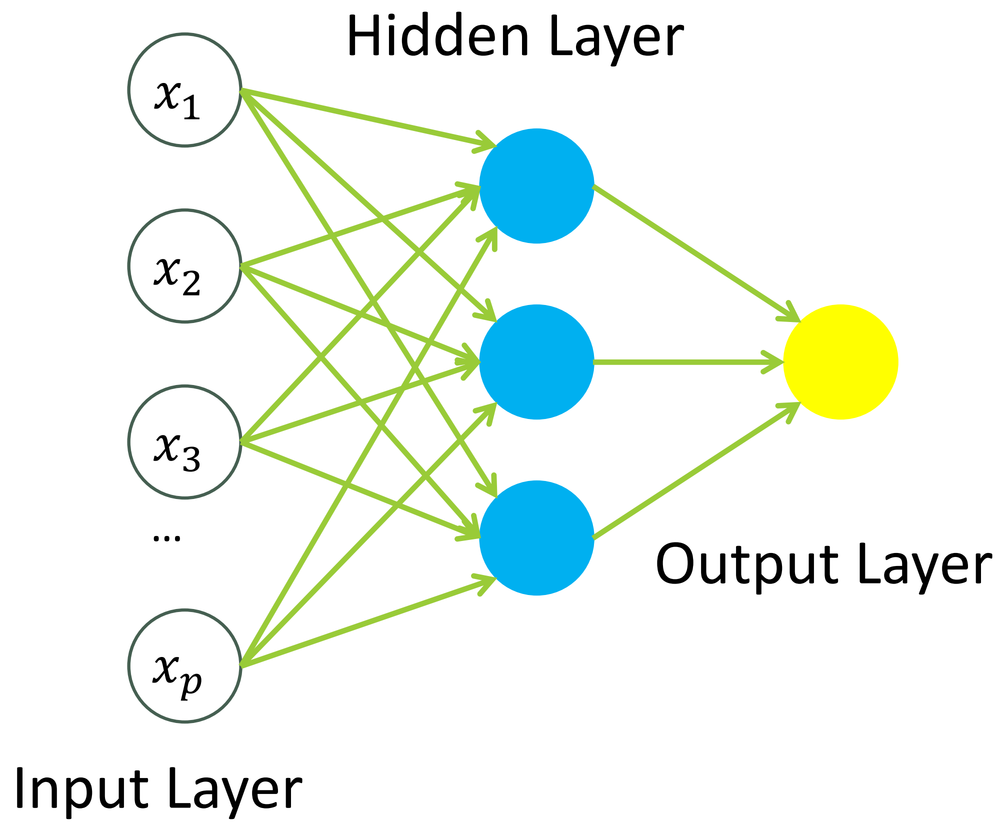
Neural Network Structure



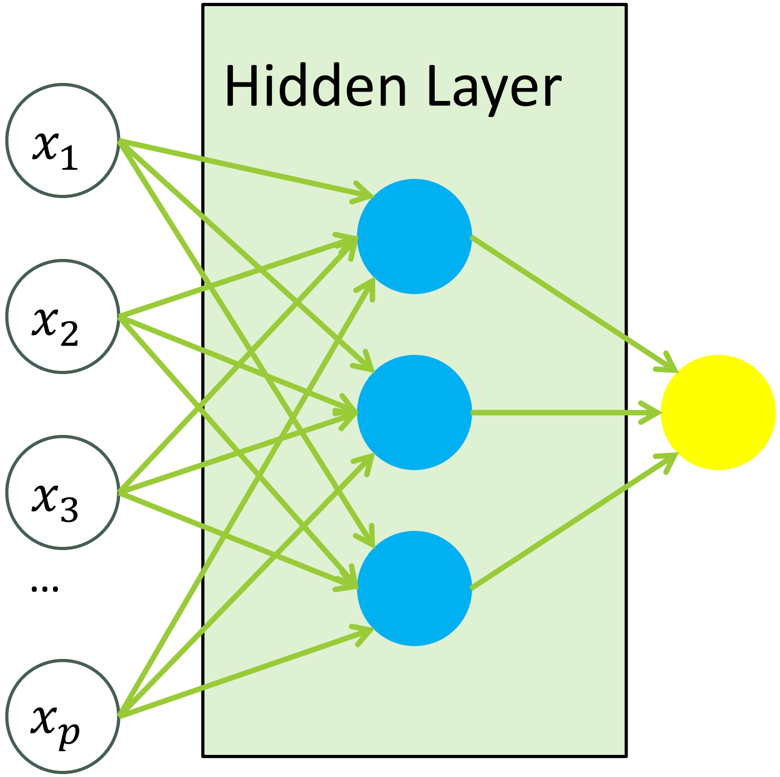
Neural Network Structure



Neural Networks

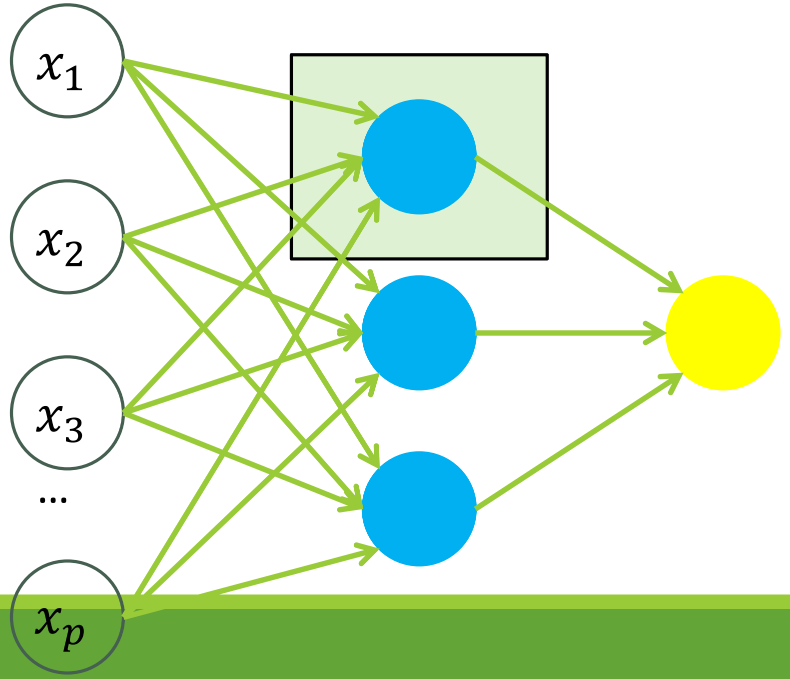


Neural Networks

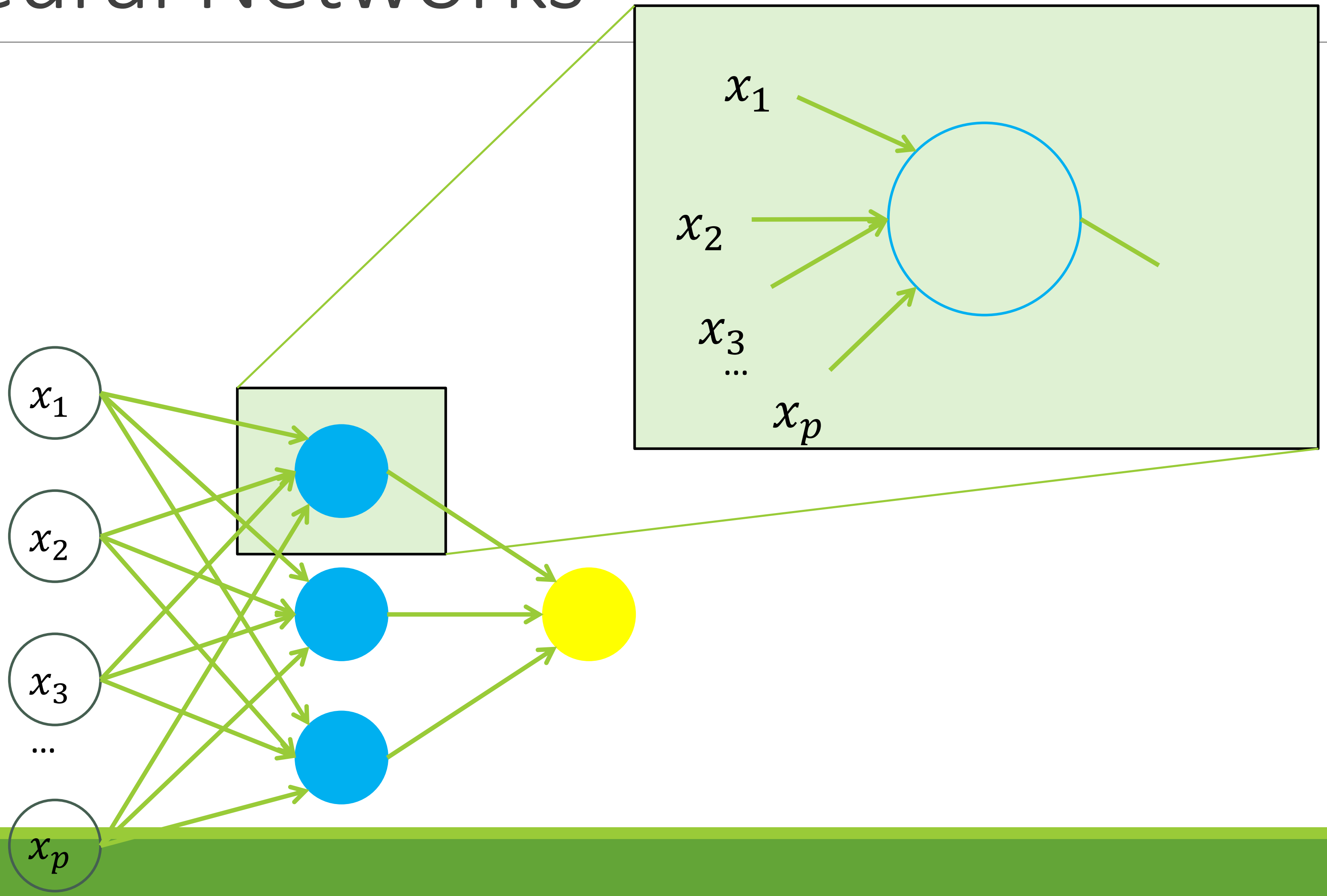


All of the nonlinearities and interactions of the variables get added to the model here.

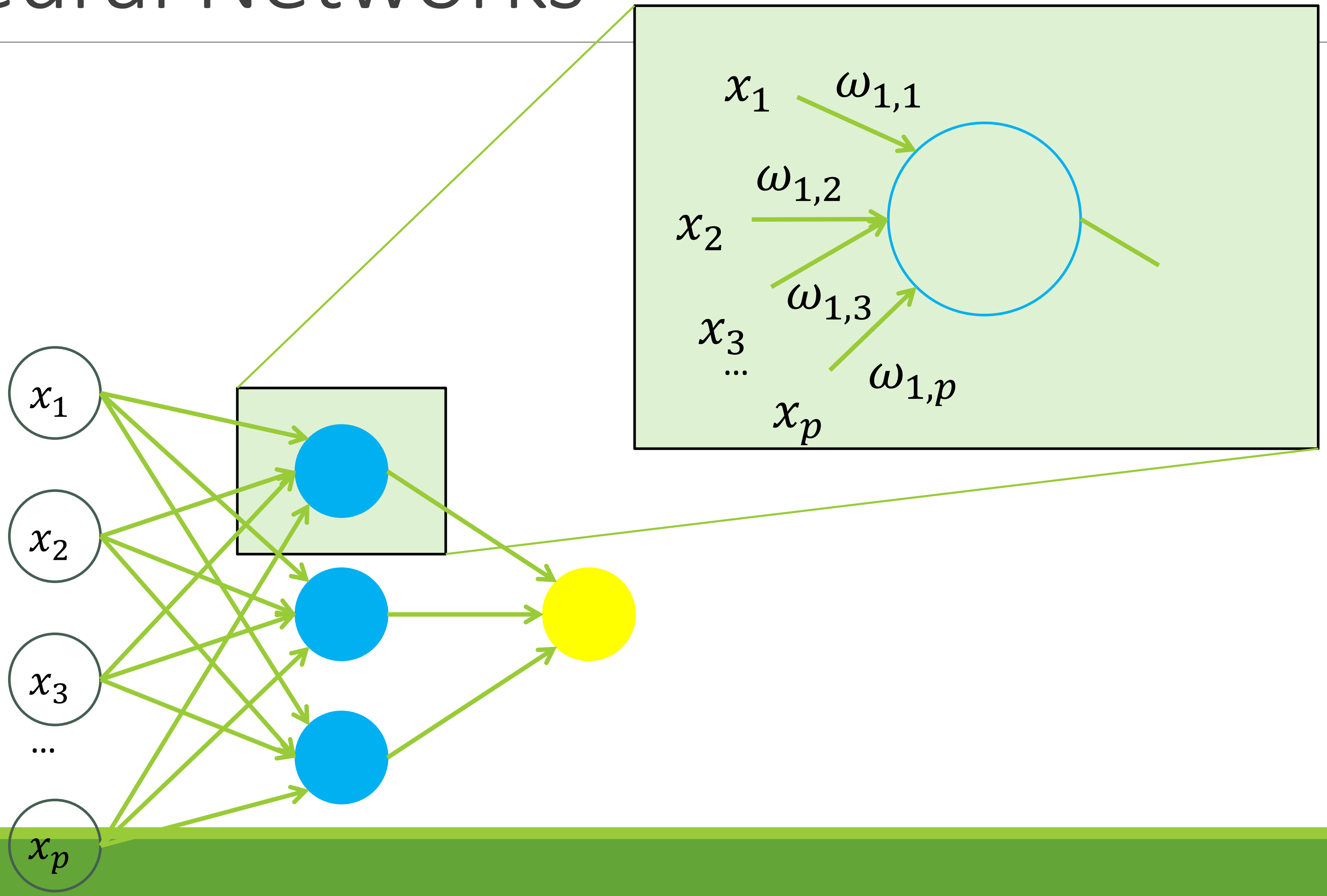
Neural Networks



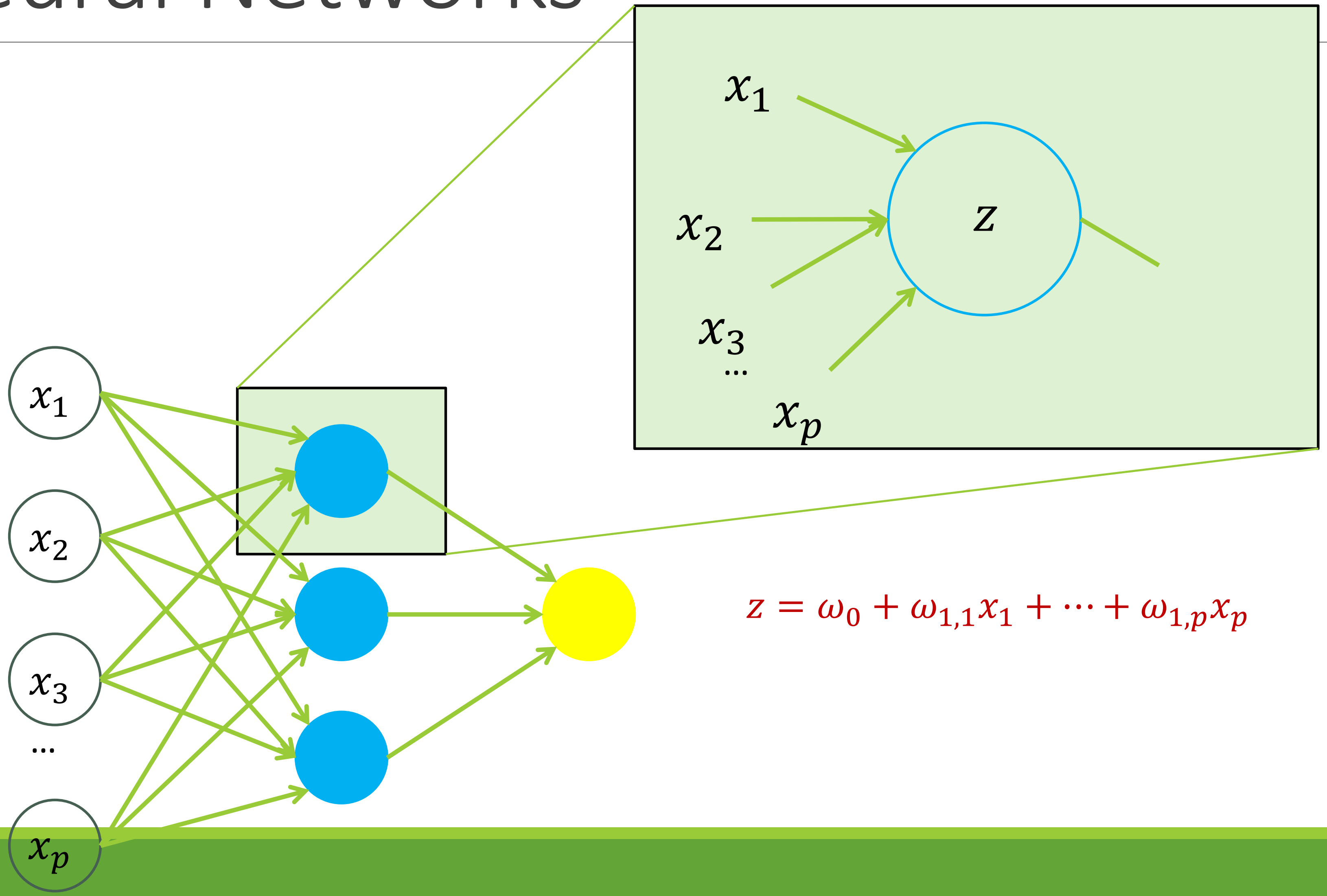
Neural Networks



Neural Networks

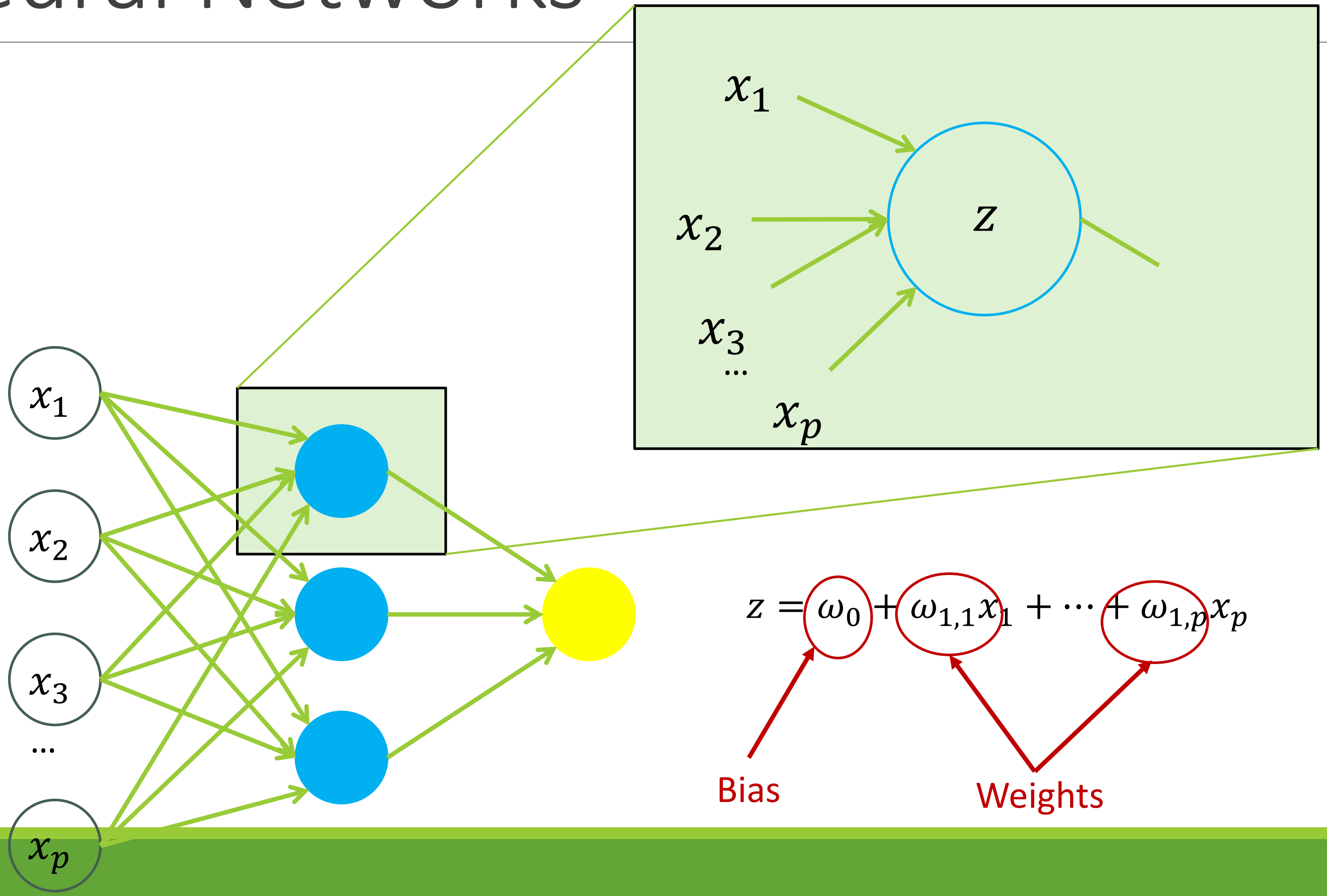


Neural Networks



$z = \omega_0 + \omega_{1,1}x_1 + \cdots + \omega_{1,p}x_p$

Neural Networks

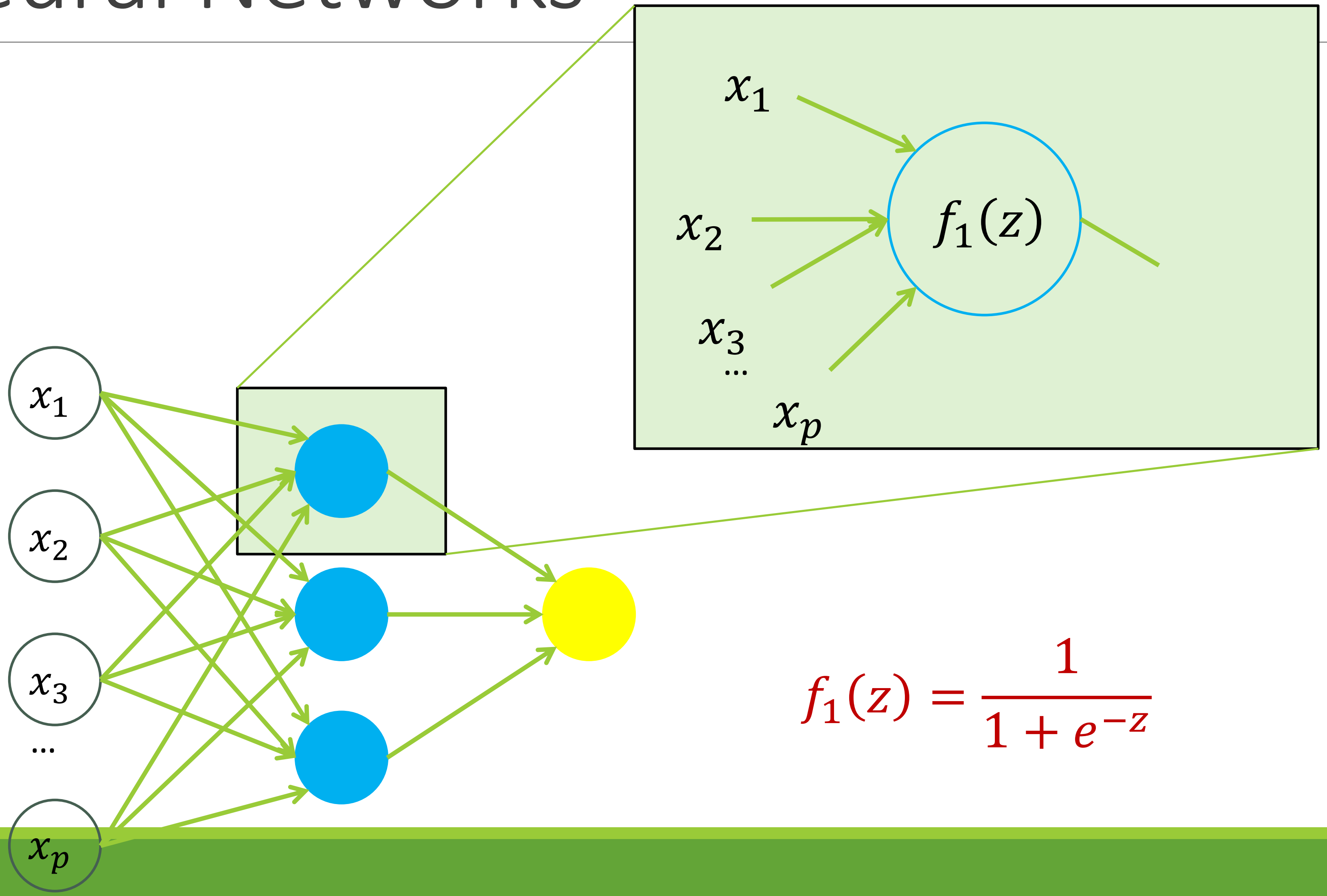


$$z = \omega_0 + \omega_{1,1}x_1 + \cdots + \omega_{1,p}x_p$$

Bias

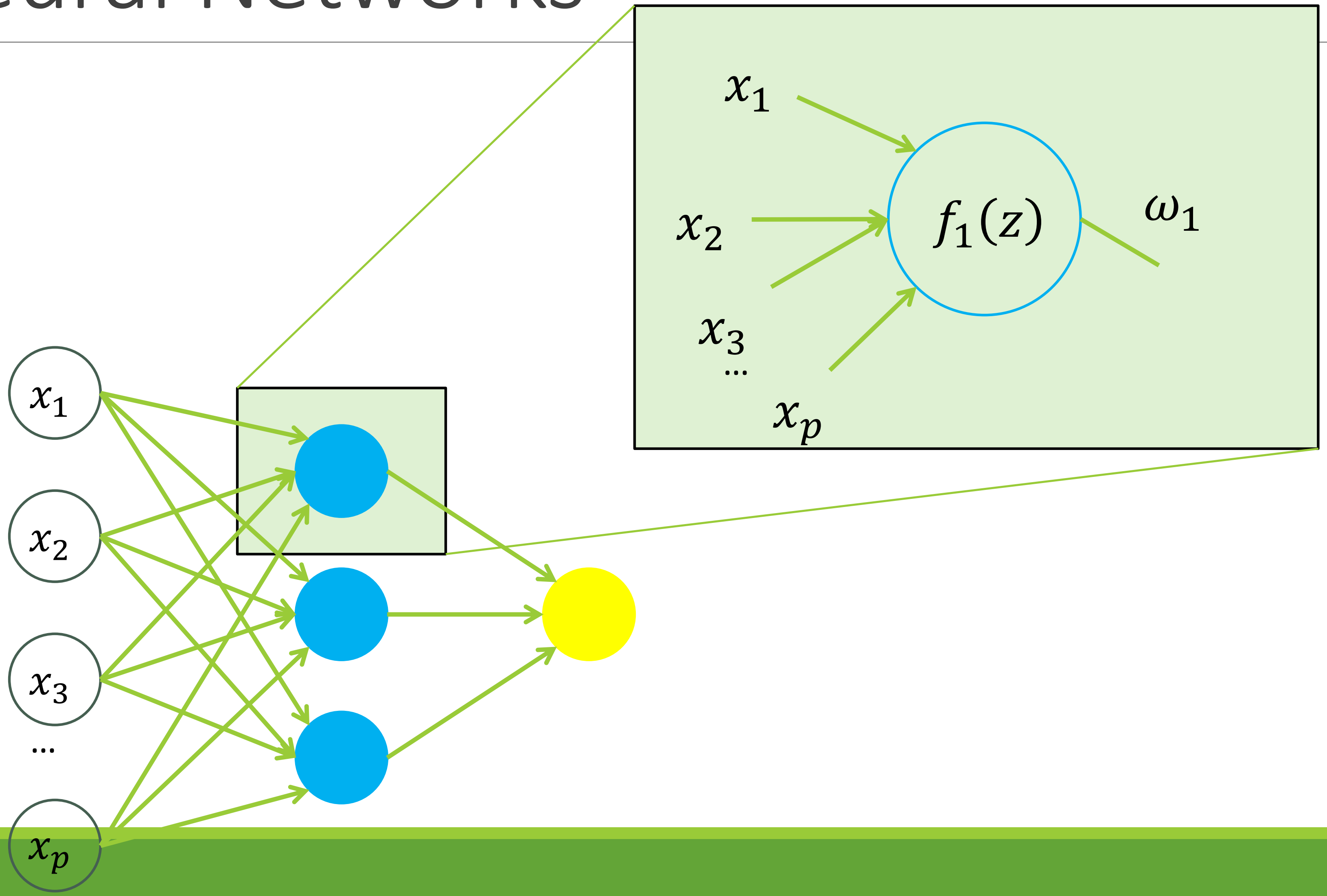
Weights

Neural Networks

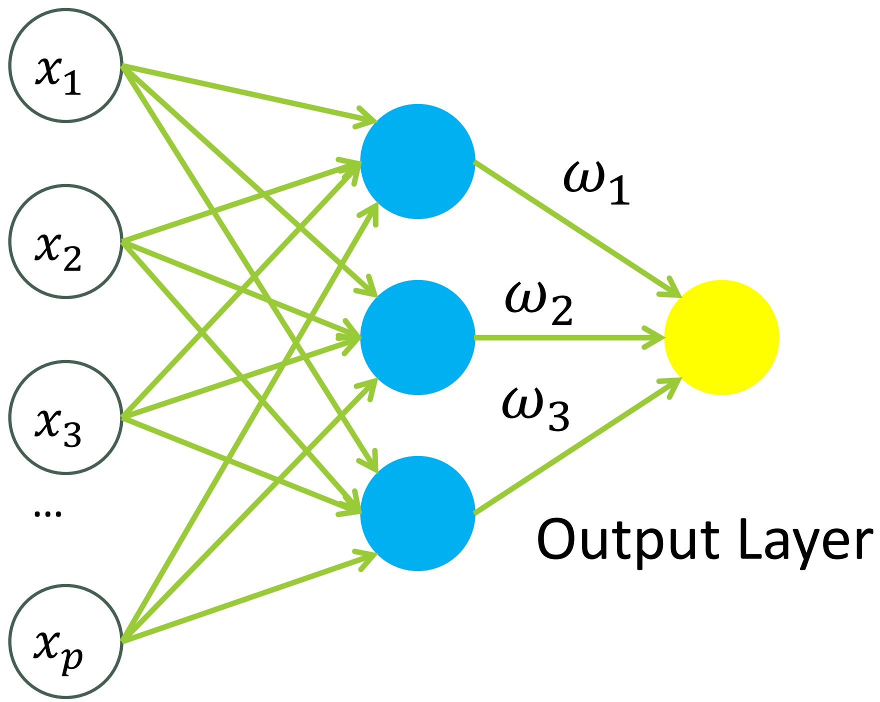


$$f_1(z) = \frac{1}{1 + e^{-z}}$$

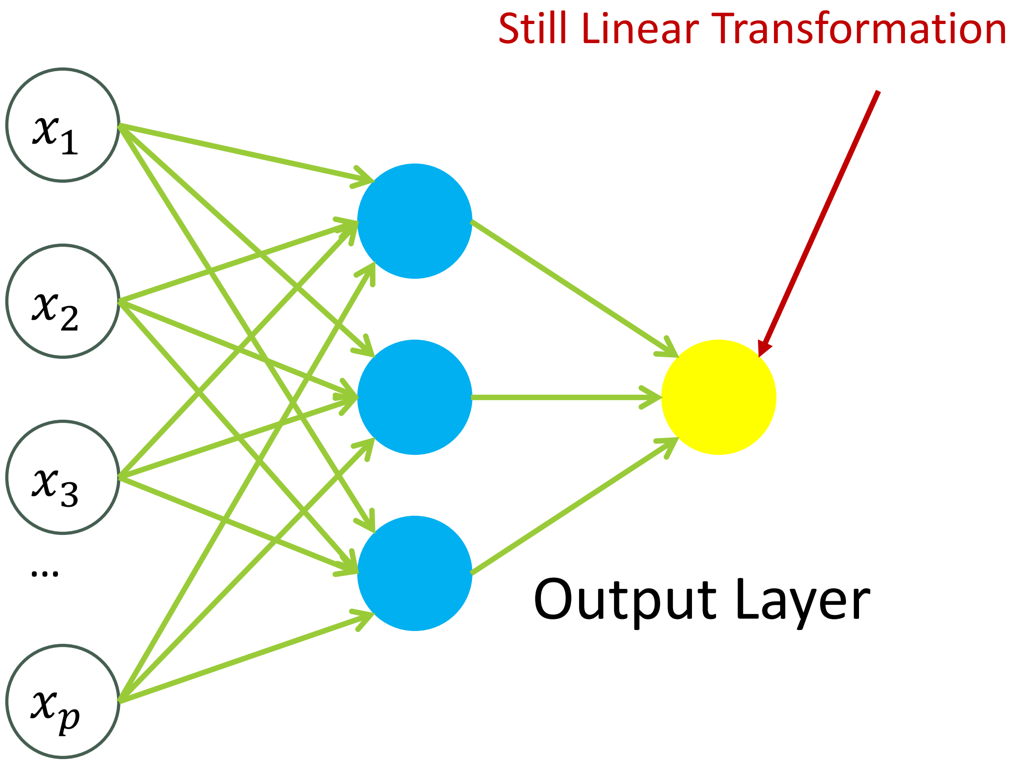
Neural Networks



Neural Networks

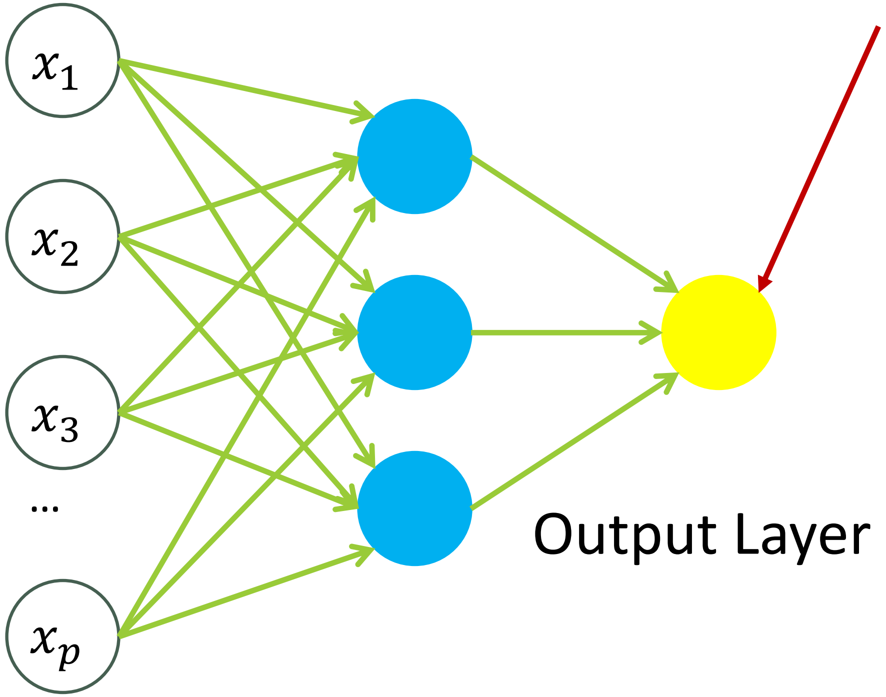


Neural Networks



Neural Networks

$$y = \omega_0 + \omega_1 f_1(z) + \omega_2 f_2(z) + \omega_3 f_3(z)$$

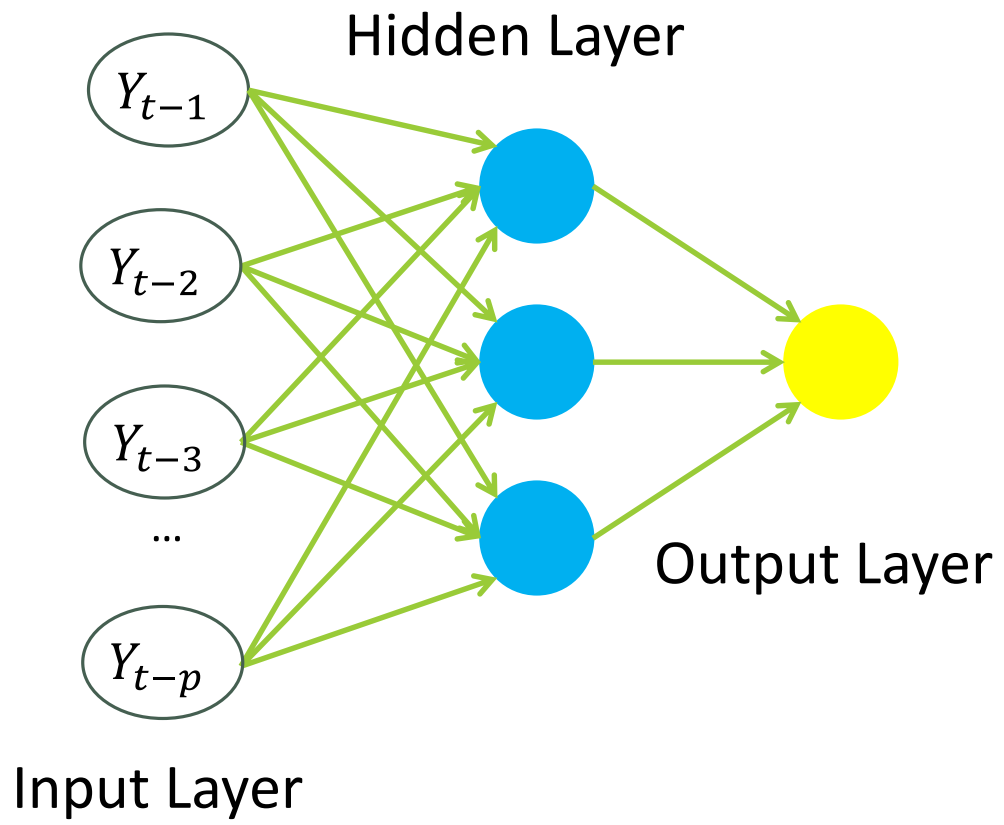


Autoregressive Neural Networks

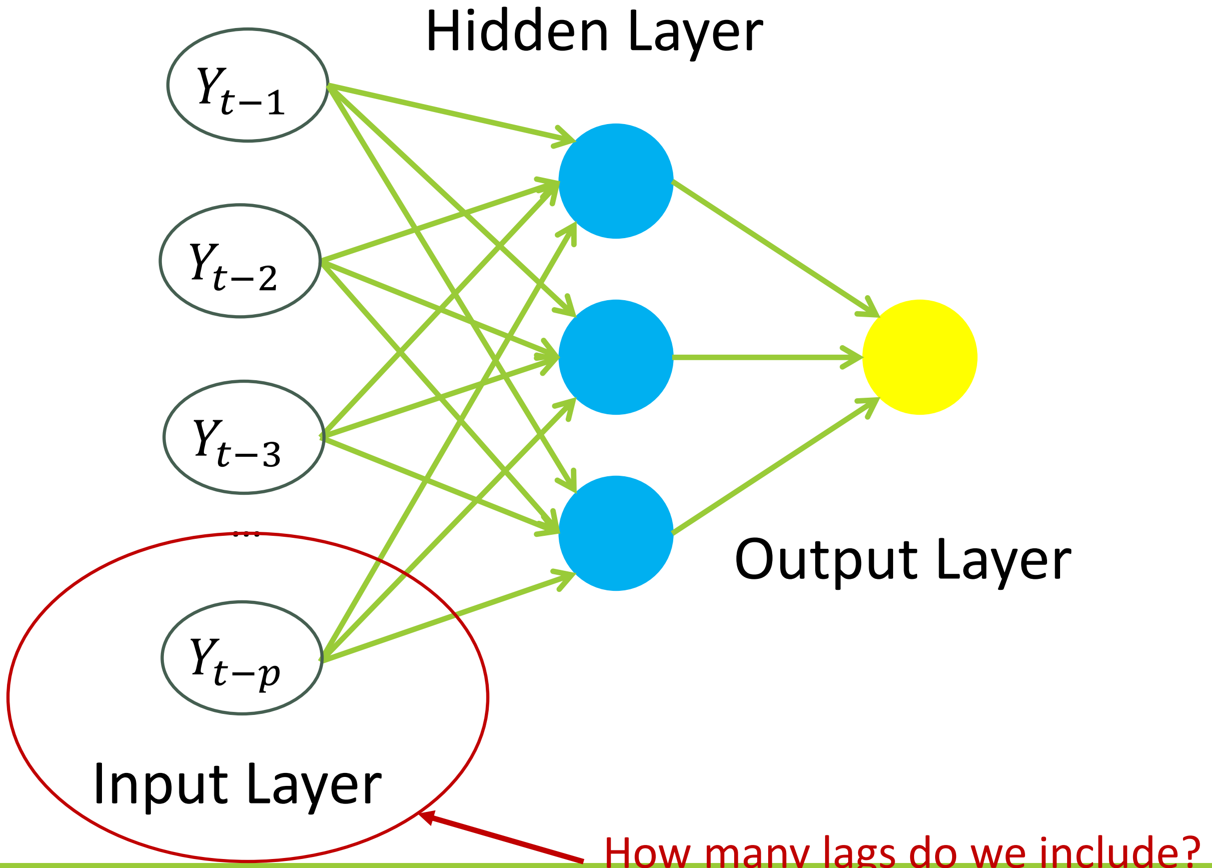
Autoregressive Terms

- Neural network models used for forecasting in time series, just have lags of Y in the bottom layer (inputs) along with (or in place of) other X variables.

Autoregressive Terms



Autoregressive Terms



Number of Autoregressive Lags

Explore with correlation plots.

Put the number of significant lags you see in the correlation plots – specifically PACF or IACF.

For **seasonal data** we typically include all lags up through one season unless correlation plots say you only need specific ones.

STILL WANT TO MAKE DATA STATIONARY FIRST!

Neural Networks in Software

Time series neural networks are a newer concept. (RNN’s and LSTM will be discussed in Spring)

R has a function that can handle lags as inputs into a neural network as well as forecast these models forward.

SAS doesn’t have any PROC that can handle this process, but it’s nodes in EM can.

- Have to add lag variables just like you would add any input variable.
- Forecasting is code heavy.

Weighted and Combined Models

Composite Forecasting

Introduction

The thought process around weighted combined forecasting (**also known as composite forecasting or ensemble models**) is **not new**.

The topic has been studied for years, and **empirical evidence indicates that the combination of forecast methods tends to outperform most single forecast methods.**

In 1969, James Bates and Clive Granger suggested that if the objective is to produce accurate forecasts, then the analyst should attempt to combine forecasts.

Combining Forecasts

It is better to **average forecasts** in hope that by so doing, the biases among the methods and/or forecasters will compensate for one another.

As a result, forecasts developed using different methods are expected to be more useful in cases of high uncertainty.

This method is **especially relevant for long-range forecasting, where uncertainty is extremely high.**

Simple Average

Simple Averaging

$$\text{Combined Forecast} = \frac{1}{\mathrm{n}}\sum_{i=1}^{n} FM_i$$

Simple Averaging

$$\text{Combined Forecast} = \frac{1}{\mathrm{n}} \sum_{i=1}^{n} FM_i$$

Number of forecasting methods

Forecasting Method