ORDINAL LOGISTIC REGRESSION – EXTRA CONTENT

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PROPORTIONAL ODDS MODEL

What if Assumption Fails?

- The proportional odds assumption may not be met for all variables.
- 2 Approaches:
  1. Partial Proportional Odds Model
  2. Multinomial Logistic Regression

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Some variables
fail assumption

All variables
fail assumption

Partial Proportional Odds – SAS

```
proc logistic data=Logistic.Wallet;
   class punish(param=ref ref='1');
   model wallet = male business punish explain /
                  unequalslopes=(business) clodds=pl;
   title 'Partial Proportional Odds Model';
run;
quit;
```

Partial Proportional Odds – SAS

Analysis of Maximum Likelihood Estimates							
Parameter		wallet	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept		1	1	-2.6695	0.4491	35.3260	<.0001
Intercept		2	1	-0.7730	0.3714	4.3318	0.0374
male			1	1.0707	0.3282	10.6426	0.0011
business		1	1	0.9722	0.4838	4.0389	0.0445
business		2	1	0.6376	0.3810	2.7996	0.0943
punish	2		1	0.6300	0.4048	2.4224	0.1196
punish	3		1	1.3956	0.4829	8.3523	0.0039
explain			1	-1.0532	0.3405	9.5660	0.0020

Partial Proportional Odds – R

```
plogit.model <- vglm(factor(wallet) ~ male + business + punish +
                     explain,
                     data = train,
                     family = cumulative(parallel = F ~ business))

summary(plogit.model)
```

Partial Proportional Odds – R

```
## Pearson residuals:
##                       Min      1Q  Median      3Q   Max
## logitlink(P[Y<=1]) -1.241 -0.4759 -0.1765 -0.1099 3.899
## logitlink(P[Y<=2]) -1.850 -0.6623 -0.3859  0.6425 2.730
##
## Coefficients:
##                Estimate Std. Error z value Pr(>|z|)
## (Intercept):1  -2.6695     0.4466  -5.978 2.26e-09 ***
## (Intercept):2  -0.7730     0.3678  -2.102  0.03557 *
## male            1.0707     0.3258   3.287  0.00101 **
## business:1      0.9722     0.4789   2.030  0.04236 *
## business:2      0.6376     0.3810   1.674  0.09423 .
## punish2         0.6300     0.4008   1.572  0.11594
## punish3         1.3956     0.4727   2.952  0.00316 **
## explain        -1.0532     0.3413  -3.086  0.00203 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

Partial Proportional Odds – R

```
## Names of linear predictors: logitlink(P[Y<=1]), logitlink(P[Y<=2])
##
## Residual deviance: 306.8392 on 382 degrees of freedom
##
## Log-likelihood: -153.4196 on 382 degrees of freedom
##
## Number of Fisher scoring iterations: 5
##
## No Hauck-Donner effect found in any of the estimates
##
##
## Exponentiated coefficients:
##       male business:1 business:2    punish2    punish3    explain
##  2.9174165  2.6438481  1.8918696  1.8776731  4.0373205  0.3488076
```

INTERPRETATION

Model Notation – SAS Option

- With cumulative logits, increasing the right-hand side of the equation leads to an increased log(odds) of **higher** outcome category:

$$\log\left(\frac{p_{i,1}}{p_{i,2}+p_{i,3}}\right) = \beta_{0,1} - \beta_1 \text{male}_i - \beta_2 \text{business}_i$$
$$-\beta_3 \text{punishM}_i - \beta_4 \text{punishH}_i - \beta_5 \text{explain}_i$$

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$$-\beta_3\text{punishM}_i-\beta_4\text{punishH}_i-\beta_5\text{explain}_i$$

Model Notation – R Default

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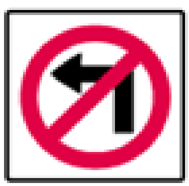
$$\log\left(\frac{p_{i,1}+p_{i,2}}{\boxed{p_{i,3}}}\right)=\beta_{0,2}-\beta_1\text{male}_i-\beta_2\text{business}_i-\beta_3\text{punishM}_i-\beta_4\text{punishH}_i-\beta_5\text{explain}_i$$

Odds Ratio Interpretation – Descending

- Interpretation is still an odds ratio: $\mathbf{100 * \left(e^{\hat{\beta}_j} - 1\right)\%}$ **higher expected odds** of being in a higher category.

No Association



Group in denominator has higher odds to be in a **higher** category.

Group in numerator has higher odds to be in a **higher** category.

0

1

∞

Odds Ratios Descending – SAS

```
proc logistic data=Logistic.Wallet desc;
   class punish(param=ref ref='1');
   model wallet = male business punish explain
                        / clodds=pl;
run;
```

Odds Ratios Descending – SAS

Type 3 Analysis of Effects			
Effect	**DF**	**Wald Chi-Square**	**Pr > ChiSq**
male	1	10.6047	0.0011
business	1	4.4167	0.0356
punish	2	9.4185	0.0090
explain	1	9.4925	0.0021

Analysis of Maximum Likelihood Estimates						
Parameter		**DF**	**Estimate**	**Standard Error**	**Wald Chi-Square**	**Pr > ChiSq**
Intercept	**3**	1	0.7890	0.3675	4.6107	0.0318
Intercept	**2**	1	2.5678	0.4169	37.9321	<.0001
male		1	-1.0598	0.3254	10.6047	0.0011
business		1	-0.7389	0.3516	4.4167	0.0356
punish	**2**	1	-0.6277	0.4005	2.4564	0.1170
punish	**3**	1	-1.4031	0.4721	8.8330	0.0030
explain		1	1.0518	0.3414	9.4925	0.0021

Odds Ratios Descending – SAS

Association of Predicted Probabilities and Observed Responses			
Percent Concordant	69.0	**Somers' D**	0.465
Percent Discordant	22.5	**Gamma**	0.508
Percent Tied	8.5	**Tau-a**	0.249
Pairs	10154	**c**	0.732

Odds Ratio Estimates and Profile-Likelihood Confidence Intervals				
Effect	**Unit**	**Estimate**	**95% Confidence Limits**	
male	1.0000	0.347	0.180	0.652
business	1.0000	0.478	0.238	0.963
punish 2 vs 1	1.0000	0.534	0.242	1.193
punish 3 vs 1	1.0000	0.246	0.094	0.632
explain	1.0000	2.863	1.469	5.612

Odds Ratios – R

```
ORtable <- data.frame(OR = exp(coef(clogit.model)),
                      lower = exp(confint(clogit.model))[,1],
                      upper = exp(confint(clogit.model))[,2])

print(ORtable)
```

Odds Ratios – R

```
##                 OR      lower     upper
## male     0.3465172 0.17995548 0.6522567
## business 0.4776512 0.23775967 0.9626137
## punish2  0.5338490 0.24246863 1.1934058
## punish3  0.2458364 0.09418942 0.6315739
## explain  2.8630214 1.46945567 5.6121973
```