BAYESIAN STATISTICS

CLASS 2

WHAT DID WE LEARN FROM CLASS 1?

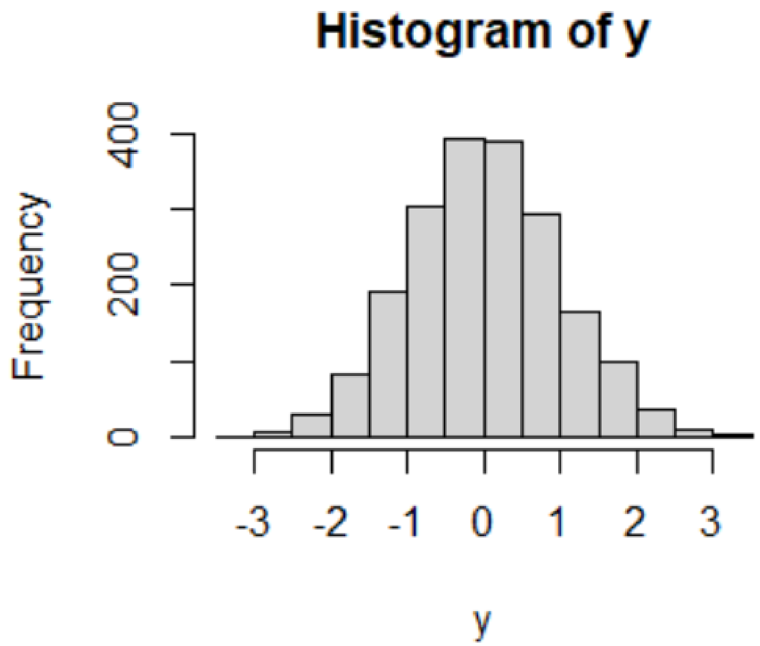
- Terminology: Prior, Sampling distribution, Posterior
- How to define problem (decide sampling distribution of data, define priors for parameters, use Stan to generate posterior distribution of parameters)
- How to use posterior to answer questions about the parameter
- How data (sample size) and prior contribute to the posterior
- Why prior is VERY important when sample size is small

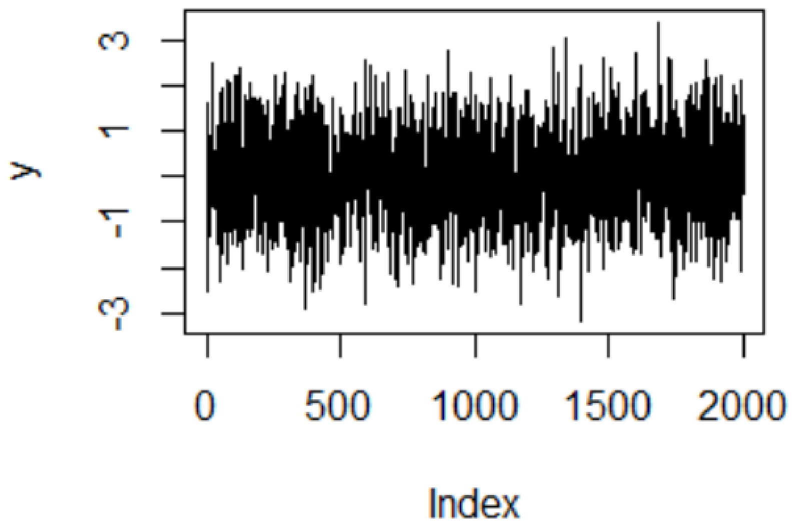
GOALS FOR TODAY

- MCMC - Markov Chain Monte Carlo
  - What it is
  - Has it converged
  - Options to help convergence
- Options in running MCMC to get posterior distribution
- Another in-class example

SIMULATING A DISTRIBUTION

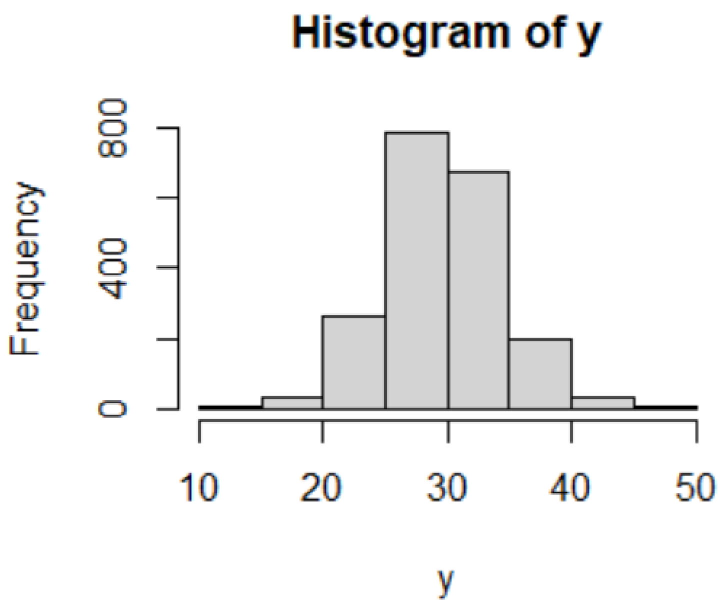
```
y=rnorm(2000)
> hist(y)
> plot(y,type='l')
```

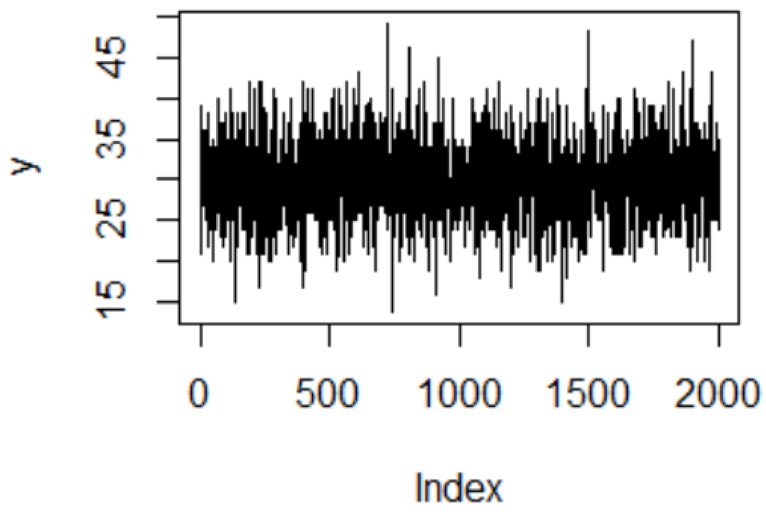




ANOTHER DISTRIBUTION

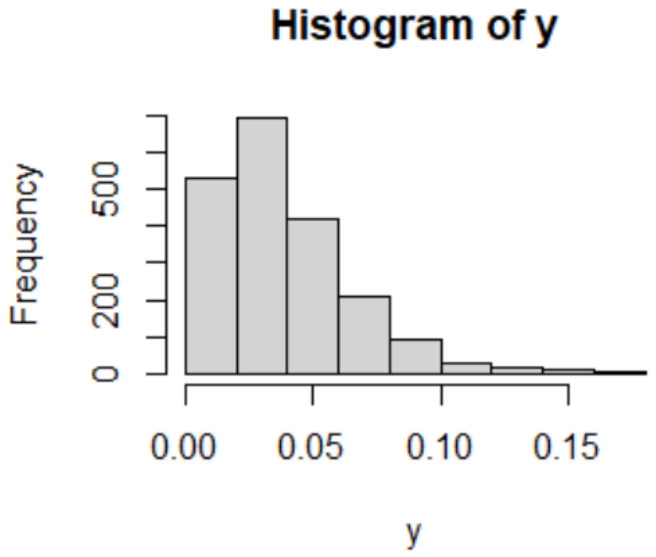
```
> y=rbinom(2000,100,0.3)
> hist(y)
> plot(y,type='l')
```

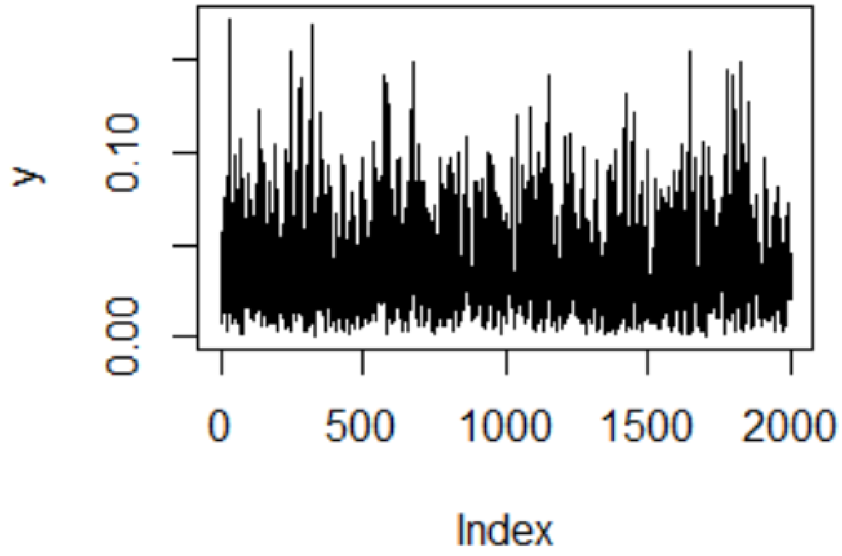




SKEWED DISTRIBUTION

```
> y=rbeta(2000,2,50)
> hist(y)
> plot(y,type='l')
```





MARKOV CHAIN MONTE CARLO

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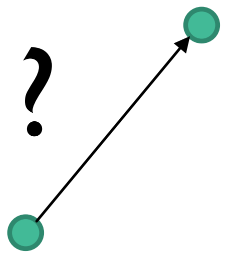
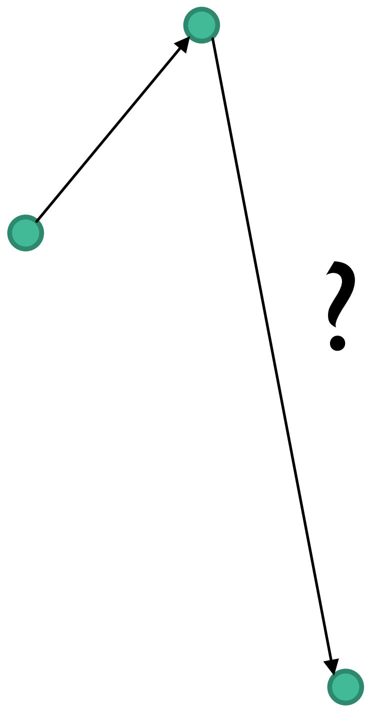
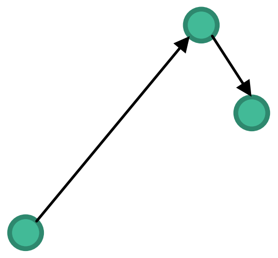


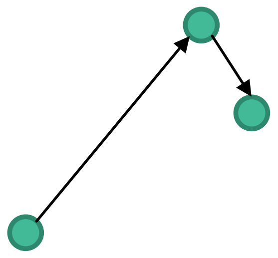
MARKOV CHAIN MONTE CARLO



MARKOV CHAIN MONTE CARLO

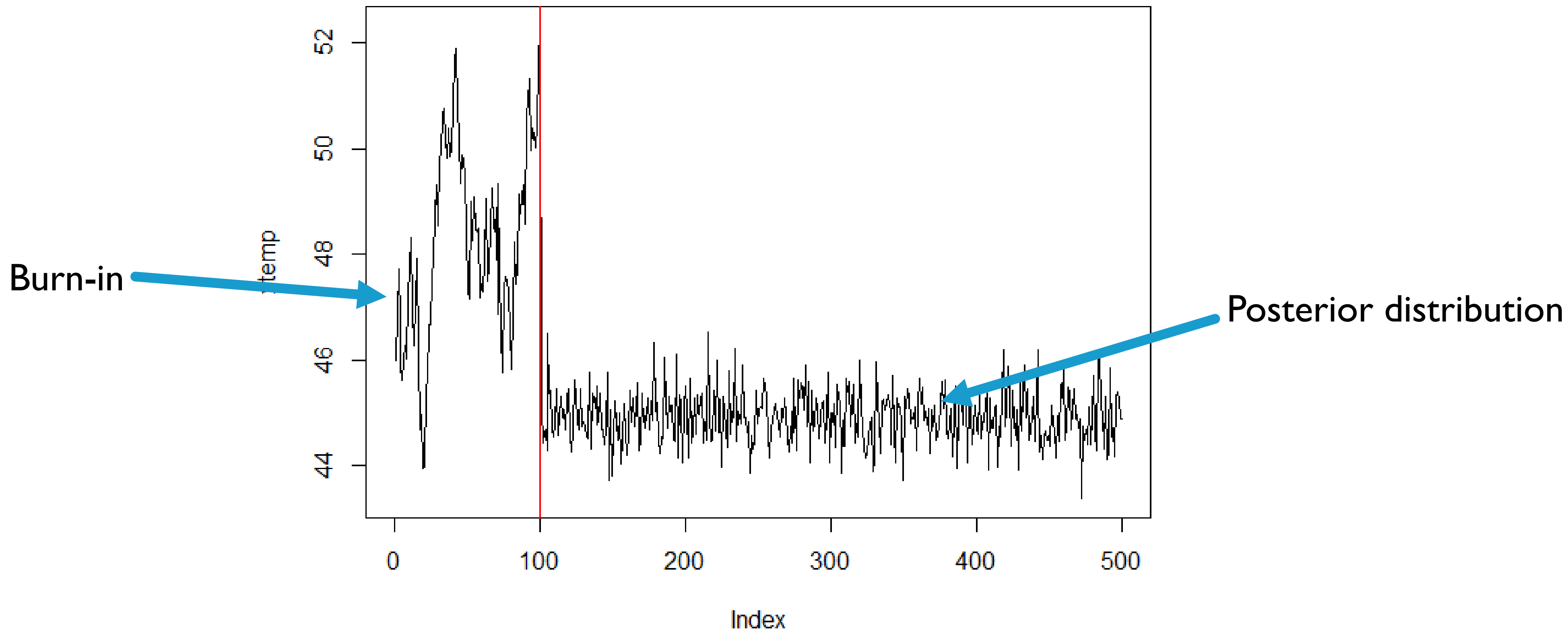


MARKOV CHAIN MONTE CARLO

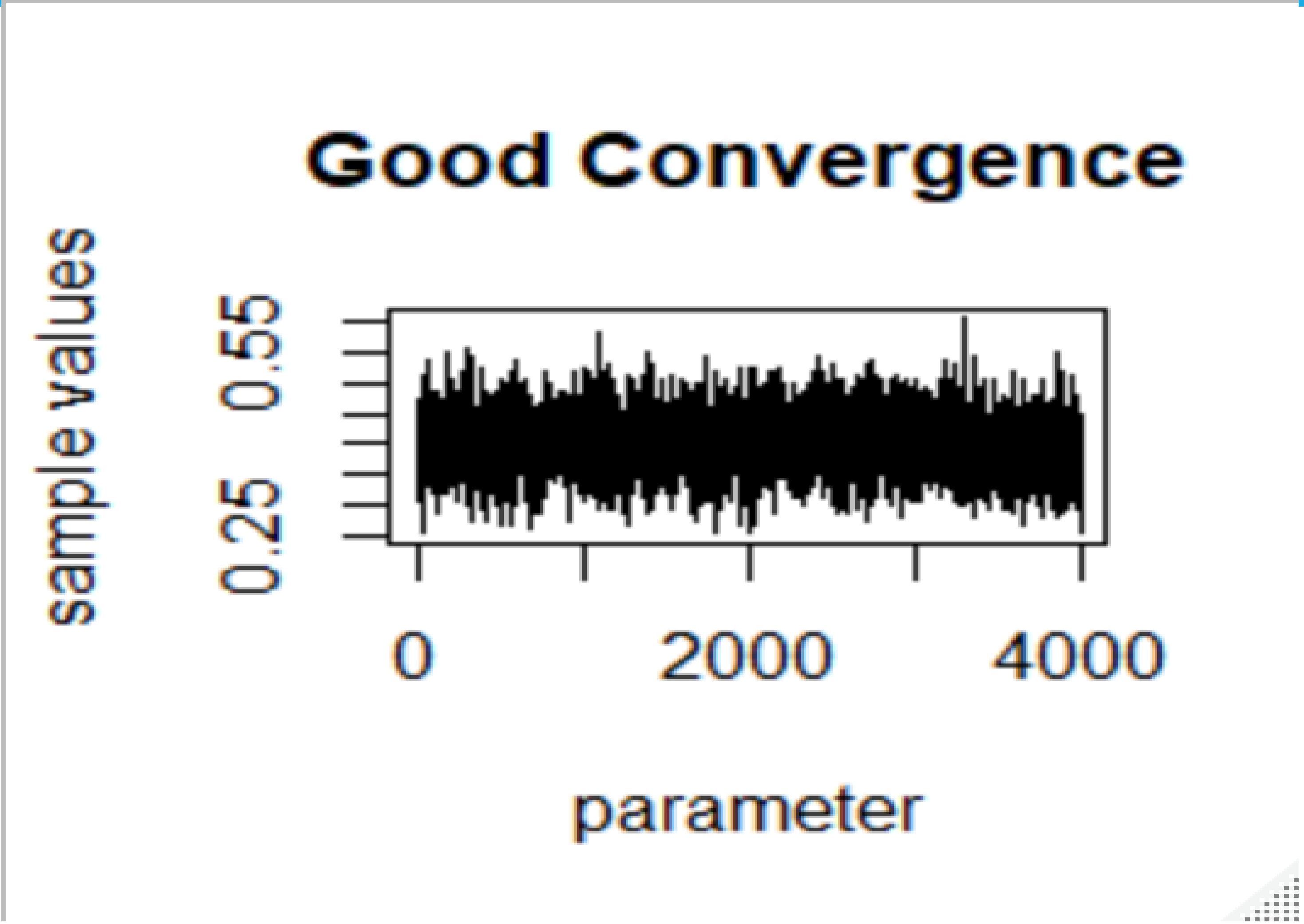


Stan uses the Hamiltonian Monte Carlo method for its Markov Chain and its adaptive variant the no U-turn sampler (NUTS). For more details, see https://mc-stan.org/docs/2_19/reference-manual/hamiltonian-monte-carlo.html

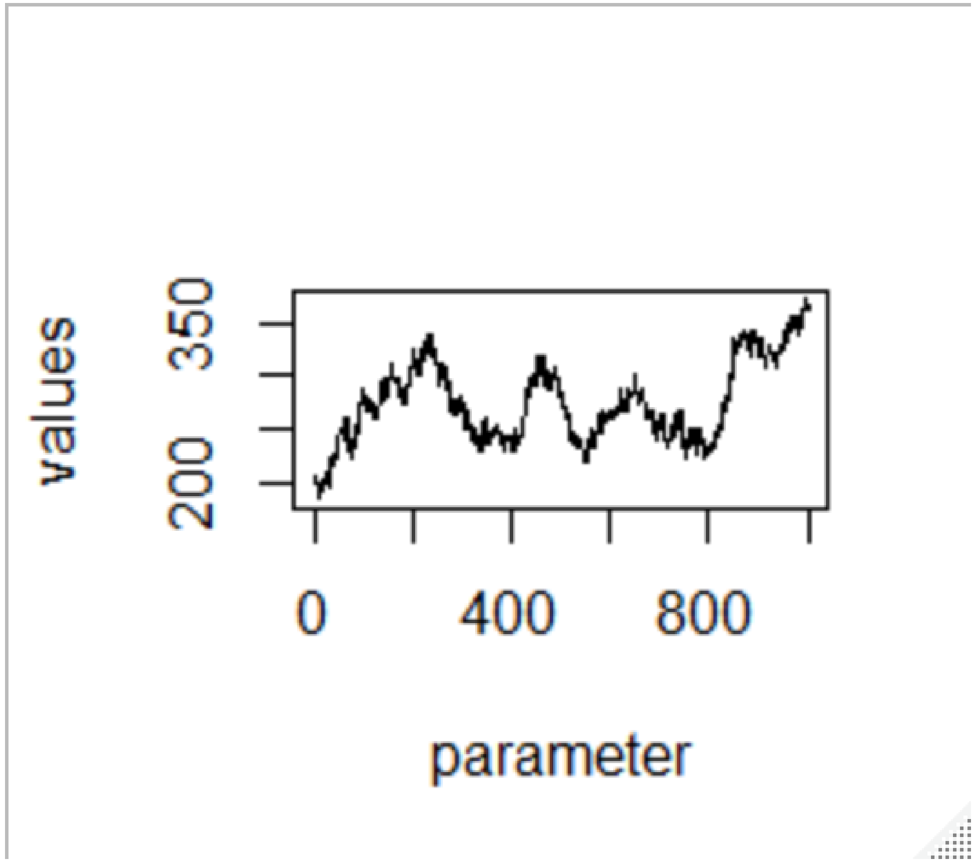
MCMC

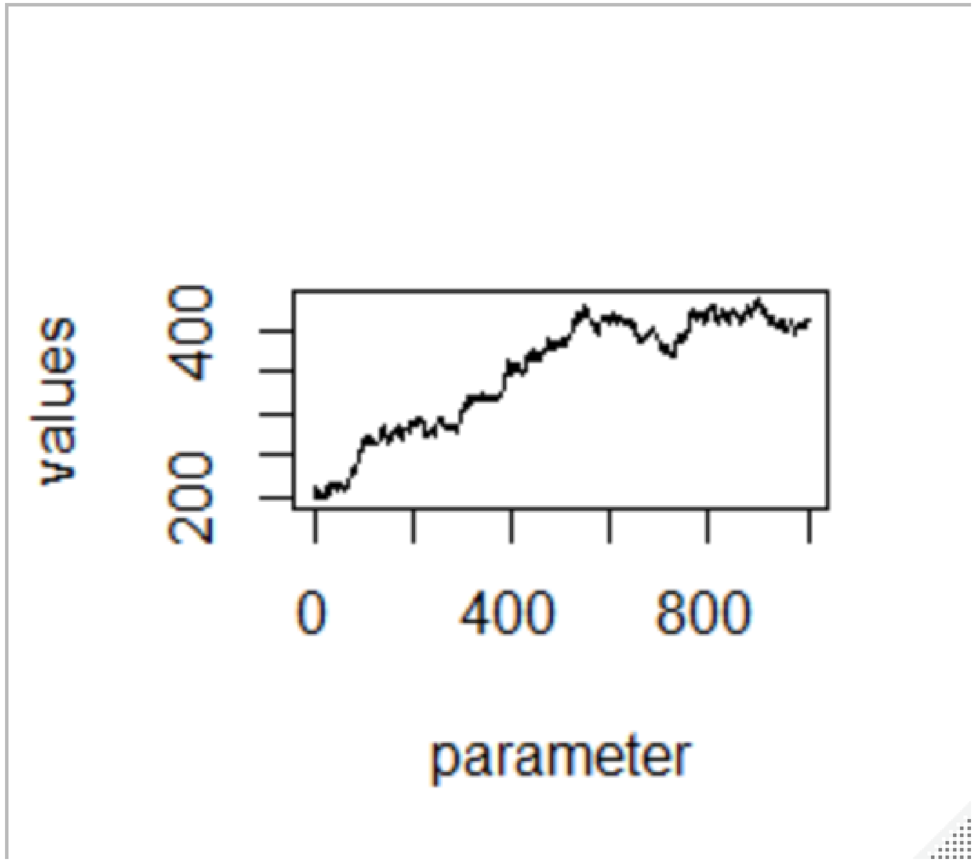

Burn-in
Posterior distribution
temp
52
50
48
46
44
0
100
200
300
400
500
Index

CONVERGENCE



NONCONVERGENCE





FIXES

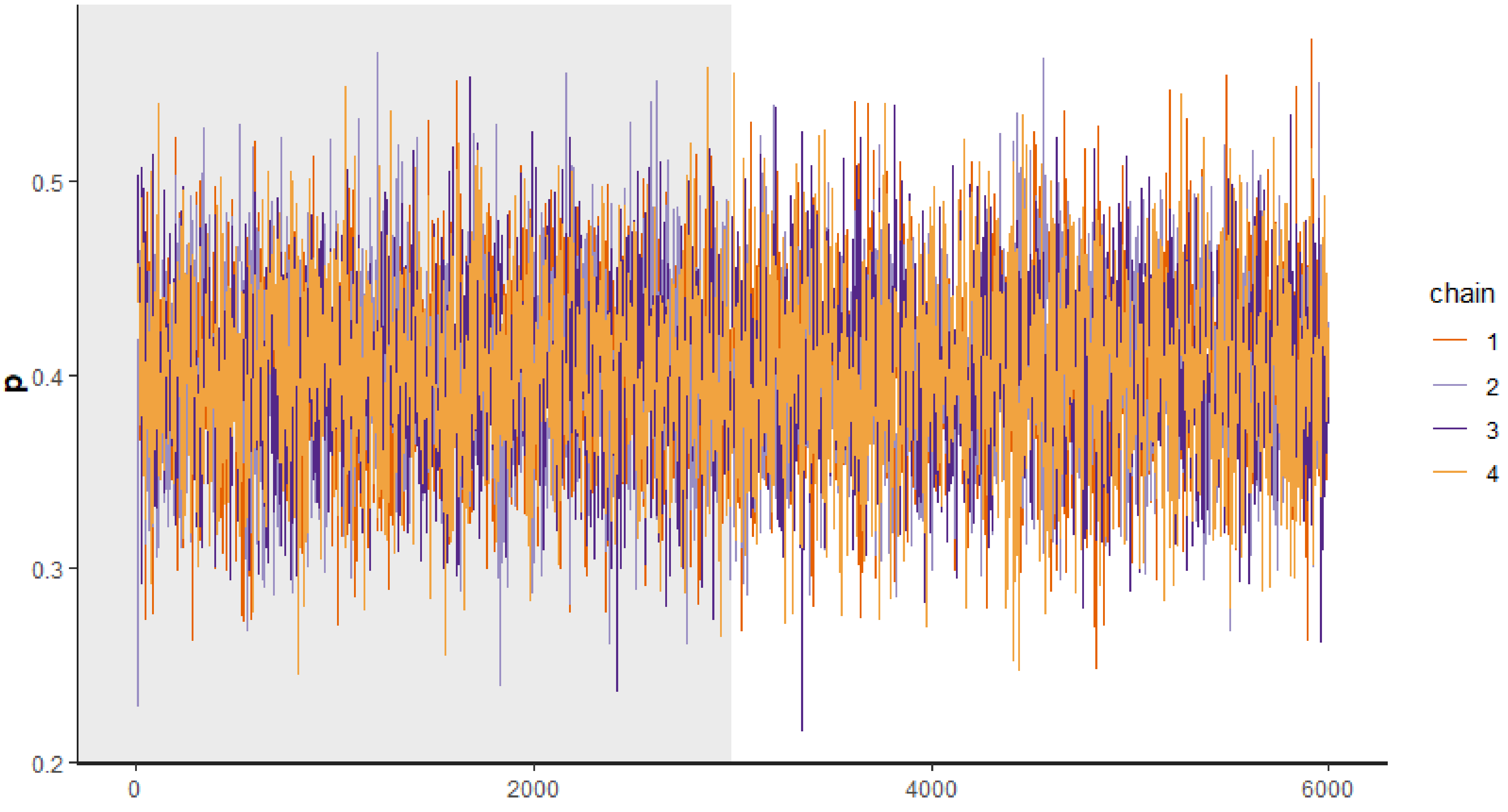
- Improper posterior or bad prior
  - Fix: New prior distribution
- Hasn't converged yet
  - Let the chain run longer
- Chain continues to increase
  - Potentially a bad starting point…provide a new starting point (or change prior)
- Too much autocorrelation in chain
  - Thin the chain

OPTIONS FOR STAN CODE

```
binom.stan=stan(file='Q:\\My Drive\\Bayesian\\Code\\Binomial_example.stan',data=binom.data,
chains = 4,           # number of Markov chains
  warmup = 3000,   # number of warmup iterations per chain
  iter = 6000,         # total number of iterations per chain
  refresh = 0,          # no progress shown
  thin=3,                 # will 'thin' the chains…help with autocorrelated posterior samples
  init=0.3                #specify initial values..I only did it for one chain
  )
```

Creates four chains; each chain has 6000 values, however, only every 3rd value is taken (now down to 2000 per chain that is useful); first 3000 (well, actually only 1000 since we are thinning) is burn-in meaning it is not used

End result will have a total of 4000 posterior values (1000 from each chain) traceplot(binom.stan, inc_warmup = TRUE)



mean se_mean sd 10% 50% 90% n_eff Rhat p 0.40 0.00 0.05 0.34 0.40 0.46 3268 1 lp__ -69.23 0.01 0.72 -70.10 -68.96 -68.74 3290 1

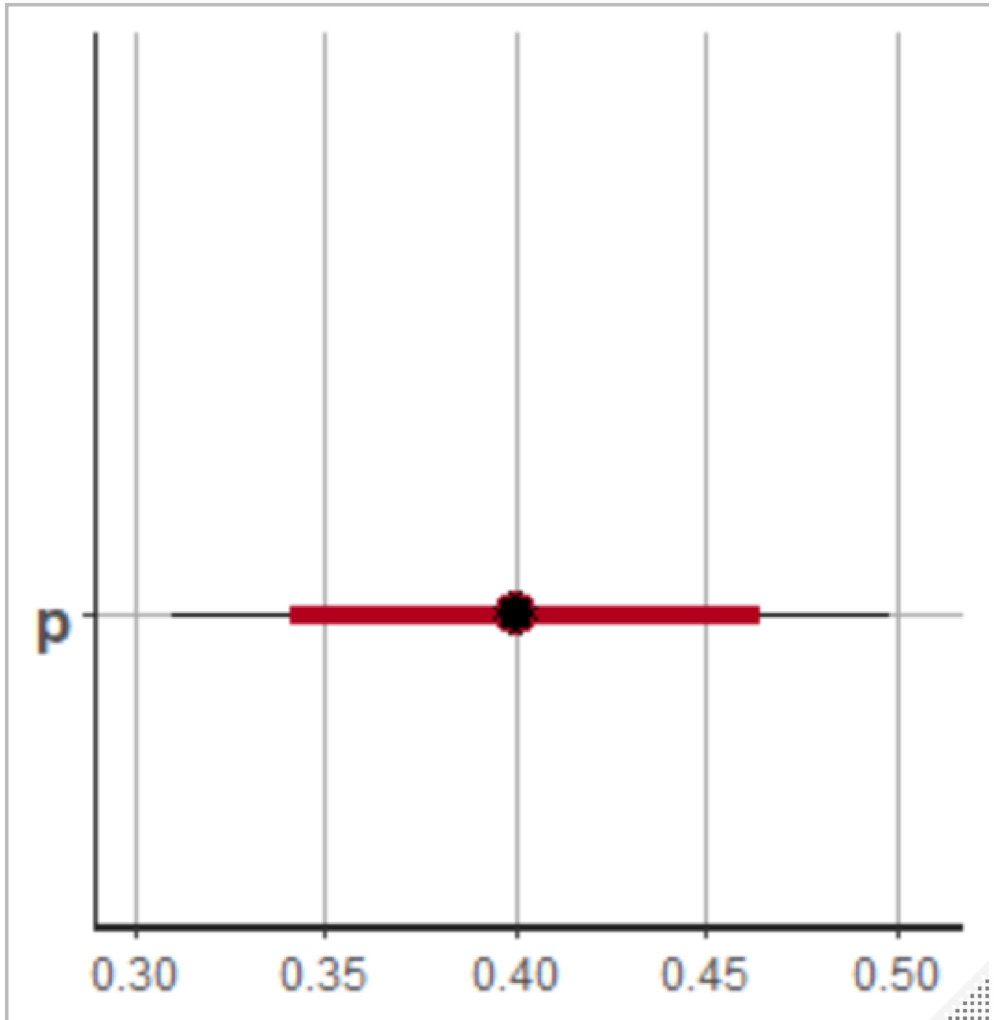
DIAGNOSTICS

print(binom.stan, pars=c("p", "lp__"), probs=c(.1,.5,.9))

	mean	se_mean	sd	10%	50%	90%	n_eff	Rhat
p	0.40	0.00	0.05	0.34	0.40	0.46	3268	1
lp__	-69.23	0.01	0.72	-70.10	-68.96	-68.74	3290	1

Log posterior value (unnormalized)

Want these to be close to 1 (this means convergence); if greater than 1.1, then there could potentially be a problem (Rhat or potential scale reduction) plot(binom.stan)



Inner is 80% probability interval
Outer is 95% probability interval

ANOTHER POTENTIAL WARNING

Warning: There were 2 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help. See http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

Fix: add this line to your options…

```
control=list(adapt_delta=0.9)
```

EMPIRICAL BAYES

- Empirical Bayes is between Full Bayesian Statistics and frequentist
- It uses data to estimate the prior distribution parameters
- However, if you ‘double-dip’ with the data, you are adding extra variability and this needs to be accounted for in the in the posterior distribution (be cautious of this!!)

NEW EXAMPLE

- WARP BREAKS per loom
- Would like to understand the mean number of warp breaks per loom (a loom corresponds to a fixed length of yarn)

Sampling distribution:

Number of parameters:

Prior distribution: