BAYESIAN STATISTICS

CLASS 1

GOALS FOR TODAY'S LECTURE

- Understand terminology
  - Prior distribution
  - Sampling distribution
  - Posterior distribution
  - Marginal distribution of the data
- Understand simple coding in Stan
- Run a simple analysis in Stan

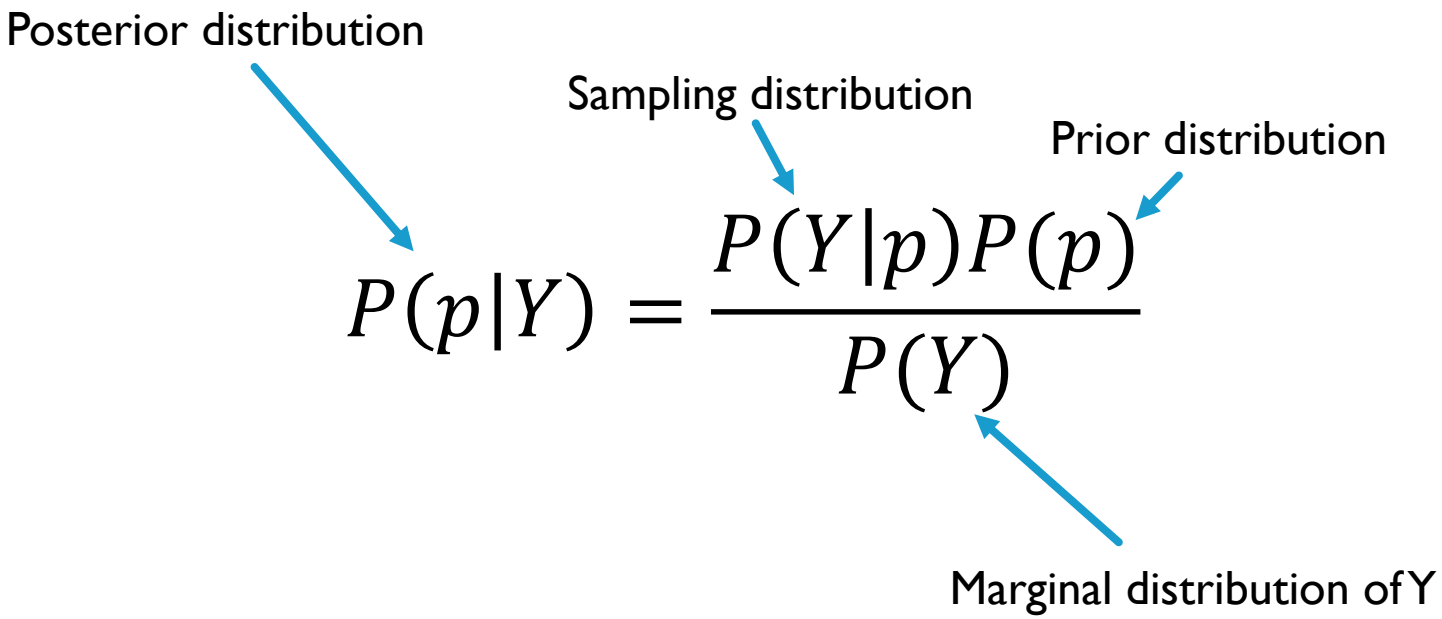
TERMINOLOGY

Sampling distribution: P(Y| parameters) (for example, $P(Y|\mu,\sigma^2)$ or P(Y|p) or $P(Y|\lambda)$)

Prior distribution: P(parameter) (for example, $P(\mu)$, $P(\sigma^2)$, P(p) or $P(\lambda)$)

Posterior distribution: P(parameters|Y) (for example, $P(\mu,\sigma^2|Y)$ or P(p|Y) or $P(\lambda|Y)$)

HOW IT ALL FITS TOGETHER (BAYES RULE)



HOW IT ALL FITS TOGETHER (BAYES RULE)

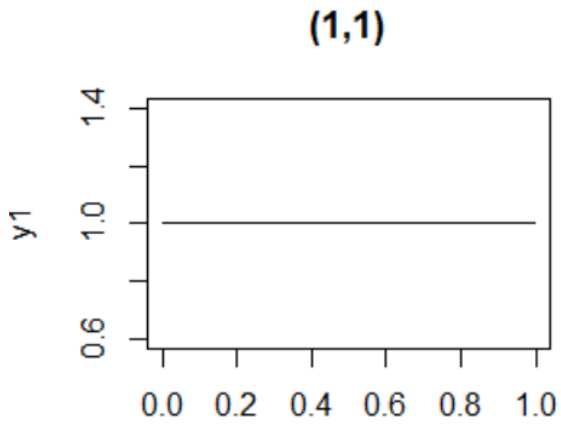
$$P(p|Y) \propto P(Y|p)P(p)$$

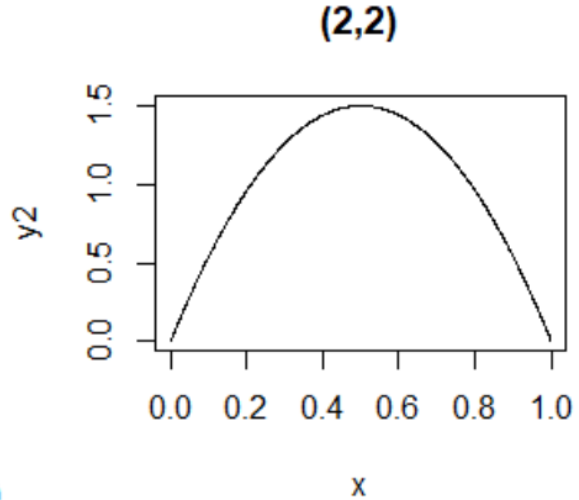
SIMPLE EXAMPLE

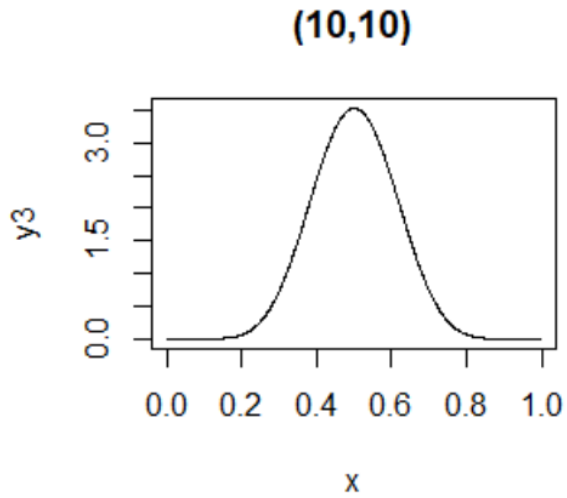
- Want to estimate the proportion of students at NCSU who voted in 2020 Democratic primary
- Sampling distribution:
- Parameters:
- Prior distribution:

EXPLORE THE BETA DISTRIBUTION

```
x<-seq(0.001,0.999,length=1000)
y1<-dbeta(x,1,1)
plot(x,y1,type='l', main='(1,1)')
y2<-dbeta(x,2,2)
plot(x,y2,type='l',main='(2,2)')
y3<-dbeta(x,10,10)
plot(x,y3,type='l',main='(10,10)')
```







MODEL INFO IN STAN

Data

Parameters

Transformed parameters (if needed)

Model

MODEL INFO IN STAN (SEPARATE FILE)

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This is where you define your data (integer, real, are there any bounds on information here?)

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If you are transforming variables…for example in a linear regression

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Transformed parameters (if needed)

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Model

This is where you will define your model

Be sure that your Stan programs ends in a blank line without any characters including spaces and comments.
Statements will end in semicolins;

CODE FOR EXAMPLE

```
data {
      int <lower=0> y;
      int <lower=0> n;
      }
parameters {
      real <lower=0, upper=1> p;
                  }
model {
      p ~ beta(1,1);
      y ~ binomial(n, p);
         }
```

NOTE: Be sure to save this file as .stan (not .txt…careful to make sure it is NOT saved as .txt)

IN CLASS EXAMPLE

- A political science student wants to estimate the proportion of students at NCSU who voted in the 2020 election. Identify the sampling distribution, the number of parameters and potential prior(s).
- The student gathered a sample of 150 students of which 100 indicated that they did vote.
- Get the posterior distribution(s) of the parameter(s) and find 95% probability interval(s). Assume a uniform prior for p.