SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

Engineering Systems and Design
40.002 Optimization

Group Project 1

0.1 Group Members

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0.2 Problem Description

Serangoon Heights is a real estate investment company with a primary focus on acquiring, selling, and renting residential properties such as HDB flats, condominiums, and landed houses in Serangoon. The company aims to **maximize overall profitability by carefully balancing short-term cash flow from property sales with long-term income from rentals and future resale.**

Given a fixed budget, the challenge is figuring out how many properties to buy across each type, and whether each should be sold immediately or rented out for five years before being sold. Selling properties brings in revenue immediately, but renting can lead to higher total returns over time if the value of property appreciates over time.

To assist them in making a strategic decision, the team built a mathematical optimization model which considers all relevant costs, rental yields, selling prices, and agent commissions. Most importantly, the model also calculates the present value[1] of future rental and resale income to ensure a fair comparison between quick returns and long-term gains. The model helps determine the most profitable combination of properties to acquire and strategies to pursue, making it a useful tool for capital allocation and long-term planning.

[1] For the reasoning behind the usage of present value, refer to Appendix part a.

0.3 Assumptions

To ensure tractability and solvability of the model, we make the following simplifying assumptions:

- The company only purchases properties that do not require renovation.
- All properties are purchased without the use of loans and the full purchase cost is paid upfront.
- Rental income, transaction costs (taxes, commissions) and property prices remain constant throughout the 5-year planning horizon, unaffected by market changes or legal updates.
- Property prices increase at a constant rate, based on 2024 data.
- Average market values are used for property costs, rental fees and resale prices with each property type treated as a homogeneous asset class
- HDB flats are assumed to be of a fixed type (e.g., 4-room flats).
- Maintenance fees are negligible and not modeled.
- Agent commission rates are constant and apply only to immediate sales.
- Properties take the same amount of time to sell and do not affect cash flow.
- All rented properties are fully occupied, and the annual rental income is constant and fully collected with no vacancy losses.
- There are no additional costs incurred (e.g., marketing cost, renovation cost, vacancies costs).
- Government regulations (eg. Additional Buyer Stamp Duty, Seller's Stamp Duty), taxes, eligibility criteria, and ownership laws are negligible or non-binding for modeling purposes.

- Minimum occupation periods, foreigner restrictions, and citizenship-specific conditions are negligible or non-binding for modeling purposes.
- Singaporeans, PRs, and foreigners have equal access and tax treatment.
- All properties are sold at today's market prices at the end of the 5-year horizon.
- The model does not account for macroeconomic shocks (eg. market crashes), interest rate hikes, or inflation fluctuations.
- Investors return requirement is negligible
- The planning horizon is limited to 5 years, with all decisions made at the start
- Property management is scaled perfectly with portfolio size, so no management capacity constraint is applied.

0.4 Model

Decision variables:

Let:

- $i \in \{1,2,3\}$
  - where $i = 1$ represents HDB, $i = 2$ represents Condo and $i = 3$ represents Landed
- x_i : Number of properties **purchased** of type i
- y_i: Number of units of type i that are **rented and sold later**
- z_i : Number of properties **sold** of type i

Let constants:

- C_i: Average **purchase cost** per unit of property type i
- S_i: Average **selling price** per unit of property type i
- R_i: Average **annual rental income** per unit of type i
- R'_i: Present value of 5 years of rent for one unit of property type i [2]
  $(R'_i = R_i \times \left(\frac{1}{r} - \frac{1}{r(1+r)^T}\right)$
- S'_i: Present value of **resale after 5 years** for one unit of property type i [3]
  $\left(S'_i = \frac{S_i}{(1+r)^5}\right)$
- A : **Agent commission rate**
- T : Investment time horizon
- B: Total **budget** available
- D_i: Maximum number of properties of type i that can be purchased
- r: Discount rate

[2] refer to Appendix part b for the reasoning behind the formula chosen.

[3] refer to Appendix part c for the reasoning behind the formula chosen.

From our data research:

Average cost of purchasing a HDB unit (C_1)	\$592,415.15
Average cost of purchasing a Condominium unit (C_2)	\$2,178,469.22
Average cost of purchasing a Landed unit (C_3)	\$4,674,196.34
Average price of selling a HDB unit (S_1)	\$649,287.00
Average price of selling a Condominium unit (S_2)	\$2,335,319.00

Average price of selling a Landed unit (S_3)	\$4,856,490.00
Annual average rent fee of a HBD unit (R_1)	\$39,432.00
Annual average rent fee of a Condominium unit (R_2)	\$47,436.00
Annual average rent fee of a Landed unit (R_3)	\$95,916.00
Present value of 5 years of rent for one unit of HDB (R'_1)	\$170,719.92
Present value of 5 years of rent for one unit of Condominium (R'_2)	\$205,373.10
Present value of 5 years of rent for one unit of Landed (R'_2)	\$415,266.10
Present value of **resale after 5 years** for one unit of HDB (S'_1)	\$508,733.35
Present value of **resale after 5 years** for one unit of Condominium(S'_2)	\$1,829,783.54
Present value of **resale after 5 years** for one unit of Landed (S'_3)	\$3,805,186.99
Agent commission rate (A)	2%
Investment time horizon	5 years
Total budget available (B)	\$50,000,000
Maximum number of HDB that can be purchased with given budget	84 units
Maximum number of Condominium that can be purchased with given budget	22 units
Maximum number of Landed properties that can be purchased with given budget	10 units
Maximum number of HDB that can be purchased in the market	21,634 units
Maximum number of Condominium that can be purchased in the market	41,536 units
Maximum number of Landed properties that can be purchased in the market	5,000 units
Discount rate	5%

Objective Function (maximize total projected profit):

$$\text{Max} \sum_{i=1}^{3} (S_i)(1-A)z_i + \sum \left(R'_i + (S'_i)(1-A)\right) y_i - \sum_i C_i x_i$$

0.5 Constraints

1. **Budget Constraint**:
    a. Ensures that the total cost of buying houses does not exceed the available budget.

$$\sum_i C_i \, x_i \leq B$$

2. **Ownership-Rental Constraint:**
    a. Houses rent out must be less than or equals to houses own

$$y_i \leq \; x_i \quad \forall i$$

3. **Sale Constraints**:
    a. Houses sold must be less than or equals to houses bought

$$z_i \leq \; x_i \quad \forall i$$

4. **Inventory Constraints:**
    a. Houses cannot be rented and sold at the same time

$$y_i + z_i \leq \; x_i \quad \forall i$$

5. **Market Demand Constraint**:
    a. Number of houses bought must be less than or equals to the maximum number of properties of type i that can be purchased/ can be purchased

$$x_i \leq \; D_i \quad \forall i$$

6. **Minimum investment:**
    a. Real estate firms diversify their portfolios to spread risk across different asset classes.
    b. HDBs offer stable rental income, but condos and landed properties offer higher potential for capital gains and brand prestige.
    c. Investing only in one segment (like just HDBs) would expose the company to regulatory or demand-specific risks.
    d. Ensuring minimum exposure in each class supports a robust, balanced investment strategy.

- Purchasing a minimum of 30% units of condominium units represents a significant capital commitment and aligns with private sector branding. Condos also offer higher capital gains and cater to mid-market demands.
- Purchasing a minimum of 10% landed units ensures the firm has a premium foothold and captures capital gains potential. They also are illiquid but offer prestige and long-term appreciation. 10% ensures exposure without straining liquidity.

$$x_2 \geq \; 0.3 \cdot \left(\sum_{i=1}^{3} x_i\right)$$

$$x_3 \geq \; 0.1 \cdot \left(\sum_{i=1}^{3} x_i\right)$$

7. **Minimum Sales Requirement:**
    a. Selling properties generates immediate liquidity, which is essential for:
       - Paying out investor dividends
       - Funding future acquisitions
       - Responding to changing market conditions
    b. A firm that only rents may face cash flow mismatches, especially if unexpected costs arise. Hence It is necessary to set a cap for minimum sales

c. Ensuring a base level of property sales across all segments mirrors common real estate practices
- Sell at least 10% of purchased HDB units — ensures fast liquidity from mass-market properties.
- Sell at least 20% of purchased Condominium units — supports stronger revenue flows to meet investor expectations.
- Sell at least 30% of Landed units — unlocks high capital value, even from fewer units.

$$z_1 \geq \lceil 0.1 \cdot x_1 \rceil$$

$$z_2 \geq \lceil 0.2 \cdot x_2 \rceil$$

$$z_3 \geq \lceil 0.3 \cdot x_3 \rceil$$

8. **Minimum Rental Participation:**
    a. Renting premium properties like condos and landed home provides recurring income from high-income tenants, strengthens long-term cash flow for the company and leverages the prestige and desirability of higher-end homes in Serangoon
    b. Firms often maintain a core rental portfolio to ensure steady returns regardless of market volatility.
    c. These percentage-based thresholds simulate active participation in the private rental market across all property segments:
    - Renting at least 20% of purchased condominiums captures growing rental demand from expats and working professionals seeking modern and conveniently located housing.
    - Renting at least 20% of purchased landed homes targets niche high-income tenant groups, such as wealthy families and corporate executives, securing premium recurring rental income.

$$y_2 \geq \lceil 0.2 \cdot x_2 \rceil$$
$$y_3 \geq \lceil 0.2 \cdot x_3 \rceil$$

9. **Rental Capacity:**
    a. Capping rentals to 70% of total properties ensure a healthy mix of rental and sales, and capital recycling to avoid over-dependence on long-term rentals
    b. 70% cap means at least 30% of the portfolio must be sold, ensuring capital recycling, operational balance and a diversified income vs, cash flow strategy
    c. 70% is high enough to allow a strong rental base, yet low enough to force strategic sales — it's a commonly used heuristic in property fund modeling

$$\sum_i y_i \leq 0.7 \cdot \sum_i x_i$$

10. **Non-negative & Integer Constraints:**

$$x_i, y_i, z_i \geq 0 \text{ , } integers$$

0.6 Correctness of model

Our model effectively identifies an optimal property investment strategy by maximizing total projected profit generated from buying, renting, and selling residential properties, subjected to a set of constraints. The objective function includes both immediate revenue from sales of the properties, the present value of future rental income and the resale value of the properties after a 5-year rental period. This structure shows the trade-off between the amount gained from selling a property, and the earnings generated by holding a property (while renting it) and selling it in the future.

To simplify this trade-off while maintaining tractability, we assume a fixed five-year rental period for all leased properties, after which they are resold. This assumption ensures that rental and resale benefits can be consistently captured and discounted to present value. It also aligns with typical investment evaluation horizons and facilitates comparability between rental and sale options within the model.

Our model also considers real-world financial and strategic factors that affect how property investment works. These include how much each type of property costs to buy, how much rental income it brings in each year, how much it could sell for in the future, and how much money we have available to invest.

To make sure the model stays realistic, every decision it makes represents an action that a real investor could take. Solutions that the model generates will also comply with the constraints set that reflect realistic financial limitations. For example, the budget constraint ensures that the total money spent on buying properties will not exceed the amount of capital we have. We also include diversification constraints to ensure that the model allocates all its capital into just one type of property, reflecting realistic circumstances, where companies spread their investments across different property types to match varying market demand and reduce risk.

However, due to unpredictable market changes, large variety of properties and complexity of a real estate project, we included some assumptions to simplify our model to ensure that the model remains efficient and solvable.

Firstly, the model assumes that all real estate market conditions, such as prices and rental rates, remain constant over the five-year period. While this allows for more efficient profit projection, it does not account for market fluctuations caused by economic or regulatory changes. However, as unforeseen circumstances and changes are difficult to predict, we ignore these possible changes to ensure that our model is solvable.

Secondly, the model uses average values for costs and income, and each category of property is treated as homogeneous. This ignores the different variations of the properties such as unit condition, floor level, or proximity to amenities, which can significantly affect profitability. Despite this, the model remains scalable and efficient.

Thirdly, the model assumes full occupancy for all rental properties and no maintenance costs. However, rental income is subjected to vacancies, tenant churn, and repair expenses, which reduce the amount of rental yield. These elements were excluded for simplicity of the model, but they could be included in future iterations to increase accuracy.

Another key assumption is that all purchases are paid fully in cash, without any financing involved. The overall impact of both mortgages and interest payments are ignored, which are important to many real-world investment decisions. Loan-to-value constraints can be included to improve realism, and incorporation of debt servicing can also reflect investor leverage strategies.

Furthermore, the model does not incorporate time-step dynamics. All the investment decisions are instead made at the beginning of the planning horizon. This means that the model cannot capture staged investments, from the reinvestment of rental income, or dynamic reallocation strategies because portfolio managers commonly practice them in the long term.

Lastly, our model also does not incorporate certain real-world policy constraints such as SSD and ABSD, and ownership eligibility regulations. Although these factors are important to consider in real-world cases, it will cause our model to be too complex.

Despite these limitations, the model produces rational, feasible, and interpretable solutions that are realistic and supports investment goals of maximizing profit and managing risk. It offers basic insight and guidance to help with decision-making, and its solutions are consistent with practical expectations of profitability, diversification, and budgetary discipline even with these simplifications. By focusing on the trade-off between immediate sales and long-term rental returns, our model strikes a balance between real life cases and feasibility.

Overall, our model is effective in highlighting an accurate estimate of the maximum total projected profit and providing us with the optimal number of properties to invest in, sell and rent.

0.7 Solve using JuMP

Code:

```
using JuMP
using HiGHS

C = [592415.15, 2178469.22, 4674196.34]
S = [649287.00, 2335319.00, 4856490.00]
R_pv = [170719.92, 205373.10, 415266.10]
S_pv = [508733.35, 1829783.54, 3805186.99]
max_units = [84, 22, 10]
market_max = [21634, 41536, 5000]
A = 0.02
B = 50_000_000.0

model = Model(HiGHS.Optimizer)

@variable(model, x[1:3] >= 0, Int)
@variable(model, y[1:3] >= 0, Int)
@variable(model, z[1:3] >= 0, Int)

@objective(model, Max, sum(z[i] * (S[i] * (1 - A)) + y[i] * (R_pv[i] + S_pv[i]) for i in 1:3))

@constraint(model, sum(C[i] * x[i] for i in 1:3) <= B)
@constraint(model, [i=1:3], y[i] <= x[i])
@constraint(model, [i=1:3], z[i] <= x[i])
@constraint(model, [i=1:3], y[i] + z[i] <= x[i])
@constraint(model, [i=1:3], x[i] <= min(max_units[i], market_max[i]))
@constraint(model, x[2] * 10 >= 3 * sum(x))
@constraint(model, x[3] * 10 >= 1 * sum(x))
@constraint(model, z[1] * 10 >= x[1])
@constraint(model, z[2] * 5 >= x[2])
@constraint(model, z[3] * 10 >= 3 * x[3])
@constraint(model, y[2] * 5 >= x[2])
@constraint(model, y[3] * 5 >= x[3])
@constraint(model, sum(y) * 10 <= 7 * sum(x))

optimize!(model)

println("\nOptimal Objective Value: ", objective_value(model))
for i in 1:3
    println("x[$i] = ", round(Int, value(x[i])),
            ", y[$i] = ", round(Int, value(y[i])),
            ", z[$i] = ", round(Int, value(z[i])))
end
```

Results for budget = $50,000,000:

```
Solving report
  Status            Optimal
  Primal bound      51662696.23
  Dual bound        51662696.23
  Gap               0% (tolerance: 0.01%)
  P-D integral      0.000480045727701
  Solution status   feasible
                    51662696.23 (objective)
                    0 (bound viol.)
                    8.881784197e-16 (int. viol.)
                    0 (row viol.)
  Timing            0.01 (total)
                    0.00 (presolve)
                    0.00 (solve)
                    0.00 (postsolve)
  Max sub-MIP depth 0
  Nodes             1
  Repair LPs        0 (0 feasible; 0 iterations)
  LP iterations     43 (total)
                    0 (strong br.)
                    20 (separation)
                    10 (heuristics)

Optimal Objective Value: 5.166269623000004e7
x[1] = 16, y[1] = 14, z[1] = 2
x[2] = 10, y[2] = 2, z[2] = 8
x[3] = 4, y[3] = 1, z[3] = 3
```

To test the sensitivity of our model, we test out the model with a different budget of $65,000,000:

For budget= $65,000,000:

```
Solving report
  Status              Optimal
  Primal bound        67068005.92
  Dual bound          67068005.92
  Gap                 0% (tolerance: 0.01%)
  P-D integral        0.000402795012095
  Solution status     feasible
                      67068005.92 (objective)
                      0 (bound viol.)
                      1.7763568394e-15 (int. viol.)
                      0 (row viol.)
  Timing              0.01 (total)
                      0.00 (presolve)
                      0.00 (solve)
                      0.00 (postsolve)
  Max sub-MIP depth 0
  Nodes               1
  Repair LPs          0 (0 feasible; 0 iterations)
  LP iterations       59 (total)
                      0 (strong br.)
                      22 (separation)
                      22 (heuristics)

Optimal Objective Value: 6.706800592e7
x[1] = 22, y[1] = 19, z[1] = 3
x[2] = 13, y[2] = 3, z[2] = 10
x[3] = 5, y[3] = 1, z[3] = 4
```

Sensitivity Analysis:

The following table summarizes the optimal solutions for two budget scenarios ($50 million and $65 million). It reflects the changes in decision variables and objective value based on variations in capital constraint.

Budget ($M)	Profit ($)	x_1	x_2	x_3	y_1	y_2	y_3	z_1	z_2	z_3
50	$51.66M	16	10	4	14	2	1	2	2	3
65	$67.07M	22	13	5	19	2	1	3	2	4

- Increasing B leads to a **proportional expansion** in all x_i, without violating binding constraints on max units or investment share.
- The model allocates additional budget toward **HDB rental units**, reflecting that renting out HDB units and sell them after 5 years has **high Net Present Value to cost ratio**.
- **Condominium and landed units increase marginally**, indicating they are profitable but capital-intensive.

0.8 Appendix

a. Usage of present values

In financial modeling, the **Present Value (PV)** concept is used to account for the time value of money. This principle states that a dollar today is worth more than a dollar in the future due to its potential earning capacity. This is a foundational concept in investment decision-making and is crucial for accurate optimization modeling.

The formula for calculating present value is:

$$PV = \frac{F}{(1+r)^t}$$

where:
PV = present value
F = Future cash flow
r = Discount rate
t = Time in years

Using present value allows us to

- Compare cash flows across different time periods on a consistent basis
- Ensure investment strategies are not biased toward higher but distant future returns
- Make sound financial decisions by weighing current costs against future gains

b. R'_i: Present Value of 5 Years of Rent (Property Type i)

$$R_i' = R_i \times \left(\frac{1}{r} - \frac{1}{r(1+r)^T}\right)$$

R_i : Annual rental income for one unit of property type i
r = Discount rate
T = Total number of years

This formula calculates the **present value of receiving constant annual rent** over 5 years. It is derived from the formula for the present value of an ordinary annuity, adjusted to match our context.
This present value term allows the optimization model to make decisions based on the **true current worth** of future rental income and resale proceeds, ensuring financially sound recommendations.

c. S'_i: Present Value of Resale after 5 Years (Property Type i)

$$R_i' = R_i \times \left(\frac{1}{r} - \frac{1}{r(1+r)^T}\right)$$

R_i : Resale value of one unit of property type i after 5 years
r = Discount rate

This formula discounts the **expected resale value after 5 years** back to its present value. This present value term allows the optimization model to make decisions based on the **true current worth** of future rental income and resale proceeds, ensuring financially sound recommendations.

d. **links & references**

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- URA Rental Market Data: https://www.ura.gov.sg/Corporate/Property/Residential/Renting-Property