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APMTH115: Mathematical Modeling
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How Much of Backgammon is Luck?

Question/Goal

The question we attempted to answer in our project was how much of backgammon is luck versus skill. We wanted to evaluate what was more important in backgammon luck or skill, and at what player skill level does luck matter more than skill. Because backgammon is an interactive two player game that relies on strategy and luck, we intend to answer our question by simulating competition with players of varied skill and games won. Our project goal is to determine how much the dice rolls of a player will affect whether or not they win a game based on how skilled they and the other player are.

Background

Backgammon is a board game that has entertained for over 5,000 years. The objective of the two-player game is to *bear off*, to move all fifteen of one's checkers off the board before the other player, by rolling two dice and moving pieces accordingly. If a player rolls doubles, the player moves their checkers equal to double the amount of the roll, essentially granting them an extra turn. The skill of the game involves hitting the other player's pieces to the bar, restarting any progress their opponent's checker made in bearing off. A player may *hit* the other player's checker if it stands alone at any of the 24 points on the board. Therefore, the game requires a combination of strategy and luck to win.

When researching how to model backgammon, the paper *Optimal Doubling in Backgammon* by Emmett B. Keeler and Joel Spencer modeled the game as a race to move all of the pieces to the end of the board. They concluded that the sum of the pieces' distances from the end is 125. Therefore, barring any interactions, a player would need to roll the dice as many times as necessary to roll 125 and the player to reach 125 first is the winner. We created our model based on this strategy while also manipulating the skill level of the players and including higher dice rolls to account for the interactions between the two players. Because there was not much research done on the significance of skill versus luck in backgammon, we researched their significance in other similar games. With most games based on skill and luck, research shows that the winner of a game between two evenly skilled players is essentially a toss-up; in a 50/50 split, luck is a much greater determinant on the winner of the game. As the difference between skills of the two players increase, luck typically becomes less of a factor as skill more greatly determines the winner of the game. If two amateurs are competing with varying low skill levels, luck is more likely to determine the winner.

Approach/Model

We modeled our game similarly to the simplified run game model from above; we define a player's skill as their *pip count,* the total number of remaining movements needed to bear off. Each player is assigned a skill that is equal to their pip count, and we simulate dice rolls until the player reaches their pip count and wins the game. Since the pip count is the number of moves required for each player to finish the game, a higher pip count and skill level means that the player is less skilled, as it takes them more moves to finish the game. In other words, a professional player will have a *lower* pip count, much like in golf. When a player's piece is moved to the bar, it takes more dice rolls for the player to bear off and they are further behind in the game then they were before. Therefore, the higher pip count accounts for players' pieces being hit and moved to the bar. Our simulated game of backgammon has Player A and Player B switch off rolling two dice back and forth until one of the players reaches their pip count or skill level. We generated two random integers between 1 and 6 to "roll the dice," and tracked the skill, turns and moves for each player. In our simulation we also account for rolling doubles by giving the player twice the amount of the dice roll.

Source of Data

Because we created our own model, we simulate a simplified version of backgammon in MATLAB. We simulated multiple versions of backgammon with varying skill levels and performed 10,000 simulations of each game to ensure enough games to have statistically accurate results. We tally the outcomes of each game to get a proportion of games won by Player A and use averages of the 10,000 simulations to compare the results for different skill levels. These simulations are the extent of our data usage. The first simulations had Player A roll first always and then held Player A's skill constant against multiple skill levels for Player B. Then, we ran simulations with increasing skill level/pip count but held the gap between the skill levels constant at 25 to evaluate at which points of skill level is luck more important. We also ran simulations when either Player A or B rolls first, because the player who goes first is decided by the luck of the die roll, and their likelihood of winning based on who rolls first.

Results

Figure 1 below shows the outcomes of the games as we hold Player A's skill constant at 125, a professional, and increase Player B's pip count. As Player B becomes less skilled, Player A's winning probability quickly approaches 1. Figure 2 shows the same analyses performed on different fixed skill levels for Player A. Both figures serve as a sort of sanity check that our model behaves as expected; as Player B becomes less skilled, Player A wins more, and vice versa.

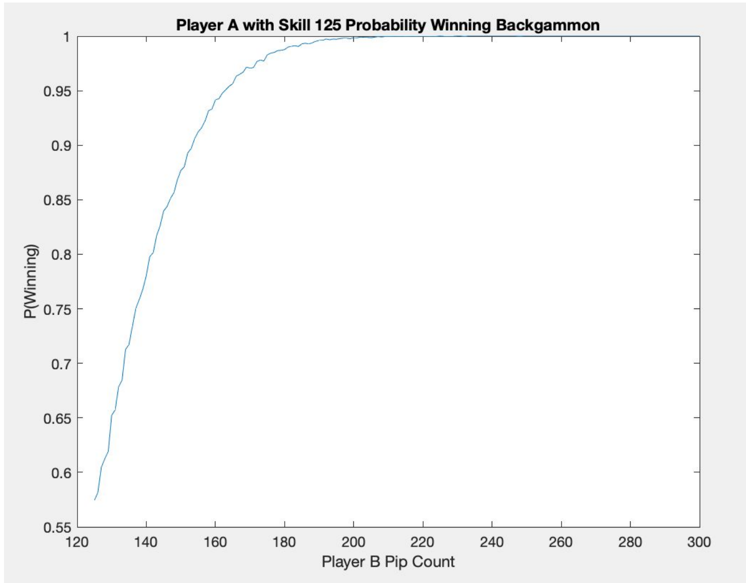

Figure 1: Player A's Winning Probability as a Function of Player B's pip count. A player with a higher pip count is less skilled.

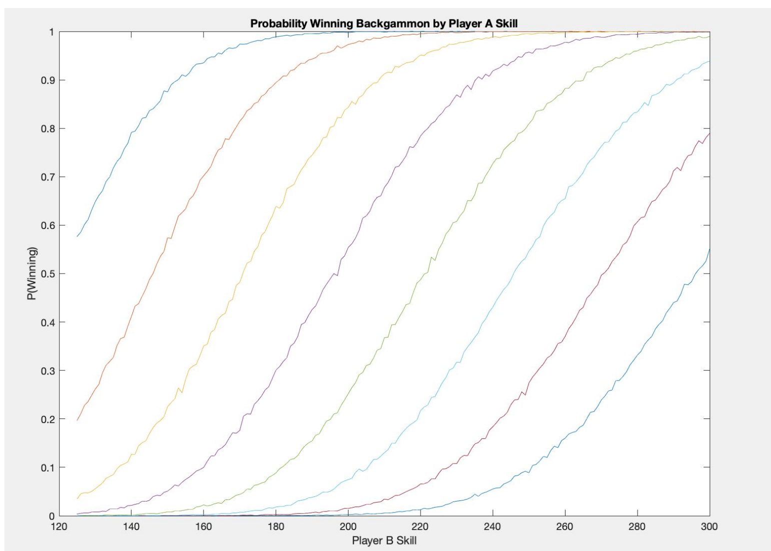

Figure 2: Player A's Winning Probability as a Function of Player B's pip count. A player with a higher pip count is less skilled. Each curve represents a different fixed pip count for Player A. Left to right, the fixed pip counts for Player A are 125, 150, 175, 200, 225, 250, 275, and 300 respectively.

We investigate the functional form that Player A's probability of winning approaches 1 by plotting the log-log and log-linear of 1 minus the win probability. We do this both as a function of the average number of turns Player B needs to win as well as the average number of moves Player B needs to win, calculated at the end of the game. We do this because we want to determine if Player A's probability of winning approaches 1 as a power law or as an exponential. As shown in Figures 3a. and 3c., we do not see a linear decrease on the log-log plot, suggesting that Player A's probability of winning does not approach 1 as a power law. However, on the log-linear plots shown in Figures 3b. and 3d. we see a linear decrease, so we know that Player A’s probability of winning approaches 1 as an exponential.

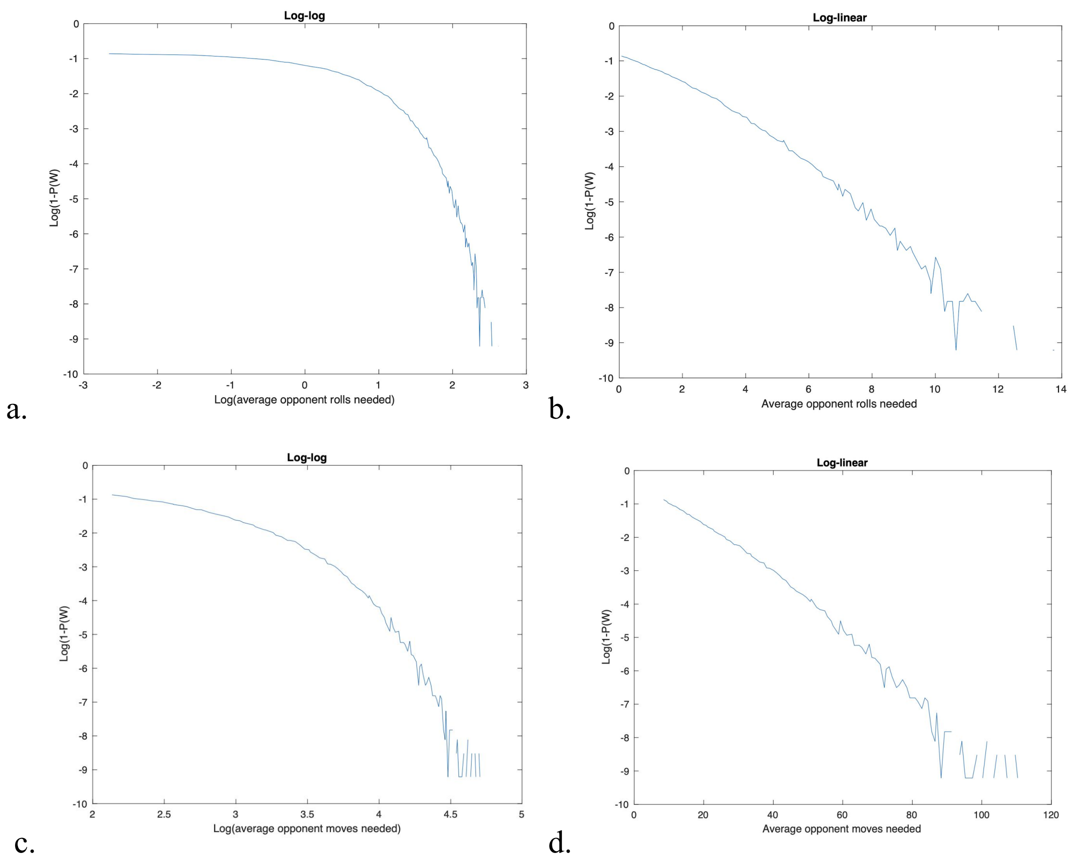

Figure 3: The log-log and log-linear plots to determine how Player A’s probability of winning approaches 1 as Player B’s skill decreases. A. and b. are a function of Player B’s turns or rolls taken, and c. and d. are functions of Player B’s moves needed to win at the end of the game.

Next, we investigated how fixed skill differentials influence the outcome of the game. We set a fixed skill gap of 25 between Player A and Player B and incrementally increased both players’ pip counts, effectively decreasing their skills. As both players become less skilled, with constant skill differentials, Player A’s win probability decreases. Player A’s win probability appears to be approaching 0.5, suggesting that skill differentials are more influential on the outcome of the game when both players are more skilled.

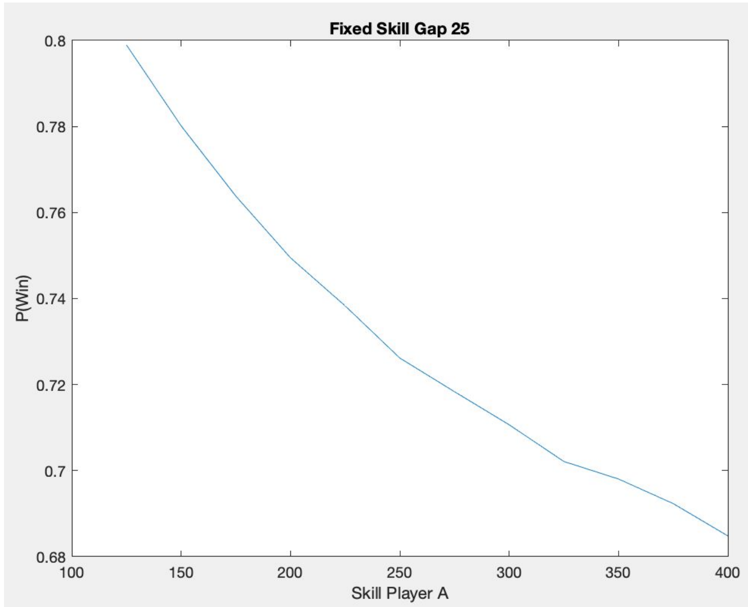

Figure 4: For players with a fixed skill gap of 25, i.e. Player B's pip count is 25 moves higher than Player A, Player A's win probability appears to approach 0.5 as both players decrease in skill level (increase pip count).

For the simulations discussed above, we arbitrarily chose Player A to roll first to simplify the game. However, in the real game of backgammon, a dice roll (luck) entirely determines who rolls first. We wanted to see how our arbitrary decision of choosing Player A to roll first had an effect on the outcome of the game, if any. We kept both players' skills fixed and plotted the win probability distribution as a function of proportion of games player A won. Figure 5 and figures in the appendix show how rolling first has a large impact on the outcome of the game, even as skill levels reach an unlikely extreme of 1000.

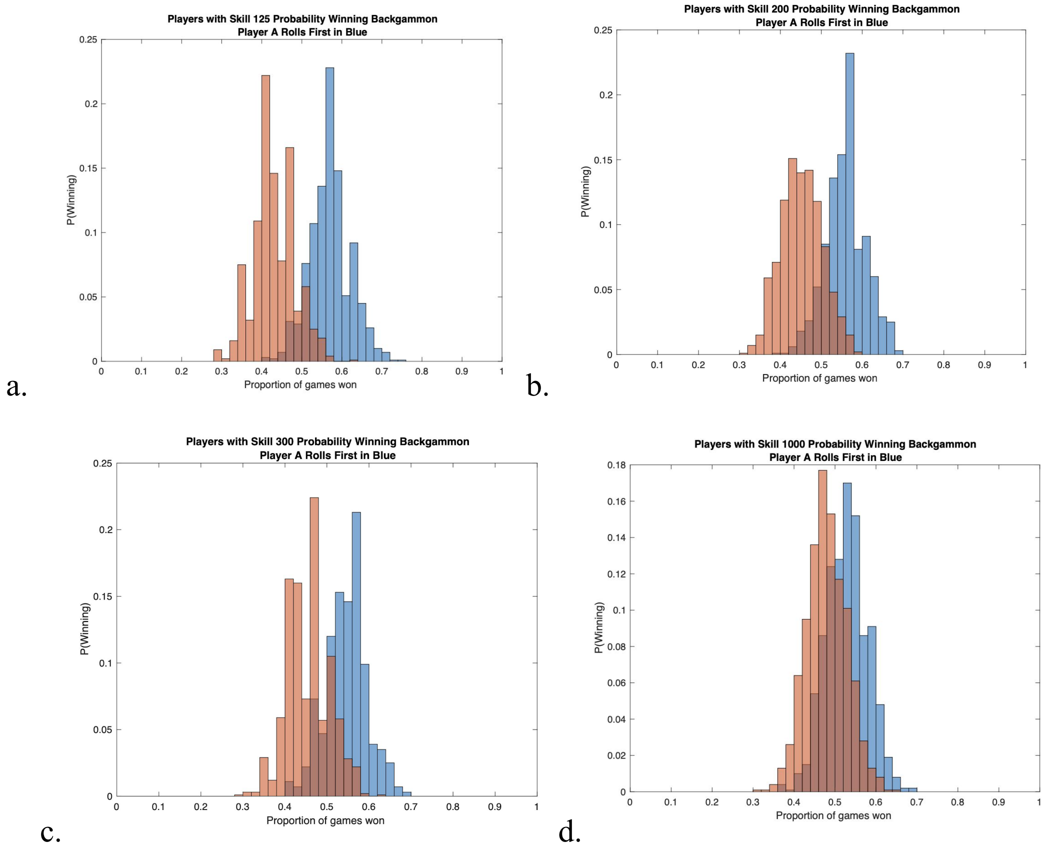

Figure 5: Probability of Player A winning a proportion of games depending on which player rolled first. The red distribution is the win probability when Player B rolls first and the blue is when Player A rolls first. Even as skill levels reach an extreme and unlikely value of 1000, which player rolls first still appears to have a large impact on the game's outcome

To further investigate the impact of rolling first on the outcome of the game, we held both players' skill levels equal and incrementally increased their pip counts to see how the win probability changes as skill decreases. As shown in Figure 6, the player who rolls first seems to have the largest impact on the outcome of the game when the players are more skilled. As players become less skilled, the win probabilities appear to be approaching 1.

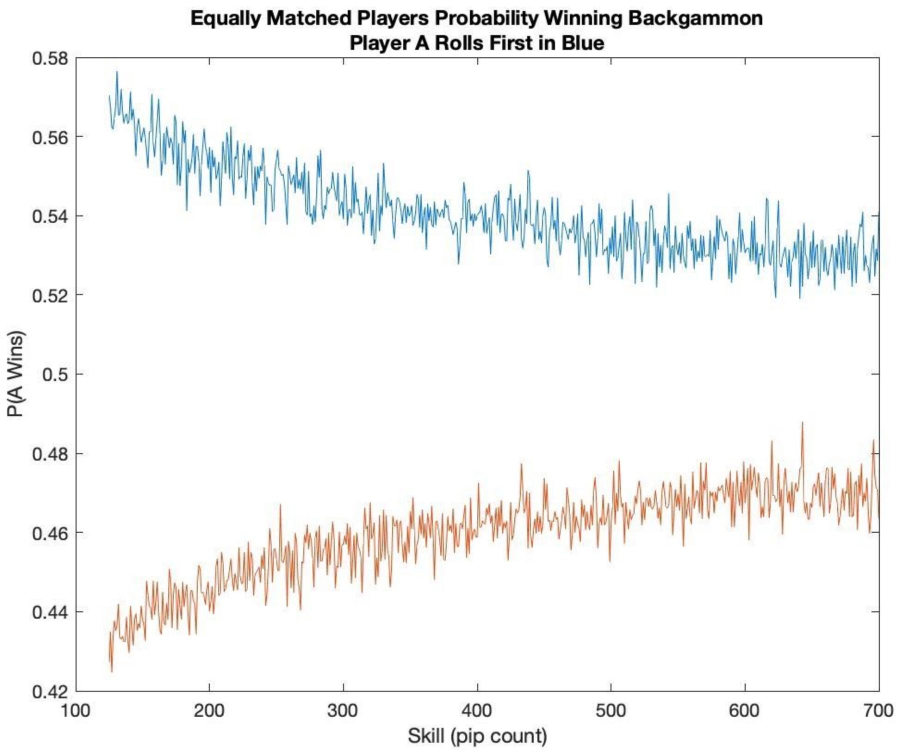

Figure 6: As players become less skilled, pip counts increase, player A's probability of winning approaches 0.5. This graph suggests that who rolls first has a more significant impact on the game when both players are highly skilled.

Conclusion

We might draw three main conclusions from our model, the first being that luck is more influential on the outcome of the game when players are highly skilled, as we saw in Figures 1 and 6. The second being that skill gaps have more of an impact on the outcome of the game when players are more skilled. In other words, the skill differential between two novice players is less noticeable than that same skill differential between two expert players. Thirdly, which player rolls first has a large impact on the game, even as skill levels reach extremes.

However, these conclusions all come with limitations because of our approach. We drastically simplified the game by assigning players' skills using pip counts. Perhaps most importantly, our model does not account for interactions between players. While higher pip counts inherently include a player's pieces getting hit by their opponent and sent back to the bar, this is a very general estimation and does not capture the interactions of *every* game. This approach assumes that players are consistent and eliminates any possibility of an oversight or an accidental mistake while playing. This approach also assumes that players' skill levels are constant. In other words, players do not get more skilled or learn from mistakes in a previous game as our simulations progress. Another limitation is that there are other strategies to backgammon besides the "run" game, or getting your pieces off the board as quickly as possible. A player may intentionally keep a very high pip count by keeping their checkers on their opponent's inner board, called the "back" game, for example. Our model assumes that the players are using the “run” game strategy and move their pieces off the board as quickly as possible. Finally, part of the appeal of backgammon is the aspect of betting and the doubling cube; for simplicity purposes we did not account for this in our model.

Future Directions

We can continue our work on this project by building off of the model what we already have and performing the same analyses but for different situations. For example, in our project we only looked at a fixed skill differential of 25 moves. In an extension of this project we could see how different sizes of skill gaps affect the win probability, if at all. Another example of an extension might include controlling for which player rolls first. In a real game of backgammon, each player rolls one die and the player with the highest roll takes these dice to be their first roll of the game. We could modify the code to behave more closely to that of a real game of backgammon where players roll to see who goes first, and see how this impacts our results.

Another future implication of our project could be to determine the exact moments at which the skill level is too high or the range is too great for luck to matter in backgammon. To improve our experiment we could run a more complicated version of our model that accounts for all the interactions between the two players instead of a simple run game. In our original proposal, we planned to train two computers in backgammon and create equally skilled and skills of different ranges and have them compete against each other. We would then introduce weighted dice to simulate “luck” and evaluate which players when luck is introduced. In our simulations, we have been assuming that the highest dice roll is the best roll for simplicity, however a better model would know the position of the piece and the best possible roll for the position and weight a dice according to the best possible roll. Another model to quantify whether luck or skill is more determinant is to have one computer trained more than the other and give each computer the same number of dice rolls and based on how many times the more skilled computer wins, it should be possible to quantify how important skill is.

Attribution of Individual Effort

Micah: I contributed to earlier research for the project and attempted to understand and code backgammon simulations found online. Also contributed to the Questions/Goal, Background, Approach Model, Data Source and Future Directions segments of the final Paper.

Ali: I wrote the project proposal, MATLAB code, Powerpoint presentation, and the Results/Conclusion segments of the final paper.

References

Levitt, Steven D, and Thomas J Miles. “The Role of Skill Versus Luck in Poker Evidence From the World Series of Poker.” *Journal of Sports Economics*, vol. 15, no. 1, 2014, pp. 31–44.

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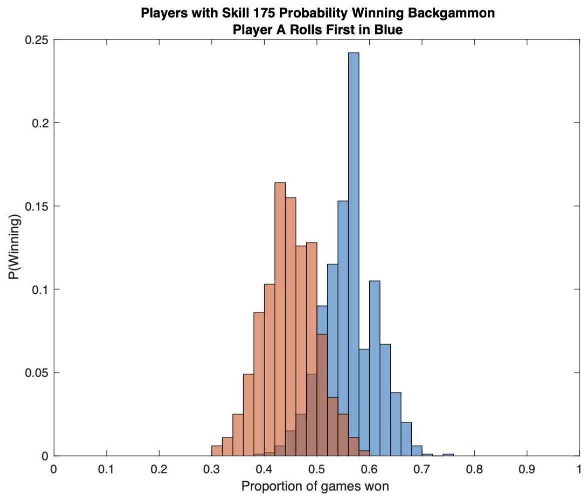
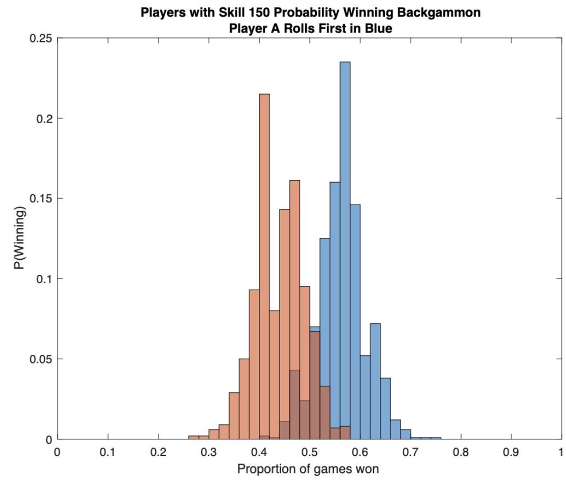
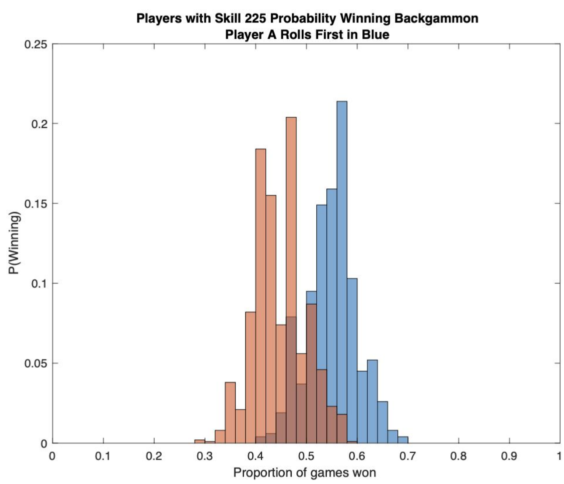
“Appendix:Glossary of Backgammon.” *Wiktionary*, en.wiktionary.org/wiki/Appendix:Glossary_of_backgammon.

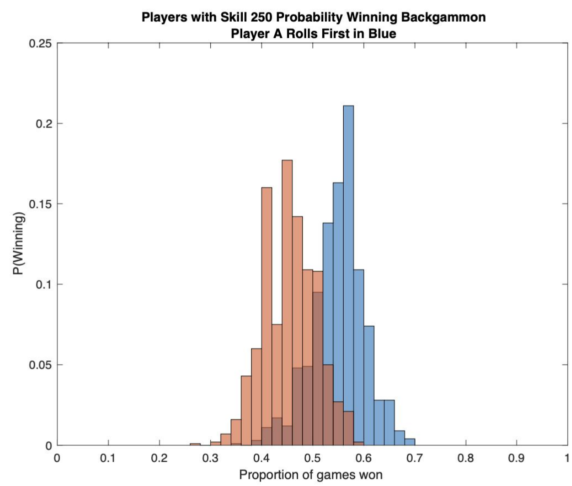
Appendix

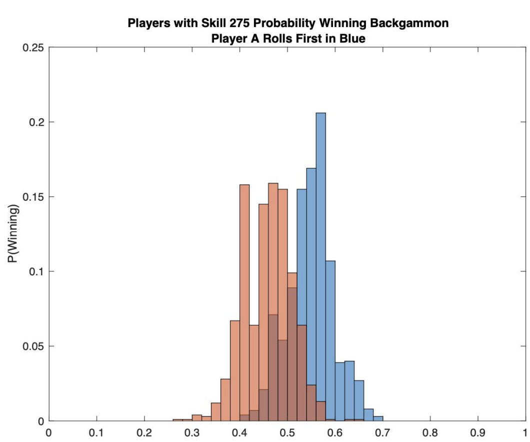
Additional plots to those shown in Figure 5.



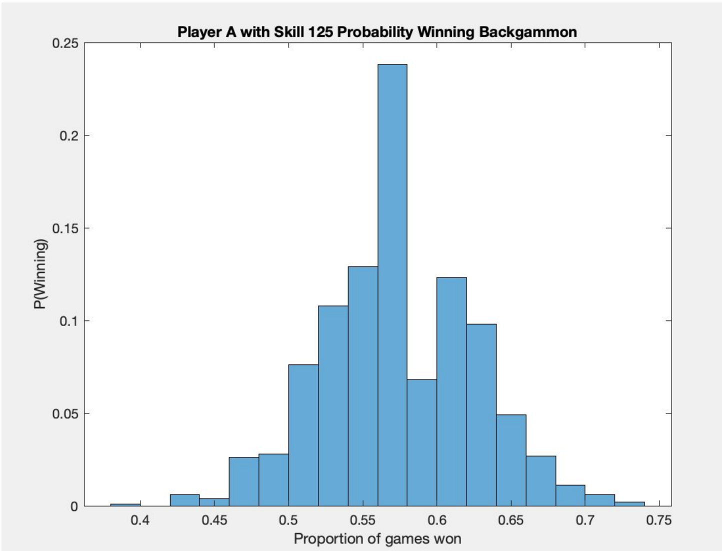








Player A rolls first:



Player B rolls first: