# Composition method

$X \quad Y \qquad 0 < \alpha < 1$

$$Z = \begin{cases} X & \alpha \\ Y & 1-\alpha \end{cases}$$

$$P_Z = \alpha P_X + (1-\alpha) P_Y$$

$$Z = \begin{cases} 1,2,3,5 & 1/10 \\ 8 & 1/10 + 1/4 \\ 4,6 & 1/4 \end{cases}$$

$$\left.\begin{aligned} X &= \text{unif}\{1,\dots,8\} \\ Y &= \text{unif}\{8,4,6\} \end{aligned}\right\} \Rightarrow Z = \begin{cases} X & 1/2 \\ Y & 1/2 \end{cases}$$

# Alias Method

$X \rightarrow$ مقدار متغیر $n$ $\rightarrow$ $P$ $X$ تابع احتمال

$1, \dots, n$

$P_1, \dots, P_n$

$$P = \frac{1}{n-1} \sum_{i=1}^{n-1} Q^{(i)}$$

متغیرها $Q^{(i)}$ حتما binary

Ex. $n=4$

P: $P_1 = \frac{7}{16}$ $P_2 = \frac{1}{4}$ $P_3 = \frac{1}{8}$ $P_4 = \frac{3}{16}$

$n-1 = 3$

$P = \frac{1}{3}(Q_1 + Q_2 + Q_3)$

در گام اول با مقدار شروع می کنیم که احتمال آن کمتر از $\frac{1}{n-1}$ باشد (به دلخواه)

$i=3 \rightarrow Q_1 = \begin{cases} 3 & \alpha_1 = \frac{3}{8} \Leftarrow \alpha_1 \times \frac{1}{3} = \frac{1}{8} \\ 1 & 1-\alpha_1 = \frac{5}{8} \end{cases}$

مقدار دوم $Q_1$ باید به گونه ای باشد که جمع احتمال آن و احتمال گام اول حداقل $\frac{1}{n-1}$ شود

$P$: $P_1 = \frac{7}{16}$ $P_2 = \frac{1}{4}$ $P_3 = \frac{1}{8}$ $P_4 = \frac{3}{16}$

$P = \frac{1}{3}(Q_1 + Q_2 + Q_3)$

$Q_1 : \begin{cases} 3 & \frac{3}{8} \\ 1 & \frac{5}{8} \end{cases}$

$$\left.\begin{aligned} P &= \tfrac{7}{16}\delta_1 + \tfrac{1}{4}\delta_2 + \tfrac{1}{8}\delta_3 + \tfrac{3}{16}\delta_4 \\ Q_1 &= \tfrac{3}{8}\delta_3 + \tfrac{5}{8}\delta_1 \end{aligned}\right\} \Rightarrow P - \tfrac{1}{3}Q_1 = \frac{3\times 7 - 2\times 5}{48}\delta_1 + \tfrac{1}{4}\delta_2 + \tfrac{3}{16}\delta_4$$

$$\tfrac{11}{48}\delta_1 + \tfrac{1}{4}\delta_2 + \tfrac{3}{16}\delta_4 - \tfrac{1}{3}(Q_2 + Q_3)$$

$Q_2 : \begin{cases} 2 & \alpha_2 = \frac{3}{4} \\ 4 & 1-\alpha_2 = \frac{1}{4} \end{cases}$ $\Leftarrow \frac{1}{4} = \frac{1}{3}\times\alpha_2$

$$\frac{11}{48}\delta_1 + \frac{1}{4}\delta_2 + \frac{3}{16}\delta_3 = \frac{1}{3}(Q_2 + Q_3)$$

$$Q_2 = \begin{cases} 2 & \alpha_2 = 3/4 \quad \Leftarrow 1/4 = 1/3 \times \alpha_2 \\ 4 & 1-\alpha_2 = 1/4 \end{cases}$$

$$\frac{11}{48}\delta_1 + \frac{1}{4}\delta_2 + \frac{3}{16}\delta_4 = \frac{1}{3}\left(\frac{3}{4}\delta_2 + \frac{1}{4}\delta_4 + Q_3\right)$$

$$\frac{11}{48}\delta_1 + \frac{9-4}{48}\delta_4 = \frac{1}{3}Q_3 \Rightarrow Q_3 = \frac{11}{16}\delta_1 + \frac{5}{16}\delta_4$$

تمرین Alias method : حالت کلی آن را بنویسید.