

$$L = \text{tr}(X) - \log \det X + v^T (XS - y)$$

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$$\nabla L = I - X^{-1} + \frac{1}{2} (v^{\odot} S^T + S^{\odot} v^T) = 0$$

$$\Rightarrow X^{-1} = I + \frac{1}{2} (v^{\odot} S^T + S^{\odot} v^T), \quad X \succ 0, \quad XS = y$$

$$S = X^{-1} y = y + \frac{1}{2} (v^{\odot} \underbrace{S^T y}_1 + S^{\odot} v^T y) = y + \frac{1}{2} (v + (v^T y) S)$$

$$\Rightarrow \underbrace{y^T S}_1 = y^T y + \frac{1}{2} (y^T v + v^T y) \Rightarrow v^T y = 1 - y^T y$$

$$\Rightarrow \cancel{v} = -2y + (1 + y^T y) S$$

$$\Rightarrow X^{-1} = I + \frac{1}{2} [-2yS^T - 2Sy^T + 2(1 + y^T y)SS^T]$$

$$= I + (1 + y^T y)SS^T - yS^T - Sy^T$$

$$X^{-1} X^* = I \Rightarrow X^* = I + yy^T - \frac{1}{S^T S} SS^T \quad \checkmark$$