دانشکده‌ی علوم ریاضی

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مسأله ۱

$Var(X) = E[X^2] - E^2[X] = \int_{-2+\theta}^{2+\theta} \frac{1}{4}x^2dx - \int_{-2+\theta}^{2+\theta} \frac{1}{4}xdx =$
$\frac{1}{12}((\theta+2)^3 - (\theta-2)^3) - \frac{1}{8}((\theta+2)^2 - (\theta-2)^2) = \frac{4}{3}$
$z = \sum_{i=1}^{n} x_i/n \Rightarrow Var(z) = \frac{1}{n}Var(x) = \frac{4}{3n}$
$\Rightarrow \theta = 30 \pm 1.96\sqrt{\frac{4}{300}} = 30 \pm 0.23\ ,\ with\ 95\%\ confidence$

مسأله ۲

$Z = \sum_{i=1}^{n} x_i/n$
$E[Z-\theta] = E[Z] - \theta = \frac{1}{n}E[\sum_i x_i] - \theta = \frac{1}{n}n\theta - \theta = 0$
$Var(X) = \theta^2 \Rightarrow \sigma_x = \theta$
$SE(Z) = \frac{\sigma_x}{\sqrt{n}} = \frac{\theta}{\sqrt{n}} = \frac{1}{n\sqrt{n}}\sum_{i=1}^{n} x_i$

مسأله‌ی ۳

الف) راه اول:

$\int_{z=-x}^{x} \frac{1}{\sqrt{2\pi}}e^{\frac{-z^2}{2}}dz = 0.9 \Rightarrow x = 1.64$
$Var(X) = \theta^2 \Rightarrow \sigma_x = \theta \Rightarrow Var(z = \sum_i x_i/n) = \frac{1}{n}\theta^2 \Rightarrow \sigma_z = \frac{1}{\sqrt{n}}\theta$
$\theta = \frac{\sum_{i=1}^{n} x_i}{n} \pm 1.64\frac{\sum_{i=1}^{n} x_i}{n\sqrt{n}}\ ,\ with\ 90\%\ confidence$

راه دوم:

First let us prove that if X follows an exponential distribution with parameter 2 λ, then Y = 2λX follows an exponential distribution with parameter 1/2, i.e. χ^2. The density function for X is f(x|λ) = $\lambda e^{-\lambda x}$ if x > 0 and 0 otherwise. It is easy to see the density function for Y is $g(y) = \frac{1}{2}e^{-y/2}$for y > 0, and g(y) = 0 otherwise. Therefore Y

has an exponential distribution with parameter 1/2, i.e. chi-square distribution with degree of freedom 2.
Now, let us first find a pivot, define

$$h(X_1, X_2, .., X_n, \lambda) = 2\lambda \sum X_i = \sum Y_i$$

and each $Y_i = 2\lambda X_i$ follows χ^2 distribution, and they are independent. Therefore h follows χ^2_n distribution. Let $\chi^2_n(\alpha/2)$ and $\chi^2_n(1-\alpha/2)$ be the $(\alpha/2) \times 100$-th and $(1-\alpha/2) \times 100$-th percentiles, respectively. Then

$$P(\chi^2_{2n}(\alpha/2) \leq 2\lambda \sum X_i \leq \chi^2_{2n}(1-\alpha/2)) = 1-\alpha$$

$$P(\frac{2\sum X_i}{\chi^2_{2n}(1-\alpha/2)} \leq \theta \leq \frac{2\sum X_i}{\chi^2_{2n}(\alpha/2)}) = 1-\alpha$$

ب)

$\theta \in (l(x), \infty),\ with\ 95\%\ confidence$
$\alpha = 0.95$
$\alpha = pr(l(x) < \theta) = pr(x < l^{-1}(\theta)) = \frac{l^{-1}(\theta)}{\theta} \Rightarrow l^{-1}(\theta) = \alpha\theta \Rightarrow l(x) = \frac{x}{\alpha} = \frac{1}{0.95}x$
$\Rightarrow \theta \geq 1.052x,\ with\ 95\%\ confidence$