



## Minimisation of Maximum Lateness in Multiple Additive Manufacturing Machine Scheduling Problem: A MILP Model and GA Approach

Journal:	<i>Production Planning &amp; Control</i>
Manuscript ID	TPPC-2018-0399
Manuscript Type:	Research paper for Regular Issue
Date Submitted by the Author:	19-Sep-2018
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Keywords:	additive manufacturing, production planning, scheduling, MILP, genetic algorithm

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## Minimisation of Maximum Lateness in Multiple Additive Manufacturing

### Machine Scheduling Problem: A MILP Model and GA Approach

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#### Abstract

Additive manufacturing is now increasingly utilised in industry especially when the batches are small but the customisation is high. The decisions on the allocation of orders to the machines used for additive manufacturing is important in terms of the completion time related measures (e.g. makespan, flow time) and delivery date related performance measures (e.g. total lateness, maximum lateness, number of tardy jobs), as well as the production cost. This research focuses on the production scheduling problem in a multiple additive manufacturing/3D printing machine environment. Parts have release dates and due dates. A genetic algorithm approach is developed to minimise the maximum lateness where there is more than one additive manufacturing machine with different capacity and speed specifications. The parts are allocated to these parallel additive manufacturing machines in batches considering their release dates, due dates as well as the resource constraints. The problem is defined and a mixed-integer linear programming model is developed. A genetic algorithm approach is also developed to solve the problem and its running mechanism is illustrated through a numerical example. Some improvements are also made to increase the performance of the proposed approach. The methodology developed in this paper can easily be adopted by practitioners to generate short term production schedules for additive manufacturing machines.

**Keywords:** Additive manufacturing, production planning, scheduling, MILP, genetic algorithm, 3D printing.

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#### Abstract

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#### 1. Introduction

Additive manufacturing (AM), also known as 3D printing (3DP), is used to describe the technologies that build 3D objects by adding layer-upon-layer of material. It is now being increasingly utilised in industries requiring small-batch but high-customisation manufacturing. Among its variants, the metallic powder-bed fusion processes (such as Selective Laser Melting and Electron Beam Melting)

1  
2 have been widely adopted as an advanced direct rapid manufacturing method particularly in aerospace  
3 and orthopaedic implant sectors (Calignano et al., 2017; Schmidt et al., 2017).  
4  
5

6 The production with metallic powder-bed fusion process is job-based where the machine can handle  
7 one job at a time and the job can consist of a batch of parts which will be processed simultaneously  
8 (Kucukkoc et al., 2016). Both the production cost and the production time of an AM/3DP job  
9 dynamically depend on the combination of parts allocated to this job (Li et al., 2017). Therefore, the  
10 decision on the allocation of parts to the machines is crucial for the scheduling in AM/3DP production,  
11 while the uncertainties caused by variations in the production cost and time make it more challenging  
12 in decision-making. This is especially important when the delivery time related performance measures  
13 and cost-based objectives must be considered together.  
14  
15

16 The literature on the additive manufacturing technology is majorly focused on the process itself and  
17 the cost structures (Calignano et al., 2017), which are out of the scope of this paper. The study belongs  
18 to Li et al. (2017) aims to minimise the average cost per volume of material through a mathematical  
19 model and two basic heuristics, namely best-fit and adapted best-fit. The model proposed by  
20 Kucukkoc et al. 2016 aimed to maximise the area utilisation of AM/3DP machines. There is no work  
21 addressing the lateness of parts concerned to the completion times and due dates. Also, release dates of  
22 parts are also incorporated in that paper.  
23  
24

25 The considered production planning problem of AM/3DP machines in this paper is a kind of batch  
26 scheduling problem (Sule and Saxena, 2007) in such an environment with multiple machines.  
27 However, it is significantly important to note that the production time of jobs in AM/3DP production  
28 planning problem changes based on the total volume and maximum height of parts assigned to the job.  
29 The decision maker needs to make a decision based on the allocation of parts to jobs on machines with  
30 different specifications, e.g. speed, maximum area, maximum height, the time needed to set-up and  
31 cleaning etc. In that sense, this paper introduces the production planning of AM/3DP problem with the  
32 aim of minimising maximum lateness. The paper also contributes to the literature considering the part  
33 release dates. So that, a job containing a set of parts to be produced cannot start before the release date  
34 of any of those parts. Also, when a job consisting a set of parts is started, it is not possible to take out  
35  
36

any of those parts until all parts finish. The first MILP model and the genetic algorithm (GA) approach, whose parameters have been tuned with preliminary tests, is proposed to solve the problem. Note that the detailed 2D or 3D nesting problem is not considered in this research.

This paper is organised as follows. The problem is defined and modelled mathematically via a mixed-integer linear programming (MILP) model in Section 2. The proposed GA is described illustratively through examples in Section 3. A medium sized numerical example consisting of three machines and eighteen parts are solved using the proposed approach in Section 4 and the paper is concluded in Section 5 with key research results and some future research directions.

## 2. Problem Statement

### 2.1. Definition of the Problem

AM/3DP problem aims to decide the allocation of part orders, where a part order (or part, shortly) is represented by  $p_i$  ( $i = 1, 2, \dots, n_i$  and  $i \in I$ ), received from customers to job batches, represented by  $b_j$  ( $j = 1, 2, \dots, n_j$  and  $j \in J$ ). Jobs are utilised on more than one AM/3DP machine, represented by  $k_m$  ( $m = 1, 2, \dots, n_m$  and  $m \in M$ ) with different specifications. Parts have different specifications, i.e. height ( $h_i$ ), area ( $a_i$ ), volume ( $v_i$ ), release date ( $r_i$ ) and due date ( $d_i$ ). Machines also have maximum supported production area ( $MA_m$ ) and height ( $MH_m$ ) specifications. Thus, it is required to ensure that the height of parts assigned to a machine must be smaller or equal to the maximum height supported by that machine. It is also necessary to ensure that the total area of parts assigned to a machine cannot exceed the production area supported by that machine. The aim is to minimise the maximum lateness. The lateness of a part ( $L_i$ ) is the time difference between the completion time of a part ( $\gamma_{mji}$ ) and its due date. As seen, a job is considered to be late if it is completed later than its due date.

$\gamma_{mji}$  is characterised by the completion time of the job ( $CT_{mj}$ ) in which the part is allocated.  $CT_{mj}$  is calculated accumulating the earliest start time of the job ( $\sigma_{mj}$ ) and its production time ( $\delta_{mj}$ ) as seen in Equation (1).

$$CT_{mj} = \sigma_{mj} + \delta_{mj} \quad (1)$$

where,  $\sigma_{mj} = \max\{CT_{m(j-1)}, \max_{i \in I_{mj}}\{r_i\}\}$ . The first expression ( $CT_{m(j-1)}$ ) here corresponds to the completion time of the previous job on the same machine. If  $b_j$  is the first job ( $j = 1$ ) on  $k_m$ ,  $CT_{m(j-1)}$  is considered to be zero as there is no job scheduled earlier than  $b_j$ .  $I_{mj}$  is the set of jobs allocated to  $b_j$  on  $k_m$ . This denotes that  $\sigma_{mj} \geq r_i$  for all  $i \in I_{mj}$ . So that a job cannot start before the release date of any part to be produced in that job.  $\delta_{mj}$  is calculated using Equation (2):

$$\delta_{mj} = SET_m + VT_m \cdot \sum_{i \in I_{mj}} v_i + HT_m \cdot \max_{i \in I_{mj}}\{h_i\} \quad (2)$$

where,  $SET_m$  is the set-up time for  $k_m$ ,  $VT_m$  is the time needed to release per volume material for  $k_m$ , and  $HT_m$  is the accumulated interval time per unit height for  $k_m$ . Thus, the production time of a job basis on the total volume and maximum height of parts assigned to that job as well as the set-up time of the machine on which the job is scheduled.

One of the basic assumptions of the problem is that, a job cannot stop until it is fully completed. Therefore, even a small and short part is completed before the completion of the whole job, it cannot be taken out until the job completely finishes. Also, it is not possible to add a part into a job after it starts. Another assumption is that all parts must be assigned to exactly one job. It is not possible to split parts or jobs on to more than one AM machine. It is also necessary to indicate that all AM machines work in parallel, independently from other(s). They may have different speed and require different amount of time for set-up.

## 2.2. Mathematical Model

This section provides a MILP model for the multiple additive manufacturing machine scheduling problem. The aim of the model is to minimise the maximum lateness of the parts, i.e. the positive difference between the completion time and due date.

The notation used in the mathematical model is presented as follows:

$n_i$  : Maximum number of parts

$I$  : Set of parts

$i$  : Part index, where  $i = 1, 2, \dots, n_i$  and  $i \in I$

1  
2        $p_i$      : Part  $i$   
3        $nj$      : Maximum number of jobs  
4        $J$      : Set of jobs  
5        $j$      : Job index, where  $j = 1, 2, \dots, nj$  and  $j \in J$   
6        $b_j$      : Job  $j$   
7  
8        $nm$      : Maximum number of machines  
9  
10       $M$      : Set of machines  
11  
12       $m$      : Machine index, where  $m = 1, 2, \dots, nm$  and  $m \in M$   
13  
14       $k_m$      : Machine  $m$   
15  
16       $h_i$      : The height of  $p_i$   
17  
18       $a_i$      : The area of  $p_i$   
19  
20       $v_i$      : The volume of  $p_i$   
21  
22       $r_i$      : The release date of  $p_i$   
23  
24       $d_i$      : The due date of  $p_i$   
25  
26       $MA_m$      : The maximum supported production area of  $k_m$   
27  
28       $MH_m$      : The maximum supported height of  $k_m$   
29  
30       $CT_i$      : The completion time of a  $p_i$   
31  
32       $d_i$      : The due date of a  $p_i$   
33  
34       $L_i$      : The lateness of  $p_i$   
35  
36       $\sigma_{mj}$      : The earliest start time of  $b_j$  on  $k_m$   
37  
38       $\delta_{mj}$      : The production time of  $b_j$  on  $k_m$   
39  
40       $CT_{mj}$      : The completion time of  $b_j$  on  $k_m$   
41  
42       $\gamma_{mji}$      : The completion time of  $p_i$  assigned to  $b_j$  on  $k_m$   
43  
44       $SET_m$      : The set-up time for  $k_m$   
45  
46       $VT_m$      : The time needed to release per volume material for  $k_m$   
47  
48       $HT_m$      : The accumulated interval time per unit height for  $k_m$   
49       $maxh_{mj}$ : The maximum height of the parts assigned to  $b_j$  on  $k_m$   
50  
51       $X_{mji}$ : 1, if  $p_i$  is assigned to  $b_j$  on  $k_m$ ; 0, otherwise  
52  
53       $Z_{mj}$ : 1, if at least one job is assigned to  $b_j$  on  $k_m$ ; 0, otherwise

54      The objective function and constraints of the MILP model are presented as follows.  
55  
56  
57  
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59  
60

3  
4  
*Objective Function*

5  
6  
Minimize  $\max_{i \in I} \{L_i\}$  (3)  
7  
8  
Subject to  
9  
10  
$$\sum_{m=1}^{nm} \sum_{j=1}^{nj} X_{mji} = 1, \quad \forall i \in I \quad (4)$$
  
11  
12  
$$\sum_{i=1}^{ni} X_{mji} a_i \leq MA_m, \quad \forall m \in M, j \in J \quad (5)$$
  
13  
14  
15  
$$X_{mji} h_i \leq MH_m, \quad \forall m \in M, j \in J, i \in I \quad (6)$$
  
16  
17  
$$CT_{mj} \geq \sigma_{mj} + \delta_{mj}, \quad \forall m \in M, j \in J \quad (7)$$
  
18  
19  
$$\sigma_{mj} \geq CT_{m(j-1)}, \quad \forall m \in M, j \in J \quad (8)$$
  
20  
21  
$$\sigma_{mj} \geq r_i X_{mji}, \quad \forall m \in M, j \in J, i \in I \quad (9)$$
  
22  
23  
$$\delta_{mj} \geq SET_m Z_{mj} + VT_m \sum_{i=1}^{ni} X_{mji} v_i + HT_m \max h_{mj}, \quad \forall m \in M, j \in J, i \in I \quad (10)$$
  
24  
25  
26  
$$Z_{mj} \geq X_{mji}, \quad \forall m \in M, j \in J, i \in I \quad (11)$$
  
27  
28  
$$\max h_{mj} \geq h_i X_{mji}, \quad \forall m \in M, j \in J, i \in I \quad (12)$$
  
29  
30  
$$\gamma_{mji} \geq CT_{mj}, \quad \forall m \in M, j \in J, i \in I \quad (13)$$
  
31  
32  
$$L_i \geq \gamma_{mji} - d_i, \quad \forall m \in M, j \in J, i \in I \quad (14)$$
  
33  
34  
$$X_{mji}, Z_{mj} \in \{0,1\} \quad (15)$$
  
35  
36  
The objective function represented with Equation (3) aims to minimize the maximum lateness of the  
37 parts. Equation (4) ensures that every part is assigned to a job on a machine exactly once. Equations  
38 (5) and (6) satisfy the capacity constraints (i.e. total area and maximum height supported by the  
39 machines). Equation (7) helps to calculate the completion times of jobs on machines. Specifically, the  
40 completion time of a job on a machine must be higher than or equal to the sum of the earliest start time  
41 of that job and its production time. Equation (8) ensures that the jobs can only start after the  
42 completion of the previous job on the same machine while Equation (9) guarantees that the jobs can  
43 start after the release date of parts assigned to that job. Equation (10) calculates the production time of  
44 the jobs (which is used in Equation (7) as well). Equation (11) confirms that a job is utilised only if at  
45 least one job is assigned to that job. The maximum height of the parts in a job is calculated using  
46 Equation (12) and the completion times of the jobs are determined using Equation (13). The lateness  
47  
48  
49  
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54  
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59  
60  
Equation (14) and the lateness of the jobs are determined using Equation (15). The lateness

1  
2 of a part is calculated via Equation (14) and finally the sign constraints are represented in Equation  
3  
4 (15).  
5  
6

7 **3. The Proposed GA**  
8

9  
10 GA is a well-known nature-based optimisation technique inspired by the natural selection process in  
11 the nature (Goldberg and Holland, 1988). GA was selected in this research due to its successful  
12 implementations on many combinatorial optimisation problems from manufacturing to transportation  
13 and design. It is a powerful method for solving sophisticated problems, and has been applied to many  
14 scheduling problems, see for example Cheng et al. (1996) for a survey of job-shop scheduling  
15 problems using genetic algorithms; Gonçalves et al. (2005) and Pezzella (2008) for GAs developed for  
16 job-shop and flexible job-shop scheduling problems; Kucukkoc and Zhang (2015) and Kucukkoc and  
17 Zhang (2016) for GA and hybrid GA algorithms for assembly line balancing problems; and Woo et al.  
18 (2017) for GA developed to solve parallel machine scheduling problem.  
19  
20

21 The general outline of the proposed GA is depicted in Figure 1. The algorithm starts with generating  
22 *popSize* number of chromosomes to form the initial population. Each chromosome is made up with a  
23 randomly permuted string of numbers corresponding to part numbers. Therefore, the length of the  
24 chromosome is equal to the number of parts. All individuals in the population are evaluated one-by-  
25 one using the procedure which will be given in Section 3.1. Genetic operators are applied to the  
26 chromosomes selected through tournament selection, with a tournament size of *popSize*/6. After the  
27 application of the genetic operators, the new generation is formed and genetic operators are applied  
28 again. This cycle continues until the maximum number of iterations (*maxIter*) is exceeded with no  
29 improvement.  
30  
31

32 **3.1. Decoding Procedure**  
33

34 The procedure used when evaluating the chromosomes is shown in Figure 2. As seen in the figure,  
35 chromosomes are decoded allocating parts to the machines in the order of their appearance on the  
36 chromosome. The decoding procedure starts with the first job on the first machine and assigns as  
37 many parts as possible starting from the first gene on the chromosome while the machine is high  
38  
39

enough to produce the part and the remaining production area ( $RA_{mj}$ ) is large enough. If any part is not assignable to a job on a specific machine, it is skipped and considered for the next job for the next machine. This procedure is continued until all parts have been assigned to a job.

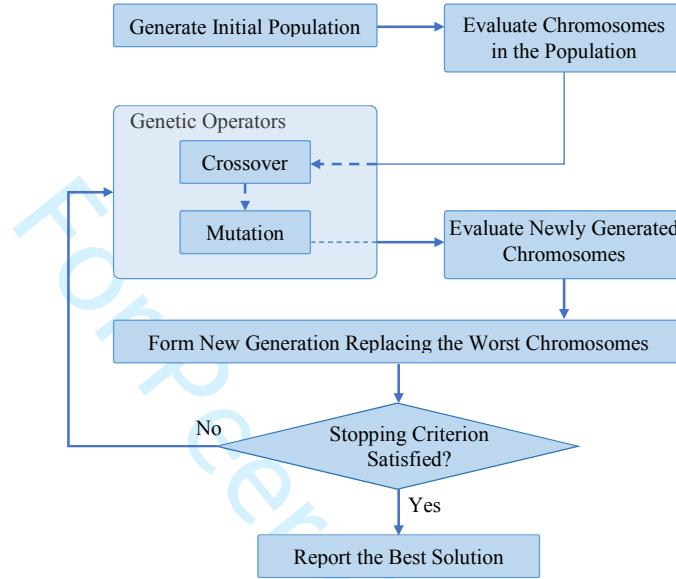


Figure 1. The general outline of the proposed algorithm

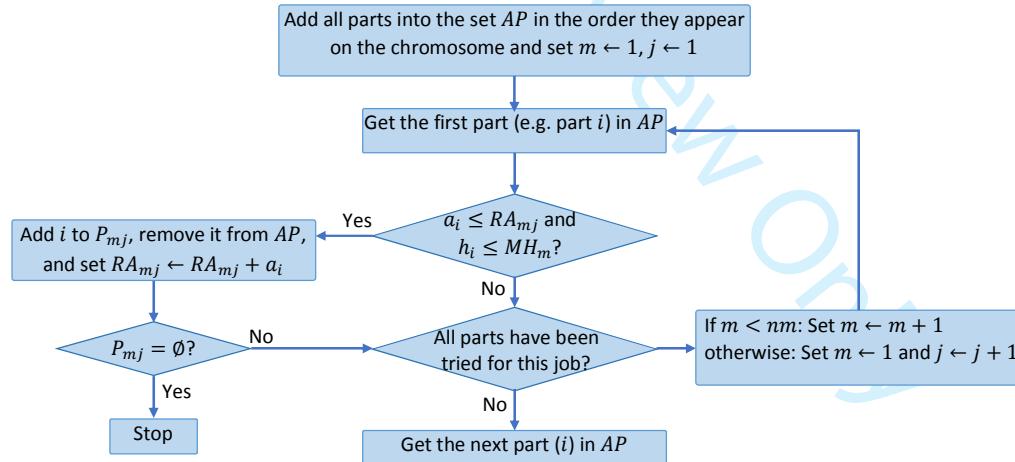


Figure 2. The flow chart of the decoding procedure

Let us assume an AM/3DP problem with two machines and ten parts, for which the detailed specifications are given in Table 1 and Table 2. Figure 3 presents a chromosome sample and illustrates its decomposition based on the decoding procedure considering the data for the numerical example given in Table 1 and Table 2. As seen in the figure, after the allocation of  $p_3$ ,  $p_6$  and  $p_1$  to job 1 on

machine 1 (i.e.  $P_{11} = \{3,6,1\}$ ), none of the other parts can be assigned due to the insufficient remaining production area. Therefore, the machine is changed and  $p_4$ ,  $p_2$  and  $p_5$  are assigned to the first job on machine 2. As the remaining production area ( $326.9 \text{ cm}^2$ ) is not large enough to assign  $p_8$ , it is skipped and  $p_7$ ,  $p_{10}$  and  $p_9$  are also assigned to the same job. Upon filling up the first two jobs on both of the machines, a new job is opened on the first machine again and  $p_8$  is assigned. Please see Figure 4 for the steps of the decoding procedure in details. The  $\sigma_{mj}$ ,  $\delta_{mj}$  and  $CT_{mj}$  values of the jobs are also calculated and presented in the same figure.

Table 1. Specifications of the AM machines

Parameters	$k_1$	$k_2$
$VT_m$ , the time consumption to form per unit volume	0.030864 (hour/cm <sup>3</sup> )	0.030864 (hour/cm <sup>3</sup> )
$HT_m$ , the accumulated time per unit height	1 (hour/cm)	1 (hour/cm)
$SET_m$ , the time consumption for setting up a new job	2 (hour)	1 (hour)
$MH_m$ , the maximum height supported	32.5 (cm)	32.5 (cm)
$MA_m$ , the maximum production area supported	625 (cm <sup>2</sup> )	625 (cm <sup>2</sup> )

Table 2. Part specifications

Part Index ( $i$ )	Height-cm ( $h_i$ )	Area-cm <sup>2</sup> ( $a_i$ )	Volume-cm <sup>3</sup> ( $v_i$ )	Release Date-hr ( $r_i$ )	Due Date-hr ( $d_i$ )
1	16.7	300.8	1573.8	6.3	305.9
2	8.8	152.8	421.5	7.3	282.2
3	20.3	19.5	147.8	9.8	378.3
4	7.4	84.2	285.2	20.9	214.7
5	27.3	61.1	583.3	36.5	149.0
6	25.8	299.3	3282.5	51.5	211.6
7	14.5	148.7	1265.5	56.3	240.4
8	3.5	376.4	723.3	69.9	576.2
9	20.4	20.5	278.5	75.0	330.9
10	23.3	91.1	1051.8	86.0	388.0

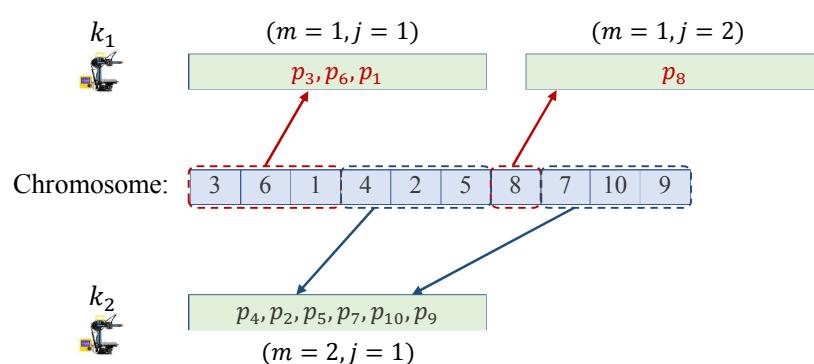


Figure 3. A chromosome sample and its decoding procedure

$(m, j)$	$CT_{m(j-1)}$	Remaining Area ( $RA_{mj}$ )	Considered Part	$h_i$ $cm$	$a_i$ $cm^2$	Fits	Assigned Parts	Total Volume $cm^3$	
(1,1)	0	625.0	3	20.3	19.5	Yes	{3}	147.8	
		605.5	6	25.8	299.3	Yes	{3,6}	3430.3	
		306.2	1	16.7	300.8	Yes	{3,6,1}	5004.1	
		5.4	4	7.4	84.2	No	{3,6,1}	5004.1	
		5.4	2	8.8	152.8	No	{3,6,1}	5004.1	
		5.4	5	27.3	61.1	No	{3,6,1}	5004.1	
		5.4	8	3.5	376.4	No	{3,6,1}	5004.1	
		5.4	7	14.5	148.7	No	{3,6,1}	5004.1	
		5.4	10	23.3	91.1	No	{3,6,1}	5004.1	
		5.4	9	20.4	20.5	No	{3,6,1}	<b>5004.1</b>	
$P_{11} = \{3,6,1\}, \delta_{11} = 2 + 0.030864 \cdot (5004.1) + 1 \cdot \max\{20.3, 25.8, 16.7\} = 182.2465 \text{ hr} \cong 182.2 \text{ hr}$									
$\sigma_{11} = \max\{CT_{10}, \max\{9.8, 51.5, 6.3\}\} = 51.5 \text{ hr}, CT_{11} = \sigma_{11} + \delta_{11} = 51.5 + 182.2 = 233.7 \text{ hr}$									
(2,1)	0	625.0	4	7.4	84.2	Yes	{4}	285.2	
		540.8	2	8.8	152.8	Yes	{4,2}	706.7	
		388.0	5	27.3	61.1	Yes	{4,2,5}	1290.0	
		326.9	8	3.5	376.4	No	{4,2,5}	1290.0	
		326.9	7	14.5	148.7	Yes	{4,2,5,7}	2555.5	
		178.2	10	23.3	91.1	Yes	{4,2,5,7,10}	3607.3	
		87.1	9	20.4	20.5	Yes	{4,2,5,7,10,9}	<b>3885.8</b>	
		$P_{21} = \{4,2,5,7,10,9\}, \delta_{21} = 1 + 0.030864 \cdot (3885.8) + 1 \cdot \max\{7.4, 8.8, 27.3, 14.5, 23.3, 20.4\} = 148.2313$							
		hr $\cong 148.2$ hr							
		$\sigma_{21} = \max\{CT_{20}, \max\{20.9, 7.3, 36.5, 56.3, 86.0, 75.0\}\} = 86.0, CT_{21} = 86.0 + 148.2 = 234.2 \text{ hr}$							
(1,2)	233.7	625.0	8	3.5	376.4	Yes	{8}	<b>723.3</b>	
		$P_{12} = \{8\}, \delta_{12} = 2 + 0.030864 \cdot (723.3) + 1 \cdot \max\{3.5\} = 27.8239 \text{ hr} \cong 27.8 \text{ hr}$							
		$\sigma_{12} = \max\{CT_{11}, \max\{69.9\}\} = 233.7 \text{ hr}, CT_{12} = \sigma_{12} + \delta_{12} = 233.7 + 27.8 = 261.5 \text{ hr}$							

Figure 4. The detailed decoding procedure of the sample chromosome

The calculations on the lateness values of parts are provided in Table 3. As the GA aims to minimise the maximum lateness, which can be formulated as  $\text{Min } OBJ = \max_{i \in I} \{L_i\}$ , the objective function value of the sample chromosome can simple be found as 85.2 hr.

Table 3. The lateness calculations for parts

$(m, j)$	$CT_i$ (hr)	Part Index ( $i$ )	$d_i$ (hr)	$L_i$ (hr)
(1,1)	233.7	3	378.3	-144.6
		6	211.6	22.1
		1	305.9	-72.2
(2,1)	234.2	4	214.7	19.5
		2	282.2	-48.0
		5	149.0	85.2
		7	240.4	-6.2
		10	388.0	-153.8
		9	330.9	-96.7
(1,2)	261.5	8	576.2	-314.7

### 3.2. Genetic Operators

Crossover and mutation are two basic operators of GA used to search the solution space through differentiating the chromosomes chosen stochastically from the population. The parents are chosen using the tournament selection (Miller and Goldberg, 1995), *i.e.* a total of  $tourSize$  (where  $tourSize = popSize/6$ ) candidates are selected from the population and the one which has the lowest objective value win the tournament. The determination of the tournament size is important to give a chance to all chromosomes in the population while favouring the better ones. This is important to keep the diversity in the population while sustaining the convergence capacity of the GA.

Crossover is applied using two chromosomes selected from the population with tournament selection. A random cutting point between  $[1, ni - 1]$ , including boundaries, is determined and two children are built as shown in when the randomly determined cutting point is assumed to be four.

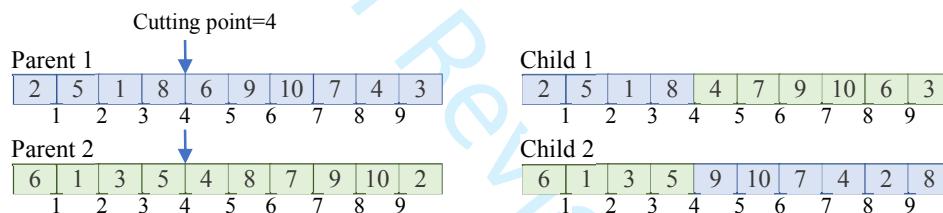


Figure 5. The application of the crossover procedure

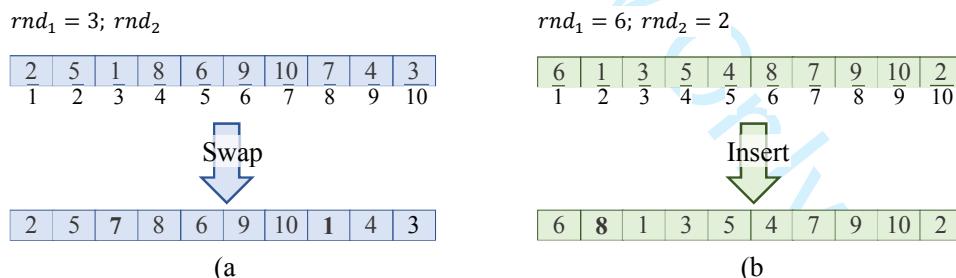


Figure 6. The application of the swap and insert mutations

Mutation helps algorithm avoid getting stuck in local optima. It plays crucial role to explore the different regions of the search space. Therefore, mutation is applied in two ways in this research, *i.e.* swap and insert. A random number ( $rnd_0$ ) is determined between  $[0,1)$ . If  $rnd_0 < 0.5$ , swap mutation is applied; otherwise, insert mutation is applied. Figure 6 illustrates the application of the mutation operator.

To apply the swap mutation, two random numbers ( $rnd_1$  and  $rnd_2$ ) are determined between  $[1,ni]$  and the genes corresponding to these numbers are exchanged (see Figure 6a). To apply the insert mutation, the gene located at the location  $rnd_1$  is removed and located at the location  $rnd_2$ , where  $rnd_1$  and  $rnd_2$  are generated randomly between  $[1,ni]$ , again (see Figure 6b).

### 3.3. Forming the New Generation

The new generation is formed replacing the worst in the population by a cyclic manner. For this aim, the fitness values of all newly generated solutions are calculated after the application of the genetic operators. The worst chromosome in the population is replaced with the best chromosome among the new individuals. This procedure is continued until there is no newly generated chromosome better than any individual in the population. It should be noted here that the duplication of the chromosomes is not allowed during the replacement process. This will be exemplified in the following subsection.

## 4. Detailed GA Solution of a Numerical Example

A numerical example consisting of three AM machines and eighteen parts is solved using the GA proposed in this research. Table 4 and Table 5 present the data related to the machine specifications and part details.

Table 4. Specifications of the AM machines

Parameters	$k_1$	$k_2$	$k_3$
$VT_m$ (hr/ cm <sup>3</sup> )	0.030864	0.030864	0.030864
$HT_m$ (hr/cm)	0.7	0.7	0.7
$SET_m$ (hr)	1.2	1.0	1.2
$MH_m$ (cm)	32.5	32.5	32.5
$MA_m$ (cm <sup>2</sup> )	625	625	625

The parameters of the algorithm have been determined as  $maxIter = 5000$ ,  $popSize = 30$ ,  $crossover rate (cr) = 0.6$ , and  $mutation rate (mr) = 0.1$  based on the common tendency in the literature and some preliminary tests as shown in Figure 7. The tournament size has been set to  $popSize/6$ .

Table 5. Parts data

$i$	$h_l \text{ (cm)}$	$a_i \text{ (cm}^2)$	$v_i \text{ (cm}^3)$	$r_i \text{ (hr)}$	$d_i \text{ (hr)}$
1	20.3	19.5	147.8	9.8	378.3
2	7.4	84.2	285.2	21.0	214.7
3	27.3	61.1	583.3	36.6	149.0
4	25.8	299.3	3282.5	51.5	576.2
5	14.5	148.7	1265.5	56.4	240.4
6	3.5	376.4	723.3	69.9	211.6
7	20.4	20.5	278.5	75.1	330.9
8	23.3	91.1	1051.8	86.0	388.0
9	26.3	20.0	201.9	93.0	447.1
10	12.8	123.9	866.1	93.1	445.1
11	28.4	333.5	7347.6	97.2	634.6
12	10.9	74.2	333.6	97.3	177.6
13	14.1	268.5	2956.0	101.8	355.3
14	3.9	11.2	32.5	107.0	258.8
15	3.2	138.9	265.6	107.6	295.6
16	24.6	92.0	1387.4	115.6	509.6
17	7.1	424.0	1086.0	128.4	294.3
18	25.2	181.4	3559.2	132.3	575.7

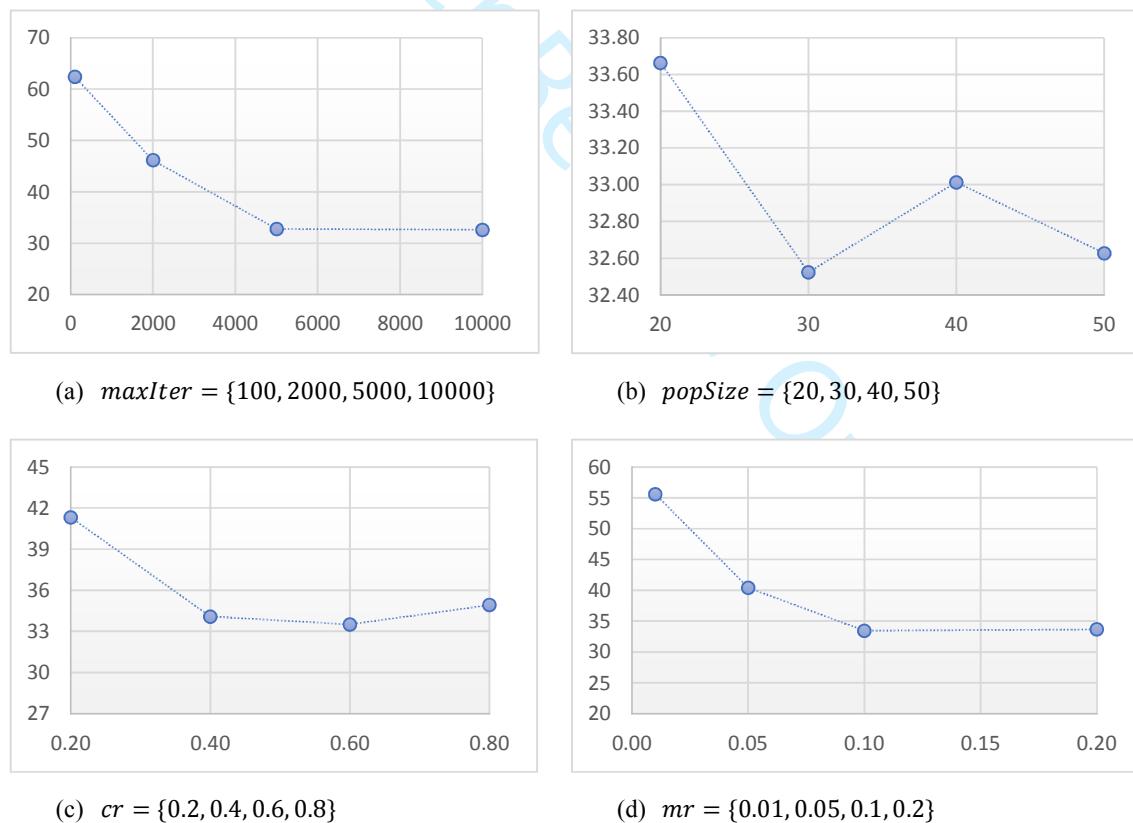


Figure 7. Parameter tuning

The algorithm was coded in Java and run on Intel Core ® i7 6700HQ CPU@2.6GHz with 16GB of RAM using the parameters determined above. Table 6 shows (a) the chromosomes in the initial

population, (b) new individuals obtained from the genetic operators, and (c) the new population after forming the new generation with fitness values. The sign “(-)” indicates that the chromosome has been replaced with that one indicated with (+) in the next generation. As seen from the table, while the best fitness value has not changed, the average fitness value was reduced from 282.30 to 231.69. This shows the convergence of the overall population in only one iteration.

Table 6. (a) The initial population, (b) new individuals obtained from the genetic operators, and (c) the new population after forming the new generation

<i>(a) Initial population</i>	<i>Fitness Value</i>
[14, 15, 4, 11, 3, 13, 16, 2, 17, 8, 9, 1, 18, 10, 6, 5, 12, 7]	340.65(-)
[18, 5, 1, 7, 16, 15, 10, 8, 4, 14, 9, 13, 11, 17, 3, 6, 2, 12]	249.07
[10, 15, 11, 3, 8, 6, 14, 16, 13, 7, 12, 18, 5, 17, 9, 1, 2, 4]	234.91
[4, 6, 14, 13, 5, 8, 9, 3, 2, 7, 18, 10, 1, 17, 15, 11, 12, 16]	262.44
[11, 8, 3, 2, 4, 17, 1, 18, 13, 5, 12, 10, 7, 15, 16, 6, 14, 9]	320.08(-)
[11, 16, 18, 1, 4, 7, 17, 14, 12, 3, 2, 15, 8, 5, 9, 10, 6, 13]	409.19(-)
[12, 3, 11, 10, 16, 4, 2, 7, 6, 13, 5, 9, 14, 18, 8, 15, 17, 1]	273.28
[16, 15, 14, 17, 12, 8, 9, 1, 13, 5, 4, 11, 2, 18, 6, 7, 3, 10]	110.83
[1, 13, 3, 15, 12, 8, 6, 18, 5, 14, 10, 4, 2, 16, 7, 9, 11, 17]	360.81(-)
[17, 1, 5, 3, 16, 10, 6, 15, 12, 11, 9, 2, 13, 8, 18, 7, 4, 14]	216.17
...	...
[9, 16, 13, 12, 17, 18, 1, 15, 4, 14, 3, 8, 5, 2, 10, 7, 11, 6]	368.24(-)
[10, 7, 3, 18, 1, 17, 9, 13, 11, 14, 16, 8, 5, 12, 6, 4, 15, 2]	274.91
[16, 2, 3, 13, 17, 8, 7, 4, 18, 12, 10, 11, 5, 14, 9, 15, 1, 6]	392.54(-)
[15, 4, 7, 5, 18, 12, 8, 9, 6, 17, 16, 14, 2, 13, 1, 10, 11, 3]	369.02(-)
[15, 10, 7, 13, 12, 17, 11, 5, 14, 2, 16, 3, 9, 18, 6, 1, 8, 4]	221.08

Best Fitness: 110.83, Average Fitness: 282.30

<i>Iteration #1</i>	
<i>(b) New chromosomes after crossover and mutation</i>	<i>Fitness Value</i>
[16, 15, 14, 17, 12, 8, 9, 1, 13, 5, 4, 11, 2, 18, 6, 7, 3, 10]	110.83
[16, 15, 14, 17, 12, 8, 9, 1, 13, 5, 4, 11, 2, 18, 6, 7, 3, 10]	110.83
[15, 10, 7, 13, 12, 17, 14, 16, 18, 5, 9, 1, 2, 4, 11, 3, 8, 6]	211.78
...	...
[2, 16, 12, 18, 3, 11, 13, 8, 6, 5, 9, 14, 15, 17, 1, 10, 4, 7]	233.08
[4, 6, 14, 13, 5, 8, 9, 3, 2, 7, 18, 10, 1, 17, 16, 15, 12, 11]	277.50
[16, 15, 14, 17, 12, 8, 9, 1, 13, 5, 4, 11, 2, 18, 6, 3, 7, 10]	110.83
[15, 10, 7, 13, 12, 17, 11, 5, 14, 2, 16, 3, 9, 18, 6, 1, 4, 8]	221.08
[14, 9, 4, 16, 15, 1, 2, 5, 11, 17, 13, 3, 12, 6, 7, 10, 18, 8]	253.21

<i>(c) The new generation after iteration #1</i>	<i>Fitness Value</i>
[16, 15, 14, 17, 12, 8, 9, 1, 13, 5, 4, 11, 2, 18, 6, 3, 7, 10]	110.83(+)
[18, 5, 1, 7, 16, 15, 10, 8, 4, 14, 9, 13, 11, 17, 3, 6, 2, 12]	249.07
[10, 15, 11, 3, 8, 6, 14, 16, 13, 7, 12, 18, 5, 17, 9, 1, 2, 4]	234.91
[4, 6, 14, 13, 5, 8, 9, 3, 2, 7, 18, 10, 1, 17, 15, 11, 12, 16]	262.44
[15, 10, 7, 13, 12, 17, 14, 16, 18, 5, 9, 1, 2, 4, 11, 3, 8, 6]	211.78(+)
[15, 10, 7, 13, 12, 17, 11, 5, 14, 2, 16, 3, 9, 18, 6, 1, 4, 8]	221.08(+)
[12, 3, 11, 10, 16, 4, 2, 7, 6, 13, 5, 9, 14, 18, 8, 15, 17, 1]	273.28
[16, 15, 14, 17, 12, 8, 9, 1, 13, 5, 4, 11, 2, 18, 6, 7, 3, 10]	110.83
[2, 16, 12, 18, 3, 11, 13, 8, 6, 5, 9, 14, 15, 17, 1, 10, 4, 7]	233.08(+)
[17, 1, 5, 3, 16, 10, 6, 15, 12, 11, 9, 2, 13, 8, 18, 7, 4, 14]	216.17
...	...

[10, 15, 11, 3, 8, 6, 5, 14, 2, 16, 9, 18, 1, 4, 7, 13, 12, 17]	247.89(+)
[10, 7, 3, 18, 1, 17, 9, 13, 11, 14, 16, 8, 5, 12, 6, 4, 15, 2]	274.91
[14, 9, 4, 16, 15, 1, 2, 5, 11, 17, 13, 3, 12, 6, 7, 10, 18, 8]	253.21(+)
[12, 3, 11, 10, 16, 4, 2, 7, 14, 6, 9, 15, 17, 1, 5, 18, 13, 8]	270.48(+)
[15, 10, 7, 13, 12, 17, 11, 5, 14, 2, 16, 3, 9, 18, 6, 1, 8, 4]	221.08

Best Fitness: 110.83, Average Fitness: 231.69

The best solution was found in the 15<sup>th</sup> iteration with the fitness value of 28.03 and the algorithm was terminated after 5000 iterations as no improvement was observed in the fitness value. The best solution obtained is shown in Table 7.

Table 7. The best solution obtained

(m,j)	P <sub>mj</sub> (hr)	δ <sub>i</sub> (hr)	CT <sub>i</sub> (hr)
(1,1)	17, 5, 14, 7, 9	108.01	236.41
(2,1)	8, 13, 16, 15, 1	197.49	313.09
(3,1)	6, 2, 3, 12	79.73	177.03
(1,2)	11, 18	357.70	594.12
(2,2)	10, 4	147.10	460.19

Chromosome: [17, 5, 8, 14, 13, 6, 7, 16, 9, 15, 2, 11, 3, 12, 1, 18, 10, 4]

Table 8. The calculation of lateness values of tasks based on the best solution obtained

(m,j)	CT <sub>i</sub> (hr)	i	d <sub>i</sub> (hr)	L <sub>i</sub> (hr)
(1,1)	236.41	17	294.3	-57.89
		5	240.4	-3.99
		14	258.8	-22.39
		7	330.9	-94.49
		9	447.1	-210.69
(2,1)	313.09	8	388.0	-74.91
		13	355.3	-42.21
		16	509.6	-196.51
		15	295.6	17.49
		1	378.3	-65.21
(3,1)	177.03	6	211.6	-34.57
		2	214.7	-37.67
		3	149.0	28.03
		12	177.6	-0.57
(1,2)	594.12	11	634.6	-40.48
		18	575.7	18.42
(2,2)	460.19	10	445.1	15.09
		4	576.2	-116.01

As seen from the table, the late parts are p<sub>15</sub> in job b<sub>1</sub> on k<sub>2</sub>, p<sub>3</sub> in job b<sub>1</sub> on k<sub>3</sub>, p<sub>18</sub> in job b<sub>2</sub> on k<sub>1</sub> and p<sub>10</sub> in job b<sub>2</sub> on k<sub>2</sub>. Among those, the maximum lateness belongs to p<sub>3</sub> with 28.03 hr, which determines the objective value. The convergence of the best fitness and average fitness values are depicted in Figure 8 for the first 511 iterations.

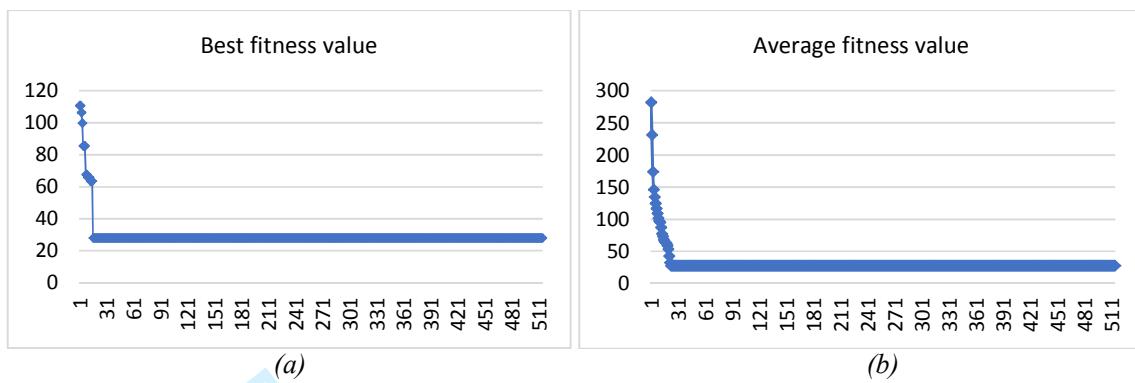


Figure 8. The convergence of the best fitness and average fitness values

## 5. Conclusions

This paper introduced the problem of scheduling additive manufacturing machines considering part release dates and due dates in a multiple machine environment with the aim of minimising maximum lateness, i.e. the maximum positive gap between the completion time and due date of jobs. The machines may have different specifications, e.g. dimensions, set up time, speed, etc. The problem has been defined and a numerical example is presented to show the production time and lateness calculations based on an example allocation of parts to machines. A GA approach, which employs a machine oriented decoding mechanism, was developed for solving the problem efficiently. Some preliminary tests have been conducted to determine the parameters of the algorithm. The steps of the algorithm have been illustrated through examples and a numerical example consisting of three machines and eighteen parts was solved using the proposed GA. The convergence of the best fitness and average fitness values were plotted and the best solution was reported. The preliminary results show that the proposed GA has a promising performance.

The methodology proposed in this research can be implemented by practitioners to schedule their production on additive manufacturing machines, especially in SLM machines. The results can also be extended to other additive manufacturing technologies. The major limitation of this work is that the performance of the proposed algorithm was not compared to any other algorithm or absolute minimum thorough a set of comprehensive computational tests. The authors' ongoing work aims to enhance the

1  
2 decoding mechanism and compare the performance of the algorithm to other heuristics and/or  
3 metaheuristics through a comprehensive computational study.  
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