Heuristic Heat-Map and Trajectory Model for Vampires vs. Werewolves

Project Notes

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1 Notation and Setting

- Grid: $G = \{(r, c) \mid 1 \le r \le R,\ 1 \le c \le C\}$.
- Players: $p \in \{\mathrm{V}, \mathrm{W}\}$ (us), opponent $\bar{p}$.
- Cell contents at time t: humans $H_{rc}(t)$, our monsters $P_{rc}(t)$, enemy monsters $\bar{P}_{rc}(t)$.
- Distance: Chebyshev $d((r, c), (i, j)) = \max(|r - i|, |c - j|)$ (8-direction moves).
- Distance kernel (choose one): $K(d) = e^{-\lambda d}$ or $K(d) = (1 + d)^{-\eta}$ with $\lambda, \eta > 0$.
- 8-neighborhood: $\mathcal{N}_8(r, c)$.
- Totals: $H_\Sigma(t) = \sum_{r,c} H_{rc}(t)$, $P_\Sigma(t) = \sum_{r,c} P_{rc}(t)$, $\bar{P}_\Sigma(t) = \sum_{r,c} \bar{P}_{rc}(t)$.

The per-cell *strategic heat* for us is

$$V_{rc}(t) = \tau_H(t)\, U^H_{rc}(t) + \tau_M(t)\, U^M_{rc}(t) + \tau_J(t)\, U^J_{rc}(t), \tag{1}$$

where U^H (humans), U^M (monsters/combat), U^J (joining/cohesion) are defined below and $\tau_\bullet(t)$ are time-/context-dependent weights (Sec. 5). When a specific number n of our units is hypothetically sent to (r, c) we write $V_{rc}(t; n)$ etc.

2 Human Term U^H: Convertible Humans vs. Distance

Let $n \in \mathbb{N}$ be the count of our units potentially dispatched to a location. The official combat/convert probability for n attackers vs. H humans is

$$P(n, H) = \begin{cases} \frac{1}{2}, & n = H, \\ \frac{n}{2H}, & n < H, \\ \frac{n}{H} - \frac{1}{2}, & n > H. \end{cases} \tag{2}$$

We map this to a smooth $[0, 1]$ *human favorability* (peaks near $n \approx H$, penalizes under/overkill):

$$g_H(n, H) = P(n, H) \cdot \sigma\Big(\kappa\, \frac{\min(n, H)}{H + \varepsilon}\Big), \qquad \sigma(x) = \frac{1}{1 + e^{-x}},\ \kappa > 0,\ \varepsilon > 0. \tag{3}$$

Aggregate human utility around (r, c) (*local subtraction choice*):

$$U^H_{rc}(t; n) = \sum_{(i,j) \in G} K\big(d((r, c), (i, j))\big)\ \Big[g_H\big(n, H_{ij}(t)\big) - g_H\big(\bar{n}, H_{ij}(t)\big)\Big], \tag{4}$$

where $\bar{n}$ is a nominal enemy packet size (e.g. median $\bar{P}$ stack). *Remark:* One may instead compute our U^H and an enemy $U^{H,\text{enemy}}$ and subtract globally at the end; both are equivalent up to scaling.

3 Monster Term U^M: Safe Kills, Avoid Being Crushed, Threat

Define local matchup utility for our n vs. enemy e:

$$h_M(n,e) = \begin{cases} +1, & n \geq 1.5\,e, \\ -1, & e \geq 1.5\,n, \\ 2\,P(n,e) - 1, & \text{otherwise}, \end{cases} \tag{5}$$

which maps $P \in [0,1]$ into $[-1,1]$. Threat from adjacent enemies (can act first next turn):

$$\text{Threat}_{rc}(n) = \sum_{(i,j)\in\mathcal{N}_8(r,c)} \gamma\ \min\big(h_M(n, \bar{P}_{ij}(t)),\, 0\big), \qquad \gamma > 0. \tag{6}$$

Monster utility:

$$U^M_{rc}(t;n) = \sum_{(i,j)\in G} K\big(d((r,c),(i,j))\big)\ \Big[h_M\big(n, \bar{P}_{ij}(t)\big) - h_M\big(\bar{n}, P_{ij}(t)\big)\Big] - \text{Threat}_{rc}(n). \tag{7}$$

4 Joining/Cohesion Term U^J

Define the *local power* of a stack of size s at (r,c) against adjacent enemies:

$$\text{pow}(r,c,s) = \sum_{(u,v)\in\mathcal{N}_8(r,c)} \mathbb{1}\{\, s \geq 1.5\,\bar{P}_{uv}(t)\,\}. \tag{8}$$

If we merge our (r,c) (size s_{rc}) with ally (u,v) (size s_{uv}), the marginal cohesion gain is

$$\Delta_J\big((r,c) \leftarrow (u,v)\big) = \text{pow}\big(r,c,\ s_{rc}+s_{uv}\big) - \text{pow}\big(r,c,\ s_{rc}\big). \tag{9}$$

Join utility within radius L (enemy-symmetric):

$$U^J_{rc}(t) = \sum_{\substack{(u,v):\,P_{uv}(t)>0 \\ d((u,v),(r,c))\leq L}} K\big(d((r,c),(u,v))\big)\,\Delta_J\big((r,c)\leftarrow(u,v)\big) - \sum_{\substack{(u,v):\,\bar{P}_{uv}(t)>0 \\ d((u,v),(r,c))\leq L}} K\big(d((r,c),(u,v))\big)\,\Delta_J^{\text{enemy}}\big((r,c)\leftarrow(\tag{10}$$

5 Time/Context Weights

Let normalized time $\theta_t = t/T_{\max} \in [0,1]$ and human abundance $q_H(t) = H_\Sigma(t)/H_\Sigma(0)$. A simple schedule (early: capture humans; late: fight/join):

$$\begin{aligned} \tau_H(t) &= \text{norm}\big(a_H\,q_H(t) + b_H(1-\theta_t)\big), \\ \tau_M(t) &= \text{norm}\big(a_M(1-q_H(t)) + b_M\,\theta_t\big), \\ \tau_J(t) &= \text{norm}\big(a_J\,\theta_t + b_J(1-q_H(t))\big), \end{aligned} \tag{11}$$

then rescale so that $\tau_H(t)+\tau_M(t)+\tau_J(t) = 1$. Small $a_\bullet, b_\bullet$ can be tuned via black-box search.

6 Area-Aware Augmentation Around Targets

Goal. If a target cell is valuable on its own, a supportive ring of nearby opportunities (humans, merges) can make it even better ("win more"), while a hostile ring reduces its value. We add a local area term that also decays when we are far from that target.

Let the map diameter be $D = \max(R, C)$ and define an area radius

$$L_A = \lfloor \alpha_A D \rfloor, \qquad \alpha_A \in (0,1).$$

Choose a local kernel $J(\Delta) = e^{-\mu\Delta}$ or $J(\Delta) = (1+\Delta)^{-\zeta}$ with $\Delta = d((r,c),(i,j))$.

Define the *area augmentation* around (r,c):

$$A_{r,c}(t) \;=\; \sum_{\substack{(i,j)\in G\\ \Delta\le L_A}} J(\Delta)\,\Big(U^H_{i,j}+U^J_{i,j}+\beta^{\text{loc}}_M\,U^M_{i,j}\Big) \;-\; \sum_{\substack{(i,j)\in G\\ \Delta\le L_A}} J(\Delta)\,\Big(U^{H,\text{enemy}}_{i,j}+\beta^{\text{loc}}_M\,U^{M,\text{enemy}}_{i,j}\Big), \quad (12)$$

with $\beta^{\text{loc}}_M \in [0,1]$ tempering combat repetition.

Distance-to-us reach scaling:

$$d^{\text{us}}_{\min}(r,c) = \min_{(u,v):\,P_{u,v}>0} d\big((u,v),(r,c)\big), \qquad S_{\text{reach}}(r,c) = \exp\big(-\lambda_A\, d^{\text{us}}_{\min}(r,c)\big).$$

Area-aware corrected value:

$$\widetilde{V}_{r,c}(t) \;=\; V_{r,c}(t) \;+\; \omega_A\, S_{\text{reach}}(r,c)\, A_{r,c}(t), \tag{13}$$

with $\omega_A > 0$ a tunable weight.

Interpretation. Favor targets surrounded by follow-up opportunities; disfavor islands in hostile rings. The farther we are, the less the area ring matters (via S_{reach}).

7 Final Heat and Path-Aware Objective

Per-cell instantaneous desirability with allocation n is $V_{rc}(t;n)$ from (1). To account for motion and on-path opportunities, for a path $\pi = [\pi_1, \dots, \pi_L]$ with $\pi_k = (r_k, c_k)$ and discount $\delta \in (0,1]$, define

$$\mathcal{V}(\pi; n) = \sum_{k=1}^{|\pi|} \delta^{k-1}\Big(\widetilde{V}_{\pi_k}(t+k-1; n_k) + \alpha\, G(\pi_k)\Big). \tag{14}$$

where $G(\pi_k)$ measures immediate gains realized at step k (e.g. converted humans or defeated enemies), $\alpha \ge 0$, and n_k is the surviving/allocated count along the path (may drop via casualties/splits).

The best trajectory for dispatch n from origin a is

$$\pi^\star(a,n) \in \arg\max_{\pi\in\Pi_L(a)} \mathcal{V}(\pi; n), \tag{15}$$

where $\Pi_L(a)$ are feasible paths of length $\le L$.

8 Detour-Limited Paths and Enemy Interference

Detour-limited enumeration. For each origin a and candidate target z among the top K cells by $\widetilde{V}$, let $L^*(a \to z)$ be the shortest Chebyshev length and allow a slack $\Delta \in \{0,1\}$. Candidate paths are

$$\Pi^{(\Delta)}(a\to z) = \Big\{\pi :\ |\pi| \in \big[L^*(a\to z),\, L^*(a\to z)+\Delta\big]\Big\}. \tag{16}$$

Enemy path set. Analogously, build enemy candidate sets $\bar{\Pi}^{(\Delta)}(\bar{a} \to \bar{z})$ for their top K targets by their $\widetilde{V}^{\text{enemy}}$.

Collision modeling. At time step k, π_k and $\bar{\pi}_k$ *collide* if they co-occupy (or nearly co-occupy) the same area. With a proximity band $B \in \{0,1\}$:

$$\mathrm{Col}(\pi, \bar{\pi}) \;=\; \sum_{k=1}^{\min(|\pi|,|\bar{\pi}|)} \mathbb{1}\big\{\, d(\pi_k, \bar{\pi}_k) \le B \,\big\}. \tag{17}$$

Define a per-collision penalty at time k that grows when our fight EV is low:

$$\mathrm{pen}_k = \delta^{\,k-1}\, \rho_{\mathrm{col}} \Big(1 - \mathrm{EV}_{\mathrm{fight}}(n_k, \bar{n}_k)\Big) \cdot \mathbb{1}\{d(\pi_k, \bar{\pi}_k) \le B\}. \tag{18}$$

Total penalty against the *worst plausible* enemy route:

$$\mathrm{Pen}(\pi) = \max_{\bar{\pi} \in \bigcup \bar{\Pi}^{(\Delta)}} \sum_k \mathrm{pen}_k. \tag{19}$$

Collision-adjusted path value:

$$\mathcal{V}^{\mathrm{adj}}_{\mathrm{ours}}(\pi; n) = \mathcal{V}(\pi; n) - \mathrm{Pen}(\pi). \tag{20}$$

9 Choosing Best Route per Target and Overall

$$\pi^*(a \to z; n) \;\in\; \arg \max_{\pi \in \Pi^{(\Delta)}(a \to z)} \; \mathcal{V}^{\mathrm{adj}}_{\mathrm{ours}}(\pi; n).$$

$$z^*,\, \pi^*(a \to z^*; n) \;\in\; \arg \max_{z \in \text{Top-}K} \; \max_{\pi \in \Pi^{(\Delta)}(a \to z)} \; \mathcal{V}^{\mathrm{adj}}_{\mathrm{ours}}(\pi; n).$$

10 Group Division (Splitting) and Multi-Route Allocation

Suppose n_0 units start at a_0. Let $F(n) = \widetilde{V}_{a_0}(t; n)$ be the diminishing-return value of keeping n at origin. Find the smallest n_1 such that

$$\widetilde{V}_{a_0}(t; n_1) \ge \rho\, \widetilde{V}_{a_0}(t; n_0), \qquad \rho \in (0,1). \tag{21}$$

Keep n_1 at a_0 and distribute the remainder among m expeditions that each follow a best collision-adjusted route from Sect. 8:

$$\max_{\{n_i\}_{i=1}^m} \; \sum_{i=1}^m \mathcal{V}^{\mathrm{adj}}_{\mathrm{ours}}(\pi_i^*; n_i) \quad \text{s.t.} \quad \sum_{i=1}^m n_i = n_0 - n_1, \;\; n_i \in \mathbb{N}. \tag{22}$$

A practical greedy approximation: iteratively assign one unit at a time to the expedition i with the largest marginal increase

$$\Delta_i = \mathcal{V}^{\mathrm{adj}}_{\mathrm{ours}}(\pi_i^*; n_i + 1) - \mathcal{V}^{\mathrm{adj}}_{\mathrm{ours}}(\pi_i^*; n_i).$$

11 Phase-2 Path Adjustment (On-Path Value)

Targets chosen purely by terminal heat may be suboptimal if alternate routes accumulate higher *en-route* value. After selecting K top targets by $V_{rc}(t)$, re-rank by an adjusted score

$$\mathcal{V}'(\pi; n) = \mathcal{V}(\pi; n) + \beta \sum_{k=1}^{|\pi|} \mathrm{HumanGain}(\pi_k) + \beta_M \sum_{k=1}^{|\pi|} \mathrm{EnemyLoss}(\pi_k), \tag{23}$$

with $\beta, \beta_M \ge 0$ controlling side-gain emphasis, then choose $\pi^\star$ maximizing $\mathcal{V}'$.

12 Algorithmic Skeleton (Per Turn)

1. Compute $H_{rc}, P_{rc}, \bar{P}_{rc}$, distances, kernels K.
2. For representative n (e.g. $n \in \{1, \lfloor s/2 \rfloor, s\}$ per stack size s):
   - Evaluate $g_H, P(n, H), h_M$, Threat (6), and Δ_J (9).
   - Build U^H, U^M, U^J via (4),(7),(10).
3. Compute weights τ_H, τ_M, τ_J via (11), form $V_{rc}(t; n)$ via (1).
4. For each of our stacks:
   (a) Determine n_1 by (21). Let remainder be $n_0 - n_1$.
   (b) Enumerate candidate targets (top-K by V), and limited-depth paths Π_L.
   (c) Score paths using $\mathcal{V}$ or $\mathcal{V}'$ (14),(23).
   (d) Allocate units by (22) (greedy or small integer program).
5. Execute resulting move set for the turn.

13 Tunable Hyperparameters (Extended)

- **Distance kernels:** $K(d) = e^{-\lambda d}$ with $\lambda \in [0.3, 0.6]$ or $K(d) = (1 + d)^{-\eta}$ with $\eta \in [1.0, 2.5]$. Larger values sharpen locality.
- **Human favorability:** Sigmoid steepness $\kappa \in [2, 6]$; $\varepsilon = 10^{-6}$. Larger κ rewards near-equal $n \approx H$ more sharply.
- **Threat weight:** $\gamma \in [0.5, 1.5]$. Larger penalizes standing near stronger enemy neighbors.
- **Joining radius:** $L \in \{1, 2, 3\}$. Bigger captures more merges but costs time.
- **Time weights:** $a_\bullet, b_\bullet$ tuned by self-play/random search; always renormalize τ's to sum to 1. Higher a_H biases early human capture; higher a_M biases late fighting.
- **Area radius:** $\alpha_A \in [0.05, 0.15]$ so $L_A = \lfloor \alpha_A D \rfloor$. Larger sees broader rings.
- **Area kernel:** $J(\Delta) = e^{-\mu\Delta}$ with $\mu \in [0.6, 1.2]$ or $(1 + \Delta)^{-\zeta}$ with $\zeta \in [1.0, 2.0]$.
- **Area combat tempering:** $\beta_M^{\text{loc}} \in [0.3, 0.8]$ to avoid double-counting combat utility in the ring.
- **Area reach scaling:** $\lambda_A \in [0.2, 0.6]$; $\omega_A \in [0.5, 1.5]$. Higher λ_A suppresses area effects when far; higher ω_A boosts the ring's influence.
- **Discount for paths:** $\delta \in [0.85, 0.98]$. Lower prefers faster routes.
- **On-path gain weight:** $\alpha \in [0.3, 1.0]$. Higher favors routes that pick up conversions/kills en route.
- **Detour slack:** $\Delta \in \{0, 1\}$. 1 lets the planner consider slightly longer but better routes.
- **Top-K per side:** $K = 5$ (typ.). Larger increases compute cost but explores more targets.
- **Collision band:** $B \in \{0, 1\}$. $B = 1$ treats near-occupancy as contesting.
- **Collision penalty:** $\rho_{\text{col}} \in [0.5, 1.5]$. Larger penalizes enemy intersections more.
- **Split retention:** $\rho \in [0.90, 0.97]$. Higher keeps a stronger anchor; lower frees more units to roam.

14 Summary and Implementation Notes

The total per-cell value V blends human, combat, and joining utilities with time-aware weights. We augment it with an area ring A that rewards "win-more" neighborhoods and penalizes hostile ones, scaled by our proximity to the target to form $\widetilde{V}$. Planning then evaluates detour-limited paths that accumulate discounted $\widetilde{V}$ plus on-path gains, and subtracts worst-case enemy path interference via a collision penalty. Finally, stacks may split, allocating units across multiple best routes by greedy marginal gain. This yields a practical, modular planner that aligns with game rules and scales with map size.