Разложить функцию $f(x)$ в тригонометричкий ряд Фурье на заданном отрезке.

14.1 $f(x) = \cos^4 x$ на $[-\pi;\pi]$.

$$f(x) = \cos^4 x = \left(\frac{1+\cos 2x}{2}\right)^2 = \frac{1}{4} + \cos 2x + \frac{\cos^2 2x}{4} = \frac{1}{4} + \cos 2x + \frac{1+\cos 4x}{8} =$$
$$= \frac{3}{8} + \cos 2x + \frac{1}{8}\cos 4x.$$

14.3 $f(x) = \sin x$ на $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$.

$$b_n = \frac{2}{\pi}\int_{-\pi/2}^{\pi/2} \sin x \cdot \sin 2nx\,dx = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2}[\cos(2n-1)x - \cos(2n+1)x]\,dx =$$
$$= \frac{1}{\pi}\left[\frac{1}{2n-1}\sin(2n-1)x - \frac{1}{2n+1}\sin(2n+1)x\right]\Bigg|_{-\pi/2}^{\pi/2} =$$
$$= \frac{1}{\pi}\left[\frac{1}{2n-1}\sin\left(\pi n - \frac{\pi}{2}\right) - \frac{1}{2n+1}\sin\left(\pi n + \frac{\pi}{2}\right) -\right.$$
$$\left. - \frac{1}{2n-1}\sin\left(-\pi n + \frac{\pi}{2}\right) + \frac{1}{2n+1}\sin\left(-\pi n - \frac{\pi}{2}\right)\right] =$$
$$= \frac{2}{\pi}\left[\frac{1}{2n-1}\sin\left(\pi n - \frac{\pi}{2}\right) - \frac{1}{2n+1}\sin\left(\pi n + \frac{\pi}{2}\right)\right] =$$
$$= \frac{2}{\pi}\left[\frac{1}{2n-1}(-1)^{n+1} - \frac{1}{2n+1}(-1)^n\right] = \frac{2}{\pi}\cdot\frac{(-1)^{n+1}4n}{4n^2-1} = \frac{8}{\pi}\cdot\frac{(-1)^{n+1}n}{4n^2-1}.$$
$$\sin x = \frac{8}{\pi}\sum_{x-1}^{\infty}\frac{(-1)^{n+1}n}{4n^2-1}\sin 2nx, \quad x \in \left(-\frac{\pi}{2};\frac{\pi}{2}\right).$$