Виноградова И.А. и др. - Математический анализ в задачах и упражнениях. Том 2 (2018)

Разложить функцию f(x) в тригонометричкий ряд Фурье на заданном отрезке.

**14.1** 
$$f(x) = \cos^4 x$$
 на  $[-\pi; \pi]$ .

$$f(x) = \cos^4 x = \left(\frac{1 + \cos 2x}{2}\right)^2 = \frac{1}{4} + \cos 2x + \frac{\cos^2 2x}{4} = \frac{1}{4} + \cos 2x + \frac{1 + \cos 4x}{8} = \frac{3}{8} + \cos 2x + \frac{1}{8}\cos 4x.$$

**14.3** 
$$f(x) = \sin x$$
 на  $\left[ -\frac{\pi}{2}; \frac{\pi}{2} \right]$ .

$$b_n = \frac{2}{\pi} \int_{-\pi/2}^{\pi/2} \sin x \cdot \sin 2nx \, dx = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} [\cos(2n-1)x - \cos(2n+1)x] \, dx =$$

$$= \frac{1}{\pi} \left[ \frac{1}{2n-1} \sin(2n-1)x - \frac{1}{2n+1} \sin(2n+1)x \right]_{-\pi/2}^{\pi/2} =$$

$$= \frac{1}{\pi} \left[ \frac{1}{2n-1} \sin\left(\pi n - \frac{\pi}{2}\right) - \frac{1}{2n+1} \sin\left(\pi n + \frac{\pi}{2}\right) -$$

$$- \frac{1}{2n-1} \sin\left(-\pi n + \frac{\pi}{2}\right) + \frac{1}{2n+1} \sin\left(-\pi n - \frac{\pi}{2}\right) \right] =$$

$$= \frac{2}{\pi} \left[ \frac{1}{2n-1} \sin\left(\pi n - \frac{\pi}{2}\right) - \frac{1}{2n+1} \sin\left(\pi n + \frac{\pi}{2}\right) \right] =$$

$$= \frac{2}{\pi} \left[ \frac{1}{2n-1} (-1)^{n+1} - \frac{1}{2n+1} (-1)^n \right] = \frac{2}{\pi} \cdot \frac{(-1)^{n+1} 4n}{4n^2 - 1} = \frac{8}{\pi} \cdot \frac{(-1)^{n+1} n}{4n^2 - 1}.$$

$$\sin x = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{4n^2 - 1} \sin 2nx, \quad x \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right).$$