

# **Determination of Pitch Diameter of Parallel Thread Gauges by Mechanical Probing**

**EURAMET cg-10**

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# **Calibration Guide**

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## DETERMINATION OF PITCH DIAMETER OF PARALLEL THREAD GAUGES BY MECHANICAL PROBING

### Purpose

This document has been produced to enhance the equivalence and mutual recognition of calibration results obtained by laboratories performing calibrations of determination of pitch diameter of parallel thread gauges by mechanical probing.

## Authorship and Imprint

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## Guidance Publications

This document gives guidance on measurement practices in the specified fields of measurements. By applying the recommendations presented in this document laboratories can produce calibration results that can be recognized and accepted throughout Europe. The approaches taken are not mandatory and are for the guidance of calibration laboratories. The document has been produced as a means of promoting a consistent approach to good measurement practice leading to and supporting laboratory accreditation.

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## Determination of Pitch Diameter of Parallel Thread Gauges by Mechanical Probing

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### 1 Introduction

- 1.1 The calibration of screw-thread gauges is both economically and technically an important issue. Although the usually achieved uncertainties (a few micrometres) appear not to be very demanding when compared to gauge blocks or ring gauges for example, a proper measurement and its evaluation as well as a reliable uncertainty analysis of the results of calibration of screw thread gauges is a difficult and complicated task. Further difficulties are caused by different definitions of parameters in written standards and in measurement and evaluation practices that have grown historically. These differences can lead to significantly different results and associated uncertainties of measurement between measurements which are each carried out and evaluated properly in their own tradition. Both for the market and for accreditation organisations this is not acceptable. This document is intended to harmonise this situation.

### 2 Scope and field of application

- 2.1 The guideline applies particularly to calibration techniques based on mechanical probing of the thread using spherical or cylindrical probing elements. It is restricted to thread gauges of the following type:
- cylindrical form, i.e. parallel thread
  - straight flanks
  - positive flank angles
  - single and multistart threads.
- 2.2 The document gives guidance on how to determine the pitch diameter and the associated uncertainty of measurement. It omits some technical aspects, which can be different for each set-up and does not consider measurement strategy. Alternative techniques such as optical measurements, two-dimensional scanning (contouring), co-ordinate measurement machines or gauging are not extensively covered. Some concepts might, however, be applicable in those cases as well.

- 2.3 Also, the document is not intended to add information to what already can be found in general practice, textbooks and written standards. However, some selected definitions and concepts are given where they are useful for understanding the document.

### 3 Terminology

- 3.1 For the sake of clarity, all relevant elements of a parallel (cylindrical) screw thread as they are further used in this document are defined hereafter. It cannot be guaranteed that these definitions are in conformance to all national or international standards applied within EA. The international vocabulary in Appendix 1 gives translations of the different terms in English, French and German.
- 3.2 Major Diameter ( $d$ ,  $D$ ): The diameter of an imaginary cylinder (termed the major cylinder) that would bound the crests of an external thread or the roots of an internal thread.
- 3.3 Minor Diameter ( $d_1$ ,  $D_1$ ): The diameter of an imaginary cylinder (termed the minor cylinder) that would bound the roots of an external thread or the crests of an internal thread.
- 3.4 Pitch ( $P$ ): The distance, measured parallel to the axis, between corresponding points on adjacent thread forms in the same axial plane and on the same side of the axis.
- 3.5 Number of grooves ( $n$ ): Number of grooves within one turn ( $n > 1$  for a multistart thread).
- 3.6 Lead ( $\ell$ ): Pitch multiplied by the number of grooves within one turn (i.e. number of starts). Axial displacement of the screw after one turn.  $\ell = n \cdot P$ .
- 3.7 Flank Angles ( $\beta$ ,  $\gamma$ ): The angles between the individual flanks (leading and trailing flank) and the perpendicular to the axis of the thread, measured in an axial plane section.
- 3.8 Thread Angle ( $\alpha$ ): Sum of the two flank angles.
- 3.9 Lead Angle ( $\psi$ ): Helix angle of the thread in the pitch diameter.
- 3.10 Pitch Diameter ( $d_2$ ,  $D_2$ ): The diameter of an imaginary cylinder (termed the pitch cylinder), the surface of which intersects the thread profile in such a manner as to make the width of the thread ridge and the thread groove equal.
- 3.11 Simple Pitch Diameter: The diameter of an imaginary cylinder, which intersects the surface of the thread profile in such a manner as to make the width of the thread groove equal to one-half of the *basic* (nominal) pitch.
- 3.12 Virtual Pitch Diameter (Functional Diameter): The pitch diameter of an imaginary thread of perfect pitch and angle, cleared at the crests and roots but having the full depth of straight flanks, which would just assemble with the actual thread over a specified length of engagement. This diameter includes the cumulative effect of variations in lead (pitch), flank angle, taper, straightness and roundness.
- 3.13 For a theoretically perfect thread, the three definitions of pitch diameter define the same quantity. The terms *simple* and *virtual* pitch diameter are used in gauging practice.
- 3.14 Indicated value ( $m$ ): Value of a quantity provided by a measuring instrument.

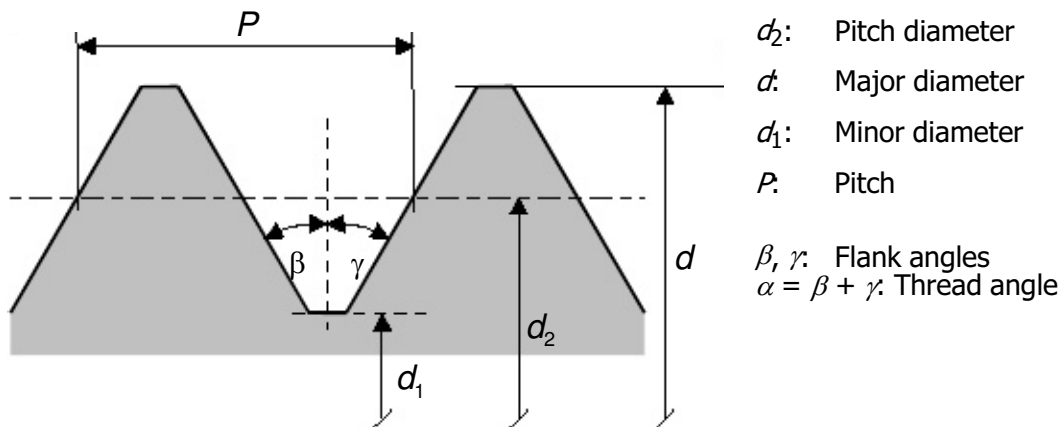


Fig.1 Elements of parallel screw threads (drawing and symbols apply to external thread. For internal diameters, the capital symbol  $D$  applies).

## 4 Categories of calibration of thread gauges

- 4.1 The calibration categories listed below refer mainly to mechanical probing techniques. They describe the extent of measurements and refer to the definition of the pitch diameter which is to be determined. Measurements and their associated uncertainties can only be properly compared when they belong to the same category mentioned below. In certain cases, for non-measured quantities (pitch and thread angle), a type B evaluation of uncertainty is made. However, all measurement results from the different categories for the same defined quantity should be consistent with each other, within some confidence intervals deduced from the respective associated uncertainties of measurement.

Note: for the first calibration of a thread gauge, the measurement of pitch  $P$  and thread angle  $\alpha$  is strongly recommended, independent on the category chosen for the calibration of the pitch diameter.

	Parameter	measured	assumed	taken into account in uncertainty analysis
1	Simple pitch diameter			
1a	$m$	✓	-	✓
	$\alpha$	-	within tolerance	✓ (tolerance zone)
	$P$	defined nominal	-	-
1b	$m$	✓	-	✓
	$\alpha$	✓	-	✓
	$P$	defined nominal	-	-
2	Pitch diameter			
2a	$m$	✓	-	✓
	$\alpha$	-	within tolerance	✓ (tolerance zone)
	$P$	✓	-	✓
2b	$m$	✓	-	✓
	$\alpha$	✓	-	✓
	$P$	✓	-	✓
3	Virtual pitch diameter			
3	$m$	✓	-	✓
	$\beta, \gamma$	✓	-	✓
	$P$	✓	-	✓

Table 1 Categories of calibration for the determination of the three different pitch diameters by defining which quantities have to be measured, which assumptions can be made and what has to be taken into account in the evaluation of the measurement uncertainty for the pitch diameter. The symbol  $m$  denotes the measured diameter defined as the distance between the centres of the probing elements touching the thread on both sides.

## 4.2 Remarks on each category

4.2.1 Measurement of diameter only (1a): The *simple pitch diameter* is calculated from this measured diameter, corrected for the rake and the measuring force, and from assumed nominal values for the pitch and thread angle.

The uncertainty analysis must consider: the uncertainty in the measurement of the diameter, the ball or wire diameter as well as the uncertainty in the thread angle. The thread angle is not measured. Instead it is assumed that it is within tolerance limits. This knowledge is best described by assigning to the thread angle its nominal value and by assuming that the distribution of possible values is rectangular. If the tolerance zone is  $\pm a$  ( $2a$  total), the standard uncertainty is  $a/\sqrt{3}$ . The value for the pitch is defined nominal and therefore considered to be a constant.



4.2.2 Measurement of diameter and thread angle (1b): Compared to 1a, a direct measurement of the thread angle can give a more reliable measurement result of the *simple pitch diameter* associated with a possibly smaller uncertainty of measurement.

4.2.3 Measurement of diameter and pitch (2a): The *pitch diameter* is calculated from this measured diameter and from the measured pitch, corrected for the rake and the measuring force, and from assumed nominal values for the thread angle.

The uncertainty analysis must consider: the uncertainty in the measurement of the diameter, the pitch, the ball or wire diameter as well as the uncertainty in the thread angle. The thread angle is not measured. Instead it is assumed that it is within tolerance limits. This knowledge is best described by assigning to the thread angle its nominal value and by assuming that the distribution of possible values is rectangular. If the tolerance zone is  $\pm a$  ( $2a$  total), the standard uncertainty is  $a/\sqrt{3}$ .

4.2.4 Measurement of diameter, pitch and thread angle (2b): Compared to 2a, a direct measurement of the thread angle can give a more reliable measurement result of the *pitch diameter* associated with a possibly smaller uncertainty of measurement.

4.2.5 Measurement of diameter, pitch and flank angles (3): The *virtual pitch diameter* is calculated from this measured diameter, corrected for the rake and the measuring force, from the measured pitch, and from the measured flank angles. For a more elaborate calibration, also the drunkenness of the thread (i.e. the variations in lead angle) would have to be taken into account.

The uncertainty analysis must consider: the uncertainty in the measurement of the diameter, the pitch, the ball or wire diameter as well as the uncertainty in the measurement of the flank angles.

### 4.3 Alternative techniques

Examples of alternative techniques are given below. What has been said in section 4.2 is largely, but not necessarily fully, applicable.

4.3.1 Measurement of a 2-D axial profile: A measurement of a complete profile gives a much more comprehensive characterisation of a thread than the few points which are measured according to categories 1 to 3.

4.3.2 Co-ordinate measuring machine techniques: The calibration of a thread gauge with a co-ordinate measuring machine (CMM) belongs usually to category 2a, with the difference, that the pitch diameter and the pitch are often determined at different heights and in different directions. Using sufficiently small probing spheres and scanning techniques, the flank angles can be determined as well.

4.3.3 Optical techniques: Normally the optical measurements are used for external threads. Measurements done with a measuring microscope or profile projector include the determination of the actual thread angle and the pitch.

4.3.4 Fitting to a screw limit gauge: The traceability of measurement results obtained when fitting a thread gauge to a limit gauge can be achieved by the calibration of the limit gauge according to virtual pitch diameter. In a measurement report of limit fitting, the values assigned to the limit gauge(s) in its calibration process (during a separate activity) including the associated uncertainties of measurement shall be given, showing that it is within specification limits, with the remark "fit" or "does not fit" for the measured gauge.

## 5 Calculation of the pitch diameter

- 5.1 The mathematical formulae given below apply to pitch diameter measurements using spherical or cylindrical probing elements and two- or three point measurements. In this chapter, the most commonly used formula is given as an example. Other approaches may be equally applicable.

The pitch diameter has to be determined from the measured diameter  $m$  (defined as the distance between the centres of the probing elements touching the thread on both sides) knowing the pitch  $P$ , the thread angle  $\alpha$  and the diameter  $d_D$  of the probing elements. From a simple geometrical analysis one obtains for a symmetrical thread with  $\beta = \gamma = \alpha/2$

$$d_2, D_2 = m \mp d_D \frac{1}{\sin(\alpha/2)} \pm \frac{P}{2} \cot(\alpha/2) \mp A_1 \pm A_2, \quad (1)$$

where the upper sign applies to external thread ( $d_2$ ) and the lower sign to internal threads ( $D_2$ ), respectively. The term  $A_1$  is the rake correction, which takes into account the progressive movement of the probing elements away from the thread axis as the lead angle increases.  $A_2$  is a correction for the measurement force as discussed in chapter 6. Eq.(1) is the solution of the two-dimensional problem and therefore only an approximation, the validity of which shall be discussed in the following sections.

### 5.2 Approximation for the rake correction

- 5.2.1 For a symmetrical thread with a small lead angle and not too steep flank angles, the following approximation for the rake correction can be used:

$$A_1 = \frac{d_D}{2} \tan^2 \psi \cdot \cos \frac{\alpha}{2} \cdot \cot \frac{\alpha}{2}, \text{ with } \tan \psi = \frac{P}{\pi \cdot D_2} \text{ or } \tan \psi = \frac{P}{\pi \cdot d_2} \quad (2)$$

$\psi$  is the lead angle. The lower table in Appendix 2 shows the effect of approximation of Eq.(2) for various threads with a symmetrical profile.

### 5.3 General case, following Berndt's theory

- 5.3.1 Equation (3) to (5) [ref. 1] are based on the theory of Berndt [ref. 2] and apply to the general case of an asymmetric multi-start thread. They provide exact values for the rake corrections:

$$d_2, D_2 = m \cdot \cos \theta \mp d_D \cdot \frac{\cos \frac{\beta - \gamma}{2}}{\sin \frac{\beta + \gamma}{2}} \cdot \sqrt{1 - \frac{m^2 \cdot \sin^2 \theta}{d_D^2 \cdot \cos^2 \left( \frac{\beta - \gamma}{2} \right)}} \pm \left( \frac{\ell}{n} - \frac{2 \cdot \ell \cdot \theta}{\pi} \right) \cdot \frac{\cos \beta \cdot \cos \gamma}{\sin(\beta + \gamma)} \quad (3)$$

The auxiliary angle  $\theta$  is calculated by iteration:

$$\theta_k = \arcsin \left( \frac{d_D \cdot l}{\pi \cdot m^2} \cdot \frac{\cos \beta \cdot \cos \gamma \cdot \cos \frac{\beta - \gamma}{2}}{\cos \frac{\beta + \gamma}{2}} \cdot \frac{\sqrt{1 - \frac{m^2 \cdot \sin^2 \theta_{k-1}}{d_D^2 \cdot \cos^2 \left( \frac{\beta - \gamma}{2} \right)}}}{\cos \theta_{k-1} \mp \sin \left( \frac{\beta + \gamma}{2} \right) \cdot \cos \left( \frac{\beta - \gamma}{2} \right) \cdot \frac{d_D}{m} \cdot \sqrt{1 - \frac{m^2 \cdot \sin^2 \theta_{k-1}}{d_D^2 \cdot \cos^2 \left( \frac{\beta - \gamma}{2} \right)}}} \right) \quad (4)$$

with the starting value

$$\theta^1 = \frac{d_D \cdot \ell}{\pi \cdot m^2} \cdot \frac{\cos \beta \cdot \cos \gamma \cdot \cos \frac{\beta - \gamma}{2}}{\cos \frac{\beta + \gamma}{2} \cdot \left( 1 \mp \sin \frac{\beta + \gamma}{2} \cdot \cos \frac{\beta - \gamma}{2} \cdot \frac{d_D}{m} \right)} \quad (5)$$

$\ell = n \cdot P$  is the lead of a multi-start thread. The upper sign again refers to external threads ( $d_2$ ), the lower sign to internal threads ( $D_2$ ). The upper table in Appendix 2 allows testing software used for pitch diameter calculation in various cases. Other mathematical approaches (e.g. numerical vectorial approaches) can be applied for exact thread calculations as well.

## 5.4 Calculation of the virtual pitch diameter

5.4.1 Corrections to be applied for pitch deviation: For the evaluation of the virtual pitch diameter, a deviation  $\delta P$  in pitch (measured value of cumulative pitch over the length of engagement) produces a correction to be applied to the *simple* pitch diameter. For a symmetrical thread, the correction equals

$$\delta D_p = \pm \frac{|\delta P|}{\tan(\alpha/2)} \quad (6)$$

The upper sign refers to external threads, the lower sign to internal threads.

5.4.2 Corrections to be applied for angle deviation: For the evaluation of the virtual pitch diameter, deviations  $\delta \alpha$  in the flank angles produce a correction to be applied to the *simple* pitch diameter. For an ISO thread ( $\alpha/2 = 30^\circ$ ), the correction equals

$$\delta D_\alpha = \pm 0.625 \cdot P \cdot (|\delta \beta| + |\delta \gamma|). \quad [\delta \beta] = [\delta \gamma] = \text{rad} \quad (7)$$

The upper sign refers to external threads, the lower sign to internal threads.

## 6 Correction for the deformation of the probing elements

6.1 The magnitude of the deformation of the probing elements due to the measuring force which has to be taken into account for pitch diameter calibration depends on the measurement procedure. In particular, the deformation encountered during the calibration

of the probing elements and during zero setting in the measurement process may partially cancel the elastic deformation during the thread gauge measurement.

- 6.2 Below an example for spherical probing elements is given with some simplifications. It still gives a good approximation of the effects for many cases. The deformation is approximated as a ball-on-flat contact with Hertz' formula

$$w_0 = \sqrt[3]{\frac{9F^2}{8d_b} \left( \frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2} \right)^2}, \quad (8)$$

where

- $w_0$  = indentation of ball-on-flat contact
- $\nu_i$  = Poisson-coefficient (0.28 for steel; 0.25 for ruby)
- $F$  = measuring force (perpendicular to the flat)
- $E_i$  = elasticity modulus ( $2 \cdot 10^{11}$  N/m<sup>2</sup> for steel;  $4 \cdot 10^{11}$  N/m<sup>2</sup> for ruby)
- $d_b$  = ball diameter

Usually the spherical probing element and the screw gauge will be of different material.

- 6.3 For a ball contacting the thread in the helix, which can be simplified by a V-groove, the measurement force perpendicular to the flanks is reduced to  $F/[2 \sin(\alpha/2)]$ . In the direction of the original measuring force (and the length measurement), the resulting indentation is by a factor  $1/\sin(\alpha/2)$  larger than the indentation perpendicular to the flank. One therefore obtains for the deformation  $w_{V0}$  of a ball in a V-groove

$$w_{V0} = (\sin(\alpha/2))^{-5/3} (1/2)^{2/3} w_0. \quad (8a)$$

- 6.4 For a metric thread with  $\alpha/2 = 30^\circ$ , a 1 mm steel ball and 1 N measuring force, the overall deformation correction for the two contacts of the probe on both sides of the thread gauge, which has to be finally taken into account, results from Eqs.(8) and (8a) and amounts to  $A_2 = 2 w_{V0} = 4 w_0 = 1.84 \mu\text{m}$ .
- 6.5 Friction effects and, for cylindrical wires and flat probes, form deviations will cause deviations from the Hertz equations. In general it is recommended to optimise the measurement strategy in order to minimise residual corrections.

## 7 Evaluation of the uncertainty of measurement

- 7.1 The uncertainty of measurement of the pitch diameter shall be discussed according to the GUM [ref. 3]. The estimation of the measurement uncertainty of the quantity  $m$ , the pitch  $P$  and the thread angle  $\alpha$  cannot be carried out without knowing the applied measurement procedure, the specific instruments used and the laboratory conditions.

### 7.2 Optimal diameter of probing element

- 7.2.1 The correct choice of the diameter  $d_b$  of the probing element (spherical or cylindrical) is very important. It should ideally contact the profile at the pitch cylinder. If the probing element diameter differs from the optimal diameter, deviations in the thread angles become increasingly important for the determination of the pitch diameter. If the thread angle is not measured, but instead the standard uncertainty of measurement is derived from the tolerance values (categories 1a and 2a), the uncertainty contribution to the (simple) pitch diameter is required to be not a major contribution in the uncertainty budget.

7.2.2 The probing elements must be calibrated not only in diameter  $d_b$ , but also in form deviation or alternatively with a 3-point measuring technique. For example wires, which are subject to wear, are usually calibrated as 2-point diameter and used as a 3-point diameter. In practice, the diameters of the three wires of a set with identical nominal diameter will be slightly different. This has to be taken into account for the uncertainty analysis.

7.2.3 Symmetric profile: For threads of moderate lead angle, a probing element (spherical or cylindrical) of the diameter

$$d_0 = \frac{P}{2} \frac{1}{\cos(\alpha/2)} \quad (9)$$

will contact the profile close to the pitch cylinder.

7.2.4 Asymmetric profile: A probing element (spherical or cylindrical) of a diameter

$$d_0 = P \cdot \frac{\tan(\alpha/2)}{\tan \beta + \tan \gamma} \cdot \frac{2}{\cos \beta + \cos \gamma} \quad (10)$$

will contact the profile symmetrically around the pitch cylinder.

### 7.3 Example 1: Internal thread calibrated with two-ball stylus

7.3.1 Internal and external threads (screw ring and plug gauges) can be measured using a length measuring machine and a double-ended spherical stylus (Fig.2). The measured displacement  $\Delta L$  shall be the average of the displacements between position 1 to 2 and 2 to 3:  $\Delta L = (\Delta L_{12} + \Delta L_{23})/2$ . If the pitch cylinder axis is properly aligned with respect to the measurement axis, a two-point measurement might be sufficient. The stylus constant  $C$  can be determined with the help of a plain ring gauge or a gauge block.

7.3.2 The quantity  $m$ , defined as the distance between the centres of the probing spheres (Fig. 2), is then for a ring gauge:  $m = \Delta L + C - d_b$   
and for a plug gauge:  $m = \Delta L - C + d_b$

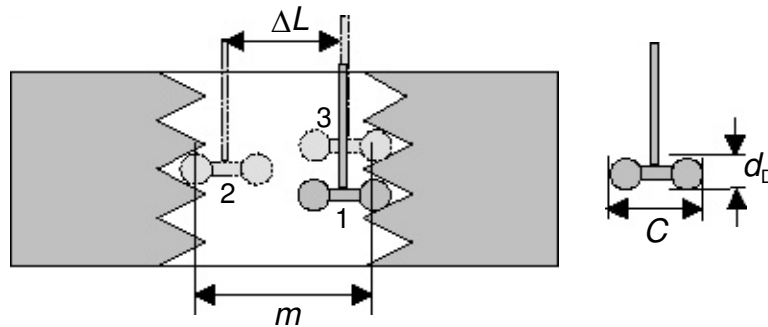


Fig. 2 Calibration of an internal thread using a double-ended spherical stylus.

7.3.3 It has to be noted, that the stylus constant  $C$  is usually not exactly the geometrical dimension sketched in Fig. 2, but includes the balls' virtual separation and the offset due to the travel of the stylus to reach the null position of the probing system. This effect, however, cancels because it occurs during both, the calibration of the stylus constant and the subsequent measurement of the thread.

7.3.4 Assuming an internal thread to be measured, the mathematical model function for the pitch diameter is obtained from Eq.(1), where  $m$  has to be replaced by  $m = \Delta L + C - d_b$ :

$$D_2 = \Delta L + C + d_b \left( \frac{1}{\sin(\alpha/2)} - 1 \right) - \frac{P}{2} \cot(\alpha/2) + A_1 - A_2 + \delta B, \quad (11)$$

where  $C$  is the stylus constant and  $\delta B$  represents additional deviations such as form deviations of the screw gauge to be calibrated.

7.3.5 Assuming the input quantities to be uncorrelated, the variance of the pitch diameter  $D_2$  is obtained from

$$u^2(D_2) = u^2(\Delta L) + u^2(C) + c_{d_0}^2 u^2(d_0) + c_P^2 u^2(P) + c_{\alpha/2}^2 u^2(\alpha/2) + u^2(A_1) + u^2(A_2) + u^2(\delta B) \quad (12)$$

where

$u(\Delta L)$  is the standard uncertainty associated with the measured displacement  $\Delta L$  (separately evaluated, similarly as for a plain ring gauge, and including contributions from the calibration of the measuring instrument, temperature effects, determination of the location of the turning points etc);

$u(C)$  is the standard uncertainty associated with the stylus constant  $C$  (contains - apart from contributions resulting directly from the measurement process - the standard uncertainty associated with the value of the reference standard (gauge block or ring gauge) used for its determination);

$u(d_0)$  is the standard uncertainty of the calibrated value of the diameter of the probing element. This uncertainty is assumed to be completely correlated for the two balls, yielding for the sensitivity coefficient  $c_{d_0} = 1/\sin(\alpha/2) - 1$ ;

$u(P)$  is the standard uncertainty associated with the pitch measurement, the associated sensitivity coefficient being  $c_P = \cot(\alpha/2)/2$ ;

$u(\alpha/2)$  is the standard uncertainty of measurement of the flank angle  $\alpha/2$ . This may in many cases - in particular for optical measurement methods - be inversely proportional to the pitch. The sensitivity factor depends on the difference of the actual diameter  $d_0$  of the probing element from the best size diameter  $d_0$ . Take care of the unit of  $\alpha$ :  $[\alpha] = \text{rad}$ .

$$c_{\alpha/2} = \frac{\cos \frac{\alpha}{2}}{\sin^2 \frac{\alpha}{2}} (d_0 - d_0); \quad (13)$$

$u(A_1)$  is the standard uncertainty associated with the error resulting from the use of an approximate equation for the rake correction;

$u(A_2)$  is the standard uncertainty associated with corrections due to the measurement force and the approximations inherent in the experimental equation used to correct for the elastic compression of the probing elements;

$u(\delta B)$  accounts for imperfections of the calibrated thread gauge, such as form deviations, and further instrument or procedure dependent contributions, which have not been taken into account so far.

7.3.6 Numeric example of an uncertainty budget:

Calibration according to category 1a of a metric screw ring gauge M36x4 with nominal values  $D_2 = 33.402 \text{ mm}$ ,  $P = 4 \text{ mm}$  and  $\alpha = 60^\circ$ . The gauge is measured with a double-ended spherical probe with  $d_0 = 2.4822 \text{ mm}$  (best size  $d_0 = 2.3094 \text{ mm}$ ), having a stylus constant  $C = 16.02 \text{ mm}$  and a measurement force of  $0.1 \text{ N}$ . Neither the pitch nor the

thread angle are measured, their nominal values are used for the determination of the *simple* pitch diameter. The sensitivity coefficient  $c_{\alpha/2}$  is linearly dependent on  $(d_D - d_0)$ , i.e. only probing elements with a diameter close to the ideal diameter should then be used, as is argued in clause 7.2.1. In this example the manufacturers tolerance for the flank angle  $\alpha/2$  shall be  $\pm 10'$ . The simple pitch diameter is calculated using Eqs.(3 - 5).

quantity	estimate	standard uncertainty	probability distribution	sensitivity coefficient	uncertainty contribution
$X_i$	$x_i$	$u(x_i)$		$c_i$	$u(y)$
$\Delta L$	18.361 mm	0.4 $\mu\text{m}$	normal	1	0.4 $\mu\text{m}$
$C$	16.02 mm	0.3 $\mu\text{m}$	normal	1	0.3 $\mu\text{m}$
$d_D$	2.4822 mm	0.3 $\mu\text{m}$	normal	1	0.3 $\mu\text{m}$
$P$	4 mm	0	-	-	0
$\alpha$	60°	5.8'=1.68mrad	rectangular	0.6 $\mu\text{m}/\text{mrad}$	1.0 $\mu\text{m}$
$A_2$	0.24 $\mu\text{m}$	0.02 $\mu\text{m}$	rectangular	1	0.02 $\mu\text{m}$
$\delta B$	0	0.3 $\mu\text{m}$	rectangular	1	0.3 $\mu\text{m}$
$D_2$	33.4018 mm				1.20 $\mu\text{m}$

Expanded uncertainty:  $U = k \cdot u(D_2) = 2 \cdot 1.2 \mu\text{m} = 2.4 \mu\text{m}$

The result is  $D_2 = (33.4018 \pm 0.0024) \text{ mm}$  for the *simple* pitch diameter.

- 7.3.7 Note that the uncertainty contribution from the flank angle is very important due to the large sensitivity coefficient (0.6  $\mu\text{m}/\text{mrad}$ ), although the manufacturer's tolerance of  $\pm 10'$  for the flank angles is not very large. In such a case, a calibration of the simple pitch diameter without measuring the flank angles is not to be recommended, unless a probing element with a diameter closer to the ideal diameter is chosen, which would reduce the sensitivity coefficient for the flank angle.

#### 7.4 Example 2: External thread calibrated with three wires

- 7.4.1 An external thread (screw plug gauge) can be calibrated between two flat probes using three wires of diameter  $d$  as probing elements (Fig.3). If the pitch cylinder is independently aligned with respect to the measurement axis, e.g. using centres, two probing wires shall be used.

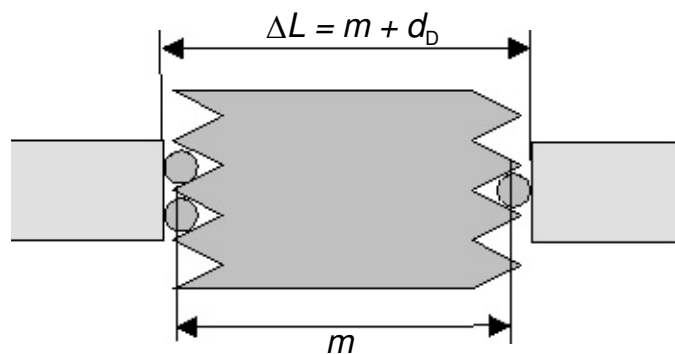


Fig. 3 Calibration of an external thread using three wires between flat probes.

- 7.4.2 Assuming an external thread to be measured as depicted in Fig. 3, the mathematical model function for the pitch diameter is obtained from Eq.(1), where  $m$  has to be replaced  $m = \Delta L - d_D$

$$d_2 = \Delta L - d_D \left( \frac{1}{\sin(\alpha/2)} + 1 \right) + \frac{P}{2} \cot(\alpha/2) - A_1 + A_2 + \delta B \quad (14)$$

7.4.3 Assuming the input quantities to be uncorrelated, the variance of the pitch diameter  $d_2$  is obtained from

$$u^2(d_2) = u^2(\Delta L) + c_{d_D}^2 u^2(d_D) + c_P^2 u^2(P) + c_{\alpha/2}^2 u^2(\alpha/2) + u^2(A_1) + u^2(A_2) + u^2(\delta B) \quad (15)$$

where

$u(\Delta L)$  is the standard uncertainty associated with the measured displacement  $\Delta L$  (separately evaluated, similarly as for a plain plug gauge, and including contributions from the calibration of the measuring instrument, temperature effects etc);

$u(d_D)$  is the standard uncertainty of the calibrated value of the diameter of the probing element. This uncertainty is assumed to be completely correlated for the three wires, yielding for the sensitivity coefficient  $c_{d_D} = 1/\sin(\alpha/2)+1$ .

(Note: This sensitivity coefficient, being different from the one calculated for the ring

gauge calibration, obviously depends on the measurement method, in particular on how the length measuring instrument was zeroed);

$u(P)$  is the standard uncertainty associated with the pitch measurement, the associated sensitivity coefficient being  $c_P = \cot(\alpha/2)/2$ ;

$u(\alpha/2)$  is the standard uncertainty of measurement of the flank angle  $\alpha/2$ . This may in many cases - in particular for optical measurement methods - be inversely proportional to the pitch. The sensitivity factor depends on the difference of the actual diameter  $d_D$  of the probing element from the best size diameter  $d_0$ . Take care of the unit of  $\alpha$ :  $[\alpha] = \text{rad}$ .

$$c_{\alpha/2} = \frac{\cos \frac{\alpha}{2}}{\sin^2 \frac{\alpha}{2}} (d_D - d_0); \quad (16)$$

$u(A_1)$  is the standard uncertainty associated with the error resulting from the use of an approximate equation for the rake correction;

$u(A_2)$  is the standard uncertainty associated with corrections due to the measurement force and the approximations inherent in the experimental equation used to correct for the elastic compression of the probing elements;

$u(\delta B)$  accounts for imperfections of the calibrated thread gauge, such as form deviations, and further instrument or procedure dependent contributions, which have not been taken into account so far.

7.4.4 Numeric example of an uncertainty budget:

Calibration according to category 2b of a metric screw plug gauge M64x6 with nominal values  $d_2 = 60.1336 \text{ mm}$ ,  $P = 6 \text{ mm}$  and  $\alpha = 60^\circ$ . The gauge is measured using 3 wires with  $d_D = 3.464 \text{ mm}$  (best size  $d_0 = 3.4641 \text{ mm}$ ) and a measurement force of 1.5 N.



quantity	estimate	standard uncertainty	probability distribution	sensitivity coefficient	uncertainty contribution
$X_i$	$x_i$	$u(x_i)$		$c_i$	$u(y)$
$\Delta L$	65.2993 mm	0.4 $\mu\text{m}$	normal	1	0.4 $\mu\text{m}$
$d_b$	3.464 mm	0.2 $\mu\text{m}$	normal	3	0.6 $\mu\text{m}$
$P$	6.004 mm	1 $\mu\text{m}$	normal	0.866	0.87 $\mu\text{m}$
$\alpha$	59.7°	1.3' = 0.38 mrad	normal	-0.35 $\mu\text{m}/\text{rad}$	-0.0001 $\mu\text{m}$
$A_2$	0.0007 mm	0.1 $\mu\text{m}$	rectangular	1	0.1 $\mu\text{m}$
$\delta B$	0	0.2 $\mu\text{m}$	rectangular	1	0.2 $\mu\text{m}$
$d_2$	60.1048 mm				1.15 $\mu\text{m}$

Expanded uncertainty:  $U = k \cdot u(d_2) = 2 \cdot 1.15 \mu\text{m} = 2.3 \mu\text{m}$

The result becomes:  $d_2 = (60.1048 \pm 0.0023) \text{ mm}$  for the pitch diameter.

- 7.4.5 The *simple* pitch diameter is not affected by the pitch measurement and its uncertainty, because the nominal pitch is to be used for calculating the simple pitch diameter, resulting in a different value and a smaller expanded uncertainty:

$$d_2 = (60.1013 \pm 0.0015) \text{ mm}$$

- 7.4.6 This example shows that the expanded uncertainty of measurement in the *simple pitch diameter* is smaller than the uncertainty in the *pitch diameter*, even when in the latter case the pitch is really measured. This situation can be accepted provided that a clear distinction between *pitch diameter* and *simple pitch diameter* is made both in certificates and in accreditation schedules.

- 7.4.7 The wire diameter used in this example corresponds almost to the ideal diameter, which results in a negligible sensitivity coefficient for the flank angle. In this case a calibration without measuring the flank angles would have been possible, even without increasing the uncertainty of measurement.

## 7.5 Additional uncertainty contributions for virtual pitch diameter

- 7.5.1 In addition to the contributions discussed in section 7.4, the corrections to be applied for determining the virtual pitch diameter are also affected by uncertainty:

$u(\delta D_p)$  takes into account the uncertainty of measurement of the pitch, contributing to the pitch correction for the virtual pitch diameter, with the sensitivity coefficient  $c_{Dp} = 1/\tan(\alpha/2)$ ;

$u(\delta D_\alpha)$  takes into account the uncertainty of measurement of the flank angle, contributing to the angle correction for the virtual pitch diameter, with the sensitivity coefficient  $c_{D\alpha} = 2 \cdot 0.625 \cdot P$ .

For the numeric example of 7.4.4, the uncertainty budget looks like:

quantity	estimate	standard uncertainty	probability distribution	sensitivity coefficient	uncertainty contribution
$X_i$	$x_i$	$u(x_i)$		$c_i$	$u(y)$
Contributions from the quantities $\Delta L, d_D, P,$			normal	1	1.15 $\mu\text{m}$
$\delta D_P^{\alpha}, A_2, \delta B$ as for the example	0.0069 $\mu\text{m}$	7.3 $\mu\text{m}$	normal	1.73	1.73 $\mu\text{m}$
$\delta D_{\alpha}$	0.0196 mm	0.38 mrad	normal	7.5 $\mu\text{m}/\text{mrad}$	2.85 $\mu\text{m}$
$d_2$	60.1278 mm				3.53 $\mu\text{m}$

The result becomes:  $d_2 = (60.1278 \pm 0.0071)$  mm for the virtual pitch diameter.

## 8 Certificate of calibration

- 8.1 The calibration certificate must make it clear what parameter has been determined and what parameters were measured or assumed. In section 4, an overview is given of what has to be taken into account. However, the categories listed are only intended for allowing a clear distinction and shall not be referred to.

## 9 References

1. M. Kochsiek und J. Lerch, "Zur Ermittlung von Bestimmungsgrößen an Gewinden", PTB-Bericht PTB-Me-4 (1974).
2. G. Berndt, "Die Anlagekorrekturen bei der Bestimmung des Flankendurchmessers von symmetrischen und unsymmetrischen Aussen- und Innengewinden nach der Dreidrahtmethode oder mittels zweier Kugeln", Zeitschrift für Instrumentenkunde 60 (1940), pp. 141ff, 177ff, 209ff, 237ff, 272ff.
3. ISO Guide to the expression of Uncertainty in Measurement, first edition 1995, ISO (Geneva)

## Appendix 1: International Vocabulary

English	French	German
pitch diameter or effective diameter	diamètre sur flancs	Flankendurchmesser
simple pitch diameter or simple effective diameter	diamètre sur flancs simple	einfacher Flankendurchmesser
virtual pitch diameter or virtual effective diameter	diamètre sur flancs virtuel	Paarungs-Flankendurchmesser
pitch	pas du profil	Teilung
lead	pas hélicoïdal	Steigung
lead angle	angle hélicoïdal	Steigungswinkel
helix	filet	abgewinkelte Schraubenlinie
multi-start thread	filetage à plusieurs filets	mehrgängiges Gewinde
flank angle	demi-angle du flanc	Teilflankenwinkel
thread angle	angle du filet	Flankenwinkel
major diameter	diamètre extérieur	Aussendurchmesser
minor diameter	diamètre intérieur	Kerndurchmesser
rake correction	correction d'obliquité	Anlagekorrektur

## Appendix 2: Examples for pitch diameter calculation

The following examples are intended to test software used to calculate the pitch diameter. The measuring force is assumed to be zero, i.e. no correction for the elastic compression was applied.

Form	Type	$d_2, D_2$ nom / mm	$P$ / mm	$l$ / mm	$n$	$\beta$	$\gamma$	$d_D$ / mm	$m$ / mm	$d_2, D_2$ ref / mm
Plug	M 64x6	60.127	6.000	6.000	1	30°	30°	3.2030	61.3458	60.1336
Ring	Tr 22x18p6	18.988	6.000	18.000	3	15°	15°	3.1058	17.6161	18.9749
Ring	Tr 22x18p6	18.988	6.000	18.000	3	15°	15°	3.2250	17.1211	18.9932
Plug	G 1	31.783	2.309	2.309	1	26°43'	27°15'	1.1549	32.0761	31.7977
Plug	Pg 48	58.56	1.580	1.580	1	40°	40°	1.1025	59.3003	58.5266
Ring	S 65x16	54.508	16.000	16.000	1	3°	30°	8.0007	52.4013	54.4872
Ring		80.8785	6.000	6.000	1	3°	30°	3.4162	79.1134	81.2846
Ring		58.7301	6.000	6.000	1	20°	30°	3.0232	57.9998	58.7551
Ring		39.7160	16.000	16.000	1	20°	30°	8.1825	37.2661	39.6890
Plug		97.9242	16.000	16.000	1	3°	30°	8.0230	100.0214	97.9304

The reference values were determined using Eqs.(3 - 5) of section 5.3.

Effects of approximations in formulas for pitch diameter calculation:

Form	$d_2, D_2$ nom / mm	$P$ / mm	$n$	$\beta$	$\gamma$	$d_D$ / mm	$m$ / mm	$d_2, D_2$ appr. <sup>1</sup> / mm	$d_2, D_2$ ref <sup>2</sup> / mm
Plug	60.127	6.000	1	30°	30°	3.2030	61.3458	60.1336	60.1336
Ring	18.988	6.000	3	15°	15°	3.1058	17.6161	19.0120	18.9749
Ring	18.988	6.000	3	15°	15°	3.2250	17.1211	19.0364	18.9932
Plug	31.783	2.309	1	26°43'	27°15'	1.1549	32.0761	31.7977	31.7977
Plug	58.56	1.580	1	40°	40°	1.1025	59.3003	58.5266	58.5266

<sup>1</sup> According to Eqs.(1 - 2) of section 5.1 and 5.2.

<sup>2</sup> According to Eqs.(3 - 5) of section 5.3