Assignment 2

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1. **Normal distribution for stock returns (50 points).** Using the Python code developed in the first lecture, download daily stock returns over the period December 31st, 1999, to December 31st, 2022 from WRDS for the following 5 companies: Gamestop, Tesla, Apple, General Electrics, and Procter & Gamble.
   (Note, that the returns of some companies may not be available for the full horizon).

   (a) For each stock compute the mean and variance of daily simple returns.

   (b) Plot the empirical density function of stock returns (i.e., a histogram estimate of the underlying density function) and compare to (i) the normal distribution with the same mean and variance as the empirical distribution and (ii) the normal distribution with mean and variance of the 'winsorized' empirical distribution. To compute the 'winsorized' empirical distribution, simply keep only the daily returns with absolute value less than 4%.

   (Effectively you should make one separate graph for each stock with the empirical histogram and two different normal densities, corresponding to the two calibrations of means and variances).

   (*Hint: If* `S` *is a* `panda` *DataFrame, then the method* `S.plot.bin` *might be useful.*)

   (c) For each stock, compute the 95% and 99% Value-at-Risk and Conditional Expected Shortfall from the empirical distribution of returns, and compare to the Value-at-Risk and Conditional Expected Shortfall, that would obtain if the distributions were normal with corresponding means and variances.

   (*Hint: If* `S` *is a* `panda` *DataFrame, then the method* `S.quantile` *might be useful. Also, you might want to import the function* `scipy.stats.norm` *which contains a lot of useful methods linked to the normal distribution.*)

(d) Based on your computations, does the normal distribution seem to describe appropriately the distribution of daily stock returns?

(e) Compare your results from the task (b) with empirical density function of 2-year and 10-year bond returns.

(*Hint:* Download from CRSP the return for 2-year and 10-year treasury bonds, using the python sql request including the variable names *caldt, b2ret, b10ret* from the database *crsp.mcti.*)

```
bonds=db.raw_sql("select caldt, b2ret, b10ret "
 "from crsp.mcti "
 "where caldt > 1999-12-31"
"and caldt < '2020-12-31",
date_cols=['caldt'])
bonds=bonds.rename(columns= "caldt":  "date")
```

Plot the empirical density function of two bonds (i.e., a histogram estimate of the underlying density function) and compare to a normal distribution with the same mean and variance as the empirical distribution (one separate graph for each bond with the empirical histogram and normal densities, corresponding to the two calibrations of means and variances). Effectively, repeat the same analysis as in (b) and compare results of bonds' analysis and stocks' analysis.)

2. **Mean-Variance Frontier with 2 risky assets (50 points).**

   Download weekly stock returns for Apple and Procter & Gamble for the period 01/01/2000-31/12/2022 from WRDS. Assume the risk-free rate over the period was 1.5%.

   (a) Estimate the expected return and standard deviation for each stock, as well as the correlation between both stocks.

   (b) Plot the mean-standard deviation frontier you can achieve by combining both risky assets.

   (c) Compute the portfolio weights of the global minimum variance portfolio, as well as its expected return, standard deviation, and Sharpe Ratio.

   (d) Plot the mean-standard deviation frontier if you can also invest in the risk-free asset (i.e., combining the two risky assets and the risk-free asset).

   (e) Recompute the correlation between Apple's and P&G's return from 2010 to 2020. Holding everything else equal, how does the different correlation change the volatility of the minimum variance portfolio?

   (f) Suppose you are targeting a standard deviation for your optimal portfolio of 20%. What is the optimal weight you should hold in the three assets? What is your implied risk-aversion coefficient? Give the mean return and Sharpe ratio of your optimal portfolio.

   (g) Suppose that you are targeting a 99% VaR of -10%. What is your optimal weight in the tangency portfolio? You can assume that the returns of the tangency portfolio are normally distributed with a mean and variance given by the historical moments computed above.