SCALING LAWS MEET MODEL ARCHITECTURE: TO-WARD INFERENCE-EFFICIENT LLMS

Song Bian*Tao Yu†Shivaram VenkataramanYoungsuk ParkUW-MadisonAmazon Web ServicesUW-MadisonAmazon Web Services

ABSTRACT

Scaling the number of parameters and the size of training data has proven to be an effective strategy for improving large language model (LLM) performance. Yet, as these models grow increasingly powerful and widely deployed, the cost of inference has become a pressing concern. Despite its importance, the tradeoff between model accuracy and inference efficiency remains underexplored. In this work, we examine how key architectural factors, hidden size, the allocation of parameters between MLP and attention (mlp-to-attention ratio), and groupedquery attention (GQA), influence both inference cost and accuracy. We introduce a conditional scaling law that augments the Chinchilla framework with architectural information, along with a search framework for identifying architectures that are simultaneously inference-efficient and accurate. To validate our approach, we train more than 200 models spanning 80M to 3B parameters and 8B to 100B training tokens, and fit the proposed conditional scaling law. Our results show that the conditional scaling law reliably predicts optimal architectural choices and that the resulting models outperform existing open-source baselines. Under the same training budget, optimized architectures achieve up to 2.1% higher accuracy and 42% greater inference throughput compared to LLaMA-3.2.

1 Introduction

Scaling law studies Kaplan et al. (2020); Hoffmann et al. (2022); Muennighoff et al. (2023); Krajewski et al. (2024); Abnar et al. (2025) have shown that increasing model parameters, training tokens, dataset quality, and compute budget consistently reduces pre-training loss, improves downstream task performance Hendrycks et al. (2021); Austin et al. (2021), and enables the emergence of novel capabilities Wei et al. (2022). These insights have driven the development of many state-of-the-art large language models Touvron et al. (2023); Yang et al. (2025); Guo et al. (2025).

However, as the field advances, it has become increasingly clear that focusing exclusively on training overlooks the practical challenges of deploying these models at scale Chien et al. (2023); Wu et al. (2024); Muhamed et al. (2023). A major limitation of existing scaling laws is their omission of inference costs, which constitute the dominant expense in deploying large models in real-world applications Sardana et al. (2023); Park et al. (2024). Moreover, the growing use of LLMs in reasoning systems highlights the need for scaling laws that account for inference costs Snell et al. (2024); Brown et al. (2024); Luo et al. (2024); Qi et al. (2024); Guan et al. (2025). Therefore, we ask the following question:

Can we explicitly capture the trade-off between inference efficiency and accuracy of large language models?

To address this question, a recent study Sardana et al. (2023) proposed scaling laws that incorporate the total FLOPs from both training and inference. However, their formulation requires estimating the total number of tokens generated over a model's entire lifespan. Because inference is performed repeatedly during deployment, this assumption renders the proposed scaling law impractical for real-world use. Another study Bian et al. (2025) extends Chinchilla scaling laws by incorporating

^{*}Work done during internship at Amazon Web Services.

[†]Correspondence to: Tao Yu (taou@amazon.com)

model architecture. However, this work has notable limitations. First, the study considers only the aspect ratio, defined as hidden size over number of layers, as the architectural factor. Yet, as shown in Figure 1, aspect ratio alone fails to capture the full range of factors that influence inference efficiency in large language models. Second, the depth of the model strongly influences accuracy: cutting layers tends to impair the model's generalization after fine-tuning Petty et al. (2023). Finally, the study lacks a general framework for incorporating broader architectural factors, including hidden size and GQA, into scaling laws.

In this work, we fix the number of layers and study the effect of other architectural factors, including GQA, hidden size, and the mlp-to-attention ratio. This design choice is motivated by recent open-weight models such as LLaMA Touvron et al. (2023), Qwen Yang et al. (2025), Gemma Team et al. (2024a), and Phi Abdin et al. (2024), which, despite having a comparable number of parameters, adopt markedly different architectural designs.

Our primary goal is to investigate how model architecture influences both inference efficiency and model accuracy. We begin by comparing the inference efficiency of models with identical parameter counts but varying architectures. Next, we train over 200 models, ranging from 80M to 297M parameters on up to 30B tokens, to systematically characterize the relationship between architectural design and accuracy. Guided by these empirical findings, we introduce a conditional extension of the Chinchilla scaling laws that incorporates architecture.

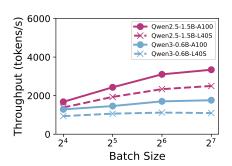


Figure 1: Although larger models generally achieve lower inference throughput than smaller ones, Qwen2.5-1.5B outperforms Qwen3-0.6B. Despite having the same number of layers, Qwen2.5-1.5B benefits from a higher hidden size, GQA, and mlp-to-attention ratio.

tural parameters, establishing a general framework for identifying model architectures that balance inference efficiency and performance.

Finally, we validate this framework by fitting the proposed scaling law on models between 80M and 297M parameters, and evaluating its predictions when scaling up to 3B-parameter models. Our results demonstrate that, under identical training setups, the derived optimal 3B-parameter architecture achieves 42% higher inference throughput than the LLaMA-3.2-3B architecture, while maintaining better accuracy.

2 BACKGROUND

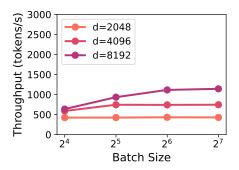
Accurately predicting the performance of large language models during scaling is essential. This enables us to answer key questions: (i) what is the optimal allocation of available resources between model size and training tokens, and (ii) what performance gains can be expected from additional resources? Fortunately, the model loss has been observed to follow a power-law relationship with respect to the number of parameters N and training tokens D Hoffmann et al. (2022); Muennighoff et al. (2023) with:

$$L(N,D) = E + \frac{A}{N^{\alpha}} + \frac{B}{D^{\beta}} \tag{1}$$

where L is the model loss, N is the number of total parameters and D is the number of tokens used for training and A, B, E, α , β are parameters to be learned.

To fit the learnable parameters in Eq. (1), Chinchilla Hoffmann et al. (2022) employs two strategies: (i) training models with a fixed number of parameters while varying the number of training tokens, and (ii) training models under a fixed compute budget¹, varying both parameters and tokens. The resulting data are combined to fit the learned parameters in Eq. (1). With the fitted scaling laws,

¹The compute cost is approximated as $FLOPs(N, D) \approx 6ND$ in Hoffmann et al. (2022); Muennighoff et al. (2023), where N denotes the number of parameters and D the number of training tokens. In this work, we adopt the same settings as prior studies.



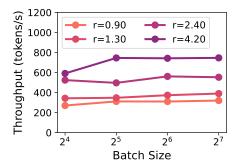


Figure 2: **Inference throughput** vs (left) hidden size $d = d_{\text{model}}$ and (right) mlp-to-attention ratio $r = r_{\text{mlp/attn}}$ on the 8B model. Under a fixed parameter budget $N_{\text{non-embed}}$, larger hidden sizes and higher mlp-to-attention ratios improve inference throughput for varying batch sizes.

Chinchilla addresses the following question to determine optimal allocation:

$$\arg\min_{N,D}L(N,D) \text{ s.t. FLOPs}(N,D) = C \tag{2}$$

where C denotes the resource constraint, N the total number of parameters, and D the number of training tokens.

In this paper, we do not address how to optimally allocate compute between model size and training data under a fixed compute budget. Instead, our focus is on identifying model architectures that optimize inference efficiency and accuracy under fixed parameter and token budgets. For example, given a model with 7B parameters trained on 14T tokens, we study how to design an architecture that satisfies both efficiency and accuracy requirements.

3 MODEL ARCHITECTURE-AWARE SCALING LAWS

3.1 Model Architecture Variations

The architecture of a decoder-only transformer is composed of a sequence of stacked decoder blocks, each sharing the same structure to facilitate model-parallel deployment across devices. Under this design, the overall architecture of dense LLMs is primarily determined by the hidden size and the MLP intermediate size, which together specify the attention and MLP layers structure. This work studies the optimal model architecture given a fixed total number of non-embedding parameters $N_{\rm non-embed}$ (at different levels). Although the number of layers $n_{\rm layer}$ also plays a critical role (closely related to aspect ratio (Petty et al., 2023)), varying $n_{\rm layer}$ under a fixed $N_{\rm non-embed}$ substantially impacts both inference cost and accuracy (Tay et al., 2021; Alabdulmohsin et al., 2023). Therefore, we fix $n_{\rm layer}$ and focus on the effects of hidden size $d_{\rm model}$ and the mlp-to-attention ratio $r_{\rm mlp/attn}$ on inference efficiency (§3.2) and accuracy (§3.3), noting that $n_{\rm layer}$ still varies across different $N_{\rm non-embed}$ levels. In §3.3, we introduce a conditional scaling law to predict the performance of architectural variants, and in §3.4, we present a lightweight framework for identifying architectures that optimally balance inference efficiency and accuracy.

Note that the number of attention parameters is primarily determined by the hidden size $d_{\rm model}$ and the attention projection dimension, since most open-weight models adopt non-square q,k,v projection matrices, as seen in Gemma (Team et al., 2024a) and Qwen3 (Yang et al., 2025). For consistency, we fix the per-head dimension $d_{\rm head}$ to 64 for models with $N_{\rm non-embed} \leq 1B$ and to 128 for models with $N_{\rm non-embed} \geq 3B$. Consequently, to maintain a constant $r_{\rm mlp/attn}$, we adjust the number of attention heads $n_{\rm head}$ rather than altering the projection dimension directly. This design choice also provides flexibility to incorporate architectural variants such as grouped-query attention.

3.2 Inference Efficiency

Inspired by the success and widespread adoption of open-weight dense models such as Qwen3 (Yang et al., 2025), LLaMA-3.2 (Dubey et al., 2024), and the Gemma-2 (Team et al., 2024b) family, we

construct architectural variants by modifying the configurations of the LLaMA-3.2 and Qwen3 dense models (Figure 11-13 in Appendix E). In addition to hidden size and the mlp-to-attention ratio, we find that group-query attention has a critical impact on inference efficiency, even though it only modestly reduces the number of attention parameters (by shrinking the key and value matrices). To disentangle these effects, we conduct controlled ablations of hidden size, MLP-to-attention ratio, and GQA under the following setups:

- hidden size d_{model} : fix $N_{\text{non-embed}}$, $r_{\text{mlp/attn}}$ and GQA= 4, vary d_{model} and number of attention heads n_{head} (Figure 2 left).
- mlp-to-attention ratio $r_{mlp/attn}$: fix $N_{non-embed}$, d_{model} and GQA= 4, vary n_{head} and intermediate size (Figure 2 right).
- GQA: fix $N_{\text{non-embed}}$, d_{model} and $r_{\text{mlp/attn}}$, vary n_{head} and number of key-value heads (Appendix E).

Figure 2 shows the ablation of varying hidden sizes $d_{\rm model}$ and mlp-to-attention $r_{\rm mlp/attn}$ on the LLaMA-3.1-8B model variants. We observe that larger hidden size (or fewer attention heads) and higher mlp-to-attention ratios improve inference throughput. Similar trends are observed in the LLaMA-3.2-1B and 3B model variants (Appendix E). These gains arise in part because larger $d_{\rm model}$ and higher $r_{\rm mlp/attn}$ reduce the total FLOPs, as detailed in the inference FLOPs analysis (Appendix H). In addition, these architectural choices shrink the KV cache, lowering I/O cost during inference and further improving throughput Adnan et al. (2024). Figure 10 in Appendix E presents the GQA ablation, confirming prior observations Ainslie et al. (2023) that increasing GQA consistently improves inference throughput. A comparable set of ablation experiments on Qwen3 models, also reported in Appendix E, further corroborates these findings.

3.3 A CONDITIONAL SCALING LAW

Improving inference efficiency should not come at the expense of significantly reducing model accuracy, making it crucial to understand how architectural choices affect accuracy and training loss. Because training large-scale language models is prohibitively expensive, a common strategy is to study smaller models and use scaling laws to extrapolate insights to larger scales, for example, the Chinchilla scaling laws (Hoffmann et al., 2022). However, incorporating multiple architectural factors into such laws remains challenging. To address this, we examine the effect of architectural choices on training loss L in a conditional manner, varying one factor at a time while keeping the others fixed.

hidden size d_{model} . We note that d_{model} generally scales linearly with $\sqrt{N_{\text{non-embed}}}$. Assuming squared attention weight matrices, the number of attention parameters N_{attn} can be expressed as

$$4d_{model}^2 \propto N_{\rm attn} = N_{\rm non-embed} \times \frac{r}{r+1},$$

where $r=r_{\text{mlp/attn}}$ is fixed, and the constant factor 4 arises from the query, key, value, and output projection layers in each attention block. To capture this scaling behavior, we normalize d_{model} by $\sqrt{N_{\text{non-embed}}}$ and examine its relation to loss L in Figure 3. The resulting U-shaped curves $L(d/\sqrt{N}\mid r,N,D)$ exhibit nearly identical optima across different model sizes. Moreover, Figure 3 confirms that excessively large hidden sizes, which reduce the number of attention heads n_{head} , can degrade accuracy—a phenomenon consistently observed in prior analyses of transformer capacity and head allocation (Kaplan et al., 2020; Hoffmann et al., 2022).

mlp-to-attention ratio $r_{\text{mlp/attn}}$. Figure 4 illustrates how the loss varies with $r_{\text{mlp/attn}}$, conditioned on d_{model} fixed at different levels, where we consistently observe a U-shaped curve $L(r \mid d/\sqrt{N}, N, D)$. While the attention mechanism is central to the success of transformers (Vaswani, 2017), recent open-weight models have allocated a progressively smaller fraction of parameters to attention as overall model size increases (e.g., LLaMA and Qwen families). Our analysis indicates that this trend is not universally optimal: there exists an interior optimum in the allocation of attention parameters, and deviating from it in either direction degrades model performance. This suggests that careful tuning of the mlp-to-attention ratio is critical for scaling transformers effectively.

As shown in Figures 3 and 4, both hidden size and the MLP-to-attention ratio exhibit U-shaped relationships with training loss. To capture these trends, we fit the function $c_0 + c_1 \log x + c_2/x$

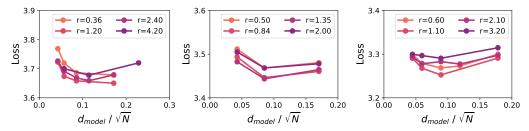


Figure 3: Loss vs. hidden size: (Left) 80M model variants; (Center) 145M model variants; (Right) 297M model variants. Across model sizes, the relationship between training loss and $d_{\rm model}/\sqrt{N}$ exhibits a consistent U-shaped curve when architectural factors such as GQA and the MLP-to-attention ratio are held fixed. The legend denotes the MLP-to-attention ratio $r = r_{\rm mlp/attn}$ for each model.

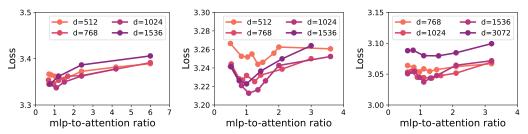


Figure 4: Loss vs. MLP-to-attention ratio: (Left) 80M model variants; (Center) 145M model variants; (Right) 297M model variants. Across model sizes, the relationship between training loss and $r_{\rm mlp/attn}$ exhibits a consistent U-shaped curve when architectural factors such as GQA and hidden size are held fixed. The legend denotes the hidden size $d = d_{\rm model}$ for each model.

separately for $x=r_{\text{mlp/attn}}$ and $d_{\text{model}}/\sqrt{N_{\text{non-embed}}}$. This formulation effectively models the U-shaped behavior while ensuring sublinear growth as x increases. However, incorporating $r_{\text{mlp/attn}}$, d_{model} , N, and D into a unified, architecture-aware scaling law remains challenging. Since fitting a single all-purpose scaling law $L(d/\sqrt{N},r,N,D)$ is unrealistic across all possible configurations, we instead propose a two-step conditional approach:

- 1. For given N and D, obtain the optimal loss $L_{\rm opt}(N,D)=\min L(N,D)=\min \left(E+\frac{A}{N^{\alpha}}+\frac{B}{D^{\beta}}\right)$ from the Chinchilla scaling law (Eq. 1) as a reference point.
- 2. Calibrate the loss of architectural variants $L(d/\sqrt{N}, r \mid N, D)$ relative to this reference.

We focus on two simple calibration schemes:

• (multiplicative)

$$L(d/\sqrt{N}, r \mid N, D) = (a_0 + a_1 \log(\frac{d}{\sqrt{N}}) + a_2 \frac{\sqrt{N}}{d}) \cdot (b_0 + b_1 \log r + \frac{b_2}{r}) \cdot L_{\text{opt}}$$
 (3)

• (additive)
$$L(d/\sqrt{N},r\mid N,D)=(a_0+a_1\log(\frac{d}{\sqrt{N}})+a_2\frac{\sqrt{N}}{d})+(b_1\log r+\frac{b_2}{r})+L_{\mathrm{opt}}$$

Here, a_i and b_i are learnable parameters that are shared across all N, D. Unlike the unified formulation, the conditional scaling law assumes that the effects of $r_{\rm mlp/attn}$ and $d_{\rm model}$ on loss are separable. We further ablate joint, non-separable formulations in Appendix G, where we find that they yield inferior predictive performance.

3.4 SEARCHING FOR INFERENCE-EFFICIENT ACCURATE MODELS

With the conditional scaling law, we can identify architectures that are both inference-efficient and accurate by solving the following optimization problem: given N, D, and a set of architectural choices P,

$$\operatorname{argmax}_{P} I_{N}(P), \quad \text{s.t.} \quad L(P \mid N, D) \leq L_{t},$$
 (4)

where $I_N(P)$ denotes the inference efficiency of an architecture P with total $N_{\text{non-embed}}$ parameters, and L_t , $(\geq L_{\text{opt}})$ is the maximum allowable training loss.

As shown in Figure 10 (Appendix E), GQA has a substantial impact on inference efficiency; However, unlike hidden size and the mlp-to-attention ratio, GQA does not exhibit a consistent relationship with loss (Figure 14) and is highly variable, making it challenging to identify settings that achieve both accuracy and efficiency. Fortunately, the search space for GQA is relatively small once $N_{\rm non-embed}$, $d_{\rm model}$, and $r_{\rm mlp/attn}$ are fixed, since GQA must be a prime factor of the number of attention heads $n_{\rm head}$. In practice, we perform a local GQA search by enumerating feasible values and applying early stopping once performance falls below that of the GQA= 4 baseline. Algorithm 1 summarizes our overall framework for identifying inference-efficient and accurate architectures.

Algorithm 1: Searching for Inference-Efficient Accurate Model

Input: Model parameters N, training tokens D, target loss L_t ; inference efficiency $I_N(\cdot)$; optional: the optimal loss $L_{\text{opt}}(N,D)$

Train smaller models to fit the Chinchilla scaling laws (Eq. 1) if $L_{\rm opt}(N,D)$ is unavailable Solve the constrained optimization (Eq. 4) for $d_{\rm model}$, $r_{\rm mlp/attn}$ and corresponding architecture P Perform a local search over GQA values with early stopping to maximize inference efficiency **return** Final model architecture $\{P, {\rm GQA}\}$

4 EXPERIMENT SETUP

We first detail the experimental setup of training, inference, and downstream task evaluation, and then describe how we derive the conditional scaling law and scale up to larger sizes.

Training Setup. We sample the training data from Dolma-v1.7 Soldaini et al. (2024), which contains data from 15 different sources. Tokens are sampled with probability proportional to each source's contribution, ensuring the sampled dataset preserves a similar distribution to Dolma-v1.7. We train decoder-only LLaMA-3.2 (Dubey et al., 2024) style transformers with $N_{\rm non-embed}$ in $\{80\mathrm{M}, 145\mathrm{M}, 297\mathrm{M}, 1\mathrm{B}, 3\mathrm{B}\}$, for each $N_{\rm non-embed}$, we obtain model architecture candidates by varying hidden size $d_{\rm model}/\sqrt{N_{\rm non-embed}}$ and mlp-to-attention ratio $r_{\rm mlp/attn}$. (changing intermediate size and number of attention heads $n_{\rm head}$) while holding other architectural factors fixed e.g. GQA= 4. A full list of over 200 model architectures used can be found in Appendix C. All models are trained on $100N_{\rm non-emb}$ tokens (5× Chinchilla optimal) to ensure convergence. We tuned training hyperparameters (mainly following prior work Chen et al. (2025)), with a full list in Appendix D.

Inference Setup. We evaluate the inference efficiency using the vLLM framework Kwon et al. (2023). By default, inputs consist of 4096 tokens and outputs of 1024 tokens. We report the averaged inference throughput (tokens/second) from 5 repeated runs. Unless otherwise specified, all experiments are conducted on NVIDIA Ampere A100 GPUs (40GB).

LLM Evaluation Setup. Following prior works Biderman et al. (2023); Zhang et al. (2024), we evaluate pretrained models in the zero-shot setting using lm-evaluation-harness² on nine benchmarks: ARC-Easy Clark et al. (2018), ARC-Challenge Clark et al. (2018), LAM-BADA Paperno et al. (2016), HellaSwag Zellers et al. (2019), OpenBookQA Mihaylov et al. (2018), PIQA Bisk et al. (2020), SciQ Welbl et al. (2017), WinoGrande Sakaguchi et al. (2021), and CoQA Reddy et al. (2019).

Fitting Scaling Laws. Following Gadre et al. (2024); Bian et al. (2025), we use the Levenberg-Marquardt algorithm to fit the conditional scaling laws (Eq. 3). The Levenberg-Marquardt algorithm does least-squares curve fitting by estimating $\hat{\beta}$ as the solution to $\arg\min_{\beta}\sum_{i=1}^{m}\left[y_i-f(x_i,\beta)\right]^2$, where (x_i,y_i) are the observed data pairs. Note that instead of fitting the Chinchilla scaling law, we empirically searched over architecture variants to find the optimal loss $L_{\rm opt}(N,D)$ for $N_{\rm non-embed}$ <1B scale.

https://github.com/EleutherAI/lm-evaluation-harness

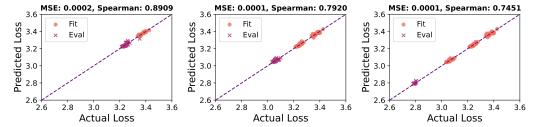


Figure 5: **Predictive performances** of the fitted conditional scaling law on: (left) Task 1: Fit on 80M, evaluate on 145M; (center) Task 2: Fit on 80, 145M, evaluate on 297M; (right) Task 3: Fit on 80, 145, 297M, evaluate on 1B. Orange dots denote fitting data points, and purple crosses indicate the test data points. We compare scaling-law predicted loss with actual pretraining loss of architectures and observed a consistently low MSE and high Spearman correlation across model scales.

We scale up the scale law fitting in the following progressive manner:

- (Task 1) fit on the 80M results and evaluate on 145M results;
- (Task 2) fit on 80, 145M results and evaluate on 297M results;
- (Task 3) fit on 80, 145, 297M results and evaluate on 1B results;

This ensures a robust and consistent way of scaling up the model sizes and evaluating our conditional scaling law. Following prior work Kumar et al. (2024), we evaluate the fitted scaling law with mean squared error (MSE) metric, defined as $\frac{1}{n}\sum_{i=1}^n(l_i-\hat{l}_i)^2$ where l_i denotes the actual loss and \hat{l}_i the predicted loss. We additionally report the Spearman's rank correlation coefficient Spearman (1961) to compare predicted and actual rankings. Both metrics are calculated on the val data points.

5 EXPERIMENT RESULTS

We begin by evaluating the predictive performances of the conditional scaling laws with multiplicative calibration. We then conduct ablation studies to assess the impact of data selection and to evaluate the performance of the scaling laws under additive calibration. Finally, we apply the fitted scaling laws to guide the training of large-scale models following the search framework (§5.1).

Predictive Accuracy. As Task 1-3 described in §4, we fit the conditional scaling laws on 80M, (80M, 145M), and (80M, 145M, 297M) loss-architecture data points, and subsequently evaluate on 145M, 297M, and 1B data, respectively. In Figure 5, the low MSE and high Spearman correlation in tasks across different model scales validate the effectiveness and strong predictive performance of the proposed conditional scaling laws.

Ablation of Outliers. The mlp-to-attention ratio $r_{\rm mlp/attn}$ of open-weights models typically fall between 0.5 and 5, for example, the mlp-to-attention ratio for LLaMA-3.2-1B, LLaMA-3.2-3B, and Qwen3-8B are 4.81, 1.5, and 4.67, respectively. In Figure 5, we fit the conditional scaling law using only model architectures with $r_{\rm mlp/attn} \in [0.5, 5]$. We ablate this choice by training model architectures with outlier $r_{\rm mlp/attn}$ below 0.5 and above 5 (such as 0.1, 12.6) in Appendix C. In Figure 15 (left) and Figure 15 (center) in Appendix G, we show on Task 3 a comparison of fitting the conditional scaling law without and with these outliers (with a clear Spearman correlation score degradation), which suggests to exclude extreme outliers for better predicted performances.

Ablation of Calibration. In Figure 15 (right), We ablate an alternative formulation of the scaling laws with additive calibration, as discussed in §3.3. The results on Task 3 show that multiplicative and additive calibrations achieve similar MSE and Spearman correlations, underscoring the robustness of our two-step reference plus calibration framework.

Table 1: **Large-Scale Model Results:** We evaluate the scaling laws and framework at the 1B and 3B scales by training Panda-1B, Surefire-1B, and Panda-3B, and compare them with LLaMA-3.2-1B and LLaMA-3.2-3B, respectively. The Avg. column reports the mean accuracy across the nine downstream tasks. Panda-1B and Panda-3B are trained using the optimal architectural configurations predicted by our scaling laws, whereas Surefire-1B and Surefire-3B satisfy the loss constraint in Eq. (4) and achieve Pareto optimality.

Models	d_{model}	$f_{ m size}$	n_{layers}	GQA	$d_{\mathrm{model}}/\sqrt{N}$	r	Loss (↓)	Avg. (†)
LLaMA-3.2-1B	2048	8192	16	4	0.066	4.80	2.803	54.9
Panda-1B	2560	4096	16	4	0.082	1.07	2.782	57.0
Surefire-1B	2560	6144	16	9	0.082	3.6	2.804	55.4
LLaMA-3.2-3B	3072	8192	28	3	0.058	4.80	2.625	61.9
Panda-3B	4096	4096	28	3	0.077	1	2.619	62.5
Surefire-3B	4096	4096	28	7	0.077	1	2.620	62.6

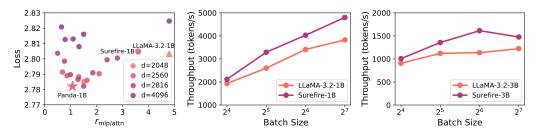


Figure 6: **Results for 1B and 3B models:** (left) Panda-1B closely follows the scaling law predictions for minimizing training loss. (center) Inference throughput comparison between LLaMA-3.2-1B and Surefire-1B, showing that Surefire-1B consistently achieves higher efficiency across batch sizes. (right) Inference throughput comparison between LLaMA-3.2-3B and Surefire-3B, demonstrating that Surefire-3B consistently delivers higher efficiency across all batch sizes.

5.1 OPTIMAL MODEL ARCHITECTURE

Validating the conditional scaling law. We validate the conditional scaling law at the 1B scale by applying multiplicative calibration on Task 3 using data from the (80M, 145M, and 297M) model variants. The learned parameters are

$$a_0 = 2.697, a_1 = 0.0974, a_2 = 0.0078, b_0 = 0.3870, b_1 = 0.0063, and b_2 = 0.0065.$$

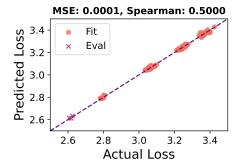
From this, we obtain the optimal architectural configuration of $d_{\rm model}/\sqrt{N}=0.08, r=1.032$ for 1B model by solving $\frac{\partial L}{\partial d_{\rm model}}=0$ and $\frac{\partial L}{\partial r}=0$. Using this configuration, we train a LLaMA-3.2-style 1B dense model on 100B tokens, denoted as Panda-1B. Panda-1B outperforms the open-weight LLaMA-3.2-1B baseline configs by 2.1% on average across downstream tasks (Table 1). Figure 6 (left) further confirms the effectiveness of the conditional scaling law by showing that Panda-1B achieves the lowest training loss among the exhaustively trained 1B variants under the same setup.

We also scale up our methodology to 3B models. Using the same approach but with data from the 80M, 145M, 297M, and 1B variants, we fit the scaling law and obtain $d_{\rm model}/\sqrt{N}=0.08$ and r=1.055 for the Panda 3B model. Trained on 100B tokens, Panda-3B outperforms the open weight LLaMA-3.2-3B configuration by 0.6% on average across downstream tasks (Table 1).

With all components in place, we apply the search framework for inference-efficient and accurate models (Alg. 1). For the $N_{\rm non-embed}=1\rm B$ and 3B setting trained on 100B tokens, we set the target loss L_t to match the training loss achieved by the LLaMA-3.2-1B and LLaMA-3.2-3B architectures, respectively. Although inference efficiency $I_N(P)$ could, in principle, be expressed analytically, it depends heavily on hardware and inference configurations. Therefore, rather than solving for $I_N(P)$ directly, we search over feasible configurations P_i that satisfy the loss constraint and select Pareto-optimal points, which we denote as Surefire-1B and Surefire-1B and Surefire-1B and Surefire-1B

Table 2: **3B Model Ablation Study:** We evaluate the robustness of scaling laws at 3B scale by training Panda-3B and Panda-3B°, and compare them with LLaMA-3.2-3B. The Avg. column reports the mean accuracy across the nine downstream tasks. Panda-3B represents the optimal architectural configuration predicted by the conditional scaling laws fitted using the 80M, 145M, and 297M model data, whereas Panda-3B° corresponds to the optimal configuration predicted from scaling laws fitted using the 1B model data.

Models	d_{model}	$f_{ m size}$	n_{layers}	GQA	$d_{\mathrm{model}}/\sqrt{N}$	r	Loss (↓)	Avg. (†)
LLaMA-3.2-3B	3072	8192	28	3	0.058	4.80	2.625	61.9
Panda-3B	4096	4096	28	3	0.077	1	2.619	62.5
Panda-3B°	4096	4608	28	3	0.076	1.23	2.606	62.5



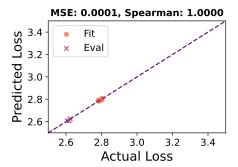


Figure 7: Effect of the Fitting Dataset on Predictive Performance vs (left) Fit on 80, 145, 297M, 1B, evaluate on 3B; (right) Fit on 1B, evaluate on 3B. Orange dots denote fitting data points, and purple crosses indicate the test data points. We compare scaling-law predicted loss with actual pretraining loss of architectures and we observe that fitting the scaling laws with only 1B model data yields lower MSE and higher Spearman correlation for the 3B model loss prediction.

3B outperform LLaMA-3.2-1B and LLaMA-3.2-3B on downstream tasks (Table 1) and deliver up to 42% higher inference throughput (Figure 6, center and right). Detailed downstream task accuracies are provided in Appendix I.

Ablation of fitting data. While we adopt a progressive strategy for selecting fitting data across tasks (§4), results from small models (e.g., 80M) may not reliably predict behavior at larger scales such as 3B. To examine this, we perform an ablation study by fitting the conditional scaling law for the 3B model using only results from the 1B variant. As shown in Figure 7, fitting with only the 1B data achieves a lower MSE and a higher Spearman correlation when predicting the 3B loss. This indicates that the coefficients of the conditional scaling law may shift as model size increases.

We therefore refit the law with multiplicative calibration using only the 1B variants, yielding the coefficients

$$a_0 = 2.319, a_1 = 0.238, a_2 = 0.0176, b_0 = 0.5104, b_1 = 0.0051, and b_2 = 0.0062.$$

This produces an alternative optimal configuration for the 3B model, with $d_{\rm model}/\sqrt{N}=0.074$ and r=1.229. We train a 3B model (Panda-3B°) under this configuration on 100B tokens and compare it with both LLaMA-3.2-3B and Panda-3B (fitted from 80M, 145M, 297M, and 1B data). As shown in Table 2, Panda-3B° achieves a lower training loss and comparable downstream accuracy to Panda-3B, with detailed results given in Appendix I. These findings suggest that when scaling up, it is often sufficient, and sometimes preferable, to fit the law using models within a closer size range to the target, such as about one third of its scale.

6 RELATED WORK

Large Language Models. Transformers Vaswani (2017) have shown strong performance across diverse downstream tasks, such as text classification Wang (2018); Sarlin et al. (2020), mathematical reasoning Cobbe et al. (2021); Hendrycks et al. (2021), and code generation Chen et al. (2021); Austin et al. (2021); Jain et al. (2024). The Transformer architecture serves as the foundation for many leading large language models, including GPT Brown et al. (2020); Achiam et al. (2023), LLaMA Touvron et al. (2023), Gemma Team et al. (2024a), Qwen Yang et al. (2025), Kimi Team et al. (2025), and DeepSeek Liu et al. (2024a); Guo et al. (2025).

Scaling Laws for Language Models. Scaling laws are powerful tools to predict the performance of large language models. Existing scaling laws Hoffmann et al. (2022); Muennighoff et al. (2023); Sardana et al. (2023); Kumar et al. (2024); Gadre et al. (2024); Ruan et al. (2024) characterize how model performance varies with model size, dataset size, data quality, and compute budget. With the rise of Mixture-of-Experts (MoE) Shazeer et al. (2017); Guo et al. (2025), a powerful architecture for large language models, recent studies Krajewski et al. (2024); Abnar et al. (2025) extend scaling laws to account for the number of experts, expert granularity, active parameters, and sparsity.

Serving Systems. Due to the increased inference cost, many inference systems have been developed to speed up model serving Yu et al. (2022); Kwon et al. (2023); Zheng et al. (2023); Ye et al. (2025). Specifically, vLLM Kwon et al. (2023) proposes PagedAttention to manage KV cache memory more effectively, thereby improving throughput. Similarly, SGLang Zheng et al. (2023) introduces RadixAttention to achieve higher throughput and lower latency.

Inference-Efficient Model Design. Efforts to improve the inference efficiency of large language models generally fall into two categories: one line of work investigates the trade-offs across different model configurations Alabdulmohsin et al. (2023); Bian et al. (2025), while the other focuses on designing more efficient model architectures Xiao et al. (2023); Gu & Dao (2023); Gao et al. (2024b); Jiang et al. (2024); Liu et al. (2024b); Dao & Gu (2024); Xiao et al. (2024); Yuan et al. (2025); Chandrasegaran et al. (2025).

7 LIMITATIONS AND FUTURE WORK

While our team has made notable progress, several open challenges remain that offer promising directions for future research. First, due to limitations in resources and time, our evaluation does not extend to 7B models. Second, our analysis is restricted to dense models, and it remains unclear whether the results extend to Mixture of Experts (MoE) architectures Shazeer et al. (2017). While we report inference efficiency measurements for MoE models under varying architectural choices in Appendix J, we have not yet established scaling laws for MoE architectures. Third, we adopt the experimental setup from Chen et al. (2025), and it is uncertain whether different model architectures warrant different hyperparameter configurations. Finally, our analysis is limited to pre-training, and it remains unclear how the results would change under post-training.

8 Conclusion

This work explores the trade-off between model accuracy and inference cost under a fixed training budget. We begin by demonstrating how architectural choices influence both inference throughput and model accuracy. Building on this, we extend Chinchilla scaling laws to incorporate architectural factors and propose a framework for optimal model architecture search. Using the fitted scaling laws and our framework, we trained models up to 3B parameters, achieving up to 42% higher inference throughput and 2.1% accuracy gains across nine downstream tasks.

REPRODUCIBILITY STATEMENT

All experiments in this work were conducted using publicly available frameworks. Section 4 provides details of our training, inference, and evaluation setups. In particular, we used

Megatron-LM (Shoeybi et al., 2019) for model training, vLLM (Kwon et al., 2023) for efficient inference, and lm-eval-harness (Gao et al., 2024a) for standardized evaluations. To facilitate reproducibility, we will release configuration files and scripts.

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A LLM USAGE

We used an LLM to improve the writing by correcting grammar in our draft. It was not used to generate research ideas.

B OPEN-WEIGHTED MODEL ARCHITECTURES

Table 3 presents an overview of the open-weight model architectures utilized in this paper.

Table 3: **Open-Weighted Model Architectures:** We list the architectural configurations of all models used in this paper. n_{layers} is the number of layers, d_{model} is the hidden size, n_{heads} is the number of attention heads, and f_{size} is the intermediate size.

Model Name	n_{layers}	d_{model}	n_{heads}	$f_{ m size}$	GQA
Qwen2.5-1.5B	28	1536	12	8960	6
Qwen3-0.6B	28	1024	16	3072	2

C MODEL ARCHITECTURES

Table 4 provides an overview of the model architectures, all configured with GQA = 4 and employing LLaMA-3.2 as the tokenizer.

Table 4: **Model Architectures:** We list the architectural configurations of all models trained in this paper. $N_{\text{non-embed}}$ is the total number of non-embedding parameters, n_{layers} is the number of layers, d_{model} is the hidden size, n_{heads} is the number of attention heads, f_{size} is the intermediate size, and $r_{\text{mlp/attn}}$ is the MLP-to-attention ratio.

$N_{ m non-embed}$	Variant	n_{layers}	d_{model}	n_{heads}	$f_{ m size}$	$d_{\mathrm{model}}/\sqrt{N}$	$r_{ m mlp/attn}$
80M	v1	12	768	16	2048	0.086	2.40
80M	v2	12	768	4	2688	0.086	12.6
80M	v3	12	768	8	2560	0.085	6.00
80M	v4	12	768	24	1536	0.087	1.20
80M	v5	12	768	32	1152	0.086	0.68
80M	v6	12	768	40	768	0.086	0.36
80M	v7	12	768	48	256	0.087	0.10
80M	v8	12	384	32	4096	0.043	2.40
80M	v9	12	384	8	5376	0.043	12.6
80M	v10	12	384	16	5120	0.042	6.00
80M	v11	12	384	48	3072	0.044	1.20
80M	v12	12	384	64	2304	0.043	0.68
80M	v13	12	384	80	1536	0.043	0.36
80M	v14	12	384	96	512	0.044	0.10
80M	v15	12	1536	8	1024	0.171	2.40
80M	v16	12	1536	4	1280	0.169	6.00
80M	v17	12	1536	12	768	0.174	1.20
80M	v18	12	1536	16	640	0.169	0.75
80M	v19	12	1536	20	384	0.171	0.36
80M	v20	12	1536	24	128	0.174	0.10
80M	v21	12	512	24	3072	0.057	2.40
80M	v22	12	512	12	3840	0.056	6.00
80M	v23	12	512	16	3584	0.057	4.20
80M	v24	12	512	36	2304	0.058	1.20
80M	v25	12	512	48	1792	0.057	0.70
80M	v26	12	512	60	1152	0.057	0.36

80M v27 12 512 72 384 0.058 0.10 80M v28 12 1024 8 1792 0.113 2.40 80M v30 12 1024 16 1280 0.115 1.50 80M v30 12 1024 24 896 0.114 0.70 80M v31 12 1024 36 256 0.114 0.73 80M v33 12 2048 8 640 0.226 4.20 80M v35 12 2048 8 640 0.231 1.50 80M v35 12 2048 16 256 0.226 4.20 80M v35 12 2048 8 640 0.231 1.50 80M v49 12 768 28 1408 0.086 0.94 80M v50 12 384 50 2816 0.043	$\overline{N_{ ext{non-embed}}}$	Variant	$n_{ m layers}$	d_{model}	$n_{ m heads}$	$f_{ m size}$	$d_{\mathrm{model}}/\sqrt{N}$	$r_{ m mlp/attn}$
80M v29 12 1024 8 1792 0.113 4.20 80M v30 12 1024 16 1280 0.115 1.50 80M v31 12 1024 24 896 0.114 0.70 80M v33 12 2048 4 896 0.226 4.20 80M v33 12 2048 8 640 0.221 1.50 80M v35 12 2048 16 256 0.226 0.30 80M v35 12 2048 16 256 0.226 0.30 80M v49 12 768 20 1792 0.086 1.68 80M v49 12 768 28 1408 0.086 1.68 80M v50 12 384 40 3584 0.043 1.68 80M v53 12 384 56 2816 0.043 <td>80M</td> <td>v27</td> <td>12</td> <td>512</td> <td>72</td> <td>384</td> <td>0.058</td> <td>0.10</td>	80M	v27	12	512	72	384	0.058	0.10
80M v30 12 1024 16 1280 0.115 1.50 80M v31 12 1024 24 896 0.114 0.70 80M v32 12 1024 36 256 0.114 0.13 80M v33 12 2048 4 896 0.226 4.20 80M v34 12 2048 8 640 0.231 1.50 80M v35 12 2048 16 256 0.226 0.30 80M v48 12 768 20 1792 0.086 1.68 80M v50 12 384 40 3584 0.043 1.68 80M v50 12 384 40 3584 0.043 1.68 80M v51 12 384 52 3072 0.043 1.11 80M v52 12 384 56 2816 0.043 0.94 80M v51 12 384 60 2560 0.058 1.50 80M v54 12 512 34 1920 0.058 8.58 80M v51 12 12 384 60 2560 0.058 1.50 80M v55 12 512 40 2176 0.057 1.02 80M v56 12 512 44 1920 0.058 0.82 80M v57 12 1024 20 1152 0.113 1.08 80M v57 12 1024 16 3072 0.085 3.60 145M v2 12 1024 8 3584 0.086 2.00 145M v4 12 1024 40 1792 0.085 0.84 145M v5 12 1024 48 1280 0.086 2.00 145M v6 12 1024 48 1280 0.086 0.50 145M v7 12 1024 48 1280 0.086 0.50 145M v7 12 1024 48 1280 0.086 0.50 145M v7 12 1024 48 1280 0.086 0.50 145M v8 12 512 32 644 0.043 3.60 145M v7 12 1024 48 1280 0.086 0.50 145M v8 12 512 32 644 0.043 0.084 1.35 145M v1 12 1024 48 1280 0.086 0.50 145M v8 12 512 32 6144 0.043 3.60 145M v9 12 512 16 7168 0.042 8.40 145M v1 12 1024 48 1280 0.086 0.50 145M v1 12 1024 48 1280 0.086 0.50 145M v1 12 512 48 5120 0.043 0.084 1.35 145M v1 12 512 48 5120 0.085 0.84 145M v1 12 512 18 64 4608 0.042 1.35 145M v1 12 512 18 80 3584 0.043 0.84 145M v1 12 512 18 80 3584 0.043 0.84 145M v1 12 512 48 8 1536 0.170 0.85 145M v1 12 512 48 8 1536 0.170 0.85 145M v1 12 512 48 8 1536 0.170 0.86 145M v1 12 512 64 4608 0.042 1.35 145M v1 12 512 64 4608 0.042 1.35 145M v1 12 512 64 4608 0.042 1.35 145M v1 12 512 768 8 56 2304 0.063 0.50 145M v2 12 1024 8 16 1152 0.168 1.35 145M v2 12 1024 8 24 600 0.172 0.00 145M v1 12 512 648 64 1152 0.168 1.35 145M v2 12 12 513 62 80 3584 0.063 0.15 145M v2 12 12 513 64 30 30 30 0.15 145M v2 12 12 513 64 30 30 30 0.15 145M v2 12 12 513 64 30 30 30 0.15 145M v2 12 12 513 64 30 30 0.172 0.064 145M v2 12 12 513 64 30 30 0.172 0.064 145M v2 12 2048 8 1536 0.170 0.168 1.35 145M v2 12 12 536 28 150 0.127 0.00	80M	v28	12	1024	12	1536	0.114	2.40
80M v31 12 1024 24 896 0.114 0.70 80M v32 12 1024 36 256 0.114 0.13 80M v33 12 2048 4 896 0.226 4.20 80M v34 12 2048 8 640 0.231 1.50 80M v35 12 2048 16 256 0.226 0.30 80M v49 12 768 28 1408 0.086 1.68 80M v49 12 768 28 1408 0.086 0.94 80M v50 12 384 56 2816 0.043 1.68 80M v53 12 384 56 2816 0.043 0.94 80M v53 12 384 56 2816 0.043 0.80 80M v56 12 512 32 22660 0.043 <td>80M</td> <td>v29</td> <td>12</td> <td>1024</td> <td>8</td> <td>1792</td> <td>0.113</td> <td>4.20</td>	80M	v29	12	1024	8	1792	0.113	4.20
80M v32 12 1024 36 256 0.114 0.13 80M v33 12 2048 4 896 0.226 4.20 80M v34 12 2048 8 640 0.221 1.50 80M v35 12 2048 16 256 0.226 0.30 80M v35 12 2048 16 256 0.226 0.30 80M v49 12 768 28 1408 0.086 0.94 80M v50 12 384 40 3584 0.043 1.68 80M v51 12 384 56 2816 0.043 0.94 80M v53 12 384 56 2816 0.043 0.80 80M v53 12 384 60 2560 0.058 1.50 80M v55 12 512 44 1920 0.058	80M	v30	12	1024	16	1280	0.115	1.50
80M v33 12 2048 4 896 0.226 4.20 80M v34 12 2048 8 640 0.231 1.50 80M v48 12 768 20 1792 0.086 1.68 80M v49 12 768 20 1792 0.086 0.94 80M v49 12 768 20 1792 0.086 0.94 80M v50 12 384 40 3584 0.043 1.68 80M v51 12 384 56 2816 0.043 0.94 80M v52 12 384 56 2816 0.043 0.94 80M v55 12 512 32 2560 0.058 1.50 80M v55 12 512 40 2176 0.057 1.02 80M v57 12 1024 20 1152 0.13	80M	v31	12	1024	24	896	0.114	0.70
80M v34 12 2048 8 640 0.231 1.50 80M v35 12 2048 16 256 0.226 0.30 80M v49 12 768 28 1408 0.086 0.94 80M v50 12 384 40 3584 0.043 1.68 80M v50 12 384 52 3072 0.043 1.11 80M v53 12 384 56 2816 0.043 0.94 80M v53 12 384 60 2560 0.043 0.80 80M v54 12 512 32 2560 0.058 1.50 80M v55 12 512 40 2176 0.058 0.82 80M v56 12 512 44 1920 0.058 0.82 80M v57 12 1024 20 1152 0.113 <td>80M</td> <td>v32</td> <td>12</td> <td>1024</td> <td>36</td> <td>256</td> <td>0.114</td> <td>0.13</td>	80M	v32	12	1024	36	256	0.114	0.13
80M v35 12 2048 16 256 0.226 0.30 80M v48 12 768 20 1792 0.086 1.68 80M v49 12 768 28 1408 0.086 0.94 80M v50 12 384 40 3584 0.043 1.68 80M v51 12 384 52 3072 0.043 1.11 80M v53 12 384 60 2560 0.043 0.80 80M v54 12 512 32 2560 0.058 1.50 80M v54 12 512 44 1920 0.058 1.50 80M v56 12 512 40 2175 0.057 1.02 80M v57 12 1024 20 1152 0.113 1.08 145M v1 12 1024 20 1152 0.113<	80M	v33	12	2048	4	896		4.20
80M v48 12 768 20 1792 0.086 1.68 80M v49 12 768 28 1408 0.086 0.94 80M v50 12 384 40 3584 0.043 1.11 80M v51 12 384 52 3072 0.043 1.51 80M v52 12 384 56 2816 0.043 0.94 80M v53 12 384 60 2560 0.043 0.80 80M v54 12 512 32 2560 0.057 1.02 80M v55 12 512 44 1920 0.058 8.2 80M v57 12 1024 20 1152 0.13 1.08 80M v57 12 1024 21 0.13 1.35 145M v1 12 1024 48 1380 0.085 3.60 <td>80M</td> <td>v34</td> <td>12</td> <td>2048</td> <td>8</td> <td>640</td> <td>0.231</td> <td>1.50</td>	80M	v34	12	2048	8	640	0.231	1.50
80M v49 12 768 28 1408 0.086 0.94 80M v50 12 384 40 3584 0.043 1.68 80M v51 12 384 52 3072 0.043 1.11 80M v53 12 384 60 2560 0.043 0.80 80M v54 12 512 32 2560 0.058 1.50 80M v55 12 512 40 2176 0.057 1.02 80M v56 12 512 44 1920 0.058 0.82 80M v57 12 1024 20 1152 0.113 1.08 145M v1 12 1024 20 1152 0.113 1.08 145M v3 12 1024 24 2560 0.086 2.00 145M v3 12 1024 24 2560 0.08	80M	v35	12	2048	16	256	0.226	0.30
80M v50 12 384 40 3584 0.043 1.68 80M v51 12 384 52 3072 0.043 0.94 80M v52 12 384 56 2816 0.043 0.94 80M v53 12 384 60 2560 0.043 0.80 80M v54 12 512 32 2560 0.058 1.50 80M v56 12 512 40 2176 0.057 1.02 80M v56 12 512 44 1920 0.058 0.82 80M v57 12 1024 20 1152 0.113 1.08 145M v1 12 1024 20 1152 0.113 1.08 145M v1 12 1024 24 2560 0.086 2.00 145M v4 12 1024 32 2304 0.08	80M	v48	12	768	20	1792	0.086	1.68
80M v51 12 384 52 3072 0.043 1.11 80M v52 12 384 56 2816 0.043 0.94 80M v53 12 384 60 2560 0.043 0.80 80M v54 12 512 32 2560 0.058 1.50 80M v55 12 512 40 2176 0.057 1.02 80M v56 12 512 44 1920 0.058 0.82 80M v57 12 1024 20 1152 0.113 1.08 145M v1 12 1024 20 1152 0.113 1.08 145M v2 12 1024 8 3584 0.084 8.40 145M v4 12 1024 22 2560 0.086 2.00 145M v4 12 1024 40 1792 0.08	80M	v49	12	768	28	1408	0.086	0.94
80M v52 12 384 56 2816 0.043 0.94 80M v53 12 384 60 2560 0.058 1.50 80M v54 12 512 32 2560 0.057 1.02 80M v56 12 512 40 2176 0.057 1.02 80M v56 12 512 44 1920 0.058 0.82 80M v56 12 512 44 1920 0.058 0.82 80M v57 12 1024 20 1152 0.113 1.08 145M v1 12 1024 40 1792 0.085 3.60 145M v3 12 1024 42 2560 0.084 8.40 145M v4 12 1024 42 2560 0.086 0.50 145M v6 12 1024 48 1280 0.0	80M	v50	12	384	40	3584	0.043	1.68
80M v54 12 384 60 2560 0.043 0.80 80M v54 12 512 32 2560 0.058 1.50 80M v55 12 512 40 2176 0.057 1.02 80M v56 12 512 44 1920 0.058 0.82 80M v57 12 1024 20 1152 0.113 1.08 145M v1 12 1024 20 1152 0.113 1.08 145M v2 12 1024 20 1152 0.013 3.60 145M v3 12 1024 42 2560 0.086 2.00 145M v4 12 1024 32 2304 0.084 1.35 145M v5 12 1024 40 1792 0.085 0.50 145M v6 12 1024 43 1280 0	80M	v51	12	384	52	3072	0.043	1.11
80M v54 12 512 32 2560 0.058 1.50 80M v56 12 512 40 2176 0.057 1.02 80M v56 12 512 44 1920 0.058 0.82 80M v57 12 1024 20 1152 0.113 1.08 145M v1 12 1024 16 3072 0.085 3.60 145M v2 12 1024 8 3584 0.084 8.40 145M v4 12 1024 24 2560 0.086 2.00 145M v4 12 1024 40 1792 0.085 0.84 145M v6 12 1024 40 1792 0.085 0.84 145M v6 12 1024 48 1280 0.086 0.50 145M v7 12 1024 48 1280 0	80M	v52	12	384	56	2816	0.043	0.94
80M v55 12 512 40 2176 0.057 1.02 80M v56 12 512 44 1920 0.058 0.82 80M v57 12 1024 20 1152 0.113 1.08 145M v1 12 1024 20 1152 0.113 1.08 145M v1 12 1024 20 1152 0.085 3.60 145M v2 12 1024 8 3584 0.084 8.40 145M v4 12 1024 22 2004 0.084 1.35 145M v4 12 1024 40 1792 0.085 0.84 145M v6 12 1024 48 1280 0.086 0.50 145M v7 12 1024 48 1280 0.086 0.50 145M v7 12 1024 48 1280	80M	v53	12	384	60	2560	0.043	0.80
80M v56 12 512 44 1920 0.058 0.82 80M v57 12 1024 20 1152 0.113 1.08 145M v1 12 1024 16 3072 0.085 3.60 145M v2 12 1024 8 3584 0.084 8.40 145M v3 12 1024 24 2560 0.086 2.00 145M v4 12 1024 32 2304 0.084 1.35 145M v6 12 1024 40 1792 0.085 0.84 145M v6 12 1024 40 1792 0.085 0.50 145M v7 12 1024 64 512 0.085 0.15 145M v8 12 512 32 6144 0.043 3.60 145M v10 12 512 48 5120 0	80M	v54	12	512	32	2560	0.058	1.50
80M v57 12 1024 20 1152 0.113 1.08 145M v1 12 1024 16 3072 0.085 3.60 145M v2 12 1024 8 3584 0.084 8.40 145M v3 12 1024 24 2560 0.086 2.00 145M v4 12 1024 32 2304 0.084 1.35 145M v5 12 1024 40 1792 0.085 0.84 145M v6 12 1024 48 1280 0.086 0.50 145M v7 12 1024 48 1280 0.086 0.50 145M v7 12 1024 48 1280 0.085 0.15 145M v8 12 512 32 6144 0.043 3.60 145M v10 12 512 48 5120 <td< td=""><td>80M</td><td>v55</td><td>12</td><td>512</td><td>40</td><td>2176</td><td>0.057</td><td>1.02</td></td<>	80M	v55	12	512	40	2176	0.057	1.02
80M v57 12 1024 20 1152 0.113 1.08 145M v1 12 1024 16 3072 0.085 3.60 145M v2 12 1024 8 3584 0.084 8.40 145M v3 12 1024 24 2560 0.086 2.00 145M v4 12 1024 32 2304 0.084 1.35 145M v5 12 1024 40 1792 0.085 0.84 145M v6 12 1024 48 1280 0.086 0.50 145M v7 12 1024 48 1280 0.086 0.50 145M v7 12 1024 48 1280 0.085 0.15 145M v8 12 512 32 6144 0.043 3.60 145M v10 12 512 48 5120 <td< td=""><td>80M</td><td>v56</td><td>12</td><td>512</td><td>44</td><td>1920</td><td>0.058</td><td>0.82</td></td<>	80M	v56	12	512	44	1920	0.058	0.82
145M v2 12 1024 8 3584 0.084 8.40 145M v3 12 1024 24 2560 0.086 2.00 145M v4 12 1024 32 2304 0.084 1.35 145M v5 12 1024 40 1792 0.085 0.84 145M v6 12 1024 48 1280 0.086 0.50 145M v7 12 1024 64 512 0.085 0.15 145M v8 12 512 32 6144 0.043 3.60 145M v10 12 512 48 5120 0.043 2.00 145M v11 12 512 64 4608 0.042 1.35 145M v11 12 512 80 3584 0.043 0.84 145M v13 12 512 80 3584					20		0.113	
145M v3 12 1024 24 2560 0.086 2.00 145M v4 12 1024 32 2304 0.084 1.35 145M v5 12 1024 40 1792 0.085 0.84 145M v6 12 1024 48 1280 0.086 0.50 145M v7 12 1024 64 512 0.085 0.15 145M v8 12 512 32 6144 0.043 3.60 145M v9 12 512 16 7168 0.042 8.40 145M v10 12 512 48 5120 0.043 2.00 145M v11 12 512 80 3584 0.043 0.84 145M v13 12 512 96 2560 0.043 0.50 145M v14 12 512 128 1024 <td< td=""><td>145M</td><td>v1</td><td>12</td><td>1024</td><td>16</td><td>3072</td><td>0.085</td><td>3.60</td></td<>	145M	v1	12	1024	16	3072	0.085	3.60
145M v4 12 1024 32 2304 0.084 1.35 145M v5 12 1024 40 1792 0.085 0.84 145M v6 12 1024 48 1280 0.086 0.50 145M v7 12 1024 64 512 0.085 0.15 145M v8 12 512 32 6144 0.043 3.60 145M v9 12 512 16 7168 0.042 8.40 145M v10 12 512 48 5120 0.043 2.00 145M v11 12 512 80 3584 0.043 0.84 145M v13 12 512 80 3584 0.043 0.84 145M v13 12 512 286 0.0043 0.15 145M v14 12 512 128 1024 0.043	145M	v2	12	1024	8	3584	0.084	8.40
145M v5 12 1024 40 1792 0.085 0.84 145M v6 12 1024 48 1280 0.086 0.50 145M v7 12 1024 64 512 0.085 0.15 145M v8 12 512 32 6144 0.043 3.60 145M v10 12 512 48 5120 0.043 2.00 145M v10 12 512 48 5120 0.043 2.00 145M v11 12 512 80 3584 0.043 0.84 145M v12 12 512 80 3584 0.043 0.50 145M v13 12 512 96 2560 0.043 0.50 145M v14 12 2048 8 1536 0.170 3.60 145M v16 12 2048 4 1792 <td< td=""><td>145M</td><td>v3</td><td>12</td><td>1024</td><td>24</td><td>2560</td><td>0.086</td><td>2.00</td></td<>	145M	v3	12	1024	24	2560	0.086	2.00
145M v6 12 1024 48 1280 0.086 0.50 145M v7 12 1024 64 512 0.085 0.15 145M v8 12 512 32 6144 0.043 3.60 145M v10 12 512 16 7168 0.042 8.40 145M v10 12 512 48 5120 0.043 2.00 145M v11 12 512 80 3584 0.043 0.84 145M v13 12 512 96 2560 0.043 0.50 145M v13 12 512 296 2560 0.043 0.15 145M v14 12 2048 8 1536 0.170 3.60 145M v15 12 2048 8 1536 0.170 3.60 145M v16 12 2048 8 1536 <t< td=""><td>145M</td><td>v4</td><td>12</td><td>1024</td><td>32</td><td>2304</td><td>0.084</td><td>1.35</td></t<>	145M	v4	12	1024	32	2304	0.084	1.35
145M v7 12 1024 64 512 0.085 0.15 145M v8 12 512 32 6144 0.043 3.60 145M v9 12 512 16 7168 0.042 8.40 145M v10 12 512 48 5120 0.043 2.00 145M v11 12 512 64 4608 0.042 1.35 145M v12 12 512 80 3584 0.043 0.50 145M v13 12 512 96 2560 0.043 0.50 145M v14 12 512 128 1024 0.043 0.15 145M v14 12 512 128 1024 0.043 0.15 145M v15 12 2048 8 1536 0.170 3.60 145M v16 12 2048 4 1792 <t< td=""><td>145M</td><td>v5</td><td>12</td><td>1024</td><td>40</td><td>1792</td><td>0.085</td><td>0.84</td></t<>	145M	v5	12	1024	40	1792	0.085	0.84
145M v8 12 512 32 6144 0.043 3.60 145M v9 12 512 16 7168 0.042 8.40 145M v10 12 512 48 5120 0.043 2.00 145M v11 12 512 64 4608 0.042 1.35 145M v12 12 512 80 3584 0.043 0.84 145M v13 12 512 96 2560 0.043 0.50 145M v14 12 512 128 1024 0.043 0.15 145M v14 12 512 128 1024 0.043 0.15 145M v14 12 2048 8 1536 0.170 3.60 145M v16 12 2048 8 1536 0.170 3.60 145M v17 12 2048 12 1280	145M	v6	12	1024	48	1280	0.086	0.50
145M v9 12 512 16 7168 0.042 8.40 145M v10 12 512 48 5120 0.043 2.00 145M v11 12 512 64 4608 0.042 1.35 145M v12 12 512 80 3584 0.043 0.84 145M v13 12 512 96 2560 0.043 0.50 145M v14 12 512 128 1024 0.043 0.15 145M v15 12 2048 8 1536 0.170 3.60 145M v16 12 2048 8 1536 0.170 3.60 145M v16 12 2048 4 1792 0.168 8.40 145M v17 12 2048 16 1152 0.168 1.35 145M v18 12 2048 20 896	145M	v7	12	1024	64	512	0.085	0.15
145M v9 12 512 16 7168 0.042 8.40 145M v10 12 512 48 5120 0.043 2.00 145M v11 12 512 64 4608 0.042 1.35 145M v12 12 512 80 3584 0.043 0.84 145M v13 12 512 96 2560 0.043 0.50 145M v14 12 512 128 1024 0.043 0.15 145M v15 12 2048 8 1536 0.170 3.60 145M v16 12 2048 8 1536 0.170 3.60 145M v16 12 2048 4 1792 0.168 8.40 145M v17 12 2048 16 1152 0.168 1.35 145M v18 12 2048 20 896	145M	v8	12	512	32	6144	0.043	3.60
145M v11 12 512 64 4608 0.042 1.35 145M v12 12 512 80 3584 0.043 0.84 145M v13 12 512 96 2560 0.043 0.50 145M v14 12 512 128 1024 0.043 0.15 145M v15 12 2048 8 1536 0.170 3.60 145M v16 12 2048 4 1792 0.168 8.40 145M v17 12 2048 12 1280 0.172 2.00 145M v18 12 2048 16 1152 0.168 1.35 145M v19 12 2048 20 896 0.170 0.84 145M v20 12 2048 24 640 0.172 0.50 145M v21 12 2048 32 256		v9	12	512		7168		8.40
145M v12 12 512 80 3584 0.043 0.84 145M v13 12 512 96 2560 0.043 0.50 145M v14 12 512 128 1024 0.043 0.15 145M v15 12 2048 8 1536 0.170 3.60 145M v16 12 2048 4 1792 0.168 8.40 145M v16 12 2048 4 1792 0.168 8.40 145M v17 12 2048 12 1280 0.172 2.00 145M v18 12 2048 16 1152 0.168 1.35 145M v19 12 2048 20 896 0.170 0.84 145M v20 12 2048 24 640 0.172 0.50 145M v21 12 2048 32 256	145M	v10		512	48		0.043	2.00
145M v13 12 512 96 2560 0.043 0.50 145M v14 12 512 128 1024 0.043 0.15 145M v15 12 2048 8 1536 0.170 3.60 145M v16 12 2048 4 1792 0.168 8.40 145M v17 12 2048 12 1280 0.172 2.00 145M v18 12 2048 16 1152 0.168 1.35 145M v19 12 2048 20 896 0.170 0.84 145M v20 12 2048 24 640 0.172 0.50 145M v21 12 2048 32 256 0.170 0.15 145M v21 12 2048 32 256 0.170 0.15 145M v22 12 768 24 3840	145M	v11	12	512	64	4608	0.042	1.35
145M v14 12 512 128 1024 0.043 0.15 145M v15 12 2048 8 1536 0.170 3.60 145M v16 12 2048 4 1792 0.168 8.40 145M v17 12 2048 12 1280 0.172 2.00 145M v18 12 2048 16 1152 0.168 1.35 145M v19 12 2048 20 896 0.170 0.84 145M v20 12 2048 24 640 0.172 0.50 145M v21 12 2048 32 256 0.170 0.15 145M v22 12 768 24 3840 0.065 3.00 145M v23 12 768 32 3584 0.063 2.10 145M v24 12 768 48 2560	145M	v12	12		80	3584	0.043	0.84
145M v15 12 2048 8 1536 0.170 3.60 145M v16 12 2048 4 1792 0.168 8.40 145M v17 12 2048 12 1280 0.172 2.00 145M v18 12 2048 16 1152 0.168 1.35 145M v19 12 2048 20 896 0.170 0.84 145M v20 12 2048 24 640 0.172 0.50 145M v21 12 2048 32 256 0.170 0.15 145M v21 12 2048 32 256 0.170 0.15 145M v22 12 768 24 3840 0.065 3.00 145M v23 12 768 32 3584 0.063 2.10 145M v24 12 768 40 3072	145M	v13	12	512	96	2560	0.043	0.50
145M v16 12 2048 4 1792 0.168 8.40 145M v17 12 2048 12 1280 0.172 2.00 145M v18 12 2048 16 1152 0.168 1.35 145M v19 12 2048 20 896 0.170 0.84 145M v20 12 2048 24 640 0.172 0.50 145M v21 12 2048 32 256 0.170 0.15 145M v21 12 2048 32 256 0.170 0.15 145M v22 12 768 24 3840 0.065 3.00 145M v23 12 768 32 3584 0.063 2.10 145M v24 12 768 40 3072 0.064 1.44 145M v25 12 768 48 2560	145M	v14	12	512	128	1024	0.043	0.15
145M v17 12 2048 12 1280 0.172 2.00 145M v18 12 2048 16 1152 0.168 1.35 145M v19 12 2048 20 896 0.170 0.84 145M v20 12 2048 24 640 0.172 0.50 145M v21 12 2048 32 256 0.170 0.15 145M v21 12 2048 32 256 0.170 0.15 145M v22 12 768 24 3840 0.065 3.00 145M v23 12 768 32 3584 0.063 2.10 145M v24 12 768 40 3072 0.064 1.44 145M v25 12 768 48 2560 0.065 1.00 145M v26 12 768 56 2304	145M	v15	12	2048	8	1536	0.170	3.60
145M v18 12 2048 16 1152 0.168 1.35 145M v19 12 2048 20 896 0.170 0.84 145M v20 12 2048 24 640 0.172 0.50 145M v21 12 2048 32 256 0.170 0.15 145M v21 12 2048 32 256 0.170 0.15 145M v22 12 768 24 3840 0.065 3.00 145M v23 12 768 32 3584 0.063 2.10 145M v24 12 768 40 3072 0.064 1.44 145M v25 12 768 48 2560 0.065 1.00 145M v26 12 768 56 2304 0.063 0.77 145M v27 12 768 64 1792	145M	v16	12	2048	4	1792	0.168	8.40
145M v19 12 2048 20 896 0.170 0.84 145M v20 12 2048 24 640 0.172 0.50 145M v21 12 2048 32 256 0.170 0.15 145M v22 12 768 24 3840 0.065 3.00 145M v23 12 768 32 3584 0.063 2.10 145M v24 12 768 40 3072 0.064 1.44 145M v25 12 768 48 2560 0.065 1.00 145M v26 12 768 48 2560 0.065 1.00 145M v26 12 768 64 1792 0.064 0.53 145M v27 12 768 64 1792 0.064 0.53 145M v28 12 1536 12 1920	145M	v17	12	2048	12	1280	0.172	2.00
145M v20 12 2048 24 640 0.172 0.50 145M v21 12 2048 32 256 0.170 0.15 145M v22 12 768 24 3840 0.065 3.00 145M v23 12 768 32 3584 0.063 2.10 145M v24 12 768 40 3072 0.064 1.44 145M v25 12 768 48 2560 0.065 1.00 145M v26 12 768 48 2560 0.065 1.00 145M v26 12 768 56 2304 0.063 0.77 145M v27 12 768 64 1792 0.064 0.53 145M v28 12 1536 12 1920 0.129 3.00 145M v29 12 1536 20 1536	145M	v18	12	2048	16	1152	0.168	1.35
145M v21 12 2048 32 256 0.170 0.15 145M v22 12 768 24 3840 0.065 3.00 145M v23 12 768 32 3584 0.063 2.10 145M v24 12 768 40 3072 0.064 1.44 145M v25 12 768 48 2560 0.065 1.00 145M v26 12 768 56 2304 0.063 0.77 145M v27 12 768 64 1792 0.064 0.53 145M v28 12 1536 12 1920 0.129 3.00 145M v29 12 1536 16 1792 0.127 2.10 145M v30 12 1536 20 1536 0.128 1.44 145M v31 12 1536 24 1280	145M	v19	12	2048	20	896	0.170	0.84
145M v22 12 768 24 3840 0.065 3.00 145M v23 12 768 32 3584 0.063 2.10 145M v24 12 768 40 3072 0.064 1.44 145M v25 12 768 48 2560 0.065 1.00 145M v26 12 768 56 2304 0.063 0.77 145M v27 12 768 64 1792 0.064 0.53 145M v28 12 1536 12 1920 0.129 3.00 145M v29 12 1536 16 1792 0.127 2.10 145M v30 12 1536 20 1536 0.128 1.44 145M v31 12 1536 24 1280 0.129 1.00 145M v32 12 1536 28 1152	145M	v20	12	2048	24	640	0.172	0.50
145M v23 12 768 32 3584 0.063 2.10 145M v24 12 768 40 3072 0.064 1.44 145M v25 12 768 48 2560 0.065 1.00 145M v26 12 768 56 2304 0.063 0.77 145M v27 12 768 64 1792 0.064 0.53 145M v28 12 1536 12 1920 0.129 3.00 145M v29 12 1536 16 1792 0.127 2.10 145M v30 12 1536 20 1536 0.128 1.44 145M v31 12 1536 24 1280 0.129 1.00 145M v32 12 1536 28 1152 0.127 0.77 145M v33 12 1536 32 896								
145M v24 12 768 40 3072 0.064 1.44 145M v25 12 768 48 2560 0.065 1.00 145M v26 12 768 56 2304 0.063 0.77 145M v27 12 768 64 1792 0.064 0.53 145M v28 12 1536 12 1920 0.129 3.00 145M v29 12 1536 16 1792 0.127 2.10 145M v30 12 1536 20 1536 0.128 1.44 145M v31 12 1536 24 1280 0.129 1.00 145M v32 12 1536 28 1152 0.127 0.77 145M v33 12 1536 32 896 0.128 0.53 145M v34 12 4096 4 768	145M					3840	0.065	3.00
145M v25 12 768 48 2560 0.065 1.00 145M v26 12 768 56 2304 0.063 0.77 145M v27 12 768 64 1792 0.064 0.53 145M v28 12 1536 12 1920 0.129 3.00 145M v29 12 1536 16 1792 0.127 2.10 145M v30 12 1536 20 1536 0.128 1.44 145M v31 12 1536 24 1280 0.129 1.00 145M v32 12 1536 28 1152 0.127 0.77 145M v33 12 1536 32 896 0.128 0.53 145M v34 12 4096 4 768 0.340 3.60 145M v48 12 1024 28 2368	145M	v23	12	768		3584	0.063	2.10
145M v26 12 768 56 2304 0.063 0.77 145M v27 12 768 64 1792 0.064 0.53 145M v28 12 1536 12 1920 0.129 3.00 145M v29 12 1536 16 1792 0.127 2.10 145M v30 12 1536 20 1536 0.128 1.44 145M v31 12 1536 24 1280 0.129 1.00 145M v32 12 1536 28 1152 0.127 0.77 145M v33 12 1536 32 896 0.128 0.53 145M v34 12 4096 4 768 0.340 3.60 145M v48 12 1024 28 2368 0.086 1.59 145M v49 12 1024 36 2048	145M					3072	0.064	1.44
145M v27 12 768 64 1792 0.064 0.53 145M v28 12 1536 12 1920 0.129 3.00 145M v29 12 1536 16 1792 0.127 2.10 145M v30 12 1536 20 1536 0.128 1.44 145M v31 12 1536 24 1280 0.129 1.00 145M v32 12 1536 28 1152 0.127 0.77 145M v33 12 1536 32 896 0.128 0.53 145M v34 12 4096 4 768 0.340 3.60 145M v48 12 1024 28 2368 0.086 1.59 145M v49 12 1024 36 2048 0.085 1.07	145M						0.065	
145M v28 12 1536 12 1920 0.129 3.00 145M v29 12 1536 16 1792 0.127 2.10 145M v30 12 1536 20 1536 0.128 1.44 145M v31 12 1536 24 1280 0.129 1.00 145M v32 12 1536 28 1152 0.127 0.77 145M v33 12 1536 32 896 0.128 0.53 145M v34 12 4096 4 768 0.340 3.60 145M v48 12 1024 28 2368 0.086 1.59 145M v49 12 1024 36 2048 0.085 1.07	145M	v26					0.063	0.77
145M v29 12 1536 16 1792 0.127 2.10 145M v30 12 1536 20 1536 0.128 1.44 145M v31 12 1536 24 1280 0.129 1.00 145M v32 12 1536 28 1152 0.127 0.77 145M v33 12 1536 32 896 0.128 0.53 145M v34 12 4096 4 768 0.340 3.60 145M v35 12 4096 16 128 0.340 0.15 145M v48 12 1024 28 2368 0.086 1.59 145M v49 12 1024 36 2048 0.085 1.07							0.064	
145M v29 12 1536 16 1792 0.127 2.10 145M v30 12 1536 20 1536 0.128 1.44 145M v31 12 1536 24 1280 0.129 1.00 145M v32 12 1536 28 1152 0.127 0.77 145M v33 12 1536 32 896 0.128 0.53 145M v34 12 4096 4 768 0.340 3.60 145M v35 12 4096 16 128 0.340 0.15 145M v48 12 1024 28 2368 0.086 1.59 145M v49 12 1024 36 2048 0.085 1.07	145M		12					
145M v31 12 1536 24 1280 0.129 1.00 145M v32 12 1536 28 1152 0.127 0.77 145M v33 12 1536 32 896 0.128 0.53 145M v34 12 4096 4 768 0.340 3.60 145M v35 12 4096 16 128 0.340 0.15 145M v48 12 1024 28 2368 0.086 1.59 145M v49 12 1024 36 2048 0.085 1.07	145M		12			1792	0.127	
145M v32 12 1536 28 1152 0.127 0.77 145M v33 12 1536 32 896 0.128 0.53 145M v34 12 4096 4 768 0.340 3.60 145M v35 12 4096 16 128 0.340 0.15 145M v48 12 1024 28 2368 0.086 1.59 145M v49 12 1024 36 2048 0.085 1.07	145M	v30	12		20	1536	0.128	1.44
145M v32 12 1536 28 1152 0.127 0.77 145M v33 12 1536 32 896 0.128 0.53 145M v34 12 4096 4 768 0.340 3.60 145M v35 12 4096 16 128 0.340 0.15 145M v48 12 1024 28 2368 0.086 1.59 145M v49 12 1024 36 2048 0.085 1.07	145M			1536			0.129	
145M v34 12 4096 4 768 0.340 3.60 145M v35 12 4096 16 128 0.340 0.15 145M v48 12 1024 28 2368 0.086 1.59 145M v49 12 1024 36 2048 0.085 1.07	145M			1536			0.127	0.77
145M v35 12 4096 16 128 0.340 0.15 145M v48 12 1024 28 2368 0.086 1.59 145M v49 12 1024 36 2048 0.085 1.07	145M	v33		1536	32	896	0.128	0.53
145M v48 12 1024 28 2368 0.086 1.59 145M v49 12 1024 36 2048 0.085 1.07	145M	v34		4096	4	768	0.340	3.60
145M v49 12 1024 36 2048 0.085 1.07	145M	v35	12	4096	16	128	0.340	0.15
	145M	v48	12	1024	28	2368	0.086	1.59
145M v50 12 512 52 5120 0.042 1.85	145M	v50	12	512	52	5120	0.042	1.85

$N_{\text{non-embed}}$	Variant	$n_{ m layers}$	d_{model}	$n_{ m heads}$	$f_{ m size}$	$d_{\mathrm{model}}/\sqrt{N}$	$r_{ m mlp/attn}$
145M	v51	12	512	60	4800	0.042	1.50
145M	v52	12	512	68	4224	0.043	1.16
145M	v53	12	512	72	3968	0.043	1.03
145M	v54	12	768	44	2944	0.063	1.25
145M	v55	12	768	52	2432	0.064	0.88
297M	v1	12	1536	24	4096	0.089	3.20
297M	v2	12	1536	8	4864	0.090	11.4
297M	v3	12	1536	16	4608	0.088	5.40
297M	v4	12	1536	32	3584	0.090	2.10
297M	v5	12	1536	48	2816	0.089	1.10
297M	v6	12	1536	64	2048	0.088	0.60
297M	v7	12	1536	80	1024	0.090	0.24
297M	v8	12	768	48	8192	0.045	3.20
297M	v9	12	768	16	9728	0.045	11.4
297M	v10	12	768	32	9216	0.044	5.40
297M	v11	12	768	64	7168	0.045	2.10
297M	v12	12	768	96	5632	0.045	1.10
297M	v13	12	768	128	4096	0.044	0.60
297M	v14	12	768	160	2048	0.045	0.24
297M	v15	12	3072	12	2048	0.178	3.20
297M	v16	12	3072	4	2432	0.180	11.4
297M	v17	12	3072	8	2304	0.177	5.40
297M	v18	12	3072	16	1792	0.180	2.10
297M	v19	12	3072	24	1408	0.178	1.10
297M	v20	12	3072	32	1024	0.177	0.60
297M	v21	12	3072	40	512	0.180	0.24
297M	v22	12	1024	36	6144	0.059	3.20
297M	v23	12	1024	12	7296	0.060	11.4
297M	v24	12	1024	24	6912	0.059	5.40
297M	v25	12	1024	48	5376	0.060	2.10
297M	v26	12	1024	72	4224	0.059	1.10
297M	v27	12	1024	96	3072	0.059	0.60
297M	v28	12	1024	120	1536	0.060	0.24
297M	v29	12	2048	12	3456	0.118	5.40
297M	v30	12	2048	24	2688	0.120	2.10
297M	v31	12	2048	48	1536	0.118	0.60
297M	v32	12	2048	60	768	0.120	0.24
297M	v45	12	1536	40	3200	0.089	1.50
297M	v46	12	1536	44	3072	0.089	1.31
297M	v47	12	1536	52	2688	0.088	0.97
297M	v48	12	1536	56	2432	0.089	0.81
297M	v49	12	768	80	6400	0.045	1.50
297M	v50	12	768	88	6016	0.045	1.28
297M	v51	12	768	104	5376	0.044	0.97
297M	v52	12	768	112	4736	0.045	0.79
297M	v53	12	3072	20	1664	0.177	1.56
297M	v54	12	3072	28	1152	0.180	0.77
297M	v55	12	1024	56	4864	0.060	1.63
297M	v56	12	1024	64	4608	0.060	1.35
297M	v57	12	1024	80	3840	0.059	0.90
297M	v58	12	1024	88	3328	0.060	0.71
297M	v59	12	2048	32	2432	0.117	1.43
297M	v60	12	2048	36	2048	0.120	1.07
297M	v61	12	2048	40	1920	0.118	0.90
297M	v62	12	2048	44	1792	0.117	0.76
1B	v1	16	2048	32	8192	0.066	4.80
1B	v2	16	2048	72	5760	0.067	1.50

$N_{ m non-embed}$	Variant	$n_{ m layers}$	d_{model}	n_{heads}	$f_{ m size}$	$d_{\mathrm{model}}/\sqrt{N}$	$r_{ m mlp/attn}$
1B	v3	16	2816	92	2432	0.089	0.50
1B	v4	16	2816	76	3072	0.091	0.76
1B	v5	16	2816	68	3584	0.090	0.99
1B	v6	16	2816	60	4096	0.090	1.28
1B	v7	16	2816	56	4480	0.089	1.50
1B	v8	16	2816	24	6144	0.089	4.80
1B	v9	16	2816	48	4736	0.090	1.85
1B	v10	16	2816	40	5120	0.090	2.40
1B	v11	16	2816	36	5376	0.090	2.80
1B	v12	16	2560	64	4480	0.082	1.31
1B	v13	16	2560	72	4096	0.082	1.07
1B	v14	16	2560	80	3648	0.082	0.86
1B	v15	16	2560	56	4864	0.082	1.63
1B	v16	16	2560	88	3200	0.082	0.68
1B	v17	16	2560	48	5376	0.082	2.10

D HYPER-PARAMETERS

Table 5 lists the detailed hyper-parameters used for training in this paper.

Table 5: **Hyper-parameters:** We show the hyper-parameters used for training in this paper.

80M	145M	297M	1B	3B	
256	256	512	512	512	
1.5e-3	1.0e-3	8.0e-4	6.0e-4	6.0e-4	
	$0.1 \times N$	Iax LR			
AdamW ($\beta_1 = 0.9, \beta_2 = 0.95$)					
0.1					
1.0					
Cosine					
500					
	20	48			
	256 1.5e-3	256 256 1.5e-3 1.0e-3 $0.1 \times M$ AdamW ($\beta_1 = 0$ 1 Cos 50	256 256 512 1.5e-3 1.0e-3 8.0e-4 $0.1 \times$ Max LR AdamW ($\beta_1 = 0.9, \beta_2 = 0.1$ 1.0 Cosine	256 256 512 512 1.5e-3 1.0e-3 8.0e-4 6.0e-4 $0.1 \times$ Max LR AdamW ($\beta_1 = 0.9, \beta_2 = 0.95$) 0.1 1.0 Cosine 500	

E ADDITIONAL INFERENCE EVALUATION RESULTS

In this section, we present additional inference efficiency results on NVIDIA A100 GPUs. Figure 10 presents that, when parameter count, MLP-to-Attention ratio, and hidden size are fixed, increasing GQA leads to higher inference throughput, consistent with the findings of Ainslie et al. (2023). We alter model configurations of LLaMA-3.2-1B, 3B, and LLaMA-3.1-8B in Figure 10.

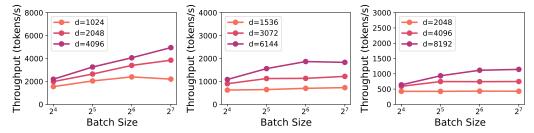


Figure 8: **Hidden size on Inference Throughput:** (left) 1B model variants; (center) 3B model variants; (right) 8B model variants. Across varying batch sizes and model scales, larger hidden sizes yield higher inference throughput under a fixed parameter budget. The legend indicates the hidden size of the models, where $d=d_{\rm model}$.

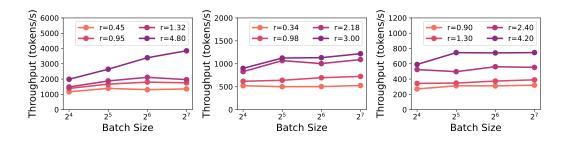


Figure 9: MLP-to-Attention ratio on Inference Throughput: (left) 1B model variants; (center) 3B model variants; (right) 8B model variants. Across varying batch sizes and model scales, a larger MLP-to-Attention ratio increases inference throughput under a fixed parameter budget. The legend indicates the MLP-to-Attention ratio of the models, where $r = r_{\rm mlp/attn}$.

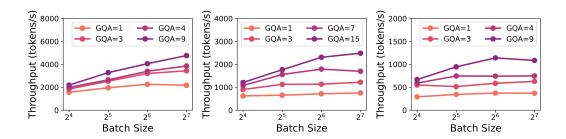


Figure 10: **GQA on Inference Throughput:** (left) 1B model variants; (center) 3B model variants; (right) 8B model variants. This figure shows the impact of GQA on inference throughput. With the total parameter count fixed, hidden size is set to 2048 (1B), 3072 (3B), and 4096 (8B), and the MLP-to-Attention ratio is 4.0, 2.67, and 4.2, respectively. Across varying batch sizes, models with larger GQA achieve higher throughput. All evaluations are performed using the vLLM framework Kwon et al. (2023) on a single NVIDIA Ampere 40GB A100 GPU with 4096 input and 1024 output tokens.

Furthermore, we derive architectural variants by altering the configurations of Qwen3-0.6B, 1.7B, and 4B to investigate the impact of model architectural factors on inference efficiency. The results are shown in Figure 11-13.

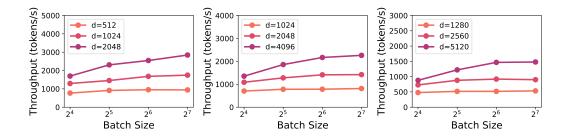


Figure 11: **Hidden size on Inference Throughput (Qwen3):** (left) Qwen3-0.6B model variants; (center) Qwen3-1.7B model variants; (right) Qwen3-4B model variants. Across varying batch sizes and model scales, larger hidden sizes yield higher inference throughput under a fixed parameter budget. The legend indicates the hidden size of the models, where $d=d_{\rm model}$. All evaluations are performed using the vLLM framework Kwon et al. (2023) on a single NVIDIA Ampere 40GB A100 GPU with 4096 input and 1024 output tokens.

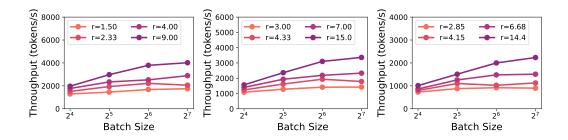


Figure 12: **MLP-to-Attention ratio on Inference Throughput (Qwen3):** (left) Qwen3-0.6B model variants; (center) Qwen3-1.7B model variants; (right) Qwen3-4B model variants. Across varying batch sizes and model scales, a larger MLP-to-Attention ratio increases inference throughput under a fixed parameter budget. The legend indicates the MLP-to-Attention ratio of the models, where $r = r_{\rm mlp/attn}$. All evaluations are performed using the vLLM framework Kwon et al. (2023) on a single NVIDIA Ampere 40GB A100 GPU with 4096 input and 1024 output tokens.

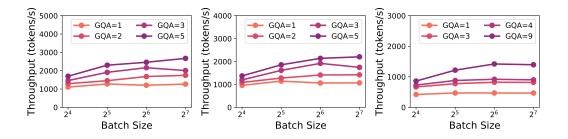


Figure 13: **GQA on Inference Throughput (Qwen3):** (left) Qwen3-0.6B model variants; (center) Qwen3-1.7B model variants; (right) Qwen3-4B model variants. This figure shows the impact of GQA on inference throughput. With the total parameter count fixed, hidden size is set to 1024 (0.6B), 2048 (1.7B), and 2560 (4B), and the MLP-to-Attention ratio is 1.5, 3.0, and 2.85, respectively. Across varying batch sizes, models with larger GQA achieve higher throughput. All evaluations are performed using the vLLM framework Kwon et al. (2023) on a single NVIDIA Ampere 40GB A100 GPU with 4096 input and 1024 output tokens.

F ADDITIONAL RESULTS: LOSS VS. MODEL ARCHITECTURE

In this section, we analyze the relationship between training loss and GQA while fixing the number of parameters, hidden size, and MLP-to-Attention ratio. As shown in Figure 14, unlike hidden size and MLP-to-Attention ratio, the relationship between loss and GQA is highly fluctuating.

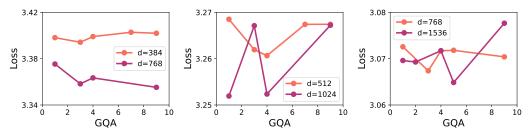


Figure 14: **Loss vs. GQA:** (left) 80M model variants; (center) 145M model variants; (right) 297M model variants. Across different model sizes, the relationship between training loss and GQA varies substantially when hidden size and the mlp-to-attention ratio are fixed. The legend denotes the hidden size of each trained model.

G MORE ABLATION STUDY

In this section, We first evaluate the impact of outlier data on the fitting of the scaling laws in Figure 15 (left) and Figure 15 (center). Then, we evaluate the fitting performance of multiplicative calibrations and additive calibrations in Figure 15 (left) and Figure 15 (right).

Finally, we evaluate the performance of Joint and non-separable calibrations shown below in Figure 16:

$$(a_0 + a_1 \log(\frac{dr}{\sqrt{N}}) + a_2/(\frac{dr}{\sqrt{N}})) \cdot L_{\text{opt}}$$

where $d = d_{\text{model}}$, $r = r_{\text{mlp/attn}}$, and $N = N_{\text{non-embed}}$. In Figure 16, we observe that the performance of joint and non-separable calibrations is significantly worse than that of multiplicative calibration, consistent with our discussion in §3.3.

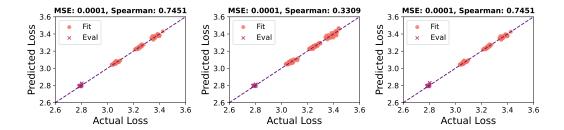
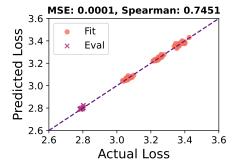


Figure 15: **Ablation Study:** (left) use multiplicative calibrations without outliers; (center) use multiplicative calibrations with outliers; (right) use additive calibrations without outliers. The outlier refers to models trained with an mlp-to-attention ratio below 0.5 or above 5. We observe that outlier data points harm the scaling law fit. Moreover, while multiplicative and additive calibrations differ in formulation, their MSE and Spearman values remain nearly identical. Dots denote the data points used for fitting, while crosses indicate the test data points.



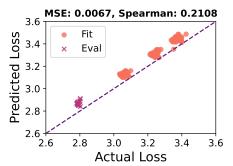


Figure 16: **Joint and non-separable calibrations:** (left) use multiplicative calibrations; (right) use joint and non-separable calibrations. We observe that joint and non-separable calibrations yield higher MSE and lower Spearman scores than multiplicative calibrations, indicating inferior performance. Dots denote the data points used for fitting, while crosses indicate the test data points.

H INFERENCE FLOPS ANALYSIS

Building on the inference FLOPs analysis from prior work Kaplan et al. (2020), we begin with the following definition:

- d_{model} : hidden size
- f_{size} : intermediate (feed-forward) size
- n_{layers} : number of layers
- A: number of query heads
- K: number of key/value heads
- d_h : per-head hidden dimension (query and value)
- T: per-head hidden dim the KV length prior to token generation

Based on the above definition, we have $d_q = Ad_h$ and $d_{kv} = Kd_h$. We focus exclusively on non-embedding FLOPs, resulting in:

Attention: QKV and Project

$$n_{\mathrm{layers}}(\underbrace{2d_{\mathrm{model}}d_q}_{Q} + \underbrace{2d_{\mathrm{model}}d_{kv}}_{K} + \underbrace{2d_{\mathrm{model}}d_{kv}}_{V} + \underbrace{2d_{\mathrm{model}}d_q}_{Q})$$

Attention: Mask

$$n_{\text{layers}}(2Td_q)$$

Feedforward:

$$n_{\text{layers}}(3 \cdot 2d_{\text{model}}f_{\text{size}})$$

Total Inference non-embedding FLOPs:

$$\text{Total-FLOPs} = n_{\text{layers}} \underbrace{\left(\underbrace{2d_{\text{model}}d_q}_{Q} + \underbrace{2d_{\text{model}}d_{kv}}_{K} + \underbrace{2d_{\text{model}}d_{kv}}_{V} + \underbrace{2d_{\text{model}}d_q}_{Q} + \underbrace{2Td_q}_{qK^\top} + \underbrace{3 \cdot 2d_{\text{model}}f_{\text{size}}}_{\text{up, gate, down}} \right)}_{\text{up, gate, down}}$$

Since $P_{\text{non-emb}} \approx n_{\text{layers}}(2d_{\text{model}}d_q + 2d_{\text{model}}d_{kv} + 3d_{\text{model}}f_{\text{size}})$. Therefore, Total-FLOPs = $2P_{\text{non-emb}} + 2n_{\text{layers}}Td_q$

we adopt the following three approaches to accelerate inference:

- Increasing the MLP-to-Attention ratio reduces the term $2Td_q$, thereby lowering the total FLOPs.
- Increasing the hidden size reduces the term $2Td_q$, thereby lowering the total FLOPs.

I MORE LARGE-SCALE TRAINING RESULTS

In this section, we first show the detailed result over downstream tasks of large-scale models in Table 6 and Table 7.

Table 6: **Detailed Results on Downstream Tasks for 1B Models:** In this table, we show detailed results of 1B models over 9 downstream tasks.

Downstream Tasks	LLaMA-3.2-1B	Panda-1B	Surefire-1B
Arc-Easy	58.8	60.9	59.7
Arc-Challenge	29.8	28.9	30.2
LAMBADA	52.8	55.1	52.0
HellaSwag	56.9	58.4	56.6
OpenBookQA	32.0	33.2	32.0
PIQA	73.6	75.2	73.0
SciQ	84.8	87.2	84.9
WinoGrande	57.1	58.6	57.5
COQA	48.7	55.3	52.7
Avg.	54.9	57.0	55.4

Table 7: **Detailed Results on Downstream Tasks for 3B Models:** In this table, we show detailed results of 3B models over 9 downstream tasks.

Downstream Tasks	LLaMA-3.2-3B	Panda-3B	Surefire-3B	Panda-3B°
Arc-Easy	66.4	65.5	67.6	66.8
Arc-Challenge	33.3	35.2	33.9	33.3
LAMBADA	60.6	61.8	61.4	61.5
HellaSwag	66.7	66.9	67.0	67.8
OpenBookQA	38.4	38.6	38.6	38.0
PIQA	76.8	76.9	77.4	76.8
SciQ	89.4	91.2	92.1	90.5
WinoGrande	62.5	63.2	60.5	62.7
COQA	63.3	63.4	65.4	64.9
Avg.	61.9	62.5	62.6	62.5

J MoE Inference

In this section, we examine how the Mixture-of-Experts (MoE) architecture affects inference efficiency. Figure 17 indicates that larger hidden sizes and higher Active-Experts-to-Attention ratios improve the inference throughput of MoE models, consistent with observations in dense models.

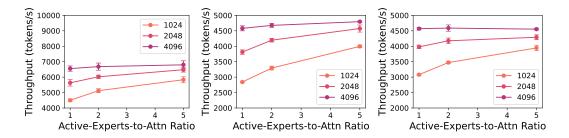


Figure 17: **Active-Experts-to-Attn on Inference Throughput:** (left) 3B-A1.1B model variants; (center) 5.3B-A1.7B model variants; (right) 8.3B-A1.5B model variants. We study the effect of the Active-Experts-to-Attention ratio on inference throughput by fixing the total number of active parameters, setting GQA to 4, and using a batch size of 2048 to reduce MoE inference variance in this figure. All evaluations are performed using the vLLM framework Kwon et al. (2023) on a single NVIDIA Ampere 40GB A100 GPU with 1024 input and 256 output tokens.