THE SIMPLE PENDULUM OSCILATIONS

Introduction

Galileo Galilei was the first to study the properties of pendulums, beginning around 1602 and this study is usually regarded as the beginning of experimental physics.

The simple pendulum consists in a small body with mass *m* (mass point) hung on a massless, inextensible string with length ℓ. (The word “simple” refers to the fact that the mass is a point mass, as opposed to an extended mass in the “physical” pendulum). If the pendulum is removed from its equilibrium position swings in a vertical plane due to the gravitational force ($G = mg$, g – the gravitational acceleration).

Let θ(t) be the angle the string makes with the vertical (Fig 1). The radial component of weight ($G_n = mg\cos\theta$) is compensated by the tension in the string (*T*). The gravitational force on the mass in the tangential direction ($G_t = mg\sin\theta$), is the restoring force that acts on the pendulum to return it to its equilibrium position.

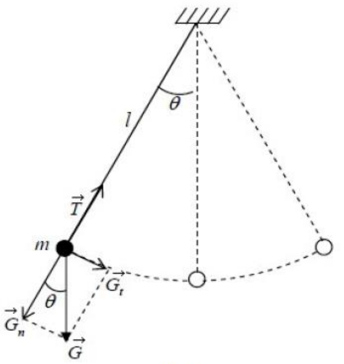

Fig.1.

For small oscillations, we have $\sin\theta \cong \theta$ in radians (For example $\theta = 5°$ = 0.0873 rad, sin 5°= 0.0872). For the arc length x of θ corresponding angle and ℓ the radius of the circle we have: $x = \ell\theta$.

$$F = G_t = mg\sin\theta \cong mg\theta = \frac{mg}{\ell}x = kx$$

where *k* represents the “elastic constant” for the system : $k = \frac{mg}{\ell}$.

We observe that the restoring force is proportional to the elongation x and therefore the pendulum undergoes simple harmonic motion with a period of

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \Leftrightarrow \quad T = 2\pi\sqrt{\frac{\ell}{g}}$$

The period of the simple pendulum’s **small** oscillations is independent of mass and amplitude. It depends only on pendulum length ℓ and gravitational acceleration *g* at the location where the measurements are done.

As we can observe we have linear dependence of the period T on $\sqrt{\ell}$, which is graphically represented by a straight line whose slope (angular coefficient) is $\frac{2\pi}{\sqrt{g}}$..

What to do

1. Remove the pendulum from the equilibrium position and, after it is left free, measure the time **t** in which **n** low amplitude oscillations are performed;
2. Calculate the period of oscillations T = t / n;
3. The length of the pendulum is successively increased by 0.05m or 0.1 m, measuring the period of the pendulums thus formed, as in step 1 and 2;

4. With the measured data fill the table below (Table1) and then plot T = f ($\sqrt{\ell}$). Find the slope of the straight line and the consequently gravitational acceleration g.

5. Using the relation (4) for each of the formed plots we calculate the gravitational acceleration g as well as its mean value;

Table 1

	ℓ (m)	$\sqrt{\ell}$ ($m^{1/2}$)	n	t (s)	T (s)	g (m/s^2)	$\bar{g}$ (m/s^2)	Δg	$\sigma_{\bar{g}}$
1									
2									
3									
4									
5									
6									
7									
8									

EX. Knowing that the period T of small oscillations of the gravitational pendulum is only a function of the length of the pendulum ℓ and the gravitational acceleration g, find the formula of the period of oscillations of the pendulum using dimensional analysis.

EX. Find the ratio between the periods of small oscillations of the same pendulum on the Moon and on the Earth. The gravitational acceleration on the surface of the Moon is about 16.6% that on Earth's surface.