Learning from Data

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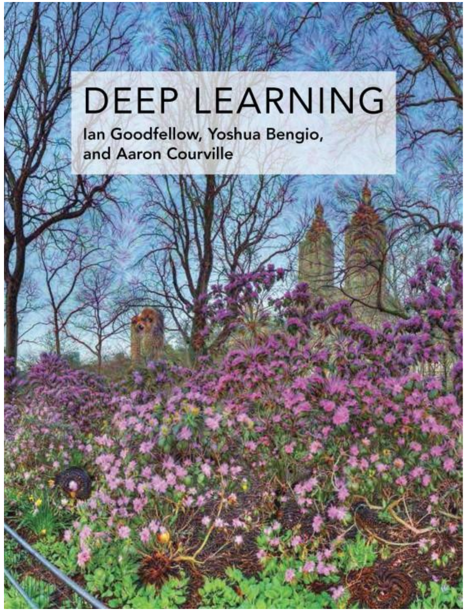
Overview of the course

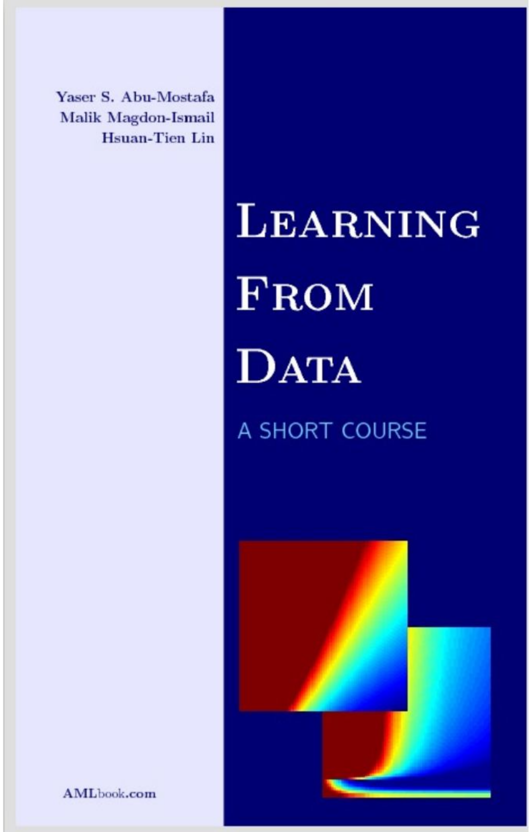
Lecture plan

	Monday	Tuesday	Wednesday	Thursday	Friday
09:00-10:30	Mon1: Overview and Denoising	Tue1: Learning from data	Wed1: Linear Models	Thu1: Knee MRI features	Fri1: Multi Layer Perceptrons
10:45-12:00	Mon2: Feature Detection	Tue2: Knowing your data	Wed2: Optimization	Thu2: Supervised Learning 1	Fri2: Convolutional Neural Networks
Lunch					
12:45-14.15	Mon3: Ridge/Root Detection	Tue3: Machine Learning Basics-1	Wed3: Unsupervised Learning 1	Thu3: Supervised Learning 2	Fri3: CNN root segmentation
14:30-16.00	Mon4: Postprocessing	Tue4: Machine Learning Basics-2	Wed4: Unsupervised Learning 2	Thu4: What is the classifier doing?	Fri4: Graph Neural Networks

Literature

1. Learning from Data by Mostafa et al.
2. Deep Learning by Goodfellow et al.

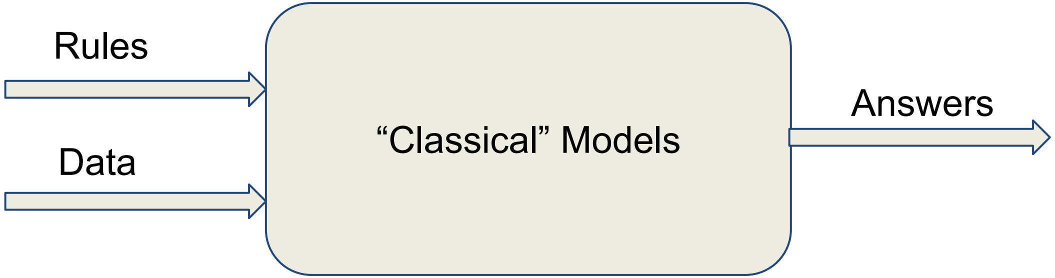




Overview

- Design-based methods
- Learning from data
- Underlying data distributions
- Perceptron
- Generalization error

Design-based methods



Design-based methods

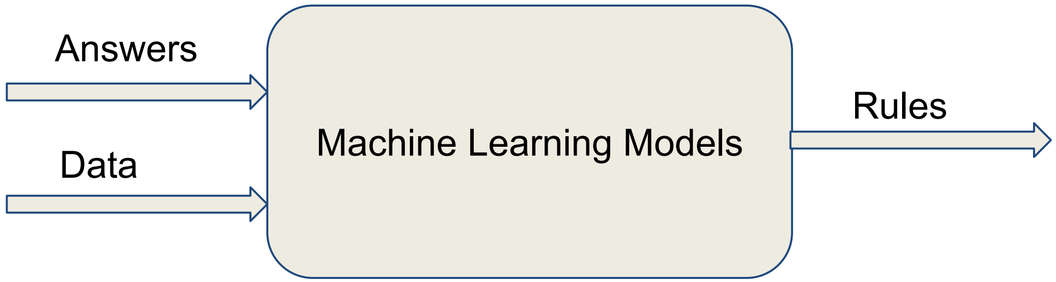
* Convert to Gray scale
* Gaussian Smoothing
* Compute Hessians
* Eigen values
* Tune thresholds

Data

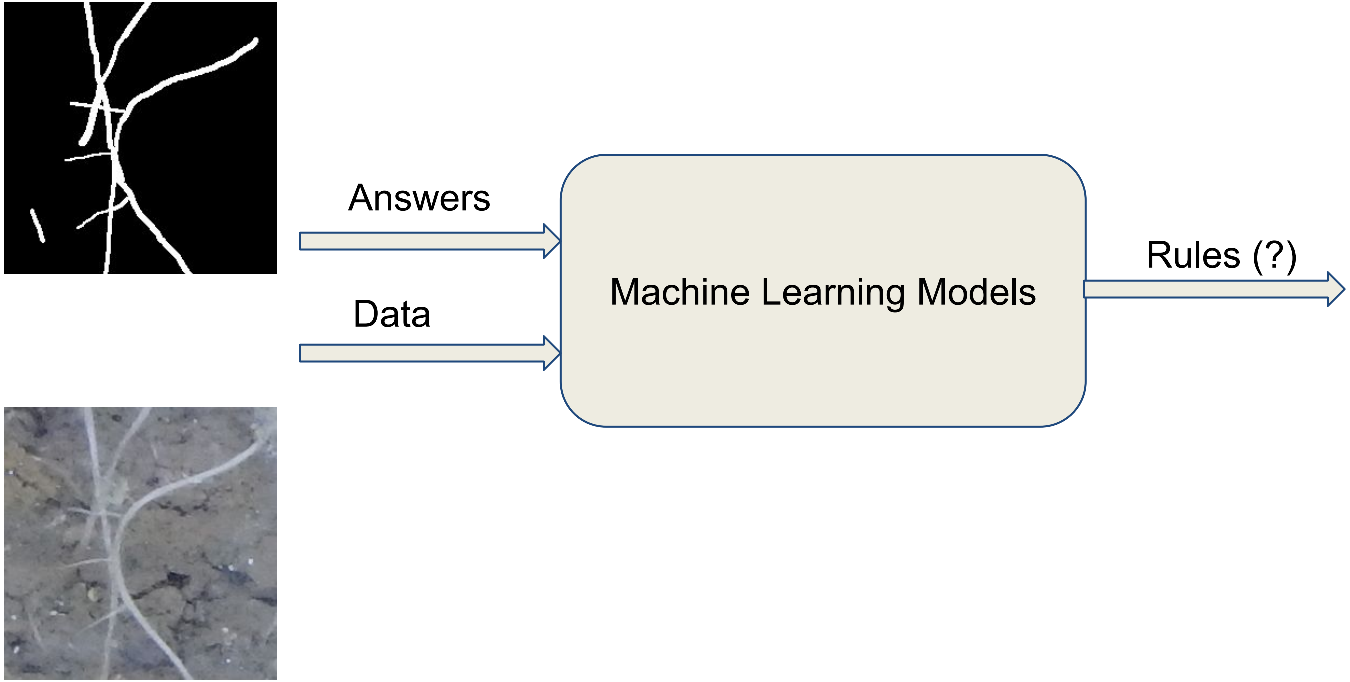
“Classical” Models:
Frangi Vesselness Filter

Answers

Machine Learning = Learning from Data



Machine Learning = Learning from Data

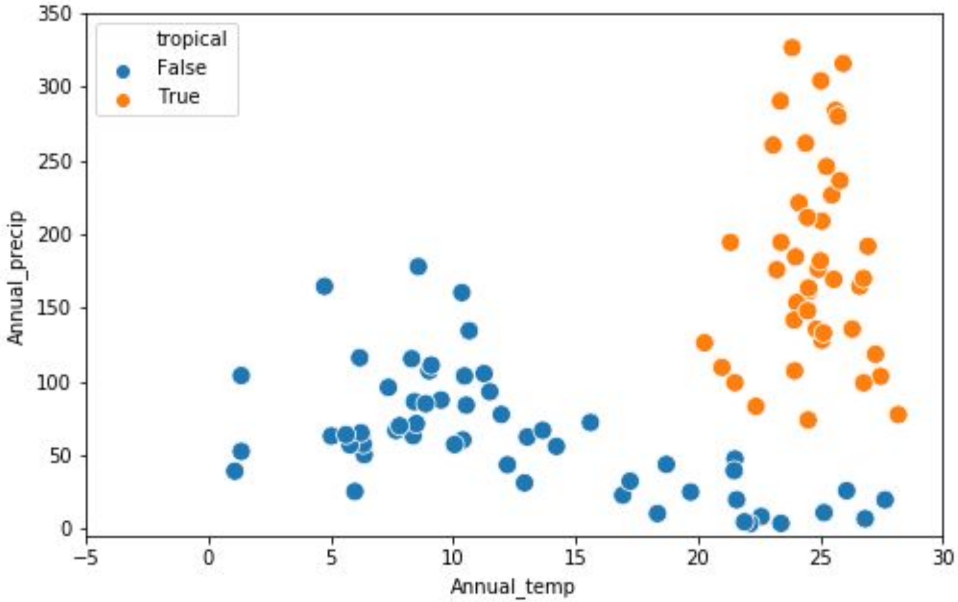


Machine Learning = Learning from Data

- ML is a lot about discovering patterns
  - Big data
  - Big computers
  - More complex patterns, than before
- Learning from examples
  - Natural to humans
  - Temptation to call it AI
- What you have is what you get (mostly)
  - Large & diverse datasets
  - Features and flaws are learned

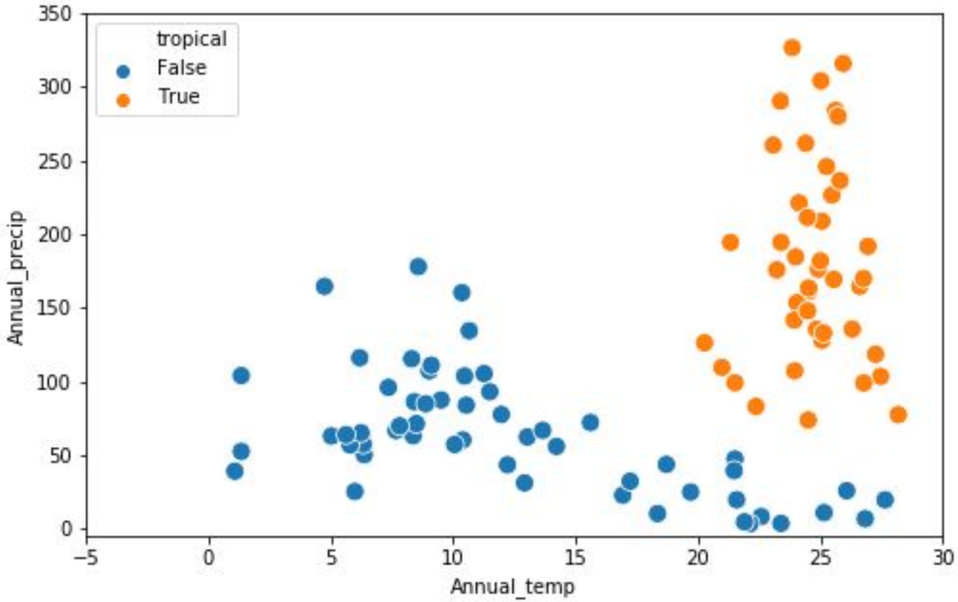
Linear Classification

- Given a training set, with binary labels
- Model a linear classifier based on the training data
- Predict classification on new data



https://datacatalog.worldbank.org/dataset/climate-change-knowledge-portal-historical-data

Linear Separability



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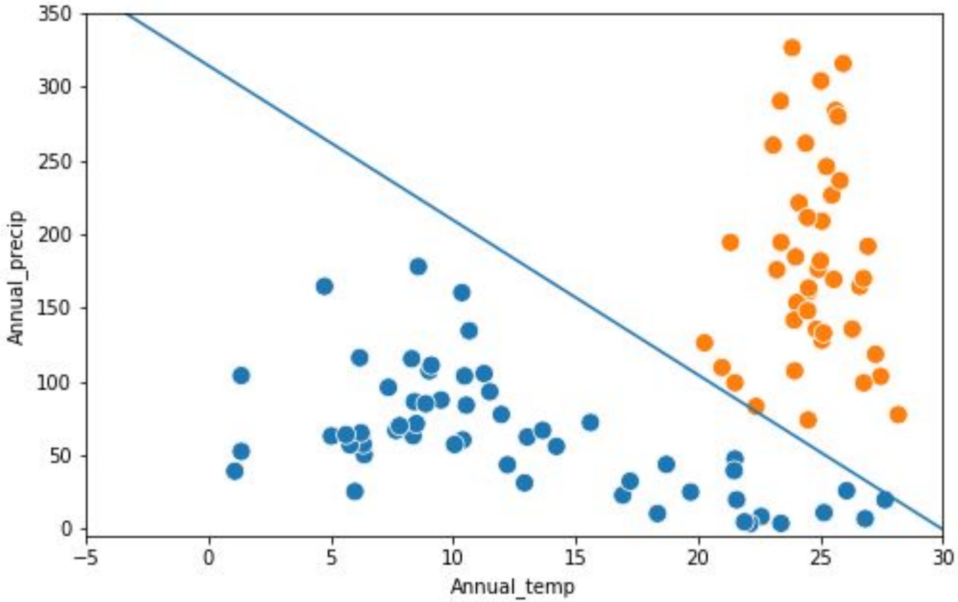
Linear Separability

If the d-dimensional data is linearly separable, then there exists at least one (d-1) dimensional hyperplane that is a classifier.



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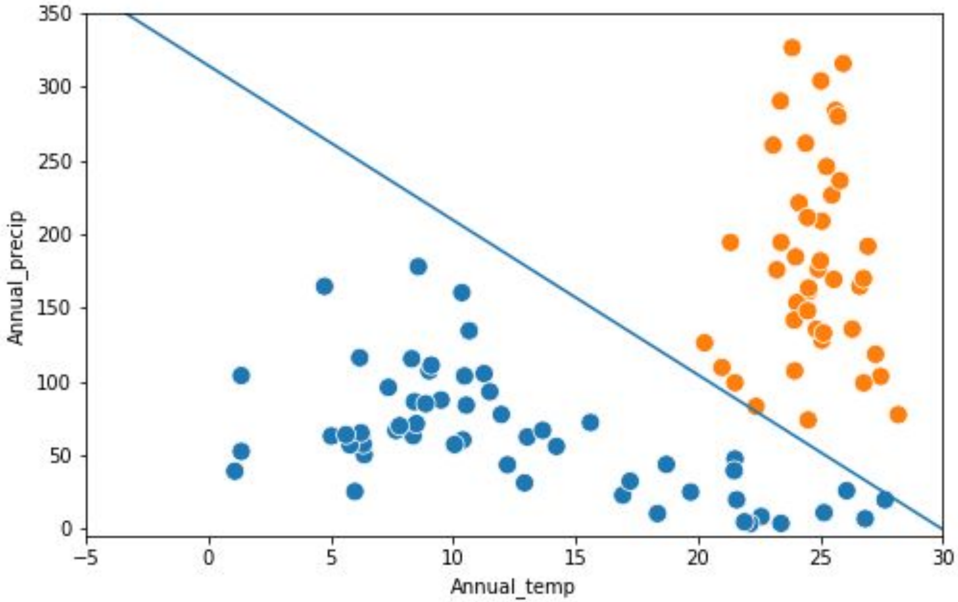
Mathematically, the hyperplane (in this case) a line is given as:

$$w_0 + w_1 x_1 + w_2 x_2 = 0$$

Or in vector form,

$$\mathbf{w}^T \mathbf{x} = 0$$



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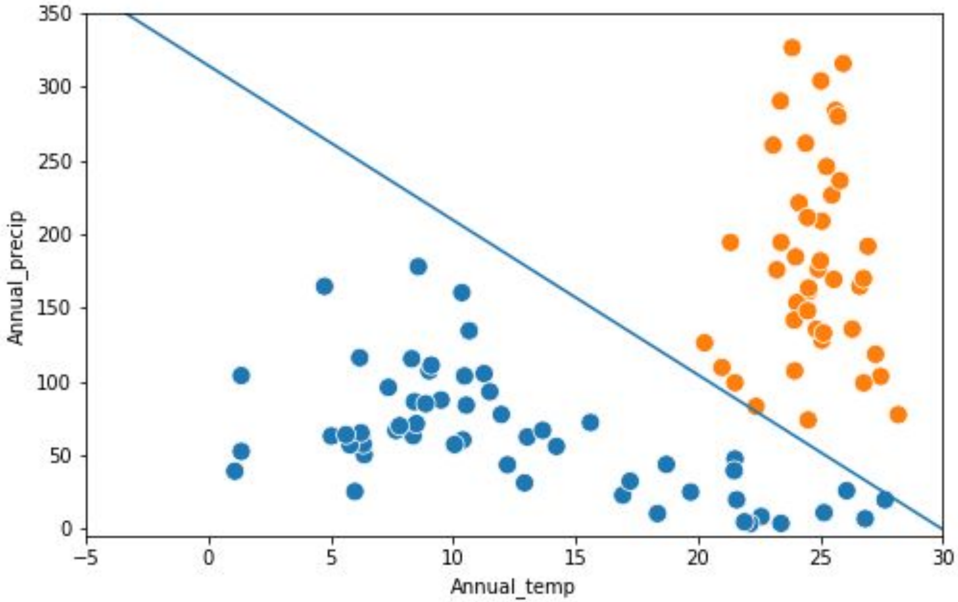
A first ML Algorithm

Given the training data,

$$\mathbf{X} = \{\mathbf{x_i}\} : \mathbf{x_i} = [x_0, \ldots, x_d]^T$$
$$\mathbf{Y} = \{y_i\} : y_i \in \{+1, -1\}$$



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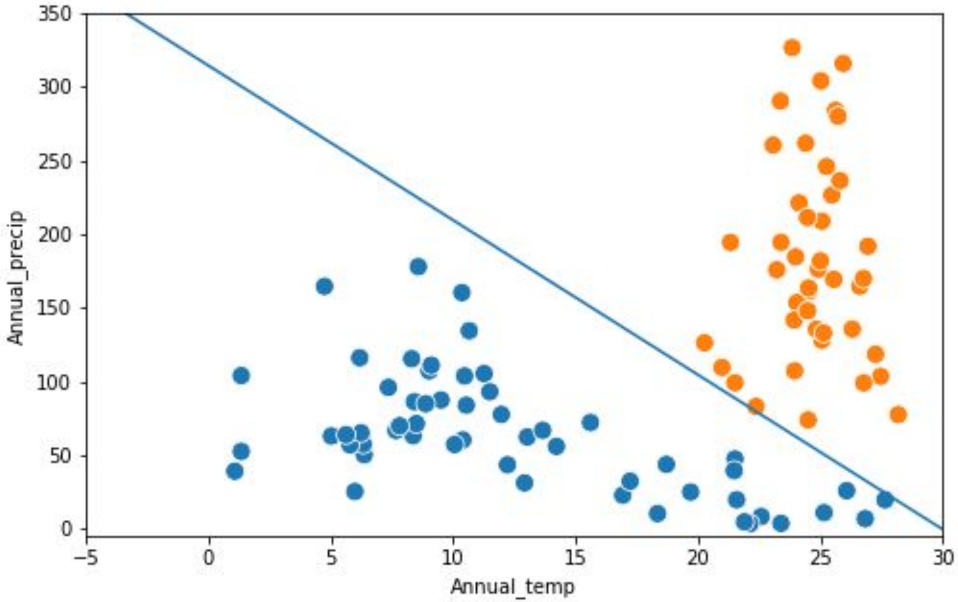
The model should be of the form:

$$h(\mathbf{x}) = \begin{cases} +1 & \text{if } \mathbf{w}^T\mathbf{x} \geq 0 \\ -1 & \text{if } \mathbf{w}^T\mathbf{x} < 0 \end{cases}$$



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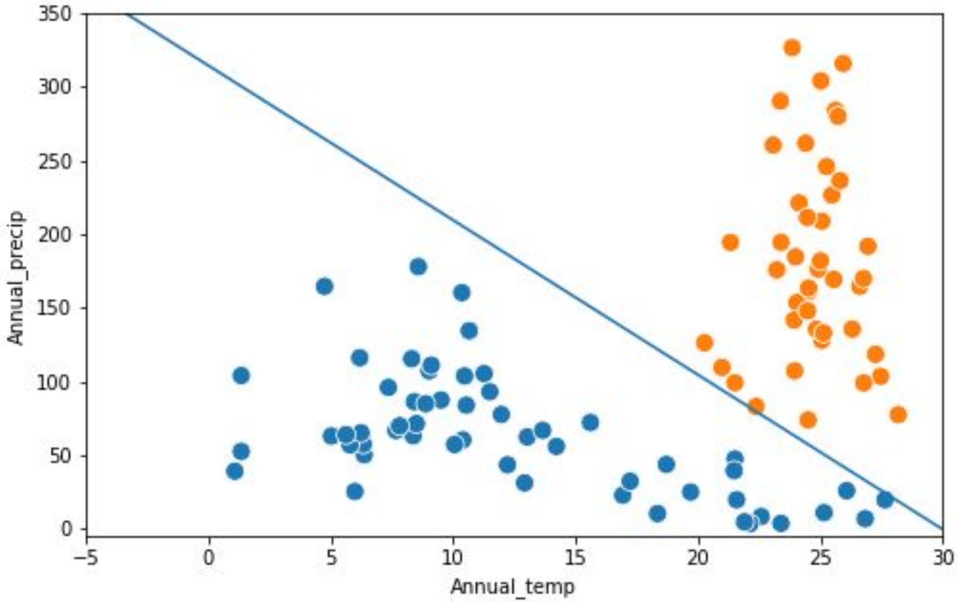
Or, more compactly:

$$h(\mathbf{x}) = \text{sign}\left(\mathbf{w}^T\mathbf{x}\right)$$



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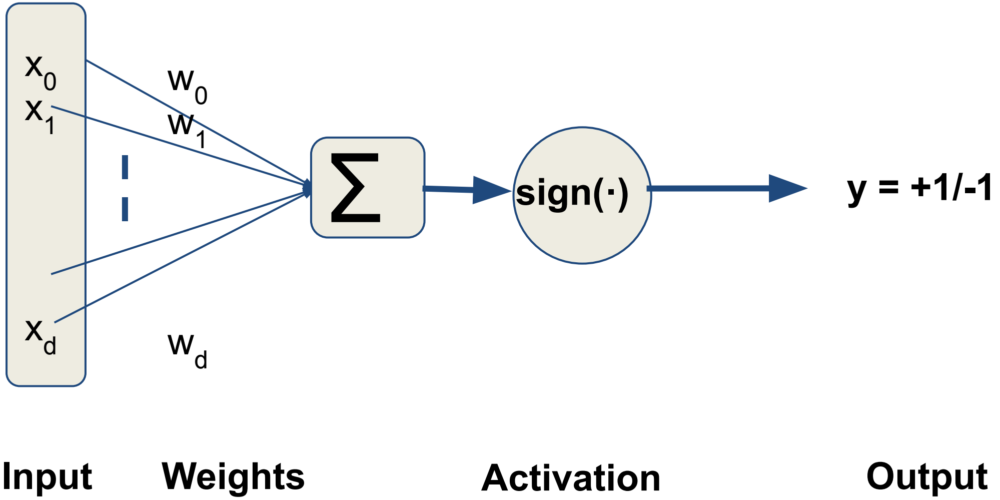
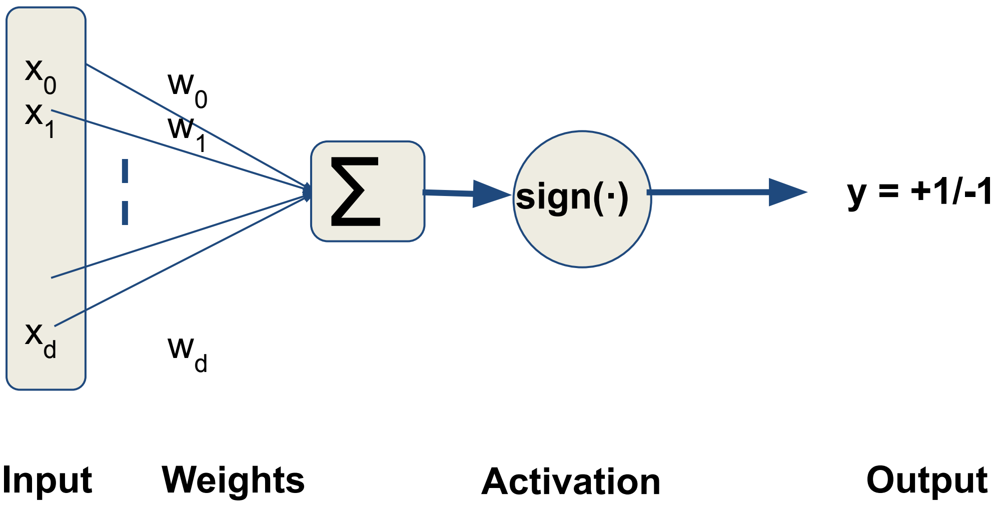


Perceptron Learning Algorithm



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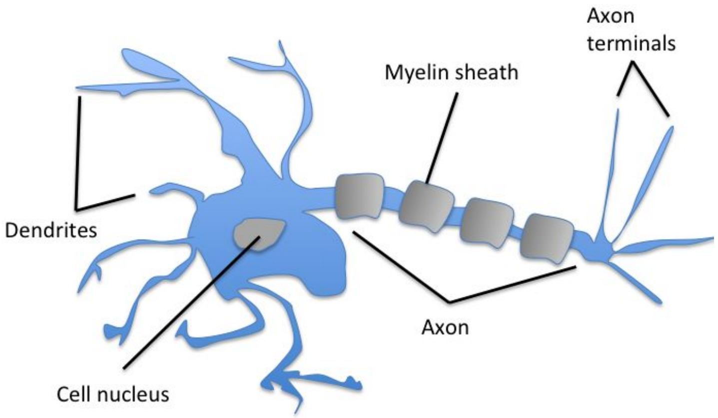
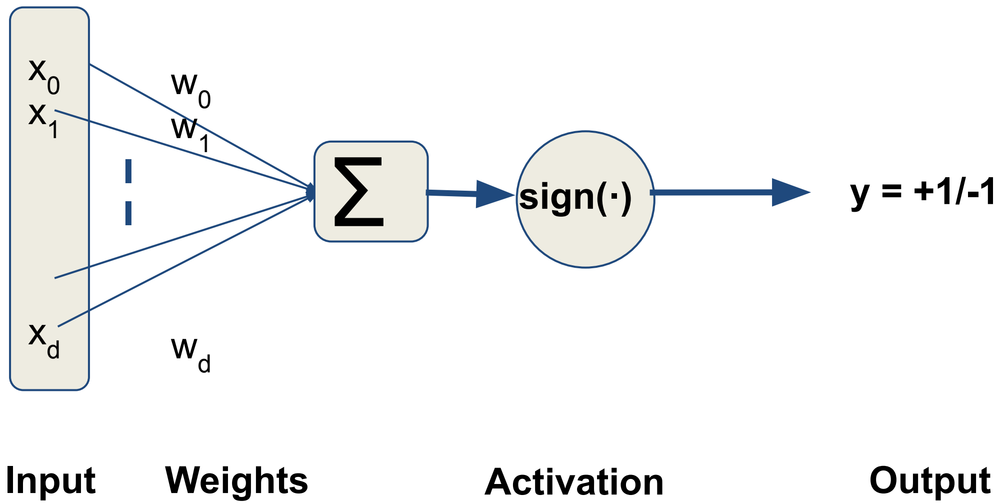


Schematic of a biological neuron.



https://sebastianraschka.com/Articles/2015_singlelayer_neurons.html

Perceptron Learning Algorithm

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$
$N \leftarrow inputs \quad with \quad label \quad 0;$
Initialize $\mathbf{w}$ randomly;
while $!convergence$ **do**
  Pick random $\mathbf{x} \in P \cup N$;
  if $\mathbf{x} \in P \quad and \quad \mathbf{w}.\mathbf{x} < 0$ **then**
    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;
  end
  if $\mathbf{x} \in N \quad and \quad \mathbf{w}.\mathbf{x} \geq 0$ **then**
    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;
  end
end
//the algorithm converges when all the inputs are classified correctly

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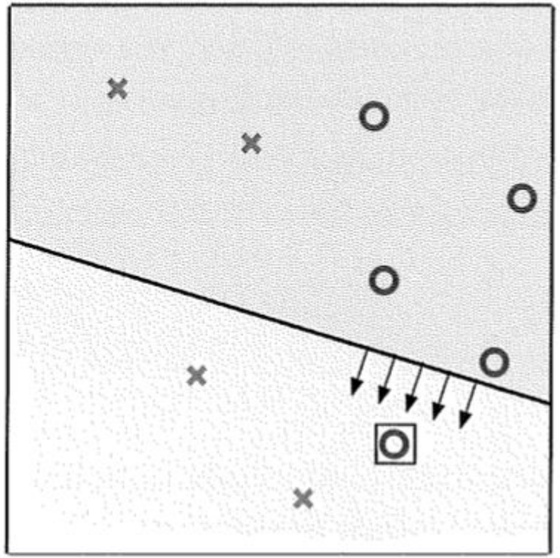
//the algorithm converges when all the inputs are classified correctly

$$\mathbf{w}(t+1) = \mathbf{w}(t) + y(t)\mathbf{x}(t).$$

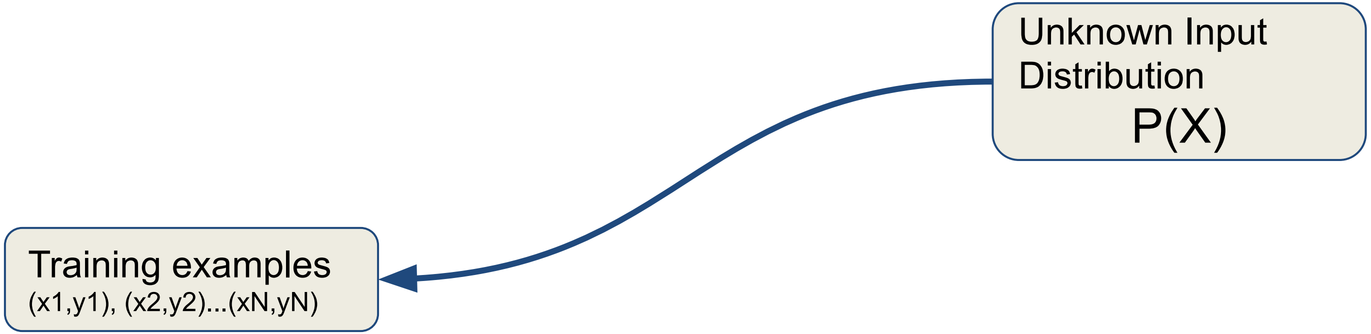


PLA Task in Notebook

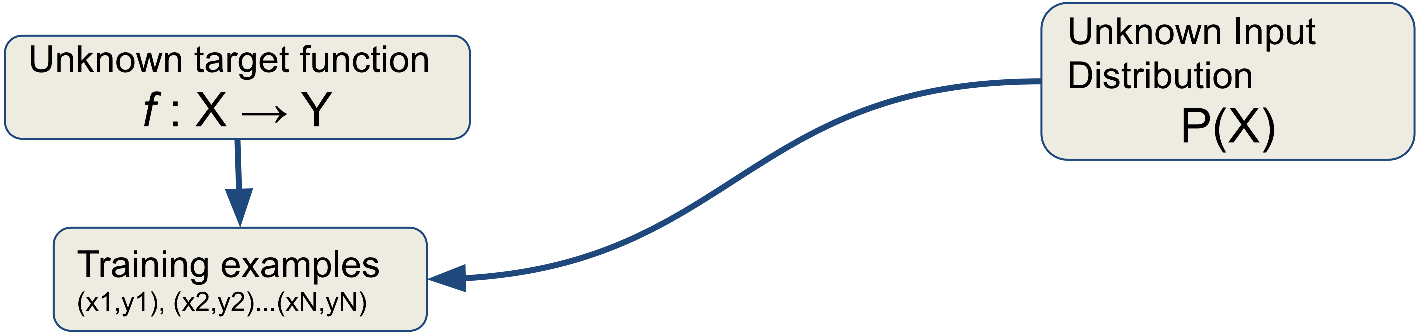
Formalising Machine Learning

Training examples
(x1,y1), (x2,y2)...(xN,yN)

Formalising Machine Learning



Formalising Machine Learning



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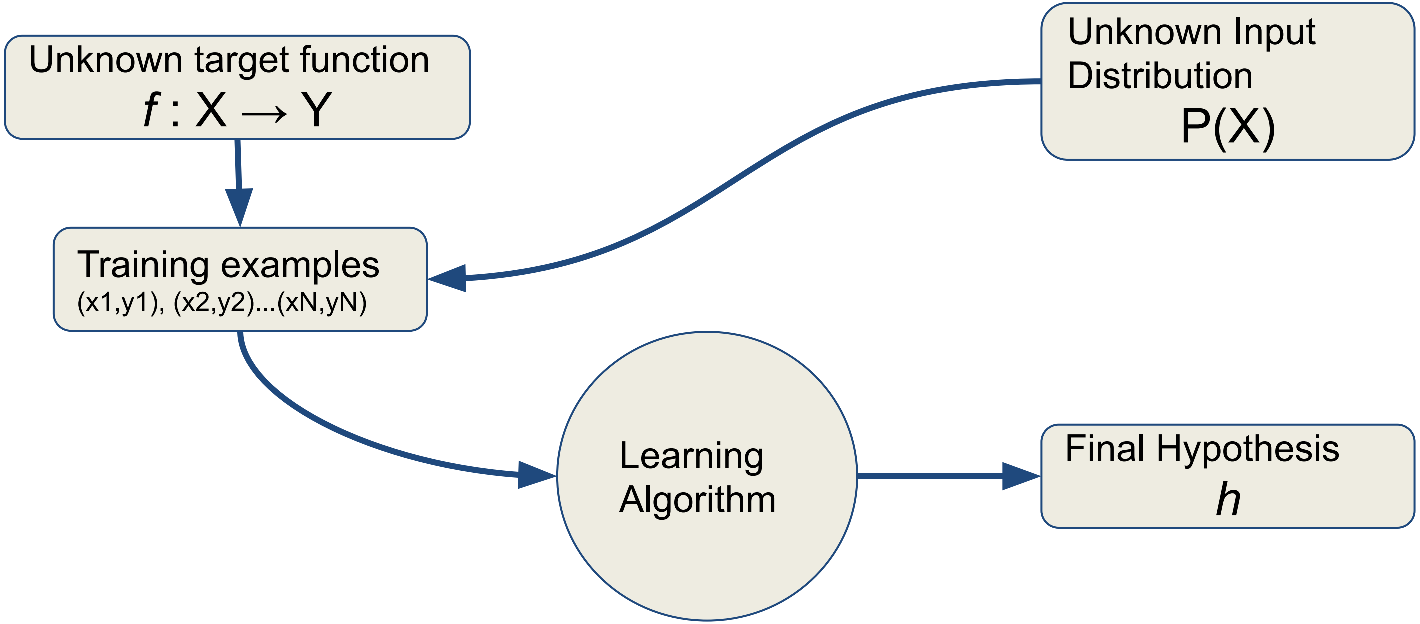
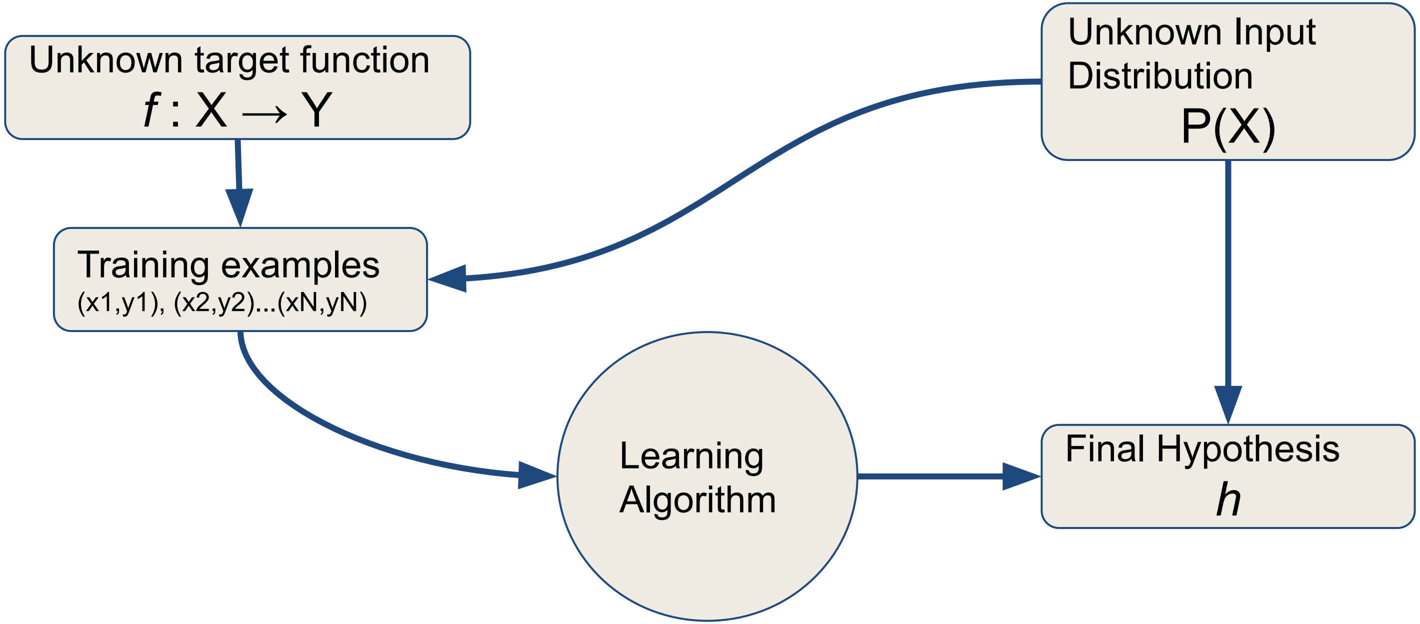


Formalising Machine Learning



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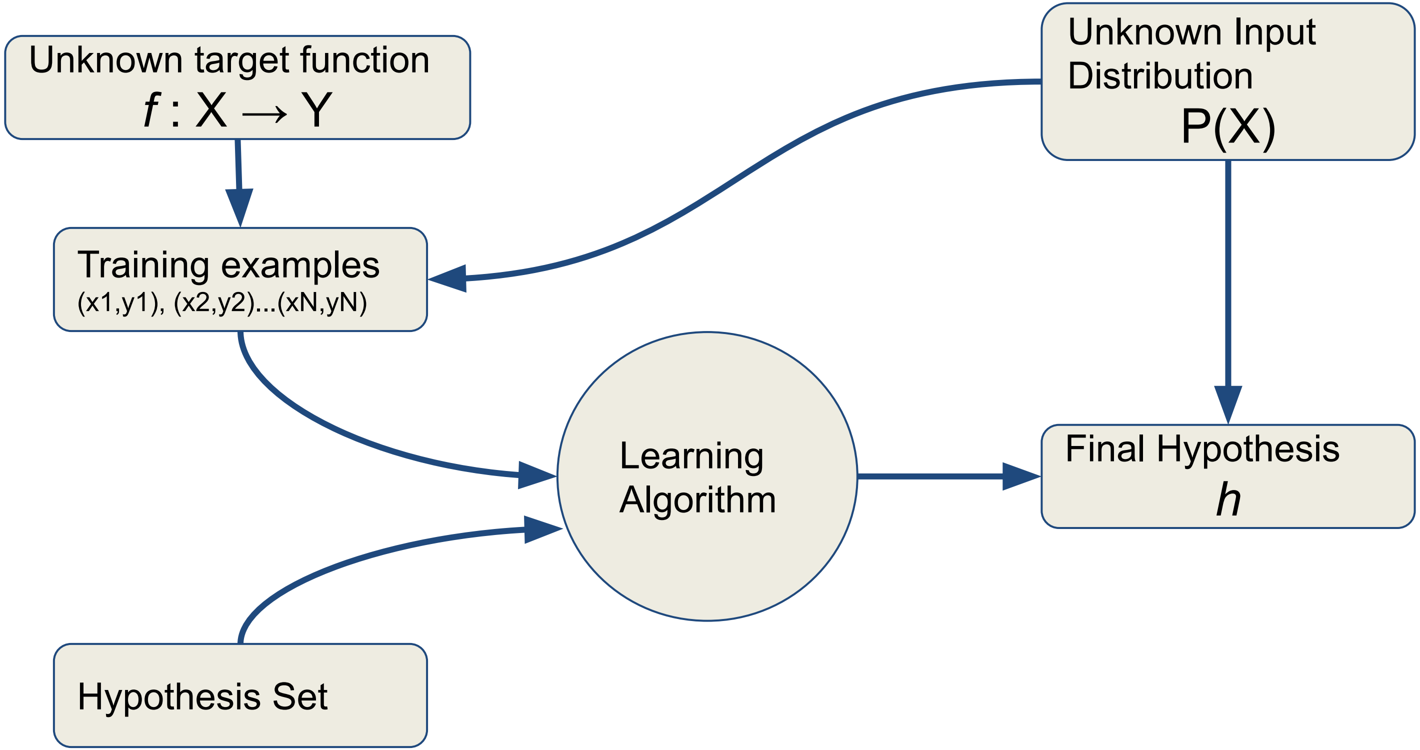

Figure based on Fig.1.9 Mostafa et al.

Generalization Error

Generalization Error

$$\mathbf{E}_{in}(h) = \frac{1}{n}\sum_{i=1}^{N} l(h(X_i), Y_i)$$

$$\mathbf{E}_{out}(h) = \mathbb{E}_{p(X,Y)}[l(h(X), Y)]$$

$$\mathcal{G}_{err} = \mathbf{E}_{out}(h) - \mathbf{E}_{in}(h)$$

Not obvious to minimize the generalization error.

Relation between data and generalization

- IID assumptions

Summary

- Can learn from data!
- Overcomes tedious model designs
- Perceptrons mimic biological neurons

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- Can learn from data!
- Overcomes tedious model designs
- Perceptrons mimic biological neurons

However,

- Depends on the data
  - Strong assumptions
  - Distribution
  - Number of samples
  - High quality labels
- Many (equally better/worse) models to choose from
- Hard to generalize