Question 1 (20%)

Restore the parentheses and dots in the following:

(a) $\lambda f \lambda g \lambda h \lambda x.(f(g(hx)))$

(b) $xxxx$

(c) $\lambda x.x \lambda y.y$

(d) $\lambda x.(x \lambda y.yxx)x$

Question 2 (20%)

Simplify the parentheses and dots in the following:

(a) (xy)

(b) $(x(yz))$

(c) $((xy)z)$

(d) $(\lambda x.x)$

(e) $(\lambda y.(\lambda x.x))$

(f) $(\lambda z.(x(\lambda y.(yz))))$

(g) $(x(\lambda z.(\lambda y.(yz))))$

(h) $(x(\lambda x.x))$

(i) $((\lambda y.(\lambda x.x))(\lambda x.x))$

(j) $(((\lambda y.(\lambda x.x))(\lambda x.x))(xy))$

(k) $((x(yz))((xy)z))$

(l) $(\lambda x.(\lambda y.(\lambda z.((xz)(yz)))))$

(m) $(((ab)(cd))((ef)(gh)))$

(n) $(\lambda x.((\lambda y.(yx))(\lambda v.v)z)u)(\lambda w.w)$

Question 3 (20%)

Reduce the following:

(a) $(\lambda f.fx)g$

(b) $(\lambda f.fx)ga$

(c) $(\lambda f.fx)(ga)$

(d) $(\lambda f \lambda x.fx)ga$

(e) $(\lambda x \lambda y \lambda z.x(yz))f$

(f) $(\lambda x.mx)j$

(g) $(\lambda y.yj)m$

(h) $(\lambda x.\lambda y.y(yx))jm$

(i) $(\lambda y.yj)(\lambda x.mx)$

(j) $(\lambda x.xx)(\lambda y.yyy)$

Question 4 (20%)

Given the formula,

$$\exists y.Fy \wedge \forall x.Fx \rightarrow Exy \tag{1}$$

where E is a predicate that stands for equality; in the semantics, it yields true if its arguments are interpreted as the same individual, and false otherwise.

(a) Give a model that makes (1) true.

(b) Give a model that makes (1) false.

Question 5 (20%)

A formula P **implies** Q, designated $P \models Q$, if and only if every model that makes P true also makes Q true.

$$\forall x.Fx \rightarrow \exists y.Gxy \models \exists x.Fx \tag{2}$$

Show that the implication in (2) does NOT hold.