Question 1

Properly parenthesize the given formulas:

(a) $\exists x.Px \wedge Qx$

(b) $\exists x.Px \wedge \forall y.Qxy \vee Qyx$

(c) $\exists x.Px \wedge \neg\forall y.Qxy \vee Qyx$

Question 2

Simplify the following formulas by eliminating as many parentheses as possible:

(a) $((\neg Qy) \vee (\exists x(Px \to Tx)))$

(b) $((\neg(Qy \wedge Pz)) \vee ((\exists xPx) \to Tx))$

(c) $(((\neg(Qy \wedge Pz)) \vee (\exists xPx)) \to Tx)$

Question 3

Express the following sentences in first-order logic:

(a) A sample was contaminated.

(b) Everything ends.

(c) Every semester ends.

(d) Every student admires some movie.

(e) If an instructor fails, every student passes.

(f) No student failed.

(g) Some humans love math, but not all who love math are humans.

(h) No book is worth reading, except if it is written before 1900.

(i) People without friends are unhappy unless they love reading.

(j) Only people without friends are happy.

(k) All but one student failed.

Question 4

Given the formula,

$$\exists y.Fy \wedge \forall x.Fx \to Exy \tag{1}$$

where E is a predicate that stands for equality; in the semantics, it yields true if its arguments are interpreted as the same individual, and false otherwise.

(a) Give a model that makes (1) true.

(b) Give a model that makes (1) false.

Question 5

A formula P **implies** Q, designated $P \models Q$, if and only if every model that makes P true also makes Q true.

$$\forall x.Fx \to \exists y.Gxy \models \exists x.Fx \tag{2}$$

Show that the implication in (2) does NOT hold.

Question 6

Given the formula,

$$\exists x.Fx \to \forall y.Fy \tag{3}$$

Can you find a model with a non-empty domain that makes (3) evaluate to 0?