

Question 1 (30%)

You have two predicates p and q . Express the following situations in first order logic:

- (a) there is no p that is not also q .

Solution: $\forall x. px \rightarrow qx$

- (b) there is exactly one p .

Solution: $\exists x. px \wedge \forall y. py \rightarrow y = x$

- (c) there are exactly two ps .

Solution: $\exists x \exists y. px \wedge py \wedge \neg(x = y) \wedge \forall z. pz \rightarrow z = x \vee z = y$

- (d) there is at most one p .

Solution: $(\neg \exists x. px) \vee \exists x. px \wedge \forall y. py \rightarrow y = x$

Question 2 (20%)

Render the following sentence in first order logic.

- (1) Every farmer who owns a donkey beats it.

If you cannot come up with a satisfactory formula, briefly comment on why your efforts have failed.

Solution: One possible attempt

$$\forall x. \text{farmer}'x \wedge (\exists y. \text{donkey}'y \wedge \text{owns}'yx) \rightarrow \text{beats}'yx$$

fails because the last occurrence of y is left unbound.

Another one

$$\forall x \forall y. \text{farmer}'x \wedge \text{donkey}'y \wedge \text{owns}'yx \rightarrow \text{beats}'yx$$

is fine with respect to binding, but implies that expressions like *a donkey* can be interpreted as involving universal quantification. We are left with no explanation why we do not have a universally quantified reading for

- (2) A donkey sleeps.

Question 3 (50%)

Specify a lexicon for all the items in the following sentences and drive their meaning specifying their order of combination, syntactic categories, semantic interpretations and semantic types in each step:

Solution: Lexicon:

Let G be an abbreviation for $(S/(S \backslash NP))$

$$\text{donkey} := N : \lambda x. \text{donkey}'x :: \langle e, t \rangle \quad (1)$$

$$\text{student} := N : \lambda x. \text{student}'x :: \langle e, t \rangle \quad (2)$$

$$\text{John} := S/(S \backslash NP) : \lambda p. \text{john}'p :: \langle \langle e, t \rangle, t \rangle \quad (3)$$

$$\text{sleeps} := S \backslash NP : \lambda x. \text{sleeps}'x :: \langle e, t \rangle \quad (4)$$

$$\text{walk(s)} := S \backslash NP : \lambda x. \text{walk}'x :: \langle e, t \rangle \quad (5)$$

$$\text{lazy} := N/N : \lambda p \lambda x. p'x \wedge \text{lazy}'x :: \langle \langle e, t \rangle, \langle e, t \rangle \rangle \quad (6)$$

$$\text{slowly} := S \backslash NP \backslash (S \backslash NP) : \lambda p \lambda x. \text{slowly}'px :: \langle \langle e, t \rangle, \langle e, t \rangle \rangle \quad (7)$$

$$\text{and} := G \backslash G/G : \lambda p \lambda q \lambda r. pr \wedge qr :: \langle \langle \langle e, t \rangle, t \rangle, \langle \langle e, t \rangle, t \rangle, \langle \langle e, t \rangle, t \rangle \rangle \rangle \quad (8)$$

$$\text{every} := S/(S \backslash NP)/N : \lambda p \lambda q \forall x. px \rightarrow qx :: \langle \langle e, t \rangle, \langle \langle e, t \rangle, t \rangle \rangle \quad (9)$$

$$a := S/(S \backslash NP)/N : \lambda p \lambda q \exists x. px \wedge qx :: \langle \langle e, t \rangle, \langle \langle e, t \rangle, t \rangle \rangle \quad (10)$$

(a) Every donkey sleeps.

Solution: D1: apply (9) to (1)
D2: apply (the result of) D1 to (4)

(b) John walks slowly.

Solution: D1: apply (7) to (5)
D2: apply (3) to D1

(c) A lazy donkey walks.

Solution: D1: apply (6) to (1)
D2: apply (10) to D1
D3: apply D2 to (5)

(d) Every student and a lazy donkey walk.

Solution: D1: apply (6) to (1)
D2: apply (10) to D1
D3: apply (9) to (2)
D4: apply (8) to D2
D5: apply D4 to D3
D6: apply D5 to (5)