# Quantification

→ every → The universal quantifier

→ some / a → Existential quantifier

→ no → Negative quantifier

Every child sleeps. $\Longrightarrow$ $\forall x.\, Cx \to Sx$

$\forall x.\, child'x \to sleeps'x$

$\forall x\,(child'x \to sleeps'x)$

$\underline{\text{Every child}}$ $\underline{\text{sleeps}}$

$S/(S\backslash NP)$ $\quad$ $\underline{S\backslash NP}$

child := $N$ : $\lambda x.child'x$
$\langle e,t\rangle$

sleeps := $\underline{S\backslash NP}$ : $\lambda x.sleeps'x$
$\langle e,t\rangle$

every child := $S/(S\backslash NP)$ : $\underline{\lambda p \forall x.child'x \to px}$
$t$ $\quad$ $\langle e,t\rangle$ $\quad$ $\langle\langle e,t\rangle,t\rangle$

every := $S/(S\backslash NP)/N$ : $\lambda q \lambda p\, \forall x.\, qx \to px$

Every child
$\to NP$
$\downarrow$
$\cancel{e}$ $\longrightarrow d_{235}$

## Model

$D = \{d_1, d_2 \ldots\}$

| Every | child | sleeps. |
|---|---|---|
| $S/(S\backslash NP)/N$ | $N$ | $S\backslash NP$ |
| $\lambda q \lambda p\, \forall x.\, qx \rightarrow px$ | $\lambda x.\mathrm{child}'x$ | $\lambda x.\mathrm{sleeps}'x$ |
| | $\mathrm{child}'$ | $\mathrm{sleeps}'$ |
| $\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$ | $\langle e,t\rangle$ | $\langle e,t\rangle$ |
| $et(ett)$ | $et$ | $et$ |

$$\xrightarrow{\hspace{6em}}$$

$S/(S\backslash NP)$

$\lambda p\, \forall x.\, \mathrm{child}'x \rightarrow px$

$\langle\langle e,t\rangle,t\rangle$

$$\xrightarrow{\hspace{6em}}$$

$S$

$\forall x.\, \mathrm{child}'x \rightarrow \mathrm{sleeps}'x$

$t$

# Quantification (cont.)

some / a

A child sleeps.

~~Reading 1~~

~~specific reading~~

ignore for now

Reading 2

existential reading

A student cheated in the exam.

↳ a student on mind.

↳ no particular student on mind.

⇒

⇒

Some

~~A~~ child sleeps. $\Longrightarrow$ $\exists x . \text{child}' x \wedge \text{sleeps}' x$

$\Longrightarrow$

model

$\text{child} := N : \lambda x . \text{child}' x$

$\langle e,t \rangle$

A child

$\cancel{NP}$

$\cancel{e}$ $\rightarrow d_{255}$

$\text{sleeps} := S \backslash NP : \lambda x . \text{sleeps}' x$

$\langle e,t \rangle$

$\underline{\underline{a}} := S/(S \backslash NP)/N : \boxed{\lambda q \lambda p . \exists x . q x \wedge p x}$

$\underline{\text{a child}} := S/(S \backslash NP) : \lambda p . \exists x . \text{child}' x \wedge p x$

$$\frac{A}{S/(S\backslash NP)/\cancel{N}} \qquad \frac{child}{\cancel{N}} \qquad \frac{sleeps.}{S\backslash NP}$$

$\lambda q \lambda p.\ \exists x\ qx \wedge px$

$\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$

$et(et\,t)$

$\lambda x.child'x$

$child'$

$\langle e,t\rangle$

$et$

$\lambda x.sleeps'x$

$sleeps'$

$\langle e,t\rangle$

$et$

$S/(S\backslash NP)$

$\Rightarrow$

$\lambda p.\ \exists x.child'x \wedge px\ :((et)t) = \langle\langle e,t\rangle,t\rangle$

$(\lambda q \lambda p\ \exists x.qx \wedge px)\ (\lambda x.child'x)$

$\lambda p.\ \exists x\ (\lambda x.child'x)x \wedge px$

$\lambda p.\exists x.\ child'x \wedge px$

$S$

$\exists x.\ child'x \wedge sleeps'x$

$t$

No child sleeps. $\Longrightarrow \neg \exists x.\ child'x \wedge sleeps'x$

$no := S/(S\backslash NP)/N : \boxed{\lambda q \lambda p.\ \neg \exists x.\ px \wedge qx}$

# Generalized Quantifiers

Ex. 'at least two'

ALT A B

$|A \cap B| \geq 2$

every : $S\backslash(S\backslash NP)/N$ : $\lambda p \lambda q. \forall x. px \to qx$

$et(ett)$

$(e \to t) \to ((e \to t) \to t)$

Every child sleeps.


Every $A$ $B$ $\equiv A \subseteq B$

Some $A$ $B$ $\equiv A \cap B \neq \emptyset$

No $A$ $B$ $\equiv A \cap B = \emptyset$

$\neg$some