Question 1   (20%)
   Restore the parentheses and dots in the following:

   (a) $\lambda f \lambda g \lambda h \lambda x.(f(g(hx)))$

   (b) $xxxx$

   (c) $\lambda x.x \lambda y.y$

   (d) $\lambda x.(x \lambda y.yxx)x$

Question 2   (20%)
   Simplify the parentheses and dots in the following:

   (a) (xy)

   (b) $(x(yz))$

   (c) $((xy)z)$

   (d) $(\lambda x.x)$

   (e) $(\lambda y.(\lambda x.x))$

   (f) $(\lambda z.(x(\lambda y.(yz))))$

   (g) $(x(\lambda z.(\lambda y.(yz))))$

   (h) $(x(\lambda x.x))$

   (i) $((\lambda y.(\lambda x.x))(\lambda x.x))$

   (j) $(((\lambda y.(\lambda x.x))(\lambda x.x))(xy))$

   (k) $((x(yz))((xy)z))$

   (l) $(\lambda x.(\lambda y.(\lambda z.((xz)(yz)))))$

   (m) $(((ab)(cd))((ef)(gh)))$

   (n) $(\lambda x.((\lambda y.(yx))(\lambda v.v)z)u)(\lambda w.w)$

Question 3   (20%)
   Reduce the following:

   (a) $(\lambda f.fx)g$

   (b) $(\lambda f.fx)ga$

   (c) $(\lambda f.fx)(ga)$

   (d) $(\lambda f \lambda x.fx)ga$

   (e) $(\lambda x \lambda y \lambda z.x(yz))f$

   (f) $(\lambda x.mx)j$

   (g) $(\lambda y.yj)m$

   (h) $(\lambda x.\lambda y.y(yx))jm$

   (i) $(\lambda y.yj)(\lambda x.mx)$

   (j) $(\lambda x.xx)(\lambda y.yyy)$

Question 4   (40%)
   In a programming language of your choice,

   (a) find a way to represent first-order logic formulas;

   (b) define a procedure that returns the list/set of free variables in the input expression.