Question 1 (20%)

Restore the parentheses and dots in the following:

(a) $\lambda f \lambda g \lambda h \lambda x.(f(g(hx)))$

Solution: $(\lambda f.(\lambda g.(\lambda h.(\lambda x.(f(g(hx)))))))$

(b) $xxxx$

Solution: $(((xx)x)x)$

(c) $\lambda x.x \lambda y.y$

Solution: $(\lambda x.(x(\lambda y.y)))$

(d) $\lambda x.(x \lambda y.yxx)x$

Solution: $(\lambda x.(x(\lambda y.((yx)x)))x)$

Question 2 (20%)

Simplify the parentheses and dots in the following:

(a) (xy)

Solution: xy

(b) $(x(yz))$

Solution: $x(yz)$

(c) $((xy)z)$

Solution: xyz

(d) $(\lambda x.x)$

Solution: $\lambda x.x$

(e) $(\lambda y.(\lambda x.x))$

Solution: $\lambda y \lambda x.x$

(f) $(\lambda z.(x(\lambda y.(yz))))$

Solution: $\lambda z.x(\lambda y.yz)$

(g) $(x(\lambda z.(\lambda y.(yz))))$

Solution: $x \lambda z \lambda y.yz$

(h) $(x(\lambda x.x))$

Solution: $x \lambda x.x$

(i) $((\lambda y.(\lambda x.x))(\lambda x.x))$

Solution: $(\lambda y \lambda x.x)\lambda x.x$

(j) $(((\lambda y.(\lambda x.x))(\lambda x.x))(xy))$

Solution: $(\lambda y \lambda x.x)(\lambda x.x)(xy)$

(k) $((x(yz))((xy)z))$

Solution: $x(yz)(xyz)$

(l) $(\lambda x.(\lambda y.(\lambda z.((xz)(yz)))))$

Solution: $\lambda x \lambda y \lambda z.xz(yz)$

(m) $(((ab)(cd))((ef)(gh)))$

Solution: $ab(cd)(ef(gh))$

(n) $(\lambda x.((\lambda y.(yx))(\lambda v.v)z)u)(\lambda w.w)$

Solution: $(\lambda x.(\lambda y.yx)(\lambda v.v)zu)(\lambda w.w)$

Question 3 (20%)

Reduce the following:

(a) $(\lambda f.fx)g$

Solution: gx

(b) $(\lambda f.fx)ga$

Solution: gxa

(c) $(\lambda f.fx)(ga)$

Solution: gax

(d) $(\lambda f \lambda x.fx)ga$

Solution: ga

(e) $(\lambda x \lambda y \lambda z.x(yz))f$

Solution: $\lambda y \lambda z.f(yz)$

(f) $(\lambda x.mx)j$

Solution: mj

(g) $(\lambda y.yj)m$

Solution: mj

(h) $(\lambda x.\lambda y.y(yx))\,jm$

Solution: $m(mj)$

(i) $(\lambda y.yj)(\lambda x.mx)$

Solution: mj

(j) $(\lambda x.xx)(\lambda y.yyy)$

Solution: This gets exponentially bigger and bigger as you reduce.

Question 4 (20%)

Reduce the following:

(a) $(\lambda p.p\,john')(\lambda x.sleeps'x)$

Solution: $sleeps'\,john'$

(b) $(\lambda p\lambda q.\forall x.p'x \rightarrow q'x)(\lambda x.student'x)(\lambda x.sleeps'x)$

Solution: $\forall x.student'x \rightarrow sleeps'x$

(c) $(\lambda p\lambda x.think'px)((\lambda p.p\,john')(\lambda x.sleeps'x))alice'$

Solution: $think'(sleeps'\,john')alice'$

(d) $(\lambda p\lambda q.\exists x.px \wedge qx)(\lambda x.student'x)((\lambda p\lambda x.think'px)((\lambda p.p\,john')(\lambda x.sleeps'x)))$

Solution: $\exists x.student'x \wedge think'(sleeps'\,john')x$

Question 5 (20%)

What should α be in the following reductions?

(a) $\alpha(\lambda x.walks'x)\,john' \equiv_\beta slow'(walks'\,john')$

Solution: $\lambda p\lambda x.slow'(px)$

(b) $\alpha(\lambda x.walks'x)\,john' \equiv_\beta slow'walks'\,john'$

Solution: $\lambda p\lambda x.slow'px$
Note that, first one obtains:
$slow'(\lambda x.walks'x)\,john'$
then, after $\eta-$reduction, you get:
$slow'walks'\,john'$

(c) $\alpha mary'\,john'(\lambda x.walks'x) \equiv_\beta walks'\,john' \wedge walks'mary'$

Solution: $\lambda x\lambda y\lambda p.py \wedge px$

(d) $\alpha(\lambda x.talks'x)(\lambda x.smiles'x)\,john' \equiv_\beta smiles'\,john' \wedge talks'\,john'$

Solution: $\lambda p\lambda q\lambda x.qx \wedge px$