Worlds

expr. $\xrightarrow{\text{intension}}$ extension.

intensionality

intension

intension vs. extension

$\rightarrow 24$

$4! = \underline{4 \times 3 \times 2 \times 1 =} 24 \Rightarrow$ extension

$\underline{62!} = ? = \underline{62 \times 61 \times 60 \cdots 2 \times 1}$

$\downarrow$

meaning

I go shopping on [the day that John does] ⇒ Tuesday

R1: (Given J. goes shopping on Tuesdays): I go shopping on Tuesdays.

R2: whenever John goes shopping, I also do so.

* Intensionality

modals: must, can, may, should, ought ...

intensional verbs: seek, want ...

John seeks / looks for a student

⇓

Terry / ?

Terry

some student

intension w/o extension

A Norwegian director with 4 fingers in her right hand.

John wants to marry a Norwegian.

R1 d

R2

⇒ Ana

some Norwegian

⇒ intension

world: a set: all the true propositions in that world

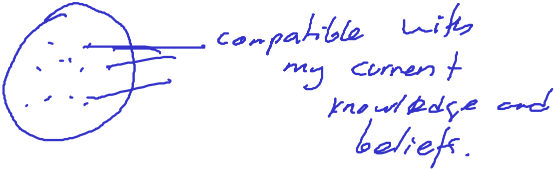
$w_0 = \{p_1, p_2, p_{447} \ldots\}$

$(p_1, p_2 \ldots)$

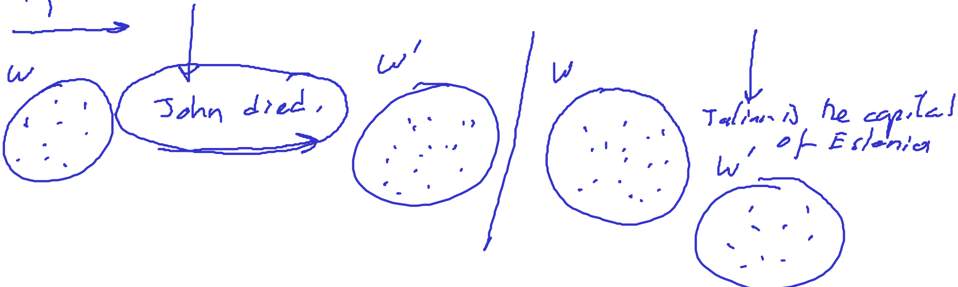
$w_4 = \{ \; | \quad | \; \neq p_{447}^{-} \ldots \}$

$\vdots$

$w_{72} = \{ \ldots \}$

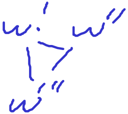


$\xrightarrow{T}$



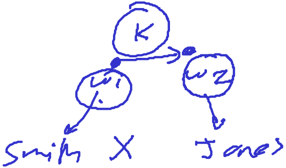
$M = \langle D, g, \{0,1\}, I, W, R \rangle$

D = desires

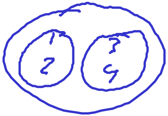


K = knowing
knowledge

aletic



O = deontic



Structuring via relations

$R = \{\langle 1,3 \rangle, \langle 2,4 \rangle, \langle 1,2 \rangle\}$

$$\begin{array}{ccc} 1 & \longrightarrow & 3 \\ \downarrow & & \\ 2 & \longrightarrow & 4 \end{array}$$

$R' = \{\langle 1,1 \rangle, \langle 2,3 \rangle\}$

$1 \circlearrowleft \quad 2 \longrightarrow 3$

loves : $\lambda x \lambda y.\ loves'\ \underset{e}{x\ y}$ :: $\langle e, \langle e, t \rangle \rangle$ $\quad s$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \downarrow \quad\quad\quad\quad\quad\quad \downarrow$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad D \quad\quad\quad\quad\quad\quad W$

sleeps : $\lambda x.\ sleeps'\ x$

e : entity

sleeps : $\lambda x \lambda s.\ sleeps'_s\ x$

t = truth values

$\lambda x \lambda s.\ sleeps' s\ x$

s = world

$\lambda \underset{e}{x}\ \lambda \underset{s}{w}.\ sleeps'_w\ x$

sleeps (old version) : $\langle e, t \rangle$

sleeps (new version) : $\langle e, \langle s, t \rangle \rangle$

John $\longrightarrow$ w_{237} $\longrightarrow$ 0 or 1

→ Sentential modal (E.g. probably)

[Probably, John sleeps.]

$\underbrace{\text{Probably,}}_{\text{operator}}$ $\underbrace{\text{John sleeps.}}_{\text{prejacent}}$

$\underline{K \text{ relation}}$ $\quad K\, w_1\, w_2$

K spk' — w

$\underline{K}\,\underline{w_3}\,\underline{spk'}$:

$\langle\langle s,t\rangle,t\rangle$ Probably	John	sleeps.
S/S	S/(S\NP)	S\NP
$\lambda p\,\exists w.\,K\,w\,spk' \wedge \underline{p}\,w$	$\lambda p.\,p\,j'$	$\lambda x \lambda s.\,\text{sleeps}'_s\,x$

$\exists w.\,K\,w\,spk' \wedge \text{sleeps}'_w\,j'$

$\langle s,t\rangle$

S : $\underline{\lambda s.\,\text{sleeps}'_s\,j'}$

S : $\underline{\exists w.\,K\,w\,spk' \wedge \text{sleeps}'_w\,j'}$

$\underline{\text{Montague}}$

Higher order Intensional Logic

[Certainly,] John sleeps.

Quantifying over the entire W

$\forall w, K\, w\, spk' \rightarrow sleeps'_w\, j'$

Certainly := S/S : $\lambda p\, \forall w\, K\, w\, spk' \rightarrow p\, w$

John may pass. "modal auxiliary"

John	may	pass.
$S/(S\backslash NP)$	$(S\backslash NP)/(S\backslash NP)$	$S\backslash NP$
$\lambda p.pJ'$	$\lambda p \lambda x \exists w. K_w spk' \wedge p\,x\,w$	$(\lambda x \lambda w. pass'_w\ x)$
$\Rightarrow$		$<e,<s,t>>$

$>$

$S\backslash NP$

$\lambda x\ \exists w. K_w spk' \wedge (\lambda x \lambda w. pass'_w\ x)\ x\ w$

$\lambda x\ \exists w. K_w spk' \wedge (\lambda w. pass'_w\ x)\ w$

$(\lambda x\ \exists w. K_w spk' \wedge pass'_w\ x)$

$>$

$(\lambda x \exists w. K_w spk' \wedge pass'_w\ x)\ J'$

$\exists w. K_w spk' \wedge pass'_w J'$

John may smoke.

$$\exists_w \; O \; w \; spk' \wedge smoke'_w \; J'$$