Question 1 (20%)

Restore the parentheses and dots in the following:

(a) $\lambda f \lambda g \lambda h \lambda x.(f(g(hx)))$

(b) $xxxx$

(c) $\lambda x.x \lambda y.y$

(d) $\lambda x.(x \lambda y.yxx)x$

Question 2 (20%)

Simplify the parentheses and dots in the following:

(a) (xy)

(b) $(x(yz))$

(c) $((xy)z)$

(d) $(\lambda x.x)$

(e) $(\lambda y.(\lambda x.x))$

(f) $(\lambda z.(x(\lambda y.(yz))))$

(g) $(x(\lambda z.(\lambda y.(yz))))$

(h) $(x(\lambda x.x))$

(i) $((\lambda y.(\lambda x.x))(\lambda x.x))$

(j) $(((\lambda y.(\lambda x.x))(\lambda x.x))(xy))$

(k) $((x(yz))((xy)z))$

(l) $(\lambda x.(\lambda y.(\lambda z.((xz)(yz)))))$

(m) $(((ab)(cd))((ef)(gh)))$

(n) $(\lambda x.((\lambda y.(yx))(\lambda v.v)z)u)(\lambda w.w)$

Question 3 (20%)

Reduce the following:

(a) $(\lambda f.fx)g$

(b) $(\lambda f.fx)ga$

(c) $(\lambda f.fx)(ga)$

(d) $(\lambda f \lambda x.fx)ga$

(e) $(\lambda x \lambda y \lambda z.x(yz))f$

(f) $(\lambda x.mx)j$

(g) $(\lambda y.yj)m$

(h) $(\lambda x.\lambda y.y(yx))jm$

(i) $(\lambda y.yj)(\lambda x.mx)$

(j) $(\lambda x.xx)(\lambda y.yyy)$

Question 4 (40%)

In a programming language of your choice,

(a) find a way to represent lambda expressions;

(b) define a procedure that returns the list/set of free variables in the input term.