Q1.

Write the meaning of these sentences in first order logic. You may ignore temporal and aspectual semantics for the moment.

(1) a. Every student who is tired is happy.
b. Every tired student is happy.
c. Every successful student is tired.
d. John ate only two apples. (Hint: you can use the two place predicate *notid*, meaning 'not identical' or 'not the same individual as'.)

Solution:

a. $\forall x.student'x \wedge tired'x \rightarrow happy'x$

b. $\forall x.student'x \wedge tired'x \rightarrow happy'x$

c. $\forall x.succ'student'x \rightarrow tired'x$
$succ'$ is a function that maps a property to another property. It's internal semantics is hard to define, but for now we make sure that it is not an intersective adjective like tired.

d. $\exists x \exists y.apple'x \wedge apple'y \wedge ate'xjohn' \wedge ate'yjohn' \wedge notid'xy \wedge \forall z.apple'z \wedge ate'zjohn' \rightarrow id'zy \vee id'zx$

Q2.

Let the operator $\circ$ be left associative. (For associativity you can consult the lambda calculus notes Section 3, item B, or the Wikipedia article on 'operator associativity'.

Eliminate all the eliminable parenthesis in the following:

(2) a. $((a \circ b) \circ c)$
b. $(a \circ (b \circ (c \circ d)))$
c. $(a \circ ((b \circ c) \circ d))$

Solution:

a. $a \circ b \circ c$

b. $a \circ (b \circ (c \circ d))$

c. $a \circ (b \circ c \circ d)$

Restore the following to fully parenthesized form.

(3) a. $a \circ b \circ c \circ d$
b. $a \circ b \circ (c \circ d)$
c. $a \circ (b \circ c) \circ d$

Solution:

a. $(((a \circ b) \circ c) \circ d)$

b. $(a \circ (b \circ (c \circ d)))$

c. $((a \circ (b \circ c)) \circ d)$