

Q 1.

Properly parenthesize the given formulas:

- (a) $\exists x.Px \wedge Qx$
- (b) $\exists x.Px \wedge \forall y.Qxy \vee Qyx$
- (c) $\exists x.Px \wedge \neg \forall y.Qxy \vee Qyx$

Q 2.

Simplify the following formulas by eliminating as many parentheses as possible:

- (a) $((\neg Qy) \vee (\exists x(Px \rightarrow Tx)))$
- (b) $((\neg(Qy \wedge Pz)) \vee ((\exists xPx) \rightarrow Tx))$
- (c) $((\neg(Qy \wedge Pz)) \vee (\exists xPx)) \rightarrow Tx$

Q 3.

Express the following sentences in first-order logic:

- (a) A sample was contaminated.
- (b) Everything ends.
- (c) Every semester ends.
- (d) Every student admires some movie.
- (e) If an instructor fails, every student passes.
- (f) No student failed.
- (g) Some humans love math, but not all who love math are humans.
- (h) No book is worth reading, except if it is written before 1900.
- (i) People without friends are unhappy unless they love reading.
- (j) Only people without friends are happy.
- (k) All but one student failed.

Q 4.

Write the meaning of these sentences in first order logic. You may ignore temporal and aspectual semantics for the moment.

- (1) a. Every student who is tired is happy.
- b. Every tired student is happy.
- c. Every successful student is tired.
- d. John ate only two apples. (Hint: you can use the two place predicate *notid*, meaning ‘not identical’ or ‘not the same individual as’.)

Q 5.

Write the meaning of these sentences in first order logic. You may ignore temporal and aspectual semantics for the moment.

- (2) a. No one speaks French. (Note: you can take *speak French* as a single lexical item. But, if you like, try to take the two words separately.)
- b. No one speaks French or Arabic.
- c. Every student is happy.
- d. Every student is happy and tired.
- e. Every student is happy, and every student is tired.
- f. Every student is happy or tired.
- g. Every student is happy, or every student is tired.

Q 6.

- (a) Do (2d) and (2e) mean the same thing? Why or why not?
- (b) Do (2f) and (2g) mean the same thing? Why or why not?

Q 7.

You have two predicates p and q . Express the following situations in first order logic:

- (a) there is no p that is not also q .
- (b) there is exactly one p .
- (c) there are exactly two ps .
- (d) there is at most one p .

Q 8.

Given the formula,

$$\exists y.Fy \wedge \forall x.Fx \rightarrow Exy \quad (1)$$

where E is a predicate that stands for equality; in the semantics, it yields true if its arguments are interpreted as the same individual, and false otherwise.

- (a) Give a model that makes (1) true.
- (b) Give a model that makes (1) false.

Q 9.

A formula P **implies** Q , designated $P \models Q$, if and only if every model that makes P true also makes Q true.

$$\forall x.Fx \rightarrow \exists y.Gxy \models \exists x.Fx \quad (2)$$

Show that the implication in (2) does NOT hold.

Q 10.

Given the formula,

$$\exists x.Fx \rightarrow \forall y.Fy \quad (3)$$

Can you find a model with a non-empty domain that makes (3) evaluate to 0?

Q 11.

Render the following sentence in first order logic.

(4) Every farmer who owns a donkey beats it.

If you cannot come up with a satisfactory formula, briefly comment on why your efforts have failed.