Q 1.

Properly parenthesize the given formulas:

(a) $\exists x.Px \wedge Qx$

(b) $\exists x.Px \wedge \forall y.Qxy \vee Qyx$

(c) $\exists x.Px \wedge \neg\forall y.Qxy \vee Qyx$

Q 2.

Simplify the following formulas by eliminating as many parentheses as possible:

(a) $((\neg Qy) \vee (\exists x(Px \rightarrow Tx)))$

(b) $((\neg(Qy \wedge Pz)) \vee ((\exists xPx) \rightarrow Tx))$

(c) $(((\neg(Qy \wedge Pz)) \vee (\exists xPx)) \rightarrow Tx)$

Q 3.

Express the following sentences in first-order logic:

(a) A sample was contaminated.

(b) Everything ends.

(c) Every semester ends.

(d) Every student admires some movie.

(e) If an instructor fails, every student passes.

(f) No student failed.

(g) Some humans love math, but not all who love math are humans.

(h) No book is worth reading, except if it is written before 1900.

(i) People without friends are unhappy unless they love reading.

(j) Only people without friends are happy.

(k) All but one student failed.

Q 4.

Write the meaning of these sentences in first order logic. You may ignore temporal and aspectual semantics for the moment.

(1) a. Every student who is tired is happy.
b. Every tired student is happy.
c. Every successful student is tired.
d. John ate only two apples. (Hint: you can use the two place predicate *notid*, meaning 'not identical' or 'not the same individual as'.)

Q 5.

Write the meaning of these sentences in first order logic. You may ignore temporal and aspectual semantics for the moment.

(2) a. No one speaks French. (Note: you can take *speak French* as a single lexical item. But, if you like, try to take the two words separately.)
b. No one speaks French or Arabic.
c. Every student is happy.
d. Every student is happy and tired.
e. Every student is happy, and every student is tired.
f. Every student is happy or tired.
g. Every student is happy, or every student is tired.

Q 6.

(a) Do (2d) and (2e) mean the same thing? Why or why not?

(b) Do (2f) and (2g) mean the same thing? Why or why not?

Q 7.

You have two predicates p and q. Express the following situations in first order logic:

(a) there is no p that is not also q.

(b) there is exactly one p.

(c) there are exactly two ps.

(d) there is at most one p.

Q 8.

Given the formula,

$$\exists y.Fy \wedge \forall x.Fx \rightarrow Exy \tag{1}$$

where E is a predicate that stands for equality; in the semantics, it yields true if its arguments are interpreted as the same individual, and false otherwise.

(a) Give a model that makes (1) true.

(b) Give a model that makes (1) false.

Q 9.

A formula P **implies** Q, designated $P \models Q$, if and only if every model that makes P true also makes Q true.

$$\forall x.Fx \rightarrow \exists y.Gxy \models \exists x.Fx \tag{2}$$

Show that the implication in (2) does NOT hold.

Q 10.

Given the formula,

$$\exists x.Fx \rightarrow \forall y.Fy \tag{3}$$

Can you find a model with a non-empty domain that makes (3) evaluate to 0?

Q 11.

Render the following sentence in first order logic.

(4) Every farmer who owns a donkey beats it.

If you cannot come up with a satisfactory formula, briefly comment on why your efforts have failed.