Question 1 (30%)

You have two predicates p and q. Express the following situations in first order logic:

(a) there is no p that is not also q.

Solution: $\forall x.px \rightarrow qx$

(b) there is exactly one p.

Solution: $\exists x.px \wedge \forall y.py \rightarrow y = x$

(c) there are exactly two ps.

Solution: $\exists x \exists y.px \wedge py \wedge \neg(x = y) \wedge \forall z.pz \rightarrow z = x \vee z = y$

(d) there is at most one p.

Solution: $(\neg\exists x.px) \vee \exists x.px \wedge \forall y.py \rightarrow y = x$

Question 2 (20%)

Render the following sentence in first order logic.

(1) Every farmer who owns a donkey beats it.

If you cannot come up with a satisfactory formula, briefly comment on why your efforts have failed.

Solution: One possible attempt

$$\forall x.farmer'x \wedge (\exists y.donkey'y \wedge owns'yx) \rightarrow beats'yx$$

fails because the last occurrence of y is left unbound.

Another one

$$\forall x \forall y.farmer'x \wedge donkey'y \wedge owns'yx \rightarrow beats'yx$$

is fine with respect to binding, but implies that expressions like *a donkey* can be interpreted as involving universal quantification. We are left with no explanation why we do not have a universally quantified reading for

(2) A donkey sleeps.

Question 3 (50%)

Specify a lexicon for all the items in the following sentences and drive their meaning specifying their order of combination, syntactic categories, semantic interpretations and semantic types in each step:

Solution: Lexicon:

Let G be an abbreviation for $(S/(S\backslash NP))$

$donkey :=$	$N :$	$\lambda x.donkey'x ::$	$\langle e,t\rangle$	(1)
$student :=$	$N :$	$\lambda x.student'x ::$	$\langle e,t\rangle$	(2)
$John :=$	$S/(S\backslash NP) :$	$\lambda p.john'p ::$	$\langle\langle e,t\rangle,t\rangle$	(3)
$sleeps :=$	$S\backslash NP :$	$\lambda x.sleeps'x ::$	$\langle e,t\rangle$	(4)
$walk(s) :=$	$S\backslash NP :$	$\lambda x.sleeps'x ::$	$\langle e,t\rangle$	(5)
$lazy :=$	$N/N :$	$\lambda p\lambda x.p'x \wedge lazy'x ::$	$\langle\langle e,t\rangle,\langle e,t\rangle\rangle$	(6)
$slowly :=$	$S\backslash NP\backslash(S\backslash NP) :$	$\lambda p\lambda x.slowly'px ::$	$\langle\langle e,t\rangle,\langle e,t\rangle\rangle$	(7)
$and :=$	$G\backslash G/G :$	$\lambda p\lambda q\lambda r.pr \wedge qr ::$	$\langle\langle\langle e,t\rangle,t\rangle,\langle\langle\langle e,t\rangle,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle\rangle$	(8)
$every :=$	$S/(S\backslash NP)/N :$	$\lambda p\lambda q\forall x.px \rightarrow qx ::$	$\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$	(9)
$a :=$	$S/(S\backslash NP)/N :$	$\lambda p\lambda q\exists x.px \wedge qx ::$	$\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$	(10)

(a) Every donkey sleeps.

Solution:
D1: apply (9) to (1)
D2: apply (the result of) D1 to (4)

(b) John walks slowly.

Solution:
D1: apply (7) to (5)
D2: apply (3) to D1

(c) A lazy donkey walks.

Solution:
D1: apply (6) to (1)
D2: apply (10) to D1
D3: apply D2 to (5)

(d) Every student and a lazy donkey walk.

Solution:
D1: apply (6) to (1)
D2: apply (10) to D1
D3: apply (9) to (2)
D4: apply (8) to D2
D5: apply D4 to D3
D6: apply D5 to (5)