Q1.

Reduce the following term

$$(\lambda f \lambda g \lambda x. fx(gx))(\lambda y.yx)(\lambda x.sx)$$

Solution: $(\lambda f \lambda g \lambda x. fx(gx))(\lambda y.yx)(\lambda x.sx)$

Observe that $(\lambda y.yx)$ is not free for f in $(\lambda f \lambda g \lambda x. fx(gx))$.

So first α-convert $(\lambda f \lambda g \lambda x. fx(gx))$ to $(\lambda f \lambda g \lambda z. fz(gz))$.

Now the form is,

$(\lambda f \lambda g \lambda z. fz(gz))(\lambda y.yx)(\lambda x.sx)$

First reduction,

$(\lambda g \lambda z.(\lambda y.yx)z(gz))(\lambda x.sx)$

inner reduction,

$(\lambda g \lambda z.zx(gz))(\lambda x.sx)$

go on,

$(\lambda z.zx((\lambda x.sx)z))$

finally,

$(\lambda z.zx(sz))$

Q2.

In the last hour we switched our perspective from sets to functions. In this setting, we can think of verb phrases like *walks* as a function from individuals to truth values, represented as $\lambda x.walk'x$, ignoring tense and aspect for now. When you apply this function to the meaning of an individual, say John, represented as $john'$, you get the logical form $walk'john'$, which stands for the proposition that John walks. In terms of the model structure, this proposition amounts to saying that the individual, named John in natural language, is a member of the set of walkers. We say that the sentence *John walks* is interpreted as $walk'john'$. To save some typing, we depict this relation as:

(1) $\|\text{John walks}\| = walk'john'$

In this framework, given a natural language expression as a sequence of words, the task of interpretation involves the following sub-tasks:

(2) a. What are the interpretations of the individual words?
    b. What is the applicative structure of the expression?

What the heck is applicative structure? Assume you are given a sequence of terms:

$$\lambda x.x^2, 7, \lambda x.x \times 3$$

and asked to compute the result. To be able to that, you need to know the applicative structure of this sequence. It has two distinct applicative structures that makes sense – certain applicative structures are meaningless.

(3) a. $(\lambda x.x \times 3)((\lambda x.x^2)7)$
    b. $(\lambda x.x^2)((\lambda x.x \times 3)7)$

Therefore, the meaning of a sequence of terms depends on the meaning of the terms and the applicative structure imposed on the sequence. In our way of thinking, the meaning of a natural language expression is no different. It depends on the meaning of the terms and their applicative structure.

What we call syntax is the process or system that maps a sequence of terms to its applicative structure.

When there are only two words in an expression, as in our example *John walks*, once the meaning of the terms are made clear, the applicative structure is automatically determined; but as the expression gets crowded, you need syntactic rules and principles that deliver the right applicative structure.

Now comes the question. Given the sentence,

(4) John walks slowly.

Give the interpretation of the terms, the applicative structure of the sentence, and comment on what these interpretations correspond to in the model of the world.

Solution: At the moment, it doesn't seem logical to take $john'$ as a function. Therefore, the two possible applicative structures are:

- $slow'(walk' john')$
- $slow' walk' john'$

The first says that the proposition or event of John walking is slow. If we take $walk' john'$ as denoting an event (or process) the first option would be fine. But it does not make sense, if we take $walk' john'$ to be denoting the proposition that John is in the set of walkers. In this latter case, the answer must be the second option, according to which, $slow' walk'$ denotes the set of slow walkers.