## **Question 1** (30%)

You have two predicates p and q. Express the following situations in first order logic:

(a) there is no p that is not also q.

**Solution:**  $\forall x.px \rightarrow qx$ 

(b) there is exactly one p.

**Solution:**  $\exists x.px \land \forall y.py \rightarrow y = x$ 

(c) there are exactly two ps.

**Solution:**  $\exists x \exists y. px \land py \land \neg(x = y) \land \forall z. pz \rightarrow z = x \lor z = y$ 

(d) there is at most one p.

**Solution:**  $(\neg \exists x.px) \lor \exists x.px \land \forall y.py \rightarrow y = x$ 

## **Question 2** (20%)

Render the following sentence in first order logic.

(1) Every farmer who owns a donkey beats it.

If you cannot come up with a satisfactory formula, briefly comment on why your efforts have failed.

**Solution:** One possible attempt

 $\forall x. farmer'x \land (\exists y. donkey'y \land owns'yx) \rightarrow beats'yx$ 

fails because the last occurrence of y is left unbound.

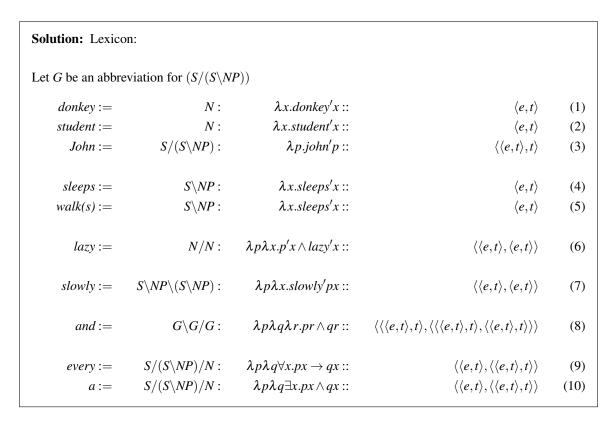
Another one

 $\forall x \forall y. farmer'x \land donkey'y \land owns'yx \rightarrow beats'yx$ 

- is fine with respect to binding, but implies that expressions like *a donkey* can be interpreted as involving universal quantification. We are left with no explanation why we do not have a universally quantified reading for
- (2) A donkey sleeps.

## **Question 3** (50%)

Specify a lexicon for all the items in the following sentences and drive their meaning specifying their order of combination, syntactic categories, semantic interpretations and semantic types in each step:



(a) Every donkey sleeps.

**Solution:** D1: apply (9) to (1)

D2: apply (the result of) D1 to (4)

(b) John walks slowly.

**Solution:** D1: apply (7) to (5)

D2: apply (3) to D1

(c) A lazy donkey walks.

**Solution:** D1: apply (6) to (1)

D2: apply (10) to D1
D3: apply D2 to (5)

(d) Every student and a lazy donkey walk.

**Solution:** D1: apply (6) to (1)

D2: apply (10) to D1
D3: apply (9) to (2)
D4: apply (8) to D2
D5: apply D4 to D3
D6: apply D5 to (5)