Advanced Machine Learning - Reinforcement Learning

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Q Learning

Policy and Q-learning functions

As a prior step, we define three functions to complete the Q-learning algorithm.

Greedy Policy

The optimal action is chosen at all times.

```
GreedyPolicy <- function(x, y){
  r <- q_table[x,y,]
  best <- which(r == max(r))
  return(sample(best, 1))
}
```

Epsilon Greedy Policy

The optimal action is chosen with probability $1 - \epsilon$ and a random action with probability ϵ.

```
EpsilonGreedyPolicy <- function(x, y, epsilon){
  r <- q_table[x,y,]
  best <- which(r == max(r))
  if(runif(1) < epsilon) {
    actions <- c(1,2,3,4)
    return(sample(actions, 1))
  }
  else {
    return(sample(best, 1))
  }
}
```

Q-Learning

Now if we assume a deterministic transition model where s and a are always followed by s' and r then the optimal policy is

$$q_\star(s,a) = r + \gamma(\max_{a'} q_\star(s',a'))$$

which is only the correction term in the **Bellman Equation** and therefore

$$0 = r + \gamma(\max_{a'} q_\star(s',a')) - q_\star(s,a)$$

Through iteration we aim to enforce the above equation to hold or in other words, we iterate until there is no correction and we reach the optimal policy and update the q_table as follows

$$q_{\star}(s,a) = q_{\star}(s,a) + \alpha(r + \gamma(\max_{a'} q_{\star}(s',a')) - q_{\star}(s,a))$$

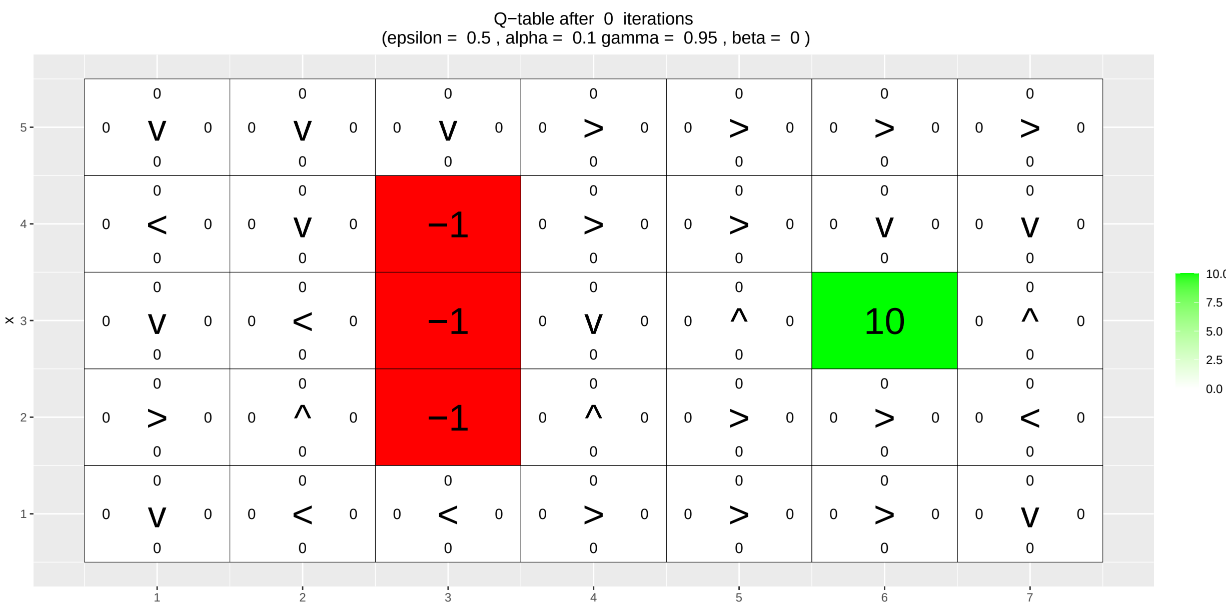
where α is the learning rate.

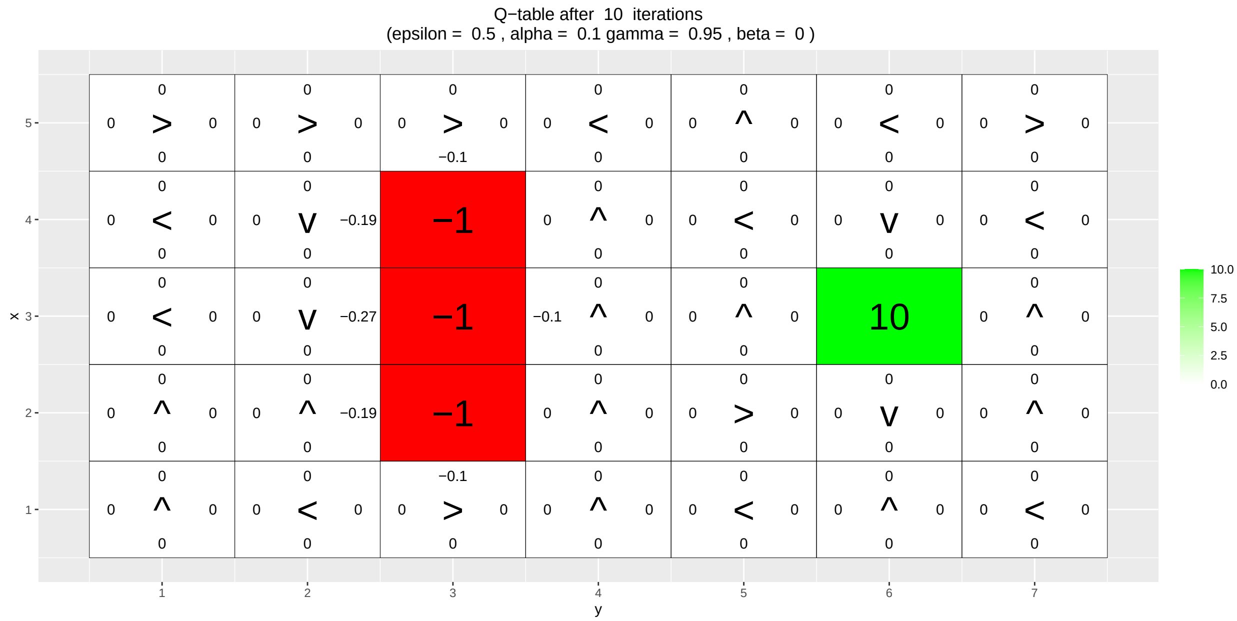
```
q_learning <- function(start_state, epsilon = 0.5, alpha = 0.1, gamma = 0.95, beta = 0){
  x <- start_state[1]
  y <- start_state[2]
  episode_correction <- 0

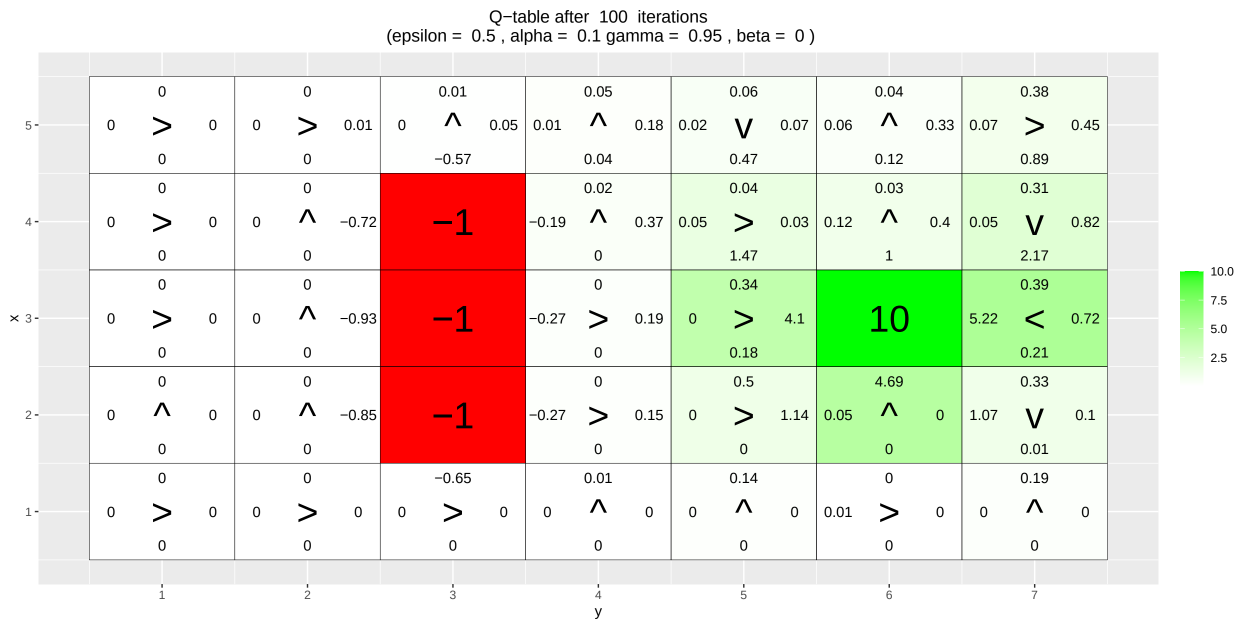
  repeat{
    # Follow policy, execute action, get reward.
    a <- GreedyPolicy(x,y)
    A <- EpsilonGreedyPolicy(x,y,epsilon)
    trstate <- transition_model(x,y,A,beta)
    reward <- reward_map[trstate[1],trstate[2]]

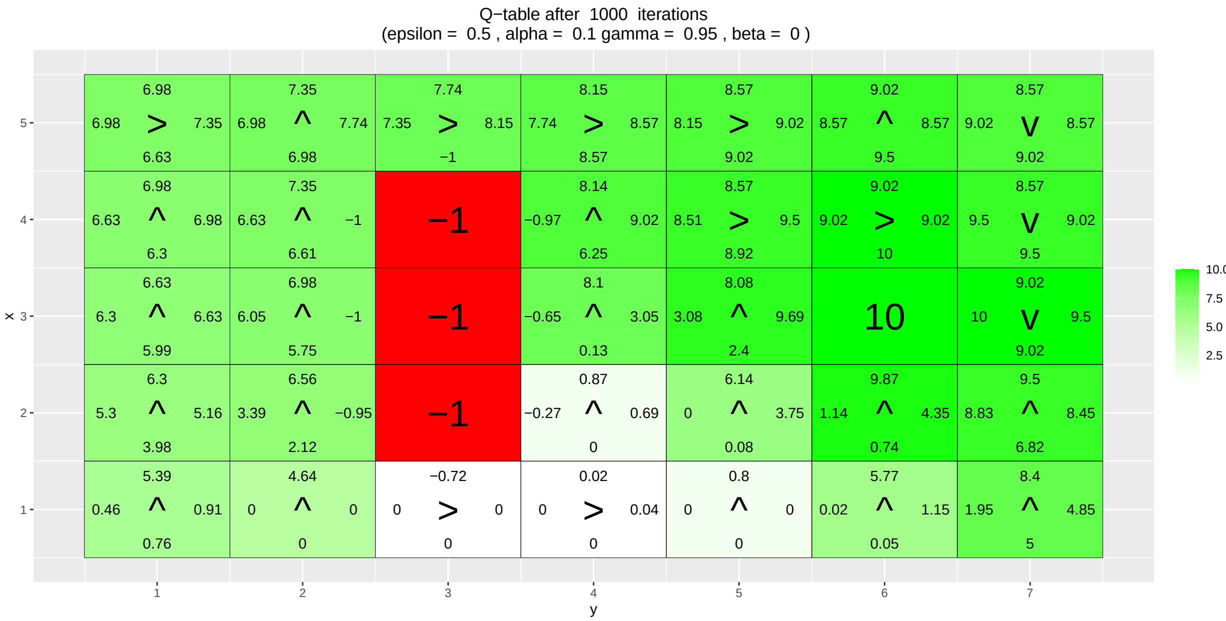
    # Q-table update.
    correction <- alpha*(reward+gamma*max(q_table[trstate[1],trstate[2],]) - q_table[x,y,A])
    episode_correction <- episode_correction + correction
    q_table[x,y,A] <<- q_table[x,y,A] + correction
    x <- trstate[1]
    y <- trstate[2]
    if(reward!=0)
      # End episode.
      return (c(reward,episode_correction))
  }
}
```

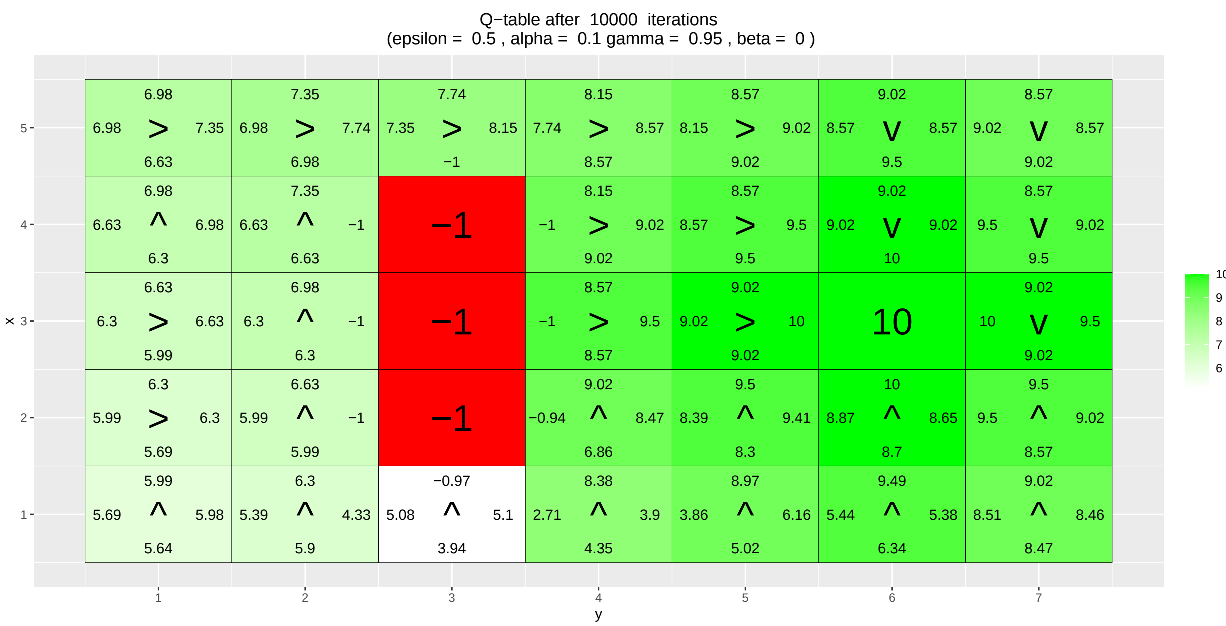
Environment A




Q−table after 10 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.95 , beta = 0)
x
y
10.0
7.5
5.0
2.5
0.0


Q−table after 100 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.95 , beta = 0)
x
y
10.0
7.5
5.0
2.5





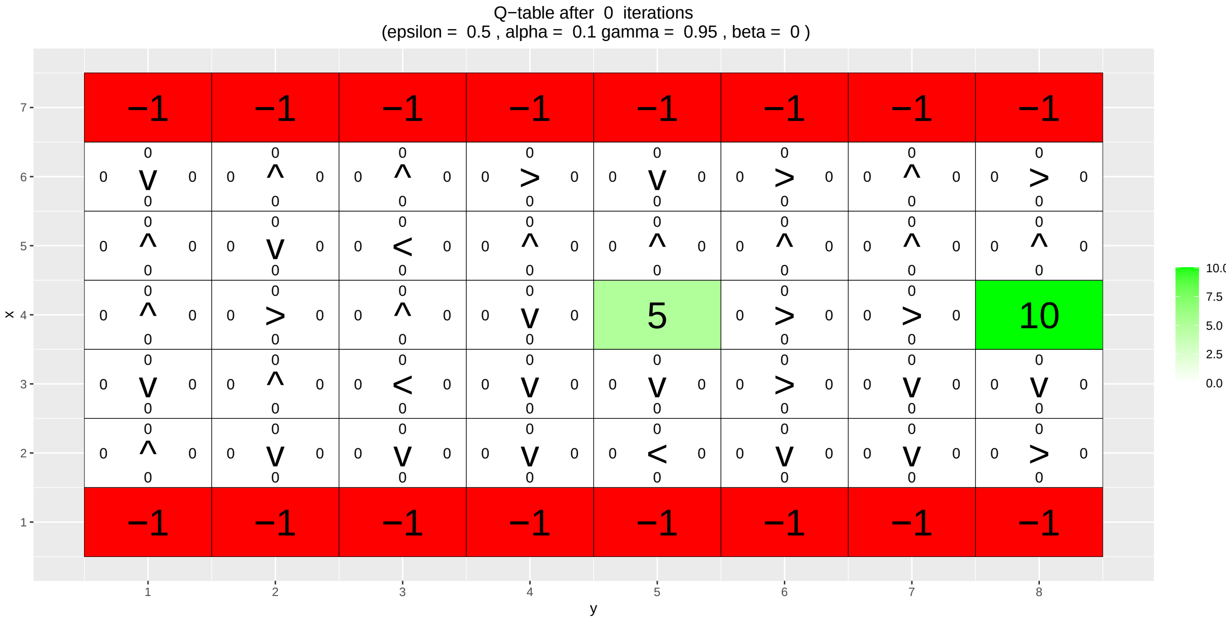
After only 10 episodes, the agent does not learn much and practically has not had the chance to explore the environment. However, it has slightly learned to avoid negative reward states.

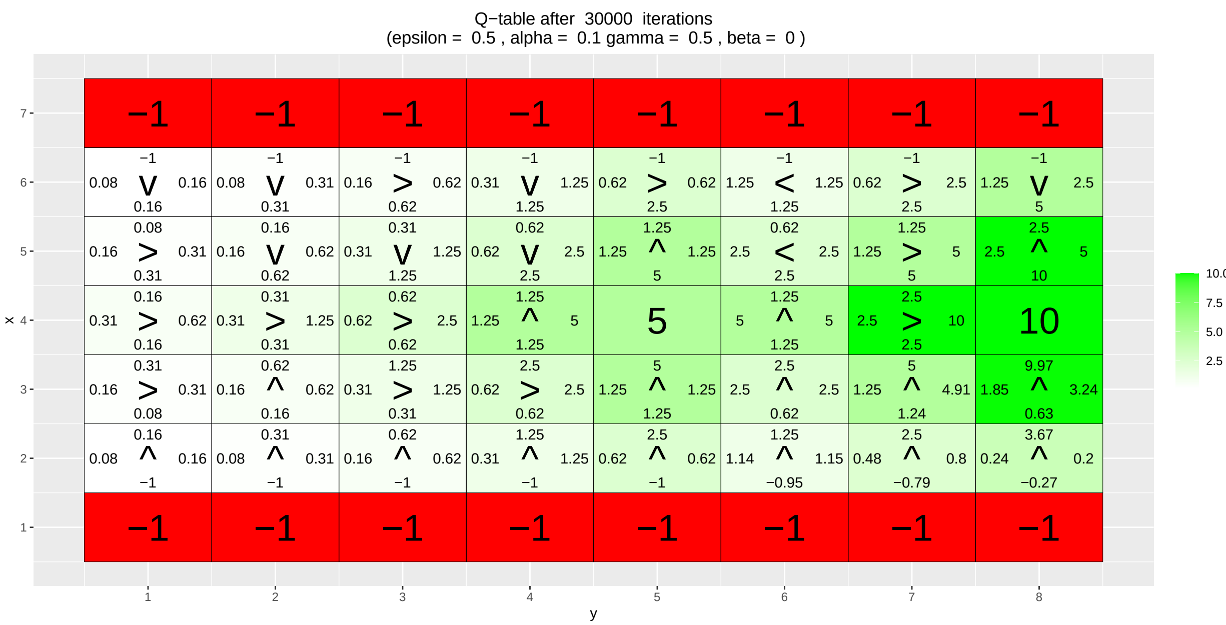
After 100 episodes, the q values for the negative states almost converge to -1 and the agent learns to ignore them. In addition, the agent has started to discover the target state.

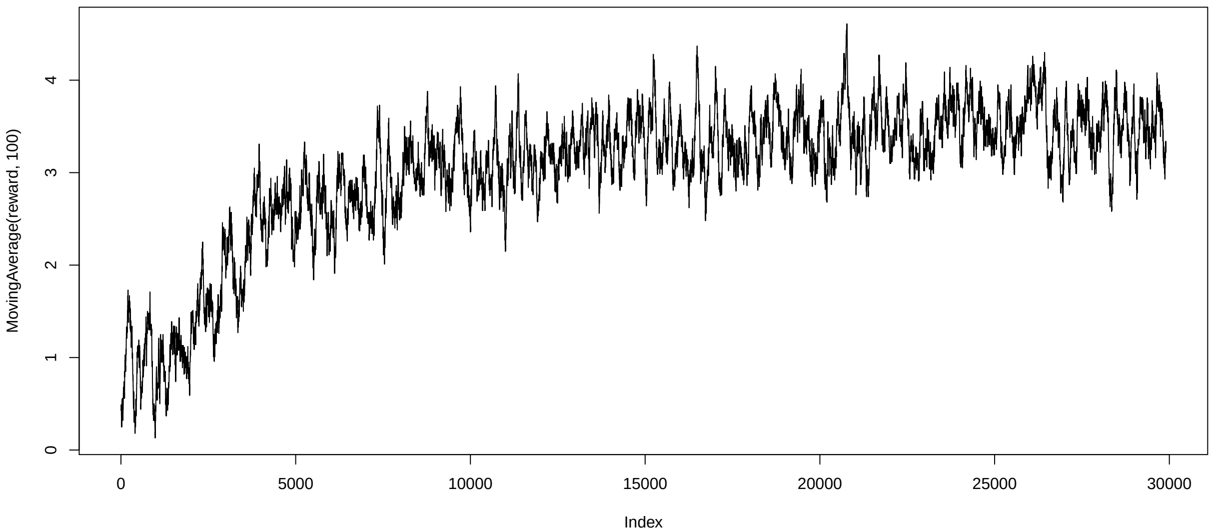
With 1000 and 10000 episodes, explores "almost" the entire map. It has tried several paths and the q values for the negative and target states have converged to the true reward value. In each episode, there is a 50% chance for the agent to prefer a random over the optimum action ($\epsilon = 0.5$). Therefore with more episodes the agent gets a bigger chance of exploring the environment.

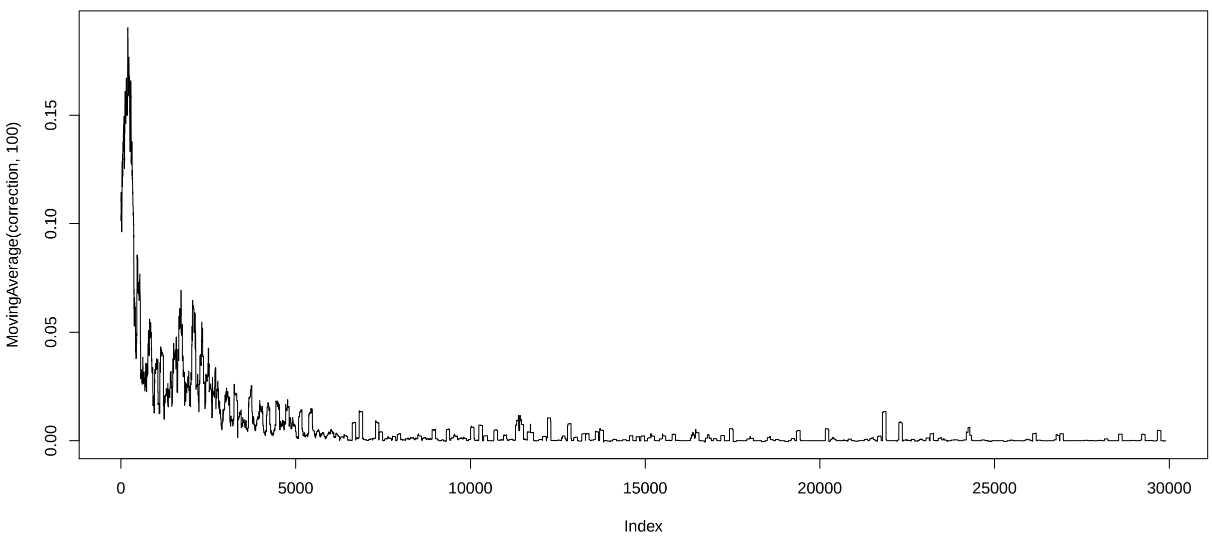
However, there is room for improvement and the obtained policy cannot be deemed optimal. In an optimal policy, the agent learns more paths and better explores the environment by increasing ϵ value. As an instance, when our agent is at any of the ([1,1],[1,2],[2,1],[2,2]) states, it takes the upper path and goes around the red states to reach the target which is not optimum.

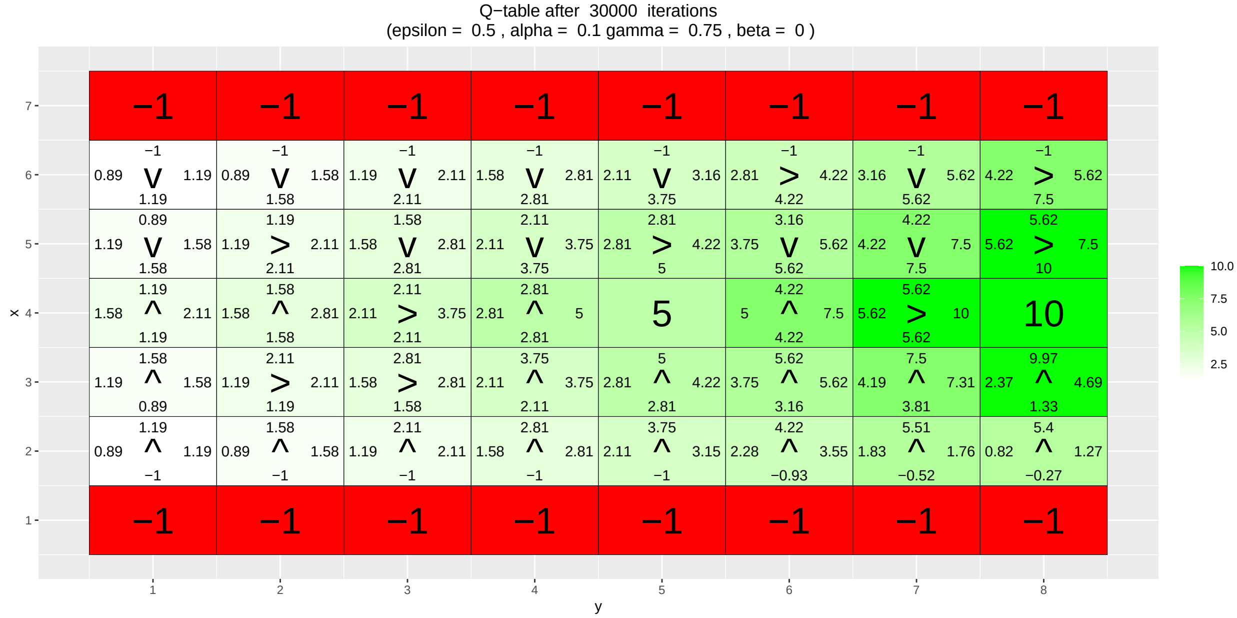
Environment B

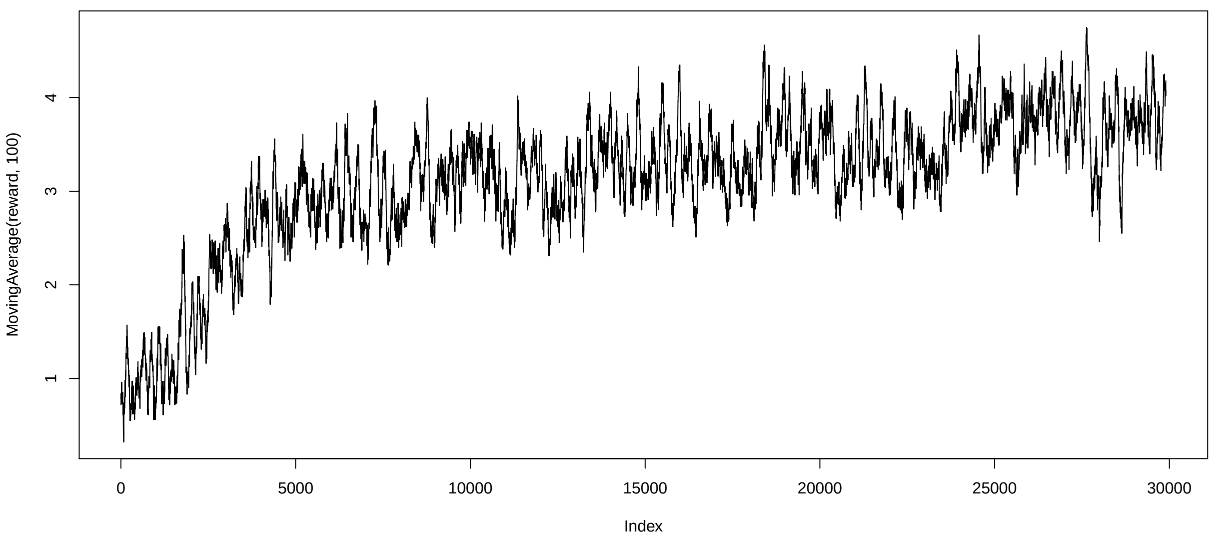


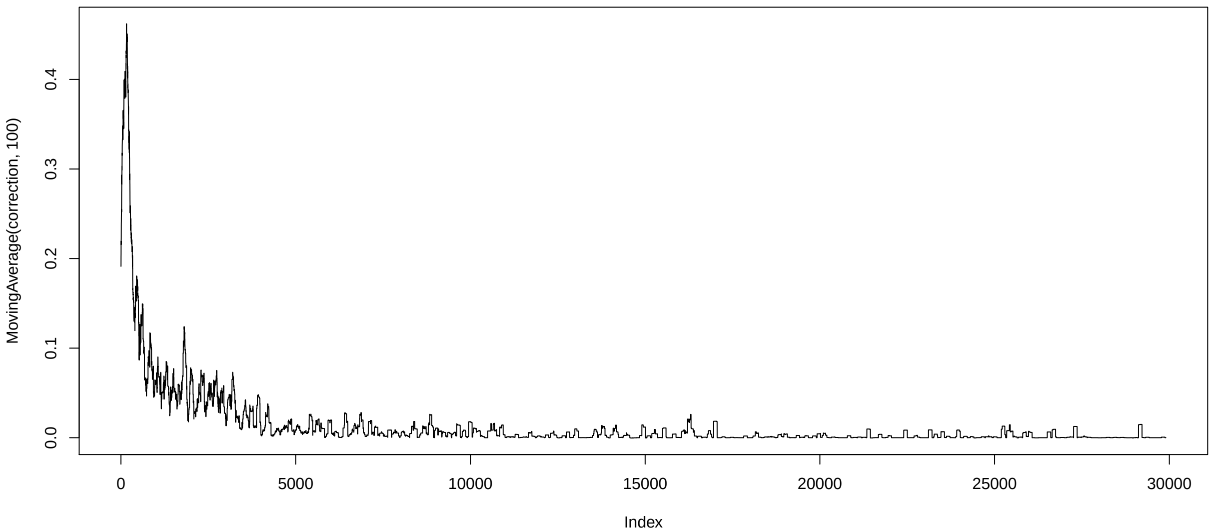


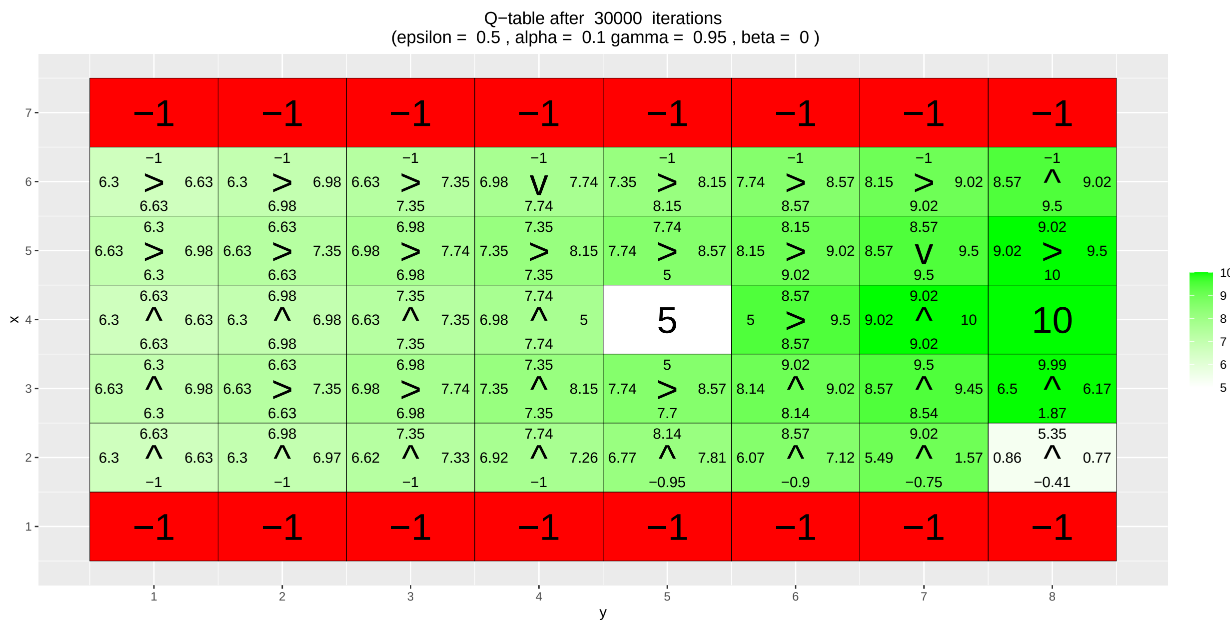

MovingAverage(reward, 100)
0
1
2
3
4
0
5000
10000
15000
20000
25000
30000
Index

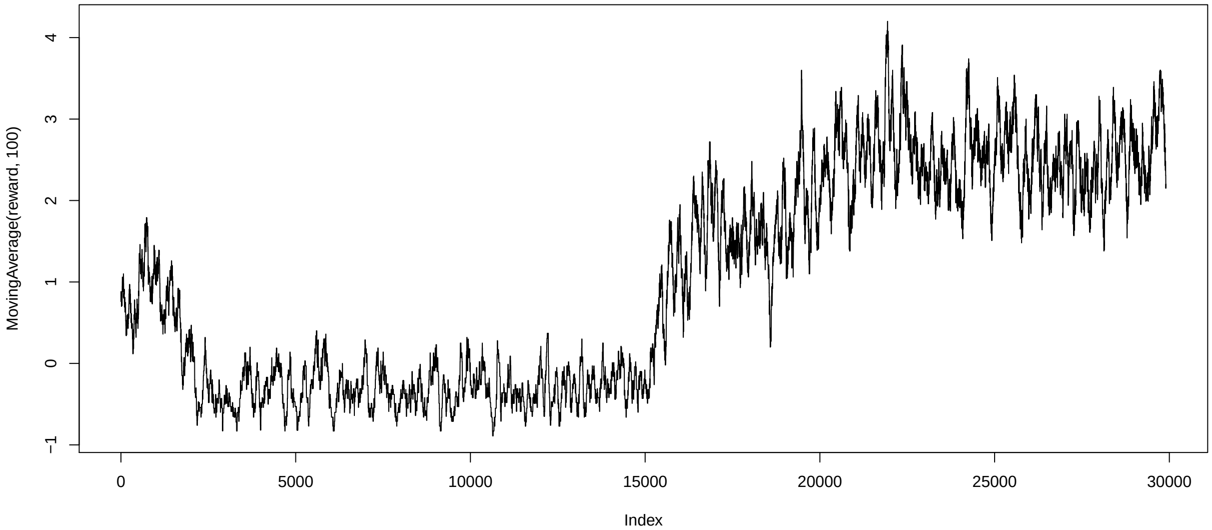

MovingAverage(correction, 100)
0.00
0.05
0.10
0.15
0
5000
10000
15000
20000
25000
30000
Index

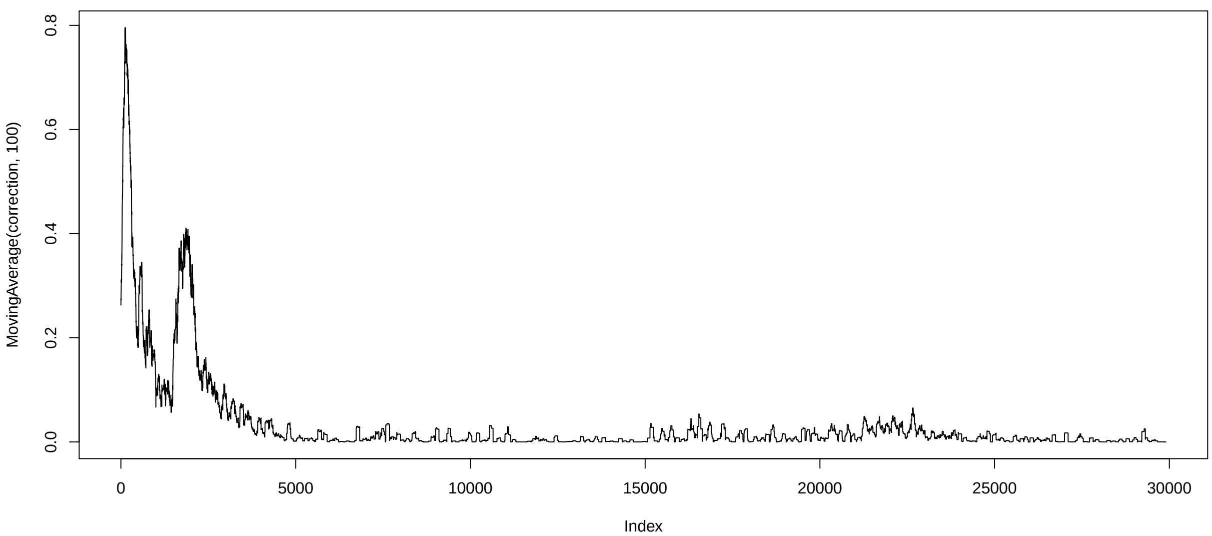

Q−table after 30000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.75 , beta = 0)
x
y
10.0
7.5
5.0
2.5

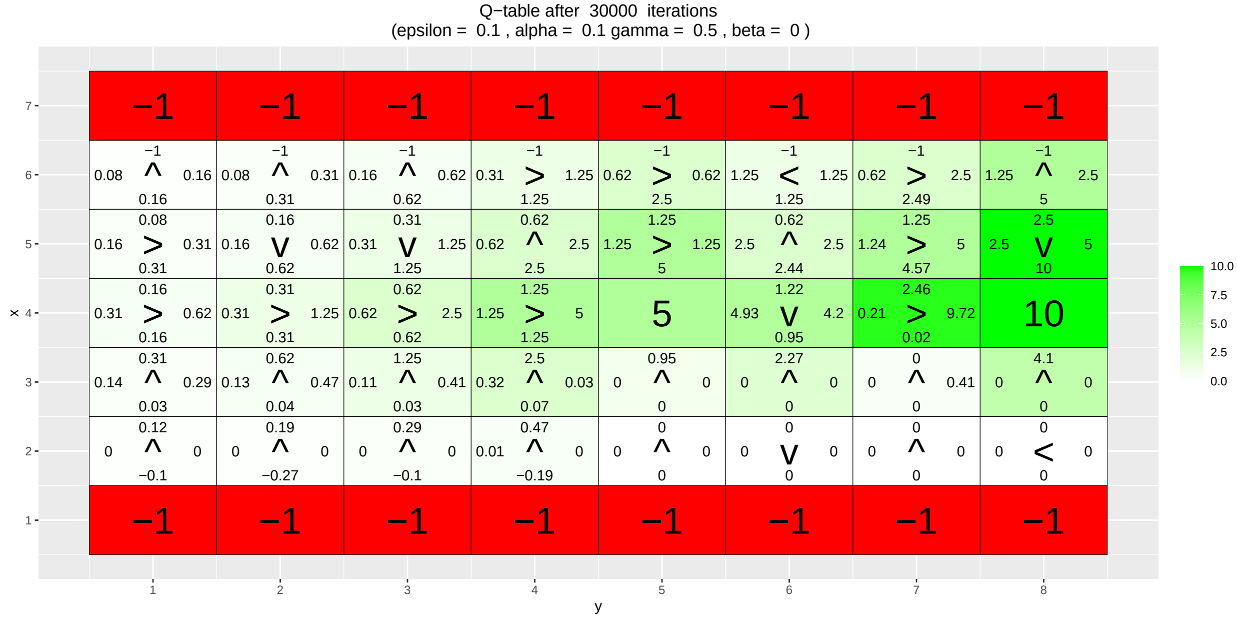

MovingAverage(reward, 100)
Index

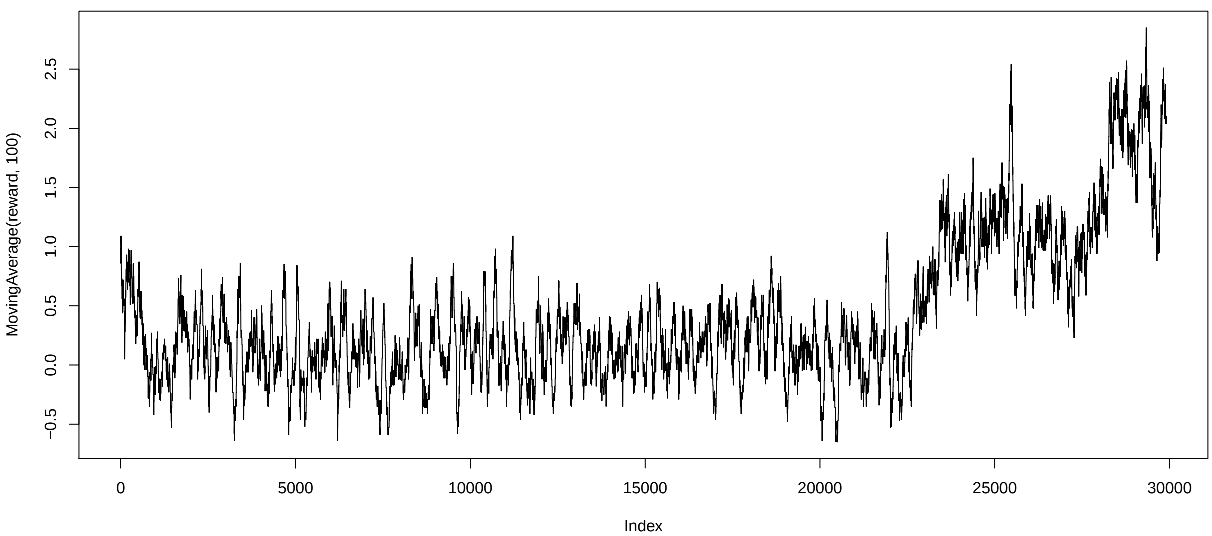

MovingAverage(correction, 100)
Index

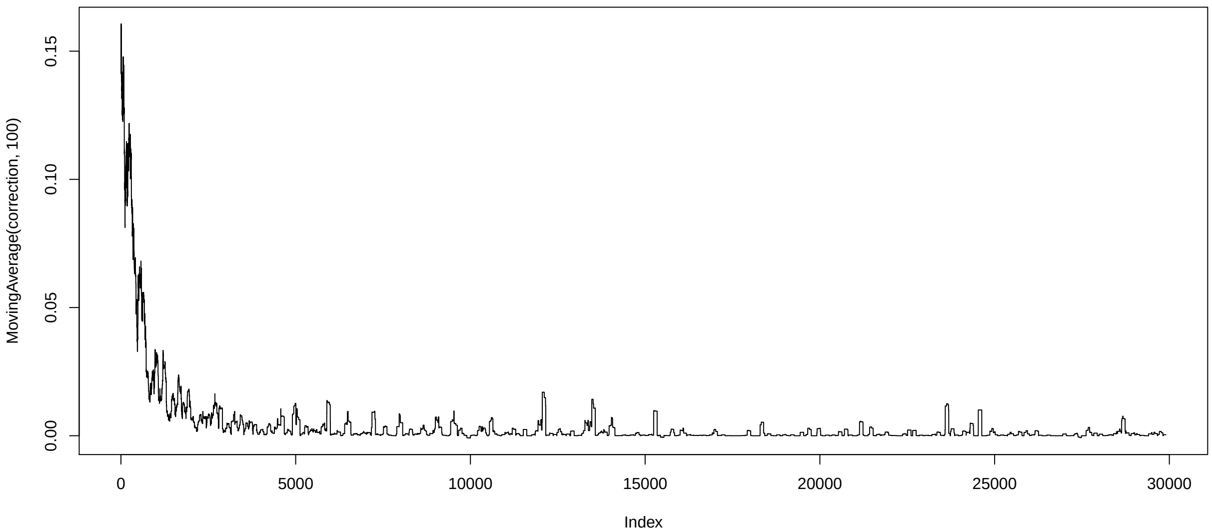

Q−table after 30000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.95 , beta = 0)
x
y


MovingAverage(reward, 100)
Index
0
5000
10000
15000
20000
25000
30000
−1
0
1
2
3
4

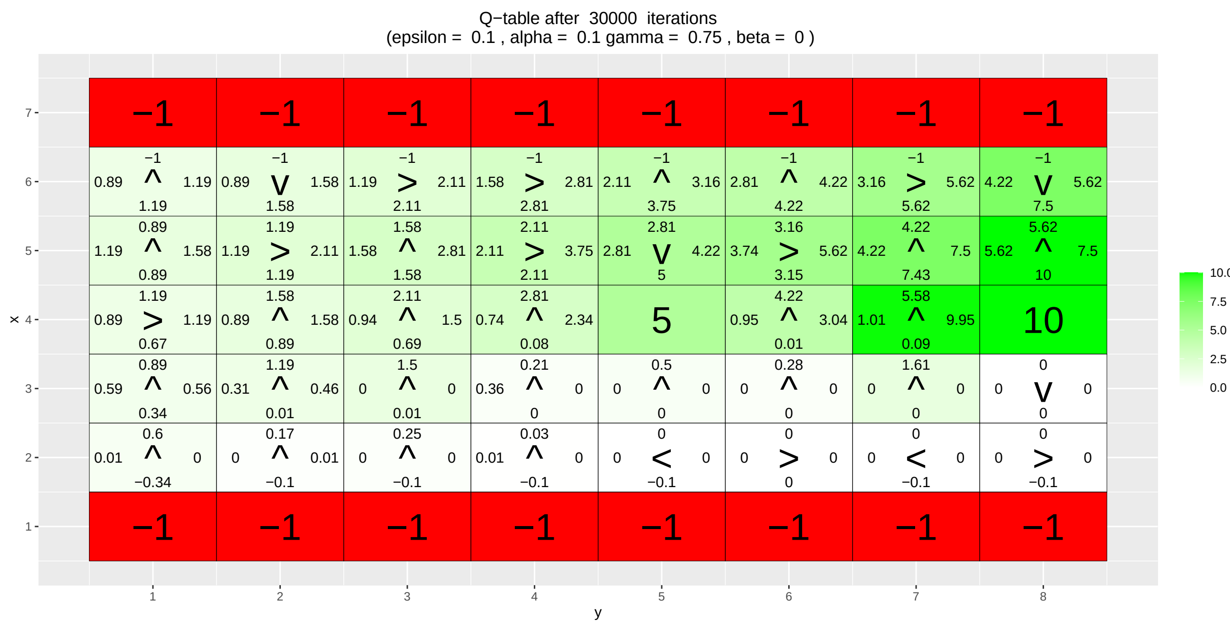

MovingAverage(correction, 100)
Index
0
5000
10000
15000
20000
25000
30000
0.0
0.2
0.4
0.6
0.8


Q−table after 30000 iterations
(epsilon = 0.1 , alpha = 0.1 gamma = 0.5 , beta = 0)
x
y

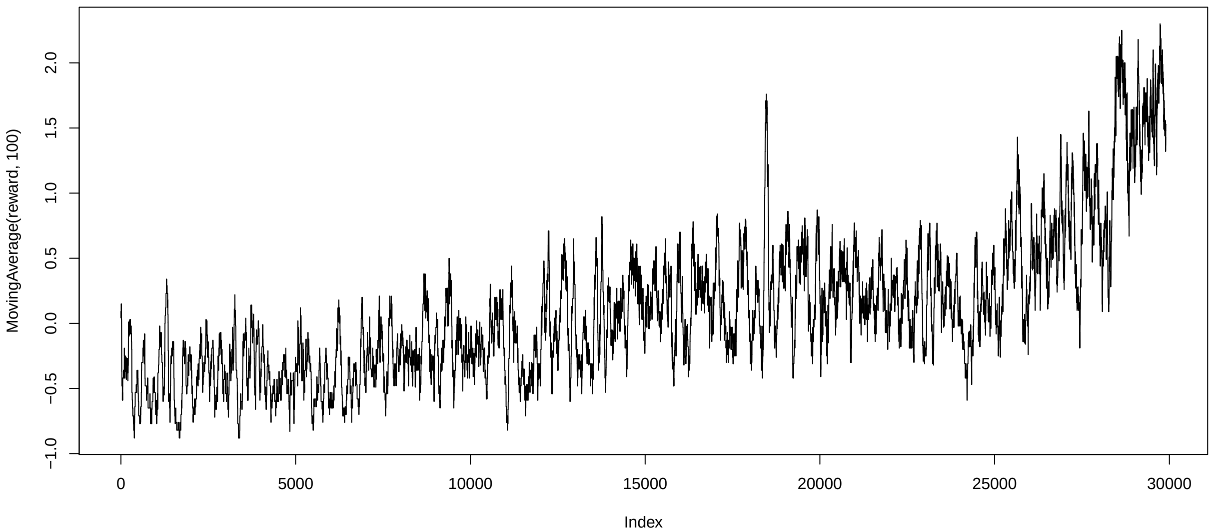

MovingAverage(reward, 100)
Index

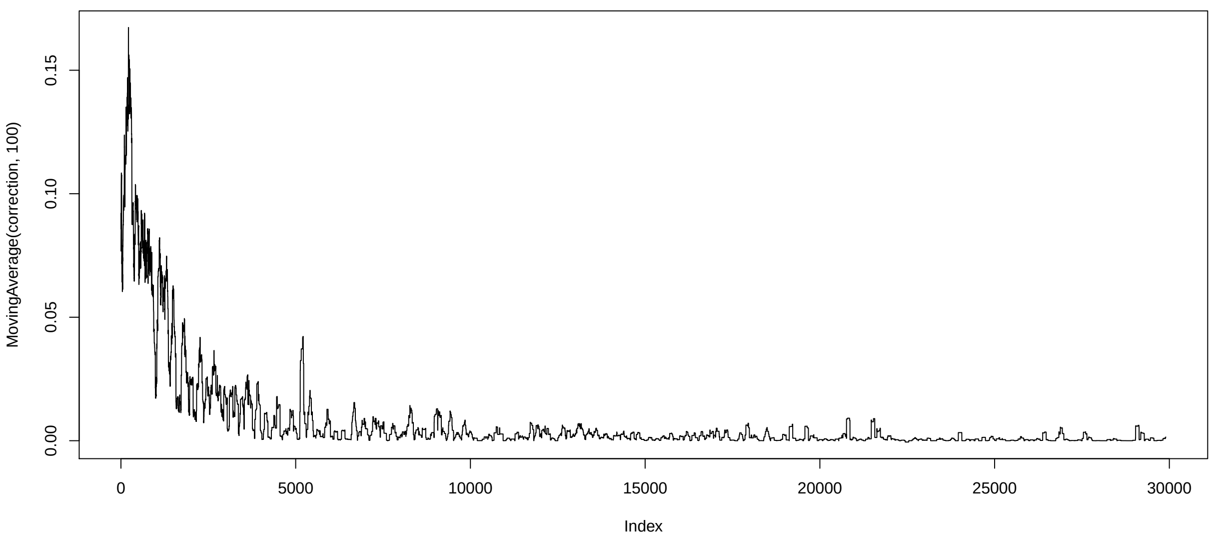


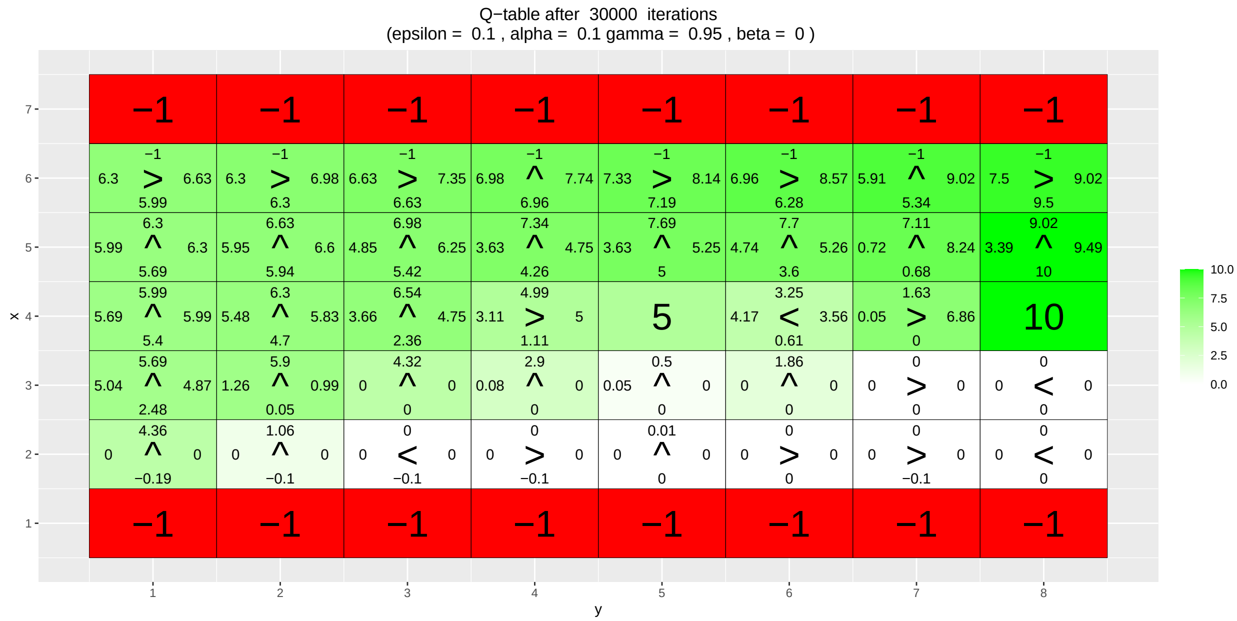
MovingAverage(correction, 100)
Index

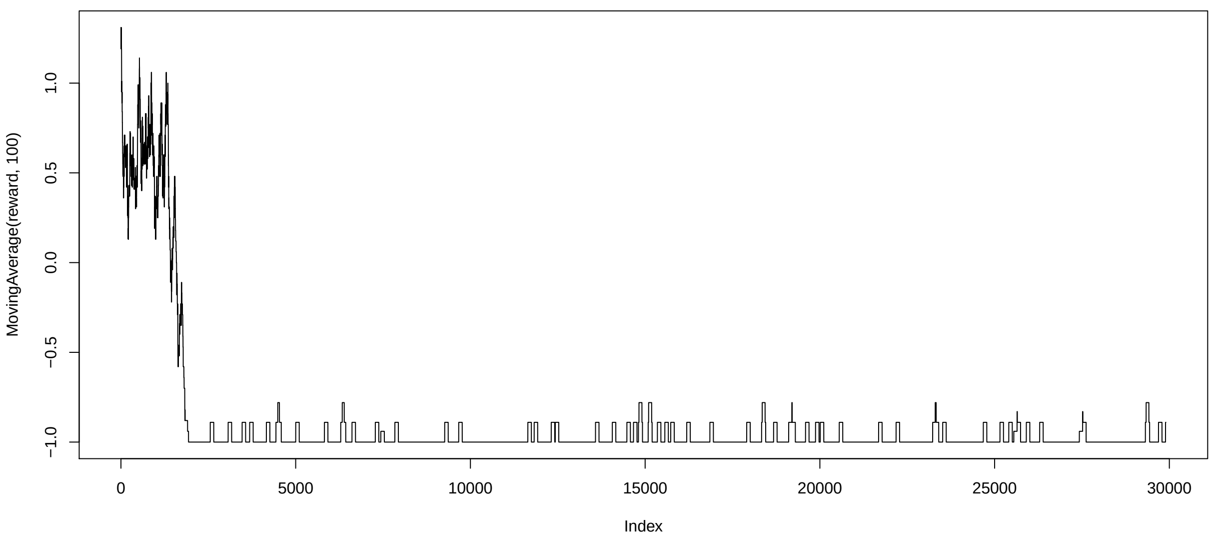


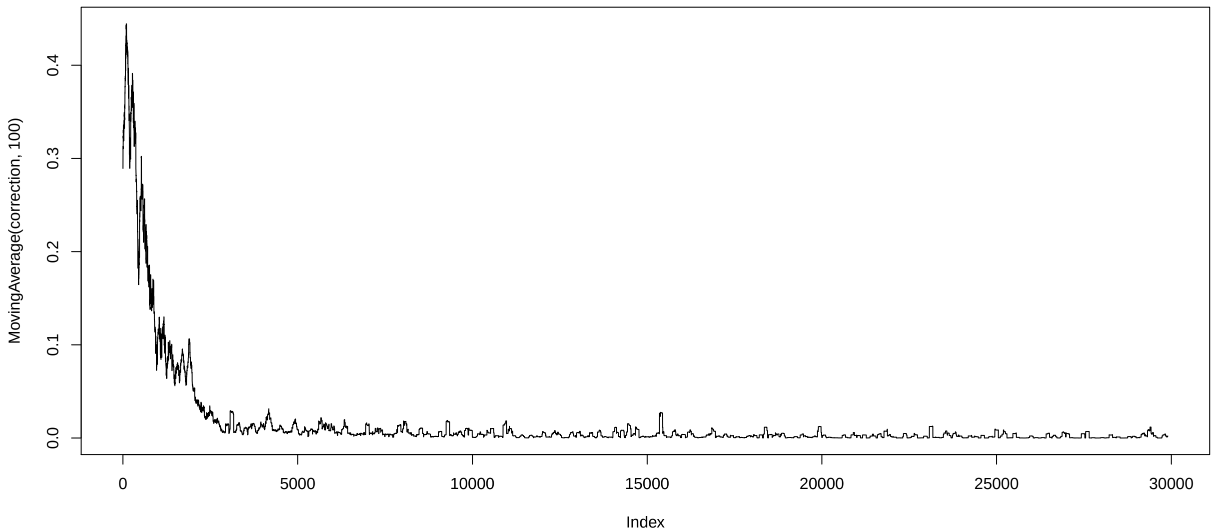
Q−table after 30000 iterations
(epsilon = 0.1 , alpha = 0.1 gamma = 0.75 , beta = 0)
x
y


MovingAverage(reward, 100)
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
0
5000
10000
15000
20000
25000
30000
Index


MovingAverage(correction, 100)
0.00
0.05
0.10
0.15
0
5000
10000
15000
20000
25000
30000
Index


Q−table after 30000 iterations
(epsilon = 0.1 , alpha = 0.1 gamma = 0.95 , beta = 0)
x
y


MovingAverage(reward, 100)
Index



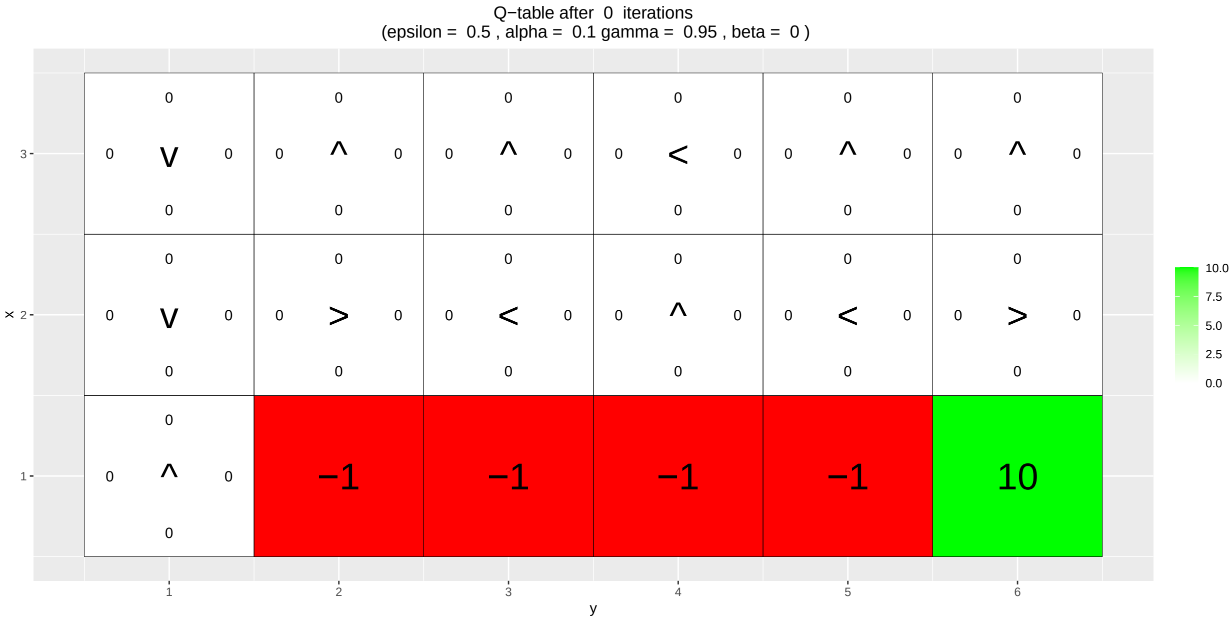
First we observe the impact of γ:

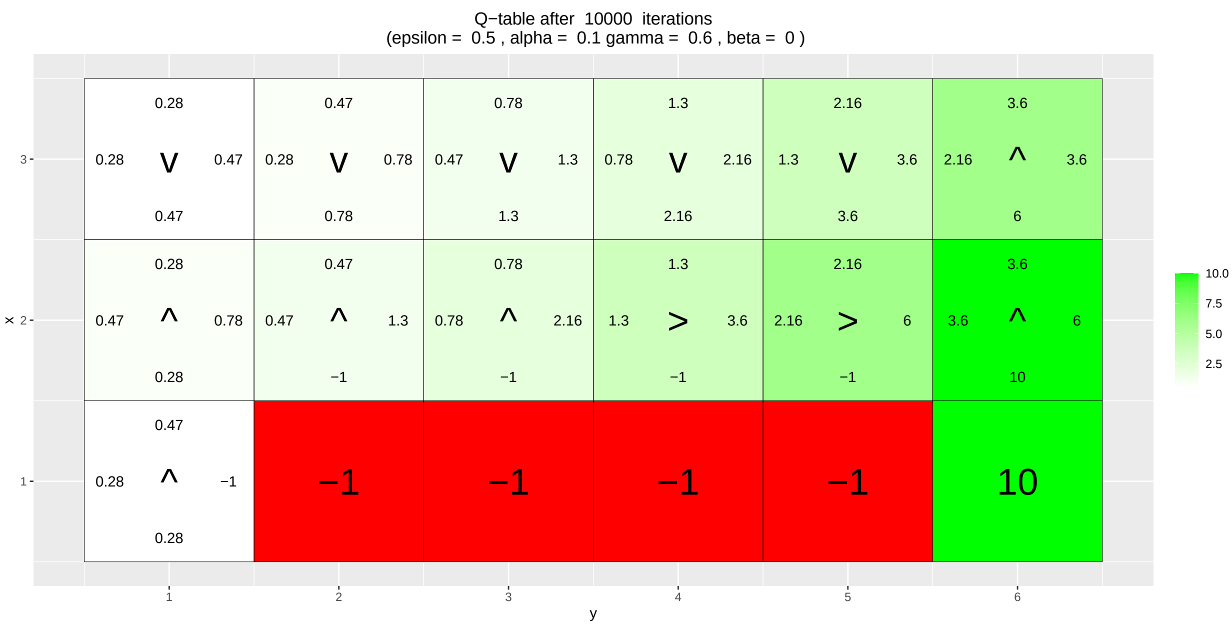
Since γ controls whether the agent prefers to get the reward immediately or later on, we can observe that when $\gamma = 0.5$ the agent tends to move near the target state with the smaller reward whereas for higher values $\gamma = 0.75$ or $\gamma = 0.95$ the agent ignores the smaller reward to get to the target state with the higher reward. In other words, when γ is close to 1, there is almost no discount on the total return and the agent acts less greedily for immediate rewards.

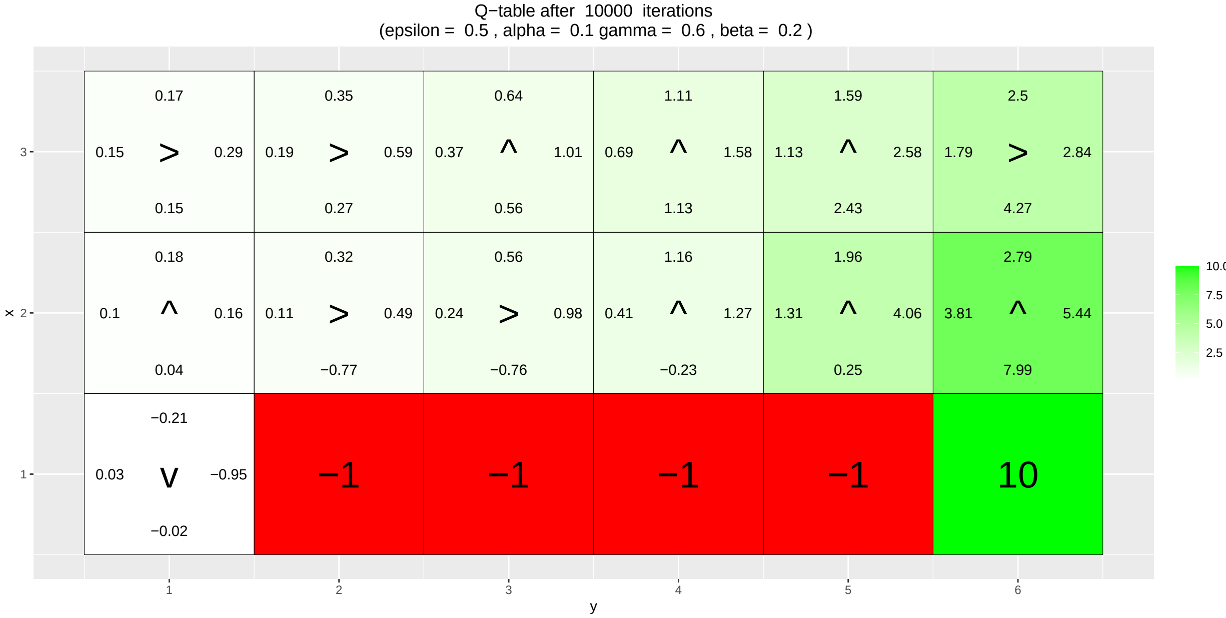
Now we observe the impact of ϵ:

As previously discussed, smaller values for ϵ means smaller probability of the agent taking a random rather than the optimal action. Here we can see that the agent explores more states when $\epsilon = 0.5$ with any given γ, compared to when $\epsilon = 0.1$. Comparing $\epsilon = 0.1$ with $\epsilon = .5$ when $\gamma = 0.95$, the agent avoids the lower states when $\epsilon = 0.1$ since there is only 10% chance for taking a random action while in both cases it reaches out for the long-term reward.

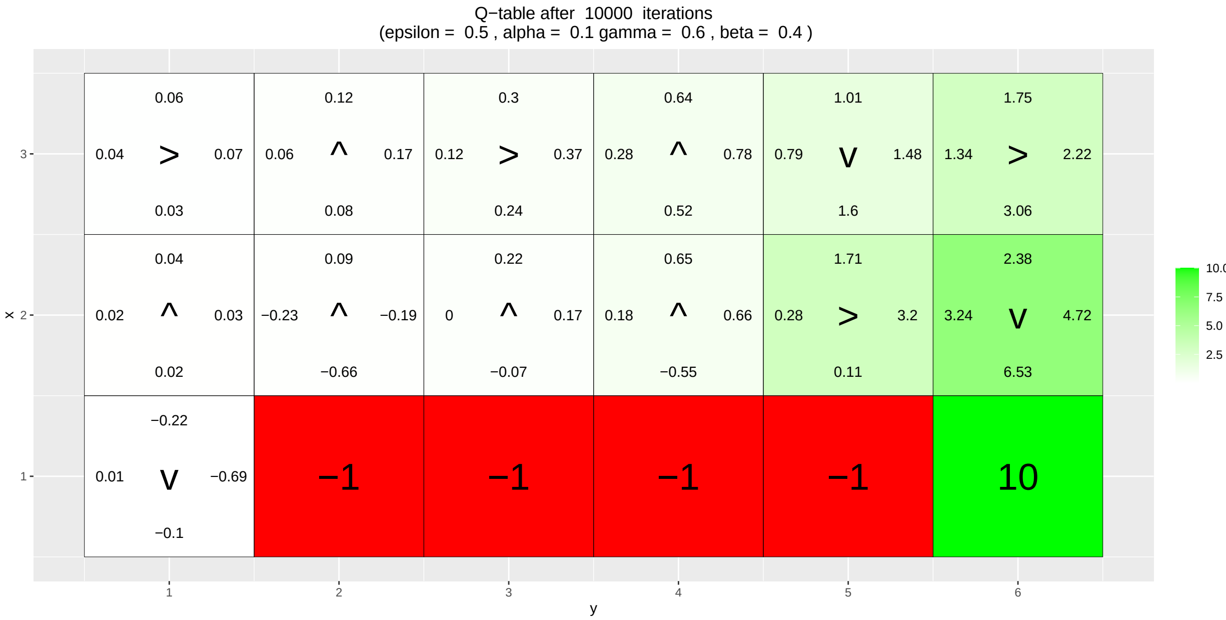
Environment C



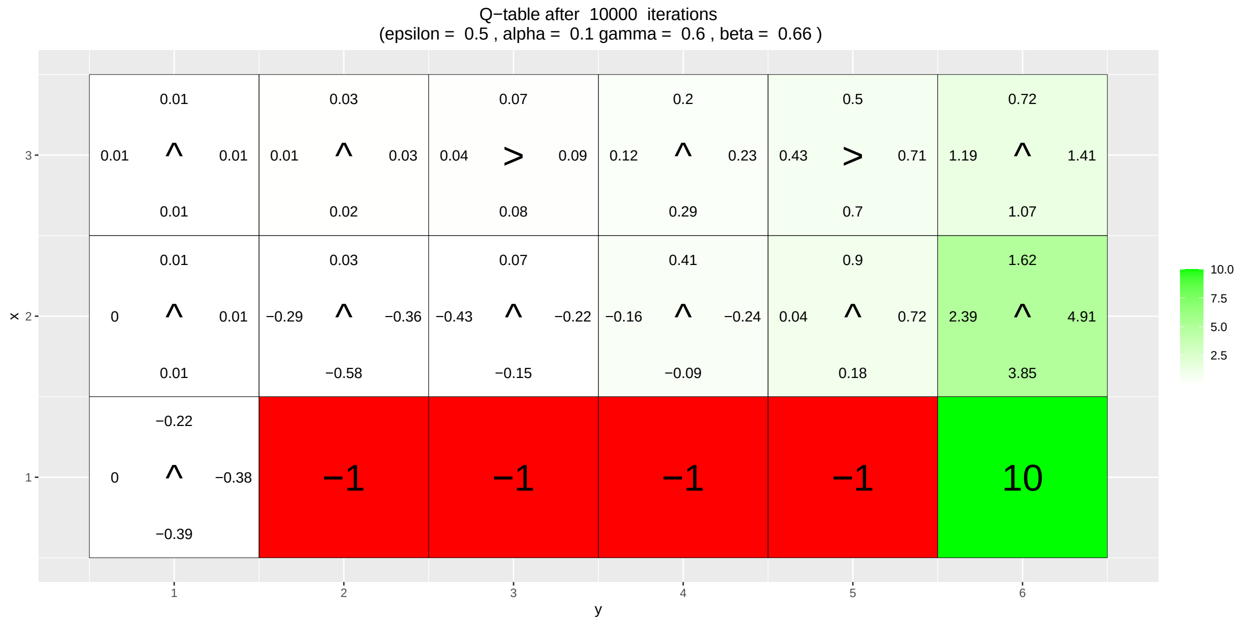




Q−table after 10000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.6 , beta = 0.2)
0.17
0.15 > 0.29
0.15
0.35
0.19 > 0.59
0.27
0.64
0.37 ^ 1.01
0.56
1.11
0.69 ^ 1.58
1.13
1.59
1.13 ^ 2.58
2.43
2.5
1.79 > 2.84
4.27
0.18
0.1 ^ 0.16
0.04
0.32
0.11 > 0.49
−0.77
0.56
0.24 > 0.98
−0.76
1.16
0.41 ^ 1.27
−0.23
1.96
1.31 ^ 4.06
0.25
2.79
3.81 ^ 5.44
7.99
−0.21
0.03 V −0.95
−0.02
−1
−1
−1
−1
10
x
y
10.0
7.5
5.0
2.5



Q−table after 10000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.6 , beta = 0.4)
0.06
0.04 > 0.07
0.03
0.12
0.06 ^ 0.17
0.08
0.3
0.12 > 0.37
0.24
0.64
0.28 ^ 0.78
0.52
1.01
0.79 V 1.48
1.6
1.75
1.34 > 2.22
3.06
0.04
0.02 ^ 0.03
0.02
0.09
−0.23 ^ −0.19
−0.66
0.22
0 ^ 0.17
−0.07
0.65
0.18 ^ 0.66
−0.55
1.71
0.28 > 3.2
0.11
2.38
3.24 V 4.72
6.53
−0.22
0.01 V −0.69
−0.1
−1
−1
−1
−1
10
x
y
10.0
7.5
5.0
2.5

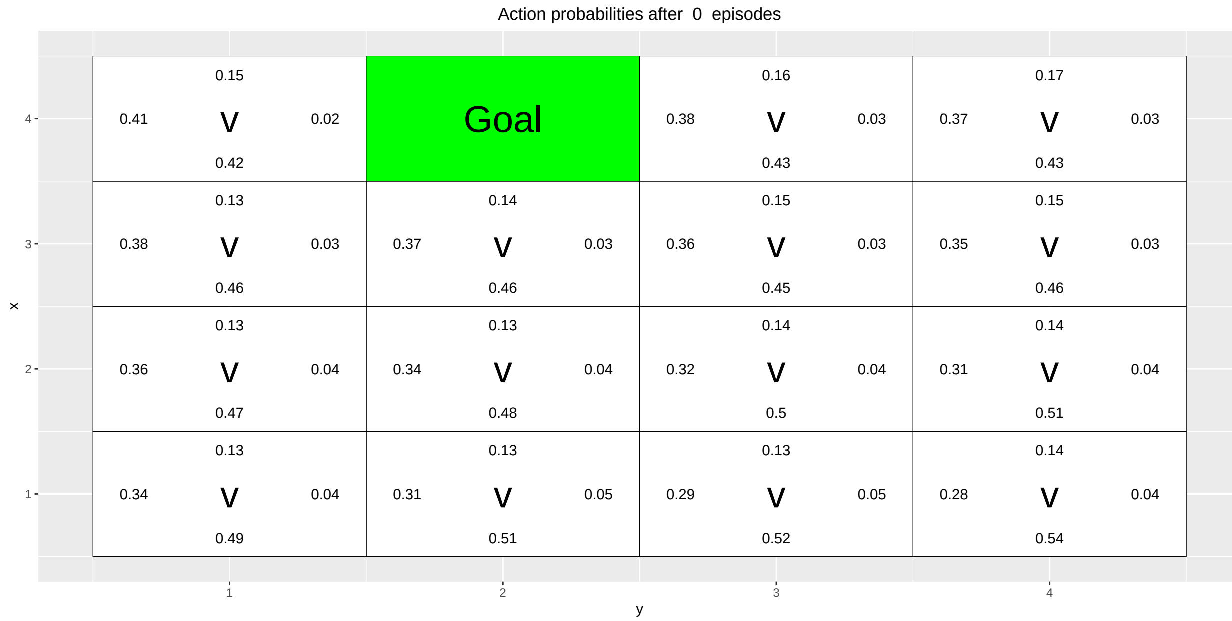


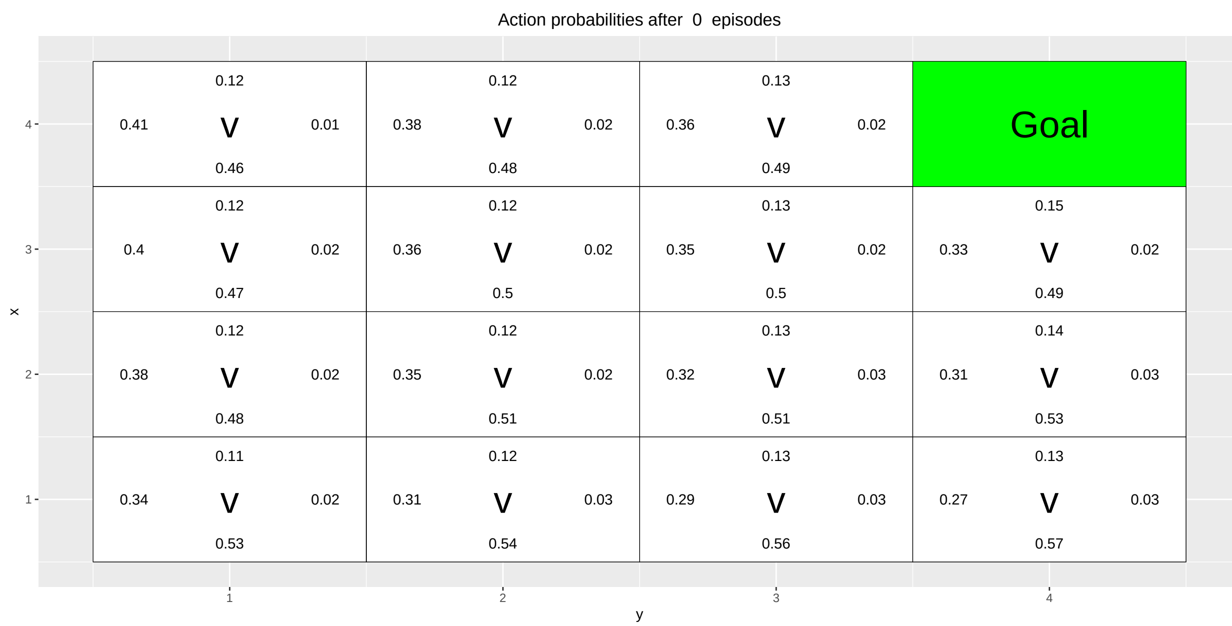
The β parameter determines the certainty of the agent moving to the intended direction. Larger β means more chance for the agent to end up in states adjacent to the intended state. When $\beta = 0$ the agent takes the shortest path to the goal state since there is no chance for it to slip into states with negative reward ([1,2],[1,3],[1,4],[1,5]). It can also be observed that the q values for all these states have converged to -1 which is the true reward.

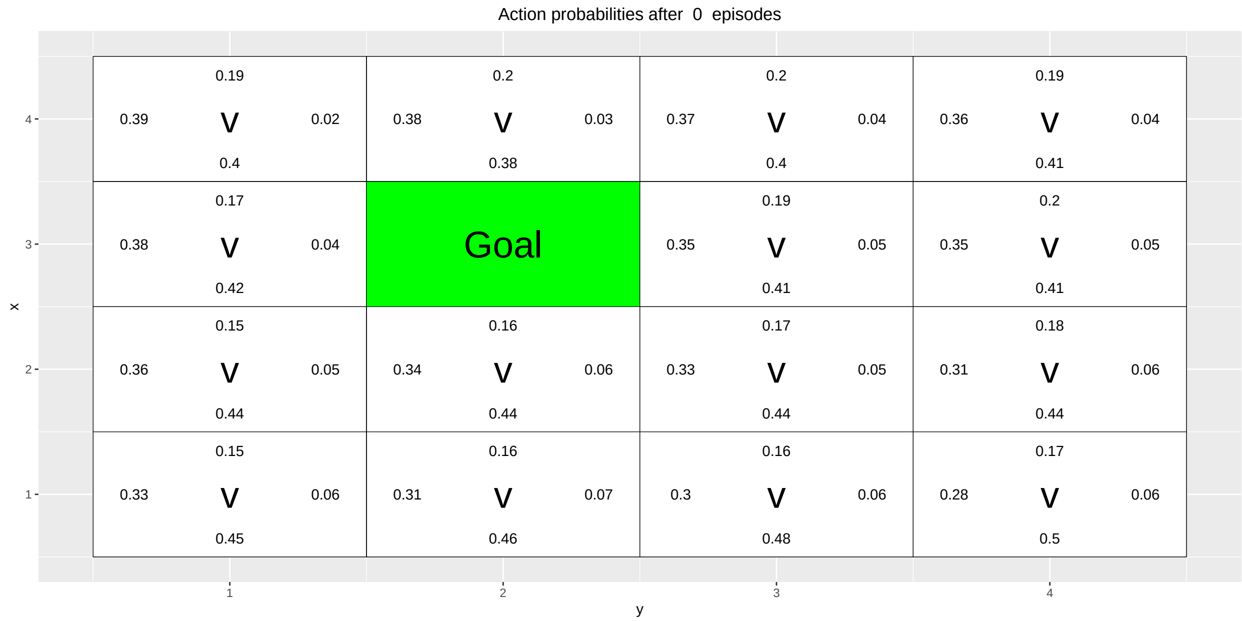
However, as β is increased, the agent learns through iterations, that there is a chance for it to end up in a state with negative reward if it takes the shortest path and therefore prefers to take the top row instead. When $\beta = 0.66$ the agent completely avoids the middle row and consecutively, the q values for the negative states grow larger (do not converge to true rewards).

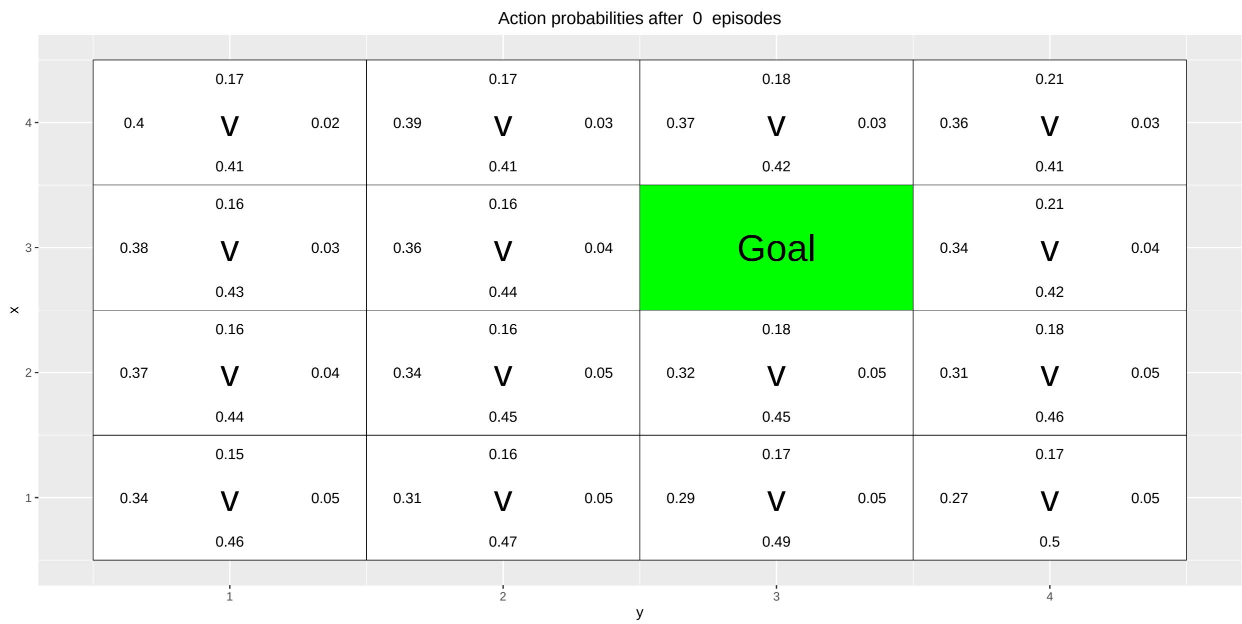
Reinforcement Learning

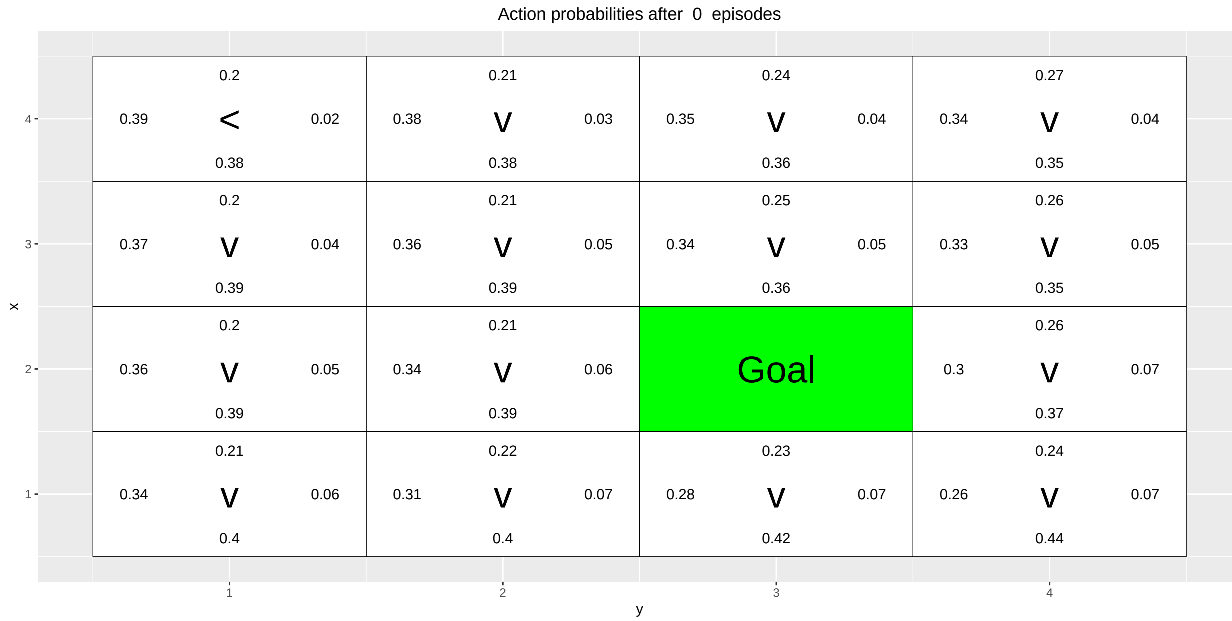
Environment D

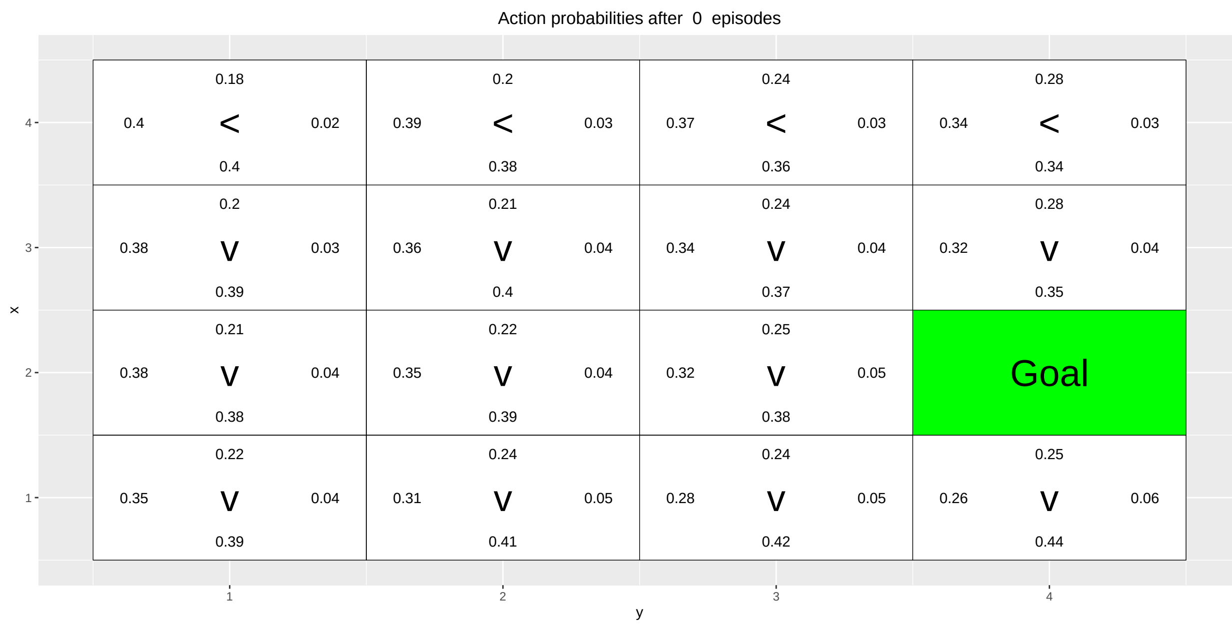


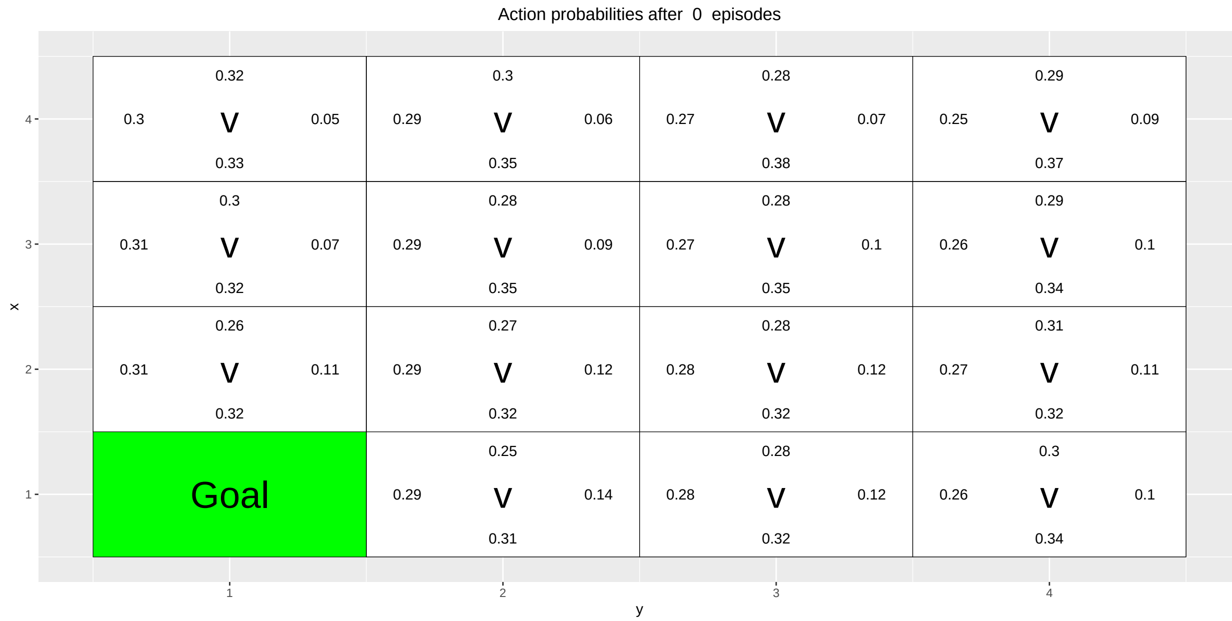


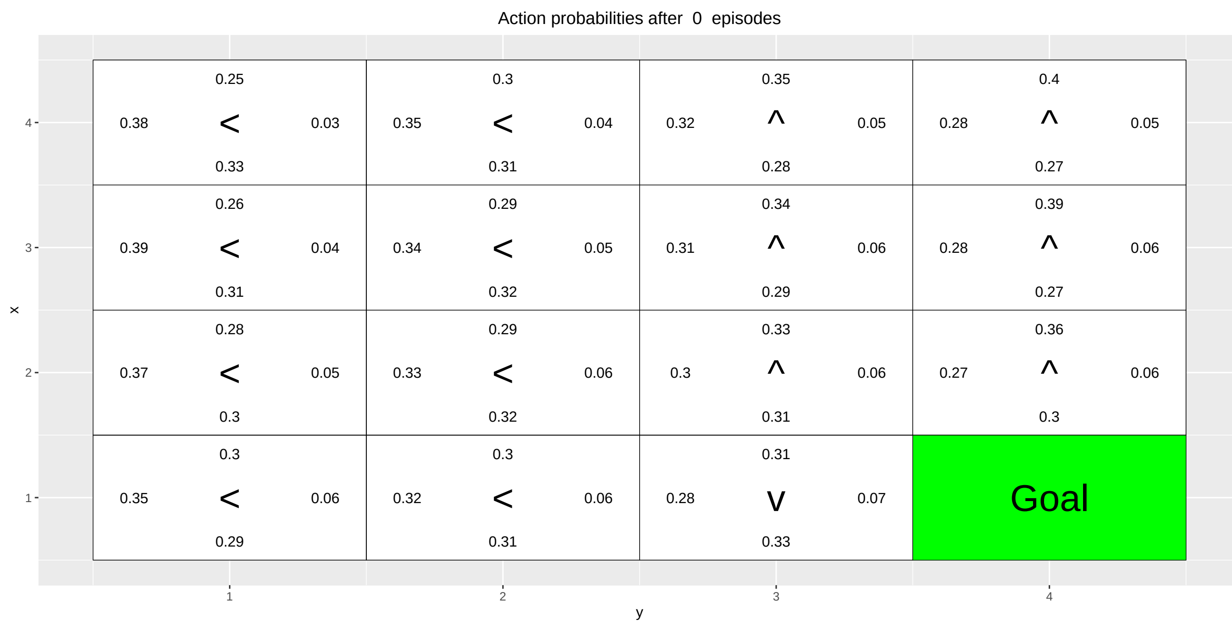

Action probabilities after 0 episodes
Goal
x
y


Action probabilities after 0 episodes
Goal
x
y


Action probabilities after 0 episodes
Goal
x
y


Action probabilities after 0 episodes
Goal
x
y






```
## episode 10
## episode 20
## episode 30
## episode 40
## episode 50
## episode 60
## episode 70
## episode 80
## episode 90
## episode 100
## episode 110
## episode 120
## episode 130
## episode 140
```

```
## episode 150
## episode 160
## episode 170
## episode 180
## episode 190
## episode 200
## episode 210
## episode 220
## episode 230
## episode 240
## episode 250
## episode 260
## episode 270
## episode 280
## episode 290
## episode 300
## episode 310
## episode 320
## episode 330
## episode 340
## episode 350
## episode 360
## episode 370
## episode 380
## episode 390
## episode 400
## episode 410
## episode 420
## episode 430
## episode 440
## episode 450
## episode 460
## episode 470
## episode 480
## episode 490
## episode 500
## episode 510
## episode 520
## episode 530
## episode 540
## episode 550
## episode 560
## episode 570
## episode 580
## episode 590
## episode 600
## episode 610
## episode 620
## episode 630
## episode 640
## episode 650
## episode 660
## episode 670
## episode 680
```

```
## episode 690
## episode 700
## episode 710
## episode 720
## episode 730
## episode 740
## episode 750
## episode 760
## episode 770
## episode 780
## episode 790
## episode 800
## episode 810
## episode 820
## episode 830
## episode 840
## episode 850
## episode 860
## episode 870
## episode 880
## episode 890
## episode 900
## episode 910
## episode 920
## episode 930
## episode 940
## episode 950
## episode 960
## episode 970
## episode 980
## episode 990
## episode 1000
## episode 1010
## episode 1020
## episode 1030
## episode 1040
## episode 1050
## episode 1060
## episode 1070
## episode 1080
## episode 1090
## episode 1100
## episode 1110
## episode 1120
## episode 1130
## episode 1140
## episode 1150
## episode 1160
## episode 1170
## episode 1180
## episode 1190
## episode 1200
## episode 1210
## episode 1220
```

```
## episode 1230
## episode 1240
## episode 1250
## episode 1260
## episode 1270
## episode 1280
## episode 1290
## episode 1300
## episode 1310
## episode 1320
## episode 1330
## episode 1340
## episode 1350
## episode 1360
## episode 1370
## episode 1380
## episode 1390
## episode 1400
## episode 1410
## episode 1420
## episode 1430
## episode 1440
## episode 1450
## episode 1460
## episode 1470
## episode 1480
## episode 1490
## episode 1500
## episode 1510
## episode 1520
## episode 1530
## episode 1540
## episode 1550
## episode 1560
## episode 1570
## episode 1580
## episode 1590
## episode 1600
## episode 1610
## episode 1620
## episode 1630
## episode 1640
## episode 1650
## episode 1660
## episode 1670
## episode 1680
## episode 1690
## episode 1700
## episode 1710
## episode 1720
## episode 1730
## episode 1740
## episode 1750
## episode 1760
```

```
## episode 1770
## episode 1780
## episode 1790
## episode 1800
## episode 1810
## episode 1820
## episode 1830
## episode 1840
## episode 1850
## episode 1860
## episode 1870
## episode 1880
## episode 1890
## episode 1900
## episode 1910
## episode 1920
## episode 1930
## episode 1940
## episode 1950
## episode 1960
## episode 1970
## episode 1980
## episode 1990
## episode 2000
## episode 2010
## episode 2020
## episode 2030
## episode 2040
## episode 2050
## episode 2060
## episode 2070
## episode 2080
## episode 2090
## episode 2100
## episode 2110
## episode 2120
## episode 2130
## episode 2140
## episode 2150
## episode 2160
## episode 2170
## episode 2180
## episode 2190
## episode 2200
## episode 2210
## episode 2220
## episode 2230
## episode 2240
## episode 2250
## episode 2260
## episode 2270
## episode 2280
## episode 2290
## episode 2300
```

```
## episode 2310
## episode 2320
## episode 2330
## episode 2340
## episode 2350
## episode 2360
## episode 2370
## episode 2380
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## episode 4980
## episode 4990
## episode 5000
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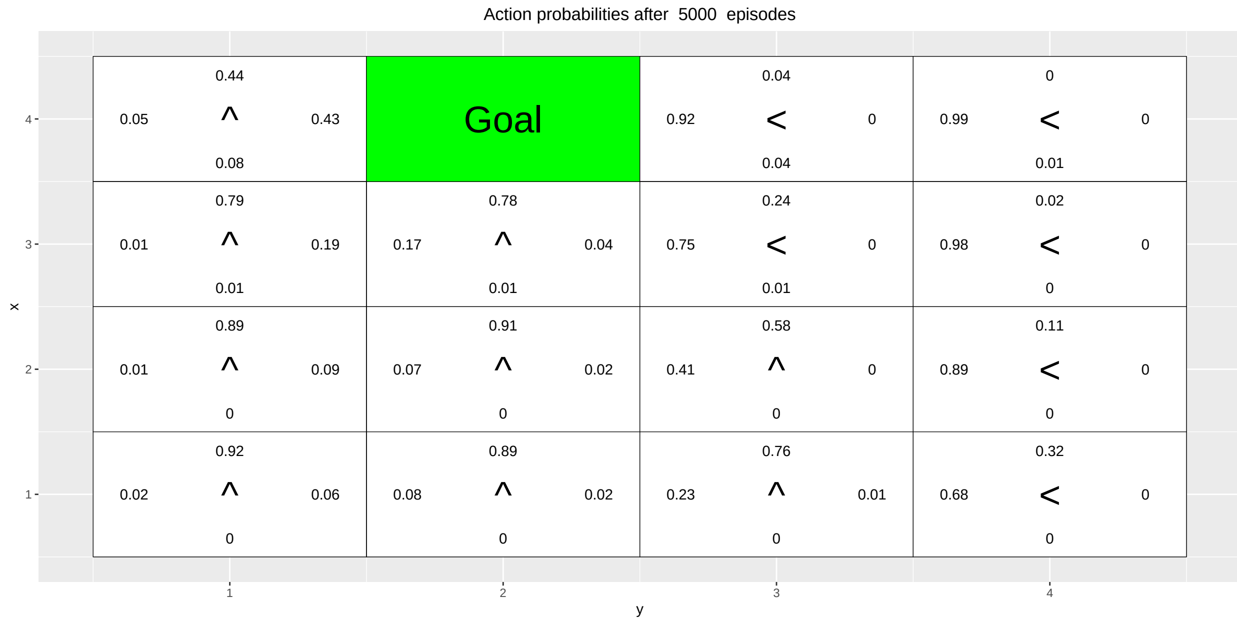
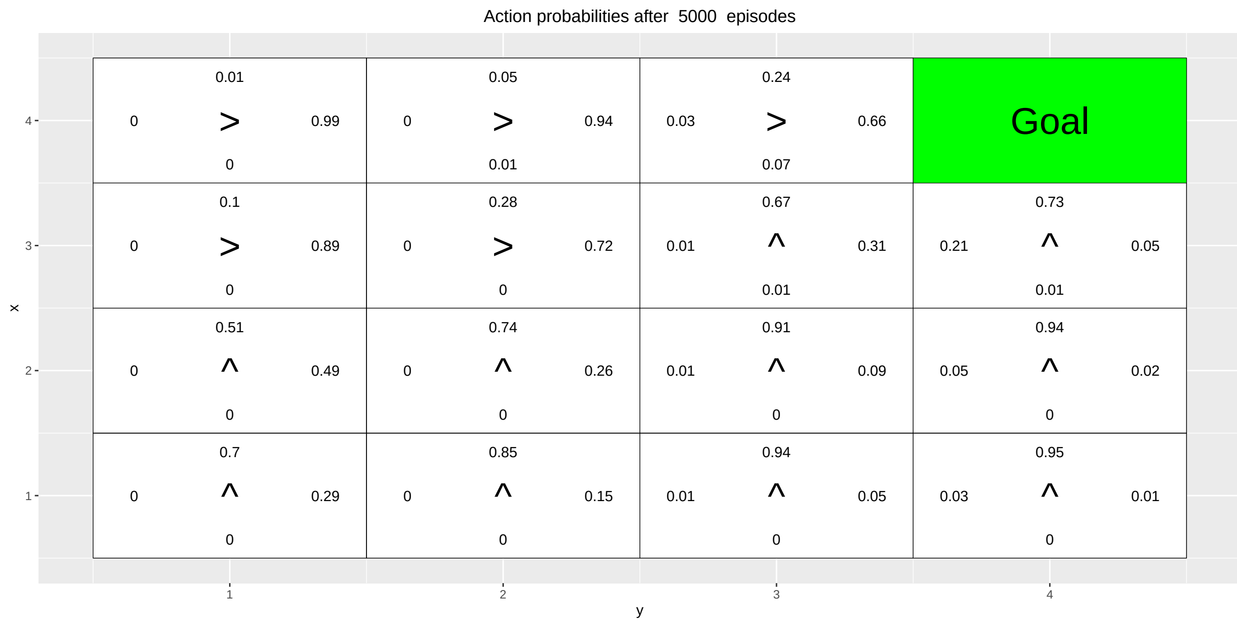
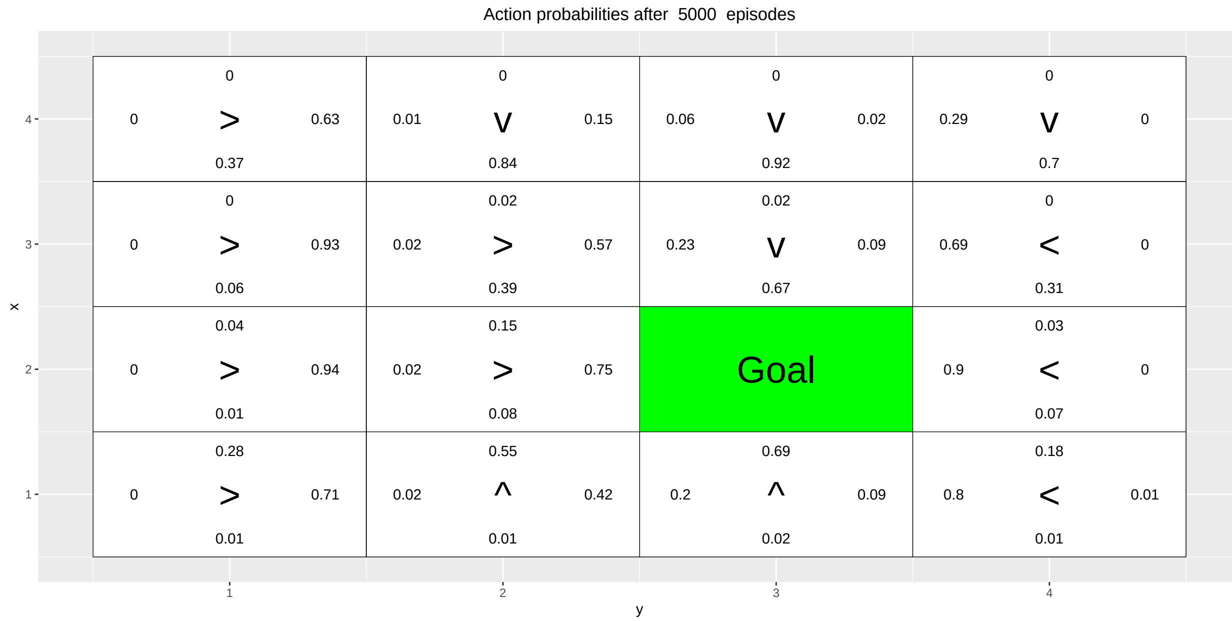
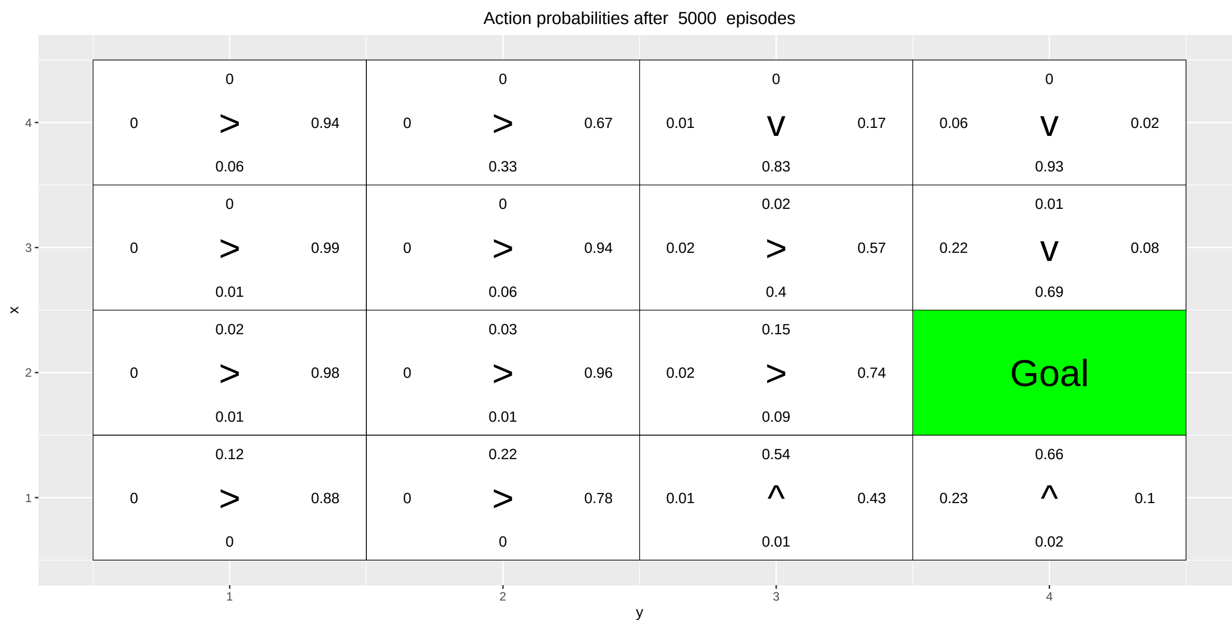

Action probabilities after 5000 episodes
0.44
0.05 ^ 0.43
0.08
Goal
0.04
0.92 < 0
0.04
0
0.99 < 0
0.01
0.79
0.01 ^ 0.19
0.01
0.78
0.17 ^ 0.04
0.01
0.24
0.75 < 0
0.01
0.02
0.98 < 0
0
0.89
0.01 ^ 0.09
0
0.91
0.07 ^ 0.02
0
0.58
0.41 ^ 0
0
0.11
0.89 < 0
0
0.92
0.02 ^ 0.06
0
0.89
0.08 ^ 0.02
0
0.76
0.23 ^ 0.01
0
0.32
0.68 < 0
0
x
4
3
2
1
y
1
2
3
4

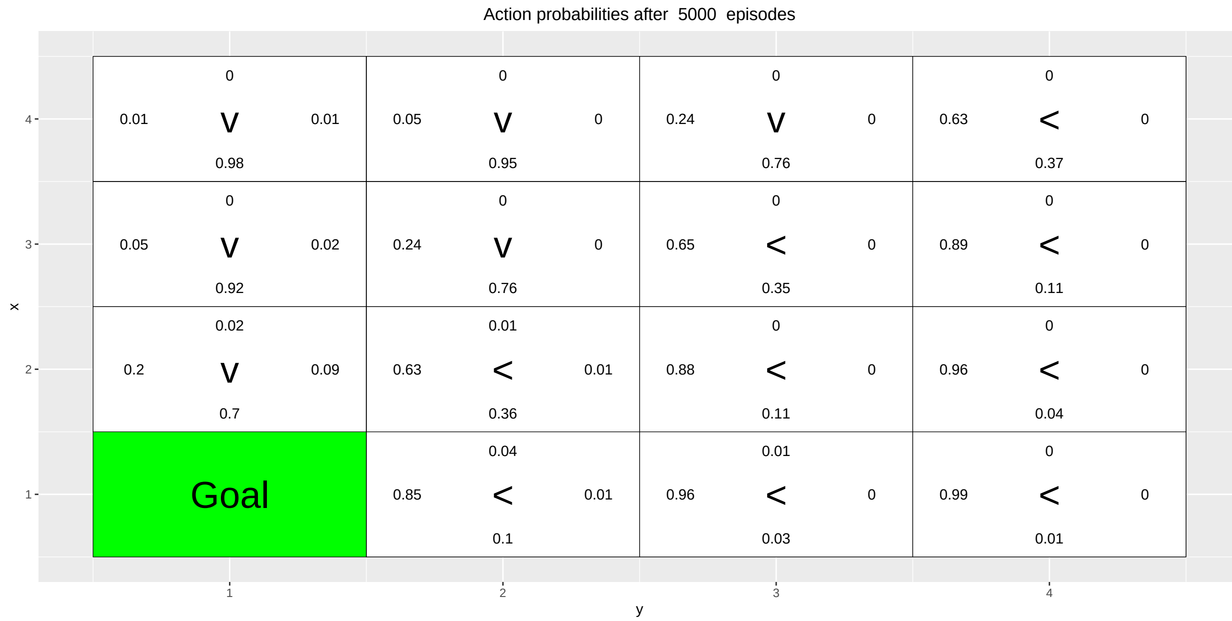

Action probabilities after 5000 episodes
0.01
0 > 0.99
0
0.05
0 > 0.94
0.01
0.24
0.03 > 0.66
0.07
Goal
0.1
0 > 0.89
0
0.28
0 > 0.72
0
0.67
0.01 ^ 0.31
0.01
0.73
0.21 ^ 0.05
0.01
0.51
0 ^ 0.49
0
0.74
0 ^ 0.26
0
0.91
0.01 ^ 0.09
0
0.94
0.05 ^ 0.02
0
0.7
0 ^ 0.29
0
0.85
0 ^ 0.15
0
0.94
0.01 ^ 0.05
0
0.95
0.03 ^ 0.01
0
x
4
3
2
1
y
1
2
3
4

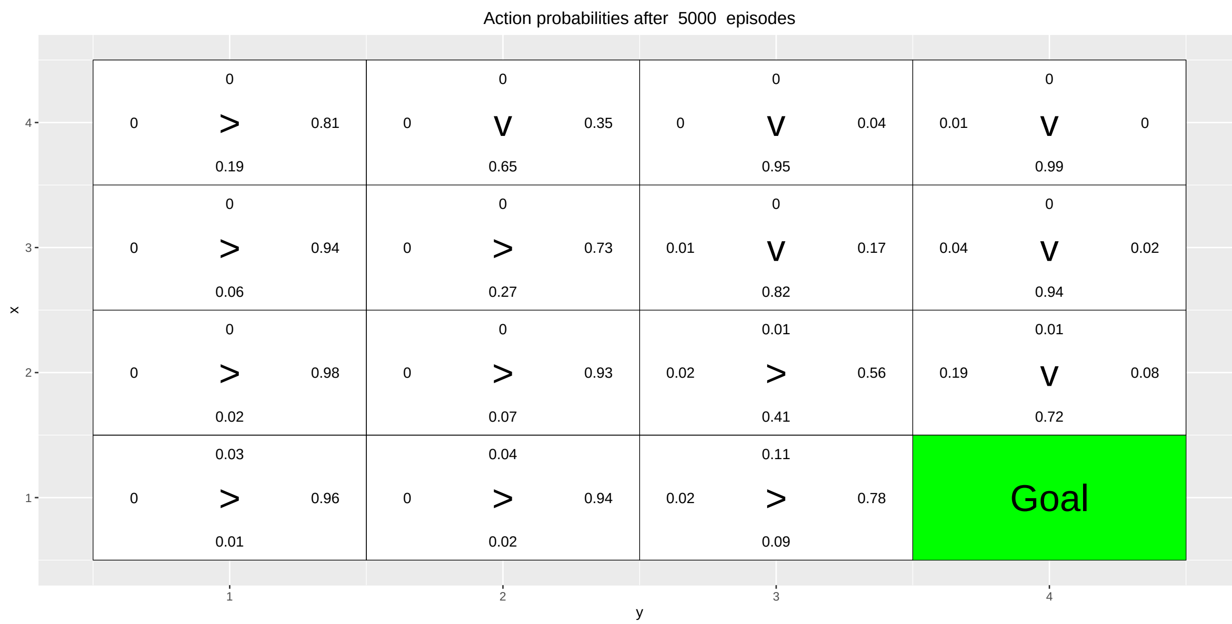

Action probabilities after 5000 episodes
0.04 0.02 0 0
0.05 V 0.4 0.32 V 0.06 0.77 < 0 0.96 < 0
0.51 0.6 0.22 0.04
0.25 0.04 0
0.03 > 0.64 Goal 0.91 < 0 0.99 < 0
0.08 0.05 0.01
0.67 0.75 0.23 0.02
0.02 ^ 0.31 0.17 ^ 0.06 0.76 < 0 0.97 < 0
0.01 0.01 0.01 0
0.82 0.85 0.55 0.09
0.02 ^ 0.16 0.1 ^ 0.04 0.43 ^ 0.01 0.9 < 0
0.01 0.01 0.01 0
x
4
3
2
1
1 2 3 4
y


Action probabilities after 5000 episodes
0.01 0.03 0.02 0
0 > 0.91 0.03 > 0.55 0.3 V 0.08 0.76 < 0
0.08 0.4 0.6 0.24
0.05 0.2 0.03
0 > 0.94 0.03 > 0.7 Goal 0.91 < 0
0.01 0.07 0.05
0.29 0.62 0.73 0.21
0 > 0.71 0.01 ^ 0.36 0.19 ^ 0.07 0.78 < 0
0 0.01 0.01 0.01
0.66 0.84 0.89 0.53
0 ^ 0.34 0.01 ^ 0.14 0.07 ^ 0.04 0.46 ^ 0.01
0 0 0 0
x
4
3
2
1
1 2 3 4
y


Action probabilities after 5000 episodes
Goal
x
y


Action probabilities after 5000 episodes
Goal
x
y

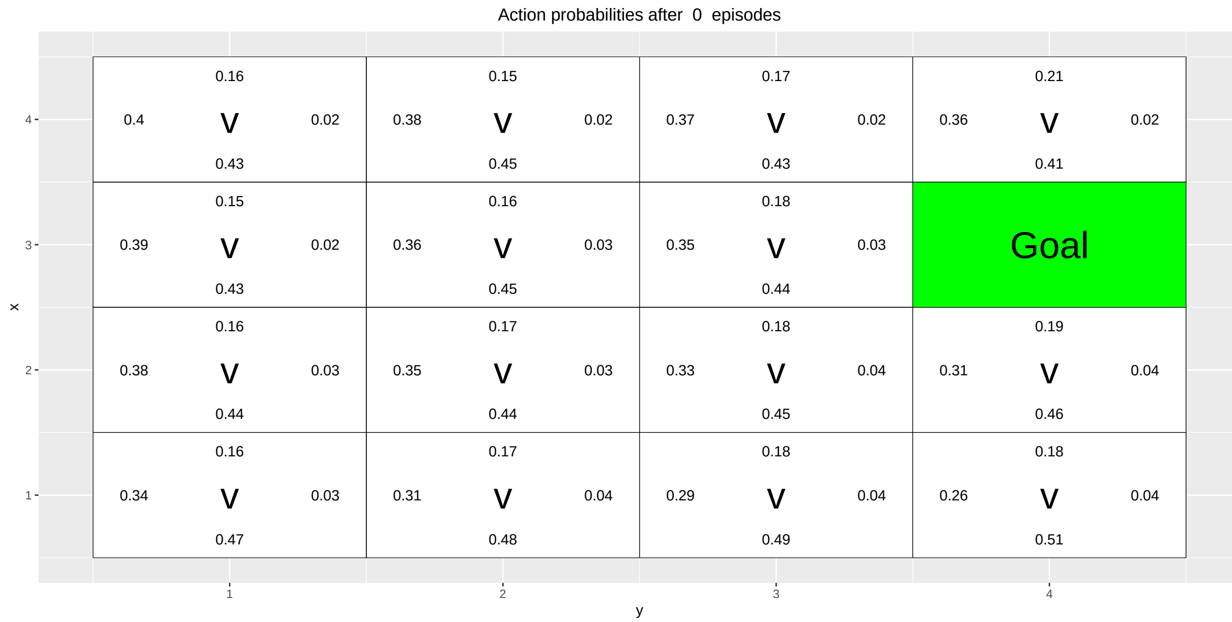


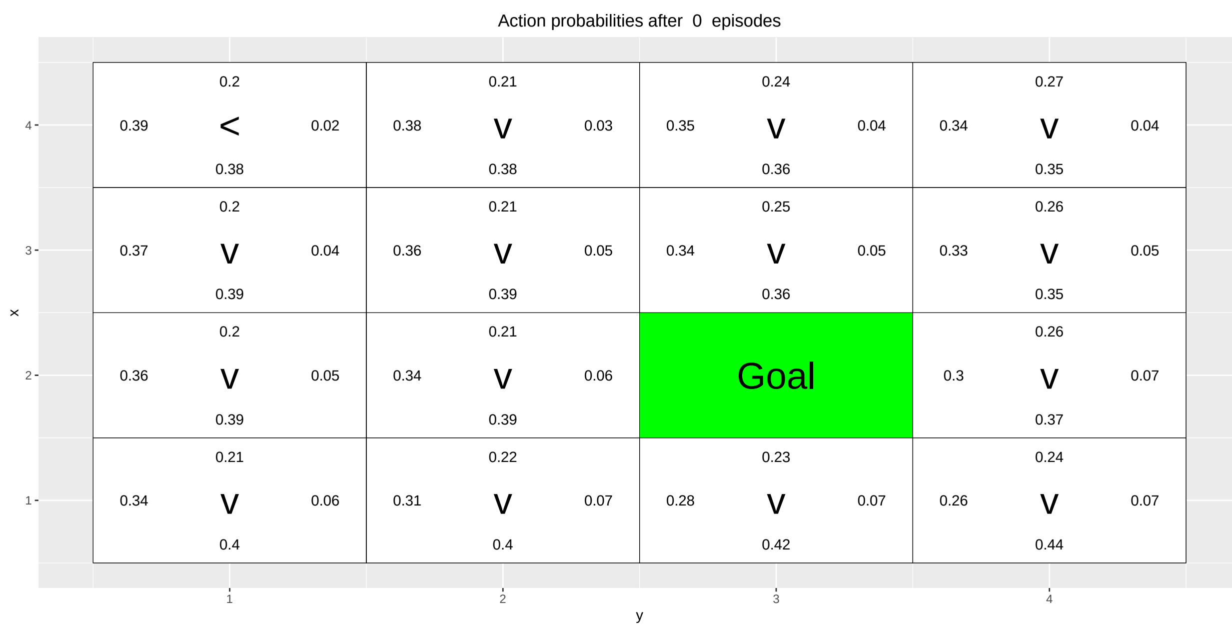


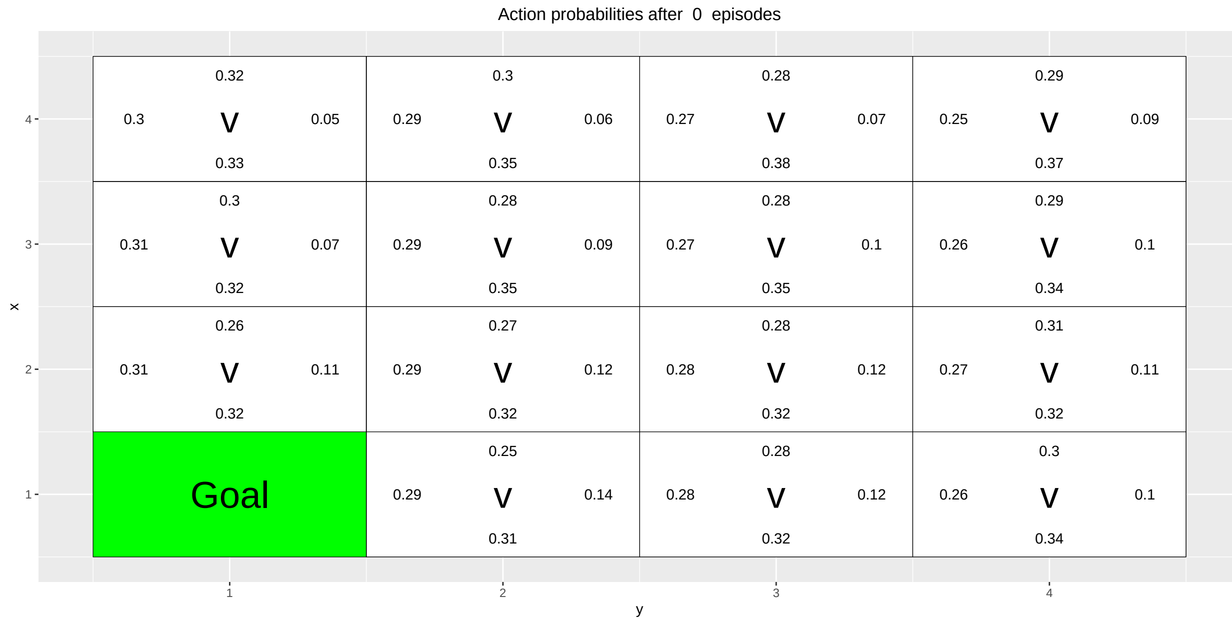
The results indicate that the learned model has a good performance since the model has been trained using all states as the *goal*. In this manner, the generalized model has accurate parameters for this environment with all states having a non-zero reward once. In every validation attempt, the arrows are pointed to the right direction which is an indication of the model accuracy.

Q-learning is not be a good approach for this task since the location of the goal keeps changing. In Q-learning we do not estimate parameters and therefore we do not have a generalized model to apply on a validation set. Q-learning works with q_tables which are deterministic to a specific environment with specific goals and so are not suitable for situations where the target state changes.

Environment E





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## episode 10
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## episode 150
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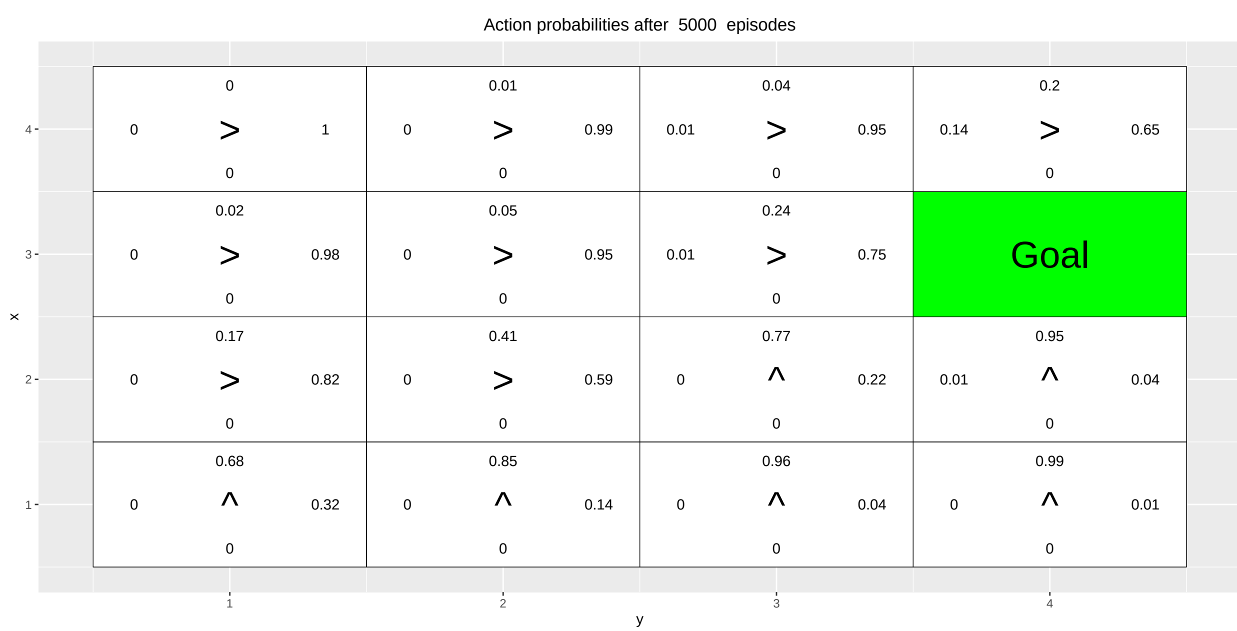

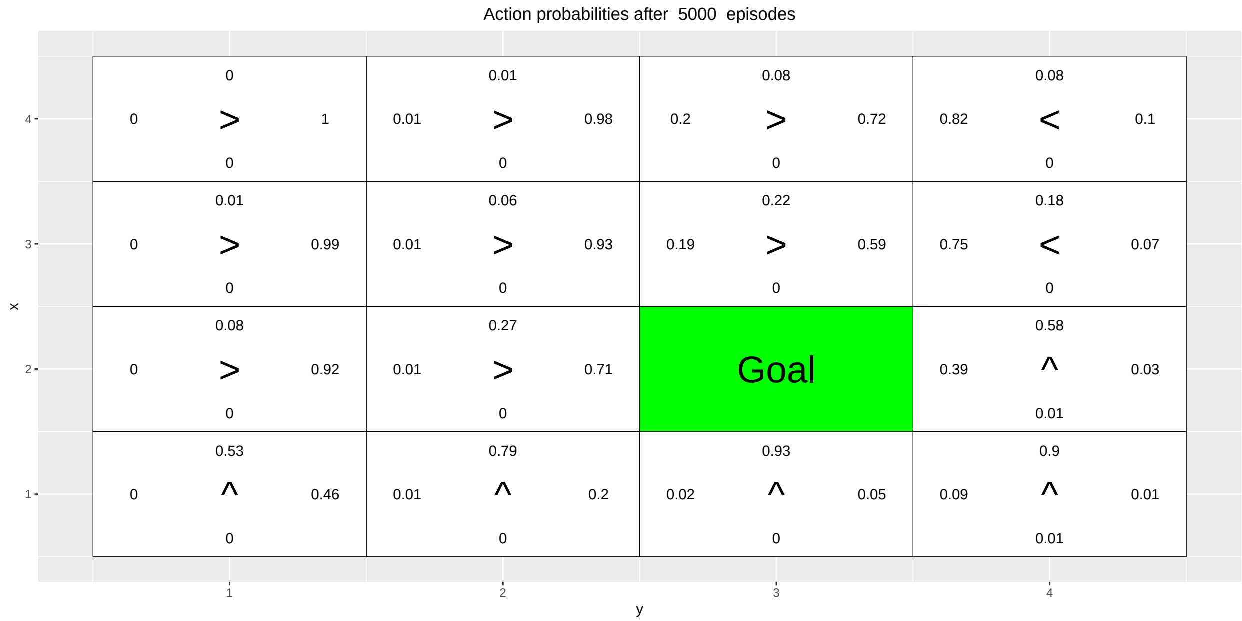

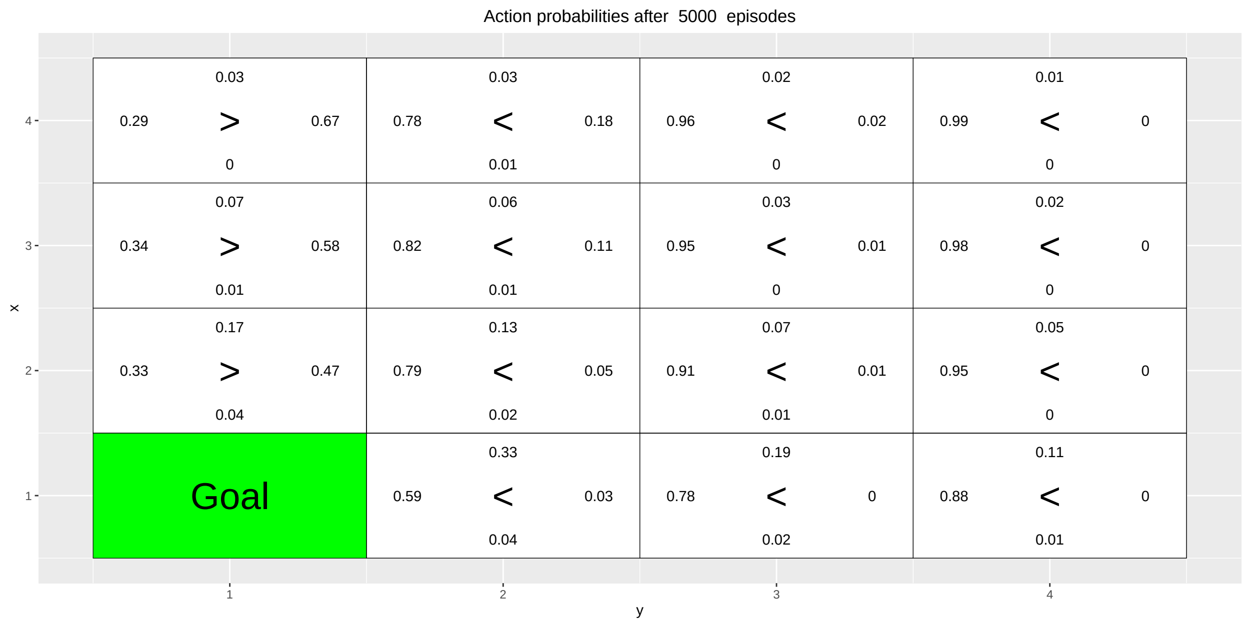

Given that the model is trained only for the first row, it means that the probabilities or the learned parameters would not yield accurate results when applied to non-visited states. The results approve this as we can see that the arrows in rows 2 or 3 are not pointed to the optimal direction and it might take the robot a long time to find the goal state.

Appendix

```
knitr::opts_chunk$set(echo = TRUE)

# By Jose M. Peña and Joel Oskarsson.
# For teaching purposes.
# jose.m.pena@liu.se.
```

```
##########################################################################################
# Q-learning
##########################################################################################

# install.packages("ggplot2")
# install.packages("vctrs")
library(ggplot2)

# If you do not see four arrows in line 16, then do the following:
# File/Reopen with Encoding/UTF-8

arrows <- c("^", ">", "v", "<")
action_deltas <- list(c(1,0), # up
                      c(0,1), # right
                      c(-1,0), # down
                      c(0,-1)) # left

vis_environment <- function(iterations=0, epsilon = 0.5, alpha = 0.1, gamma = 0.95, beta = 0){

  # Visualize an environment with rewards.
  # Q-values for all actions are displayed on the edges of each tile.
  # The (greedy) policy for each state is also displayed.
  #
  # Args:
  #   iterations, epsilon, alpha, gamma, beta (optional): for the figure title.
  #   reward_map (global variable): a HxW array containing the reward given at each state.
  #   q_table (global variable): a HxWx4 array containing Q-values for each state-action pair.
  #   H, W (global variables): environment dimensions.

  df <- expand.grid(x=1:H,y=1:W)
  foo <- mapply(function(x,y) ifelse(reward_map[x,y] == 0,q_table[x,y,1],NA),df$x,df$y)
  df$val1 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(reward_map[x,y] == 0,q_table[x,y,2],NA),df$x,df$y)
  df$val2 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(reward_map[x,y] == 0,q_table[x,y,3],NA),df$x,df$y)
  df$val3 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(reward_map[x,y] == 0,q_table[x,y,4],NA),df$x,df$y)
  df$val4 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y)
    ifelse(reward_map[x,y] == 0,arrows[GreedyPolicy(x,y)],reward_map[x,y]),df$x,df$y)
  df$val5 <- as.vector(foo)
  foo <- mapply(function(x,y) ifelse(reward_map[x,y] == 0,max(q_table[x,y,]),
                                     ifelse(reward_map[x,y]<0,NA,reward_map[x,y])),df$x,df$y)
  df$val6 <- as.vector(foo)

  print(ggplot(df,aes(x = y,y = x)) +
          scale_fill_gradient(low = "white", high = "green", na.value = "red", name = "") +
          geom_tile(aes(fill=val6)) +
          geom_text(aes(label = val1),size = 4,nudge_y = .35,na.rm = TRUE) +
          geom_text(aes(label = val2),size = 4,nudge_x = .35,na.rm = TRUE) +
          geom_text(aes(label = val3),size = 4,nudge_y = -.35,na.rm = TRUE) +
          geom_text(aes(label = val4),size = 4,nudge_x = -.35,na.rm = TRUE) +
          geom_text(aes(label = val5),size = 10) +
```

```r
          geom_tile(fill = 'transparent', colour = 'black') +
          ggtitle(paste("Q-table after ",iterations," iterations\n",
                        "(epsilon = ",epsilon,", alpha = ",alpha,"gamma = ",gamma,", beta = ",beta,")")) +
          theme(plot.title = element_text(hjust = 0.5)) +
          scale_x_continuous(breaks = c(1:W),labels = c(1:W)) +
          scale_y_continuous(breaks = c(1:H),labels = c(1:H)))

}

GreedyPolicy <- function(x, y){

  # Get a greedy action for state (x,y) from q_table.
  #
  # Args:
  #   x, y: state coordinates.
  #   q_table (global variable): a HxWx4 array containing Q-values for each state-action pair.
  #
  # Returns:
  #   An action, i.e. integer in {1,2,3,4}.
  # Your code here.
  r <- q_table[x,y,]
  best <- which(r == max(r))
  return(sample(best, 1))
}

EpsilonGreedyPolicy <- function(x, y, epsilon){

  # Get an epsilon-greedy action for state (x,y) from q_table.
  #
  # Args:
  #   x, y: state coordinates.
  #   epsilon: probability of acting greedily.
  #
  # Returns:
  #   An action, i.e. integer in {1,2,3,4}.

  # Your code here.
  r <- q_table[x,y,]
  best <- which(r == max(r))
  if(runif(1) < epsilon) {
    actions <- c(1,2,3,4)
    return(sample(actions, 1))
  }
  else {
    return(sample(best, 1))
  }
}

transition_model <- function(x, y, action, beta){

  # Computes the new state after given action is taken. The agent will follow the action
  # with probability (1-beta) and slip to the right or left with probability beta/2 each.
  #
```

```r
  # Args:
  #   x, y: state coordinates.
  #   action: which action the agent takes (in {1,2,3,4}).
  #   beta: probability of the agent slipping to the side when trying to move.
  #   H, W (global variables): environment dimensions.
  #
  # Returns:
  #   The new state after the action has been taken.

  delta <- sample(-1:1, size = 1, prob = c(0.5*beta,1-beta,0.5*beta))
  final_action <- ((action + delta + 3) %% 4) + 1
  foo <- c(x,y) + unlist(action_deltas[final_action])
  foo <- pmax(c(1,1),pmin(foo,c(H,W)))

  return (foo)
}

q_learning <- function(start_state, epsilon = 0.5, alpha = 0.1, gamma = 0.95,
                       beta = 0){

  # Perform one episode of Q-learning. The agent should move around in the
  # environment using the given transition model and update the Q-table.
  # The episode ends when the agent reaches a terminal state.
  #
  # Args:
  #   start_state: array with two entries, describing the starting position of the agent.
  #   epsilon (optional): probability of acting greedily.
  #   alpha (optional): learning rate.
  #   gamma (optional): discount factor.
  #   beta (optional): slipping factor.
  #   reward_map (global variable): a HxW array containing the reward given at each state.
  #   q_table (global variable): a HxWx4 array containing Q-values for each state-action pair.
  #
  # Returns:
  #   reward: reward received in the episode.
  #   correction: sum of the temporal difference correction terms over the episode.
  #   q_table (global variable): Recall that R passes arguments by value. So, q_table being
  #   a global variable can be modified with the superassigment operator <<-.

  # Your code here.
  x <- start_state[1]
  y <- start_state[2]
  episode_correction <- 0

  repeat{
    # Follow policy, execute action, get reward.
    a <- GreedyPolicy(x,y)
    A <- EpsilonGreedyPolicy(x,y,epsilon)
    trstate <- transition_model(x,y,A,beta)
    reward <- reward_map[trstate[1],trstate[2]]

    # Q-table update.
    correction <- alpha*(reward+gamma*max(q_table[trstate[1],trstate[2],]) - q_table[x,y,A])
```

```
    episode_correction <- episode_correction + correction
    q_table[x,y,A] <<- q_table[x,y,A] + correction
    x <- trstate[1]
    y <- trstate[2]
    if(reward!=0)
      # End episode.
      return (c(reward,episode_correction))
  }
}


GreedyPolicy <- function(x, y){
  r <- q_table[x,y,]
  best <- which(r == max(r))
  return(sample(best, 1))
}


EpsilonGreedyPolicy <- function(x, y, epsilon){
  r <- q_table[x,y,]
  best <- which(r == max(r))
  if(runif(1) < epsilon) {
    actions <- c(1,2,3,4)
    return(sample(actions, 1))
  }
  else {
    return(sample(best, 1))
  }
}


q_learning <- function(start_state, epsilon = 0.5, alpha = 0.1, gamma = 0.95, beta = 0){
  x <- start_state[1]
  y <- start_state[2]
  episode_correction <- 0

  repeat{
    # Follow policy, execute action, get reward.
    a <- GreedyPolicy(x,y)
    A <- EpsilonGreedyPolicy(x,y,epsilon)
    trstate <- transition_model(x,y,A,beta)
    reward <- reward_map[trstate[1],trstate[2]]

    # Q-table update.
    correction <- alpha*(reward+gamma*max(q_table[trstate[1],trstate[2],]) - q_table[x,y,A])
    episode_correction <- episode_correction + correction
    q_table[x,y,A] <<- q_table[x,y,A] + correction
    x <- trstate[1]
    y <- trstate[2]
    if(reward!=0)
      # End episode.
      return (c(reward,episode_correction))
  }
```

```
}


# Environment A (learning)

H <- 5
W <- 7

reward_map <- matrix(0, nrow = H, ncol = W)
reward_map[3,6] <- 10
reward_map[2:4,3] <- -1

q_table <- array(0,dim = c(H,W,4))

vis_environment()

for(i in 1:10000){
  foo <- q_learning(start_state = c(3,1))

  if(any(i==c(10,100,1000,10000)))
    vis_environment(i)
}


H <- 7
W <- 8

reward_map <- matrix(0, nrow = H, ncol = W)
reward_map[1,] <- -1
reward_map[7,] <- -1
reward_map[4,5] <- 5
reward_map[4,8] <- 10

q_table <- array(0,dim = c(H,W,4))

vis_environment()

MovingAverage <- function(x, n){

  cx <- c(0,cumsum(x))
  rsum <- (cx[(n+1):length(cx)] - cx[1:(length(cx) - n)]) / n

  return (rsum)
}

for(j in c(0.5,0.75,0.95)){
  q_table <- array(0,dim = c(H,W,4))
  reward <- NULL
  correction <- NULL

  for(i in 1:30000){
    foo <- q_learning(gamma = j, start_state = c(4,1))
    reward <- c(reward,foo[1])
```

```r
    correction <- c(correction,foo[2])
  }

  vis_environment(i, gamma = j)
  plot(MovingAverage(reward,100),type = "l")
  plot(MovingAverage(correction,100),type = "l")
}

for(j in c(0.5,0.75,0.95)){
  q_table <- array(0,dim = c(H,W,4))
  reward <- NULL
  correction <- NULL

  for(i in 1:30000){
    foo <- q_learning(epsilon = 0.1, gamma = j, start_state = c(4,1))
    reward <- c(reward,foo[1])
    correction <- c(correction,foo[2])
  }

  vis_environment(i, epsilon = 0.1, gamma = j)
  plot(MovingAverage(reward,100),type = "l")
  plot(MovingAverage(correction,100),type = "l")
}



# Environment C (the effect of beta).

H <- 3
W <- 6

reward_map <- matrix(0, nrow = H, ncol = W)
reward_map[1,2:5] <- -1
reward_map[1,6] <- 10

q_table <- array(0,dim = c(H,W,4))

vis_environment()

for(j in c(0,0.2,0.4,0.66)){
  q_table <- array(0,dim = c(H,W,4))

  for(i in 1:10000)
    foo <- q_learning(gamma = 0.6, beta = j, start_state = c(1,1))

  vis_environment(i, gamma = 0.6, beta = j)
}


# By Jose M. Peña and Joel Oskarsson.
# For teaching purposes.
# jose.m.pena@liu.se.
```

```
##############################################################################################
# REINFORCE
##############################################################################################
library(tensorflow)
#install_tensorflow()
library(keras)

# install.packages("keras")


# install.packages("ggplot2")
# install.packages("vctrs")
library(ggplot2)

# If you do not see four arrows in line 19, then do the following:
# File/Reopen with Encoding/UTF-8

arrows <- c("^", ">", "v", "<")
action_deltas <- list(c(1,0), # up
                      c(0,1), # right
                      c(-1,0), # down
                      c(0,-1)) # left

vis_prob <- function(goal, episodes = 0){

  # Visualize an environment with rewards.
  # Probabilities for all actions are displayed on the edges of each tile.
  # The (greedy) policy for each state is also displayed.
  #
  # Args:
  #   goal: goal coordinates, array with 2 entries.
  #   episodes, epsilon, alpha, gamma, beta (optional): for the figure title.
  #   H, W (global variables): environment dimensions.

  df <- expand.grid(x=1:H,y=1:W)
  dist <- array(data = NA, dim = c(H,W,4))
  class <- array(data = NA, dim = c(H,W))
  for(i in 1:H)
    for(j in 1:W){
      dist[i,j,] <- DeepPolicy_dist(i,j,goal[1],goal[2])
      foo <- which(dist[i,j,]==max(dist[i,j,]))
      class[i,j] <- ifelse(length(foo)>1,sample(foo, size = 1),foo)
    }

  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),NA,dist[x,y,1]),df$x,df$y)
  df$val1 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),NA,dist[x,y,2]),df$x,df$y)
  df$val2 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),NA,dist[x,y,3]),df$x,df$y)
  df$val3 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),NA,dist[x,y,4]),df$x,df$y)
  df$val4 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),NA,class[x,y]),df$x,df$y)
```

```r
  df$val5 <- as.vector(arrows[foo])
  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),"Goal",NA),df$x,df$y)
  df$val6 <- as.vector(foo)

  print(ggplot(df,aes(x = y,y = x)) +
          geom_tile(fill = 'white', colour = 'black') +
          scale_fill_manual(values = c('green')) +
          geom_tile(aes(fill=val6), show.legend = FALSE, colour = 'black') +
          geom_text(aes(label = val1),size = 4,nudge_y = .35,na.rm = TRUE) +
          geom_text(aes(label = val2),size = 4,nudge_x = .35,na.rm = TRUE) +
          geom_text(aes(label = val3),size = 4,nudge_y = -.35,na.rm = TRUE) +
          geom_text(aes(label = val4),size = 4,nudge_x = -.35,na.rm = TRUE) +
          geom_text(aes(label = val5),size = 10,na.rm = TRUE) +
          geom_text(aes(label = val6),size = 10,na.rm = TRUE) +
          ggtitle(paste("Action probabilities after ",episodes," episodes")) +
          theme(plot.title = element_text(hjust = 0.5)) +
          scale_x_continuous(breaks = c(1:W),labels = c(1:W)) +
          scale_y_continuous(breaks = c(1:H),labels = c(1:H)))

}

transition_model <- function(x, y, action, beta){

  # Computes the new state after given action is taken. The agent will follow the action
  # with probability (1-beta) and slip to the right or left with probability beta/2 each.
  #
  # Args:
  #   x, y: state coordinates.
  #   action: which action the agent takes (in {1,2,3,4}).
  #   beta: probability of the agent slipping to the side when trying to move.
  #   H, W (global variables): environment dimensions.
  #
  # Returns:
  #   The new state after the action has been taken.

  delta <- sample(-1:1, size = 1, prob = c(0.5*beta,1-beta,0.5*beta))
  final_action <- ((action + delta + 3) %% 4) + 1
  foo <- c(x,y) + unlist(action_deltas[final_action])
  foo <- pmax(c(1,1),pmin(foo,c(H,W)))

  return (foo)
}

DeepPolicy_dist <- function(x, y, goal_x, goal_y){

  # Get distribution over actions for state (x,y) and goal (goal_x,goal_y) from the deep policy.
  #
  # Args:
  #   x, y: state coordinates.
  #   goal_x, goal_y: goal coordinates.
  #   model (global variable): NN encoding the policy.
  #
  # Returns:
```

```
  #   A distribution over actions.

  foo <- matrix(data = c(x,y,goal_x,goal_y), nrow = 1)

  # return (predict_proba(model, x = foo))
  return (predict_on_batch(model, x = foo)) # Faster.

}

DeepPolicy <- function(x, y, goal_x, goal_y){

  # Get an action for state (x,y) and goal (goal_x,goal_y) from the deep policy.
  #
  # Args:
  #   x, y: state coordinates.
  #   goal_x, goal_y: goal coordinates.
  #   model (global variable): NN encoding the policy.
  #
  # Returns:
  #   An action, i.e. integer in {1,2,3,4}.

  foo <- DeepPolicy_dist(x,y,goal_x,goal_y)

  return (sample(1:4, size = 1, prob = foo))

}

DeepPolicy_train <- function(states, actions, goal, gamma){

  # Train the policy network on a rolled out trajectory.
  #
  # Args:
  #   states: array of states visited throughout the trajectory.
  #   actions: array of actions taken throughout the trajectory.
  #   goal: goal coordinates, array with 2 entries.
  #   gamma: discount factor.

  # Construct batch for training.
  inputs <- matrix(data = states, ncol = 2, byrow = TRUE)
  inputs <- cbind(inputs,rep(goal[1],nrow(inputs)))
  inputs <- cbind(inputs,rep(goal[2],nrow(inputs)))

  targets <- array(data = actions, dim = nrow(inputs))
  targets <- to_categorical(targets-1, num_classes = 4)

  # Sample weights. Reward of 5 for reaching the goal.
  weights <- array(data = 5*(gamma^(nrow(inputs)-1)), dim = nrow(inputs))

  # Train on batch. Note that this runs a SINGLE gradient update.
  train_on_batch(model, x = inputs, y = targets, sample_weight = weights)

}
```

```r
reinforce_episode <- function(goal, gamma = 0.95, beta = 0){

  # Rolls out a trajectory in the environment until the goal is reached.
  # Then trains the policy using the collected states, actions and rewards.
  #
  # Args:
  #   goal: goal coordinates, array with 2 entries.
  #   gamma (optional): discount factor.
  #   beta (optional): probability of slipping in the transition model.

  # Randomize starting position.
  cur_pos <- goal
  while(all(cur_pos == goal))
    cur_pos <- c(sample(1:H, size = 1),sample(1:W, size = 1))

  states <- NULL
  actions <- NULL

  steps <- 0 # To avoid getting stuck and/or training on unnecessarily long episodes.
  while(steps < 20){
    steps <- steps+1

    # Follow policy and execute action.
    action <- DeepPolicy(cur_pos[1], cur_pos[2], goal[1], goal[2])
    new_pos <- transition_model(cur_pos[1], cur_pos[2], action, beta)

    # Store states and actions.
    states <- c(states,cur_pos)
    actions <- c(actions,action)
    cur_pos <- new_pos

    if(all(new_pos == goal)){
      # Train network.
      DeepPolicy_train(states,actions,goal,gamma)
      break
    }
  }

}


# Environment D (training with random goal positions)

H <- 4
W <- 4

# Define the neural network (two hidden layers of 32 units each).
model <- keras_model_sequential()
model %>%
  layer_dense(units = 32, input_shape = c(4), activation = 'relu') %>%
  layer_dense(units = 32, activation = 'relu') %>%
  layer_dense(units = 4, activation = 'softmax')
```

```r
compile(model, loss = "categorical_crossentropy", optimizer = optimizer_sgd(lr=0.001))

initial_weights <- get_weights(model)

train_goals <- list(c(4,1), c(4,3), c(3,1), c(3,4), c(2,1), c(2,2), c(1,2), c(1,3))
val_goals <- list(c(4,2), c(4,4), c(3,2), c(3,3), c(2,3), c(2,4), c(1,1), c(1,4))

show_validation <- function(episodes){

  for(goal in val_goals)
    vis_prob(goal, episodes)

}

set_weights(model,initial_weights)

show_validation(0)

for(i in 1:5000){
  if(i%%10==0) cat("episode",i,"\n")
  goal <- sample(train_goals, size = 1)
  reinforce_episode(unlist(goal))
}

show_validation(5000)


# Environment E (training with top row goal positions)

train_goals <- list(c(4,1), c(4,2), c(4,3), c(4,4))
val_goals <- list(c(3,4), c(2,3), c(1,1))

set_weights(model,initial_weights)

show_validation(0)

for(i in 1:5000){
  if(i%%10==0) cat("episode", i,"\n")
  goal <- sample(train_goals, size = 1)
  reinforce_episode(unlist(goal))
}

show_validation(5000)
```