NUMBER SYSTEM CONVERSIONS

NUMBER SYSTEMS

- The system used to count discrete units is called number system.

- There are four systems of arithmetic which are often used in digital electronics.

  - Decimal Number System
  - Binary Number System
  - Octal Number System
  - Hexa Decimal System

DECIMAL NUMBER SYSTEM

- Decimal number system contains 10 digits: 0,1,2,3,4,5,6,7,8,9; and that is why its base or radix is 10.

- Here radix means total number of digits used in any system.

- To indicate digits greater than 9 (nine) we shall use the sum of these digits times power of 10.

FRACTIONAL DECIMAL SYSTEM

- Decimal System The decimal system is composed of 10 numerals or symbols. These 10 symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9; using these symbols as digits of a number, we can express any quantity. The decimal system, also called the base-10 system because it has 10 digits.

---------	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	---------

DECIMAL NUMBER SYSTEM

- The decimal number system is a positional number system.

- Example:

5 6 2 1	$1 \times 10^0 = 1$
10^3 10^2 10^1 10^0	$2 \times 10^1 = 20$
	$6 \times 10^2 = 600$
	$5 \times 10^3 = 5000$

BINARY NUMBER SYSTEM

- The binary number system is also a positional numbering system.

- Instead of using ten digits, 0 - 9, the binary system uses only two digits, 0 and 1.

BINARY NUMBER SYSTEM

- The binary number system is also known as **base 2**. The values of the positions are calculated by taking 2 to some power.

- Why is the base 2 for binary numbers?
  - Because we use 2 digits, the digits 0 and 1.

FRACTIONAL BINARY SYSTEM

- In the binary system, there are only two symbols or possible digit values, 0 and 1. This base-2 system can be used to represent any quantity that can be represented in decimal or other number system.

2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}

- In digital systems the information that is being processed is usually presented in binary form. Binary quantities can be represented by any device that has only two operating states or possible conditions. Eg. a switch has only open or closed. We arbitrarily (as we define them) let an open switch represent binary 0 and a closed switch represent binary 1. Thus we can represent any binary number by using series of switches.

OCTAL NUMBER SYSTEM

- Also known as the Base 8 System
- Uses digits 0 - 7
- Readily converts to binary
- Groups of three (binary) digits can be used to represent each octal digit
- Also uses multiplication and division algorithms for conversion to and from base 10

FRACTIONAL OCTAL NUMBER SYSTEM

- The octal number system has a base of eight, meaning that it has eight possible digits: 0,1,2,3,4,5,6,7.

8^4	8^3	8^2	8^1	8^0	8^{-1}	8^{-2}	8^{-3}	8^{-4}	8^{-5}

- Use to represent long binary numbers in computers and microprocessors.

HEXADECIMAL NUMBER SYSTEM

- Base 16 system
- Uses digits 0-9 & letters A,B,C,D,E,F
- Groups of four bits represent each base 16 digit

Decimal	Hexadecimal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

FRACTIONAL HEXADECIMAL NUMBER SYSTEM

- The hexadecimal system uses base 16. Thus, it has 16 possible digit symbols. It uses the digits 0 through 9 plus the letters A, B, C, D, E, and F as the 16 digit symbols.

16^4	16^3	16^2	16^1	16^0	16^{-1}	16^{-2}	16^{-3}	16^{-4}	16^{-5}

- Use to represent long binary numbers in computers and microprocessors.
- These digits can use to program machine language.

BINARY NUMBERING SCALE

Base 2 Number	Base 10 Equivalent	Power	Positional Value
000	0	2^0	1
001	1	2^1	2
010	2	2^2	4
011	3	2^3	8
100	4	2^4	16
101	5	2^5	32
110	6	2^6	64
111	7	2^7	128

SIGNIFICANT DIGITS

Binary: 11101101

Most significant digit ***Least significant digit***

Hexadecimal: 1D63A7A

Most significant digit ***Least significant digit***

CONVERTING FROM DECIMAL TO BINARY

- Make a list of the binary place values up to the number being converted.
- Perform successive divisions by 2, placing the remainder of 0 or 1 in each of the positions from right to left.
- Continue until the quotient is zero.
- Example: 42_{10}

2^5	2^4	2^3	2^2	2^1	2^0
32	16	8	4	2	1
1	0	1	0	1	0

DECIMAL TO BINARY CONVERTION

- There are two methods to convert it:-

i. Revese of Binary-To-Digital Method

- ✓ Decimal number write as the sum of square
- ✓ 0 & 1 is write on the byte

Example 1: Convert 45_{10} to the binary value

Solve = $45_{10} = 32 + 8 + 4 + 1$

$$= 2^5 \quad 0 \quad 2^3 \quad 2^2 \quad 0 \quad 2^0$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$= 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1_2$$

ii. Repeat division method

- ✓ The numbers is divide by 2.
- ✓ Balance for the question is written until the last answer.

Example : convert 25_{10} to binary

Solve $= 25_{10} = ?_2$

$$= \frac{25}{2} \longrightarrow 12 \text{ balance } 1 \longrightarrow \text{LSB}$$

$$= \frac{12}{2} \longrightarrow 6 \text{ balance } 0$$

$$= \frac{6}{2} \longrightarrow 3 \text{ balance } 0$$

$$= \frac{3}{2} \longrightarrow 1 \text{ balance } 1$$

$$= \frac{1}{2} \longrightarrow 0 \text{ balance } 1 \longrightarrow \text{MSB}$$

$\therefore$ Answer = $\underline{11001_2}$

DECIMAL TO BINARY CONVERSION

- The easiest way to convert a decimal number to its binary equivalent is to use the ***Division Algorithm***

- This method repeatedly divides a decimal number by 2 and records the quotient and remainder

  - ***The remainder digits (a sequence of zeros and ones) form the binary equivalent in least significant to most significant digit sequence***

DIVISION ALGORITHM

Convert 67 to its binary equivalent:

$67_{10} = x_2$

Step 1: 67 / 2 = 33 R 1 — *Divide 67 by 2. Record quotient in next row*

Step 2: 33 / 2 = 16 R 1 — ***Again divide by 2; record quotient in next row***

Step 3: 16 / 2 = 8 R 0 — ***Repeat again***

Step 4: 8 / 2 = 4 R 0 — ***Repeat again***

Step 5: 4 / 2 = 2 R 0 — ***Repeat again***

Step 6: 2 / 2 = 1 R 0 — ***Repeat again***

Step 7: 1 / 2 = 0 R 1 — **STOP *when quotient equals 0***

$$\boxed{1\,0\,0\,0\,0\,1\,1_2}$$

DECIMAL TO BINARY (FRACTIONAL)

- Examples:
- Convert the decimal number $(12.0625)_{10}$ into binary number.

Solution:

```
2| 12    0
 2| 6    0
  2| 3   1
   |1    1
    0
```

Fractional part:

0.0625 x 2 = 0.1250 0

0.1250 x 2 = 0.2500 0

0.2500 x 2 = 0.500 0

0.500 x 2 = 1.000 1

$$(12.0625)_{10} = (1100.0001)_2$$

DECIMAL TO OCTAL CONVERSION

Examples:

- $(315)_{10} = (473)_8$

$$
\begin{array}{r|l}
8 & 315 \\
8 & 39 \\
8 & 4 \\
 & 0
\end{array}
\qquad
\begin{array}{l}
3 \quad \text{LSD} \\
7 \\
4 \quad \text{MSD}
\end{array}
$$

DECIMAL TO OCTAL CONVERTION

- Convert from **decimal to octal** by using the repeated division method used for decimal to binary conversion.
- Divide the decimal number by 8
- The first remainder is the LSB and the last is the MSB.

Example : convert 359_{10} to Decimal Value

Solve = $359_{10} = ?_8$

$$= \frac{359}{8} \longrightarrow 44 \text{ balance } 7 \longrightarrow \text{LSB}$$

$$= \frac{44}{8} \longrightarrow 5 \text{ balance } 4$$

$$= \frac{5}{8} \longrightarrow 0 \text{ balance } 5 \longrightarrow \text{MSB}$$

$\therefore$ Answer = $\underline{547_8}$

DECIMAL TO OCTAL CONVERSION

Convert 427_{10} to its octal equivalent:

427 / 8 = 53 R3	Divide by 8; R is LSD
53 / 8 = 6 R5	Divide Q by 8; R is next digit
6 / 8 = 0 R6	Repeat until Q = 0

$$\boxed{653_8}$$

DECIMAL TO OCTAL (FRACTIONAL)

- Examples:
- Convert the decimal number $(225.225)_{10}$ into octal number.

Solution:

$$
\begin{array}{r|l}
8 & 225 \\
\hline
8 & 28 \quad 1 \\
\hline
8 & 3 \quad 4 \\
\hline
 & 0 \quad 3
\end{array}
\quad \uparrow
$$

Fractional part:

$0.225 \times 8 = 1.800 \quad 1$

$0.800 \times 8 = 6.400 \quad 6$

$0.400 \times 8 = 3.200 \quad 3$

$0.200 \times 8 = 1.600 \quad 1$

$0.600 \times 8 = 4.800 \quad 4$

$\downarrow$

$$(225.225)_{10} = (341.16314)_8$$

DECIMAL TO HEXADECIMAL CONVERSION

– ***Examples***

- $(315)_{10} = (13B)_{16}$

16	315	B	LSD
16	19	3	
16	1	1	MSD
	0		

DECIMAL TO HEXADECIMAL CONVERSION

Convert 830_{10} to its hexadecimal equivalent:

830 / 16 = 51 R14 ← = E in Hex

51 / 16 = 3 R3

3 / 16 = 0 R3

$\boxed{33E_{16}}$

DECIMAL TO HEXADECIMAL (FRACTIONAL)

- Examples:
- Convert the decimal number $(225.225)_{10}$ into hexadecimal number.

Solution:

```
16|225
  16|14    1
     0    14
          (or E)
```

Fractional part:

0.225 x 16 = 3.600 3

0.600 x 16 = 9.600 9

$$(225.225)_{10} = (E1.39)_{16}$$

CONVERTING FROM BINARY TO DECIMAL

- Example of a binary number and the values of the positions:

1	0	0	1	1	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

CONVERTING FROM BINARY TO DECIMAL

1	0	0	1	1	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

$2^0 = 1$ $2^4 = 16$

$2^1 = 2$ $2^5 = 32$

$2^2 = 4$ $2^6 = 64$

$2^3 = 8$

$1 \times 2^0 = 1$

$0 \times 2^1 = 0$

$1 \times 2^2 = 4$

$1 \times 2^3 = 8$

$0 \times 2^4 = 0$

$0 \times 2^5 = 0$

$1 \times 2^6 = \underline{64}$

77_{10}

BINARY TO DECIMAL CONVERSION

- The easiest method for converting a binary number to its decimal equivalent is to use the ***Multiplication Algorithm***

- Multiply the binary digits by increasing powers of two, starting from the right

- Then, to find the decimal number equivalent, sum those products

MULTIPLICATION ALGORITHM

Convert $(10101101)_2$ to its decimal equivalent:

Binary →	1	0	1	0	1	1	0	1
	x	x	x	x	x	x	x	x
Positional Values →	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Products →	128 + 32 + 8 + 4 + 1							

$$\boxed{173_{10}}$$

CONVERTING FROM BINARY TO DECIMAL

Practice conversions:

Binary	Decimal
11101	
1010101	
100111	

CONVERTING FROM DECIMAL TO BINARY

Practice conversions:

Decimal	Binary
59	
82	
175	

BINARY TO OCTAL CONVERSION

- Can be converted by grouping the binary bit in group of three starting from LSB
- Octal is a base-8 system and equal to two the **power of three**, so a digit in Octal is equal to three digit in binary system.

BINARY TO HEXADECIMAL SYSTEM

1 Group the digits of the binary number by **four** starting from the right.

2 Replace each group of **four** digits by an equivalent hexadecimal digit.

Convert 10110101_2 into a hexadecimal number.

$$\underbrace{1011}_{B}\ \underbrace{0101_2}_{5} = B5_{16}$$

EXERCISE

1 Convert 101000_2 into a hexadecimal number.

2 Convert 11101111_2 into a hexadecimal number.

OCTAL TO DECIMAL CONVERSION

Convert 653_8 to its decimal equivalent:

Octal Digits →	6	5	3
	x	x	x
Positional Values →	8^2	8^1	8^0
Products →	384 +	40 +	3

$$\boxed{427_{10}}$$

OCTAL TO BINARY CONVERTION

- Convert from **octal to binary** by converting each octal digit to a three bit binary equivalent

Octal digit	0	1	2	3	4	5	6	7
Binary Equivalent	000	000	000	000	000	000	000	000

- Convert from **binary to octal** by grouping bits in threes starting with the LSB.
- Each group is then converted to the octal equivalent
- Leading zeros can be added to the left of the MSB to fill out the last group.

OCTAL TO BINARY CONVERSION

Each octal number converts to 3 binary digits

Code
0 - 000
1 - 001
2 - 010
3 - 011
4 - 100
5 - 101
6 - 110
7 - 111

To convert 653_8 to binary, just substitute code:

6	5	3
↓	↓	↓
110	101	011

HEXADECIMAL TO DECIMAL CONVERSION

Convert 3B4F16 to its decimal equivalent:

Hex Digits →	3	B	4	F
	x	x	x	x
Positional Values →	16^3	16^2	16^1	16^0
Products →	12288 +	2816 +	64 +	15

$$\boxed{15{,}183_{10}}$$

HEXADECIMAL TO BINARY

- To convert a hexadecimal to binary number, convert each hexadecimal digit to its 4 bit equivalent using the hexa number.

- Example: $(23.AB)_{16} = ()_2$

Solution: $(23.AB)_{16} =$ 2 3 . A B

0010 0011 1010 1011

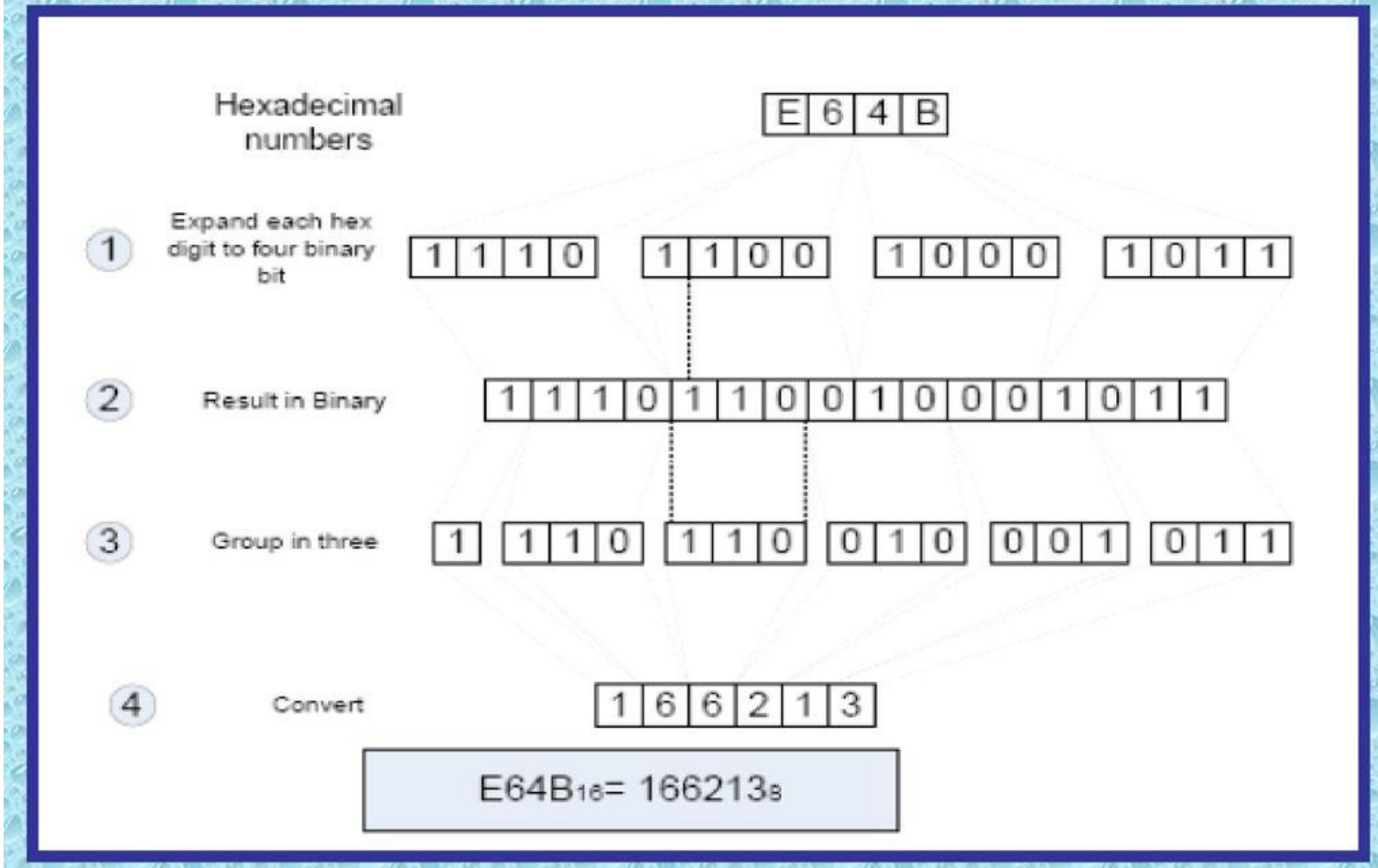
$(23.AB)_{16} = (00100011.10101011)_2$

HEXADECIMAL TO OCTAL CONVERTION

- There is two ways to convert it:-
  i. Hexadecimal – Decimal – Octal
  ii. Hexadecimal – Binary – Octal

i. Hexadecimal – Decimal – Octal

i. Hexadecimal – Binary – Octal



HEXADECIMAL NUMBER SYSTEM

Binary	Decimal	Hexadecimal	Binary	Decimal	Hexadecimal
0	0	0	1010	10	A
1	1	1	1011	11	B
10	2	2	1100	12	C
11	3	3	1101	13	D
100	4	4	1110	14	E
101	5	5	1111	15	F
110	6	6			
111	7	7			
1000	8	8			
1001	9	9			