

# **The minimum cut is the smallest number of edges whose removal disconnects the graph into two parts.**

## Step-by-Step Process (Matching the Diagram)

1. Take Edge Input
  - The program reads the number of vertices  $n$  and edges  $m$ .
  - All edges  $(u, v)$  are stored in a vector.
2. Initialize Disjoint Sets
  - Each vertex starts as its own parent (separate set)
  - A size array is used for efficient union operations.
3. Random Edge Selection
  - A random edge  $(u, v)$  is selected from the edge list
  - This matches the “for each random edge  $\rightarrow$  find  $(u, v)$ ” step in the picture.
4. Find Super Nodes
  - Using `find_set()`, the algorithm finds the current super-nodes of  $u$  and  $v$
  - These represent contracted components
5. Contract the Edge
  - If  $u$  and  $v$  belong to different components:
    - They are merged using `union_sets()`
    - The number of vertices is reduced by 1
  - This corresponds to creating a \*super node  $(n+1)$ \* and connecting edges to it.
6. Ignore Self-Loops
  - If both endpoints belong to the same component, the edge is skipped.
  - This matches the “remove  $(u-u)$  edge” logic in the diagram.
7. Repeat Until Two Super Nodes Remain
  - The contraction continues until only \*two components\* are left.
  - This matches the “break if edge vector size is 1 / vertices = 2” step

## 8. Count Crossing Edges

- All remaining edges that connect the two super-nodes are counted
- This count represents the \*cut size\*.

## 9. Repeat Multiple Times

- Since the algorithm is randomized, it runs \*100 times\*
- The minimum cut found across all runs is selected as the final answer.

## Final Output

- The program prints the \*minimum cut value\* found after multiple iterations
- Repetition increases the probability of finding the true minimum cut

## Conclusion

This implementation follows the exact idea shown in the picture:

- Random edge selection
- Super-node creation
- Edge contraction
- Removal of self-loops
- Repetition for accuracy