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THE UNREASONABLE EFFECTIVENSS OF MATHEMATICS IN THE NATURAL SCIENCES

Eugene Wigner

Mathematics, rightly viewed, possesses not only truth, but supreme beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as in poetry.

- BERTRAND RUSSELL, Study of Mathematics

The Unreasonable Effectiveness of Mathematics in Molecular Biology*

My title is an emulation of that of the well-known paper by E.P. Wigner, "The unreasonable effectiveness of mathematics in the natural sciences [1]." Of course the irony cuts in opposite ways in physics and molecular biology. In physics, mathematics is obviously effective—

The Mathematical Universe

Max Tegmark

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Abstract I explore physics implications of the *External Reality Hypothesis* (ERH) that there exists an external physical reality completely independent of us humans. I argue that with a sufficiently broad definition of mathematics, it implies the *Mathematical Universe Hypothesis* (MUH) that our physical world is an abstract mathematical structure. I discuss various implications of the ERH and MUH, ranging from standard physics topics like symmetries, irreducible representations, units, free para-

THE UNREASONABLE EFFECTIVENESS OF MATHEMATICS

R. W. HAMMING

Prologue. It is evident from the title that this is a philosophical discussion. I shall not apologize for the philosophy, though I am well aware that most scientists, engineers, and mathematicians have little regard for it; instead, I shall give this short prologue to justify the approach.

Man, so far as we know, has always wondered about himself, the world around him, and what life is all about. We have many myths from the past that tell how and why God, or the gods, made man and the universe. These I shall call *theological explanations.* They have one principal characteristic in common—there is little point in asking why things are the way they are, since we are given mainly a description of the creation as the gods chose to do it.

Philosophy started when man began to wonder about the world outside of this theological framework. An early example is the description by the philosophers that the world is made of earth, fire, water, and air. No doubt they were told at the time that the gods made things that way and to stop worrying about it.

Solving Wigner's Mystery: The Reasonable (Though Perhaps Limited) Effectiveness of Mathematics in the Natural Sciences

IVOR GRATTAN-GUINNESS

The Viewpoint column offers mathematicians the opportunity to write about any issue of interest to the international mathematical community. Disagreement and controversy are welcome. The views

In 1960 the physicist Eugene Wigner published an influential article on 'The unreasonable effectiveness of mathematics in the natural sciences'. I counter the claim stated in its title with an interpretation of science in which many of the uses of mathematics are shown to be quite reasonable, even rational, although maybe somewhat limited in content and indeed not free from ineffectiveness. The alternative view emphasizes two factors that Wigner largely ignores: the effectiveness of the natural sciences in mathematics, in that much mathematics has been motivated by interpretations in the sciences, and still is; and the central place of theories in both mathematics and the sciences, especially theory-building, in which analogies drawn from other theories play an important role. A major related feature is the desimplification of theories, which attempts to reduce limitations on their effectiveness. Significant also is the ubiquity and/or generality of many topics and notions in mathematics. It emerges that the connections between mathematics and the natural sciences are, and always have been, rationally although fallibly forged links, not a collection of mysterious parallelisms.

Wigner's Thesis

Wigner states as his main thesis 'that the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and that there is no rational explanation for it'; for example, 'The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand or deserve' [1960, 2,14]. By way of illustration he recalls a story about two friends studying population statistics by means of the normal (or Gaussian) distribution and being bewildered by the presence in the analysis of π: 'surely the population has nothing to do with the circumference of the circle' [p. 1]. He judges this mystery to be 'plain common sense' and does not discuss it again in the article.

Wigner's article has been cited especially by scientists and mathematicians on many occasions, with approval or at least without demur; some related articles have appeared.[1] Philosophers have also considered the article, and some have largely accepted the force of the argument.[2] One should note that most of the established philosophies of mathematics favoured by philosophers have aimed to grasp mathematical theories *already developed* rather than to address theory-building. There [Pólya 1954a, 1954b] is much more promising, with his masterly survey of 'plausible reasoning' and the dynamic relationships between theorems and proofs; however, he focusses largely upon pure mathematics. In my approach, which in general terms follows Pólya, the unreasonableness will largely disappear, but doubts are raised over effectiveness. The discussion is set at the level of formed cognition and theory-building; I