Assignment 1

Computational Plasticity (SoSe25)

Bagus Alifah Hasyim
108023246468
Last Three Digits: 468

Given Data

In this section we need to define every data that is given in the assignment, such as the material properties, geometry, and other relevant parameters that are needed for the analysis. Therefore, following data shall we define using the author's "immatrikulation nummer":

- $E_a = 200 + (10 \times 4) = 240$ GPa
- $\sigma_a = 300 + (10 \times 6) = 360$ MPa
- $\sigma_{yb} = 200 + (10 \times 8) = 280$ MPa

Introduction

In this assignment, we will analyze the mechanical response of a baseline plate and a plate with a circular inclusion under uniaxial loading. The analysis will include comparison of obtaining stress-strain curves between the force-displacement and direct stress-strain curves, differentiating plane stress and plane strain conditions, comparing local stress field distribution based on different constitutive models, and evaluating stress at specific points based on mesh sizes.

For the following sections we will systematically address each question in the assignment, providing detailed explanations, derivations, and relevant figures or tables to support the analysis. All calculations and results are based on the provided data and the author's unique identification number. Numerical analysis will be performed using Abaqus CAE with an appropriate boundary condition and settings.

Set up of the Model

For the model setup, we will use Abaqus CAE to create a 2D model of the plate with the specified geometry and material properties.

Geometry

1. **Baseline Plate:** A rectangular plate with length $L = 40$ mm, width $W = 10$ mm, and thickness $t = 1$ mm.

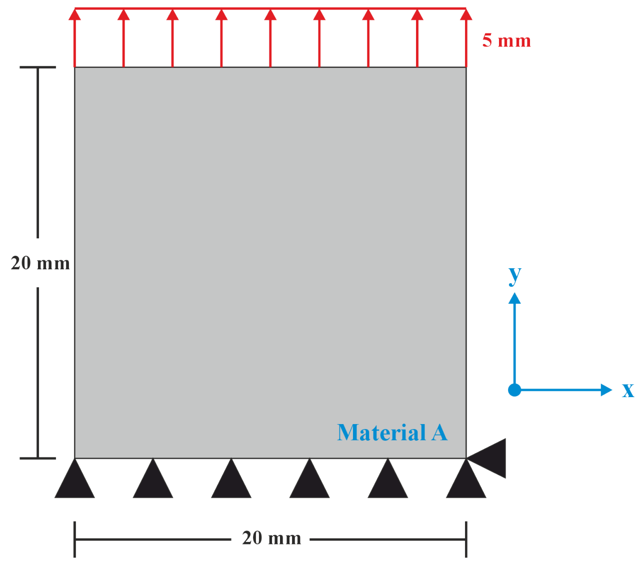

Figure 1: Baseline plate geometry with defined material A. Two boundary conditions are applied, which are the fixed support restriction in y direction on the bottom line of the plate, fixed support point on the bottom right point, and constant distribution of displacement in y direction on the top line of the plate.

2. **Plate with Circular Inclusion:** Same as the baseline plate, but with a circular inclusion of radius r located at the center or specified position.

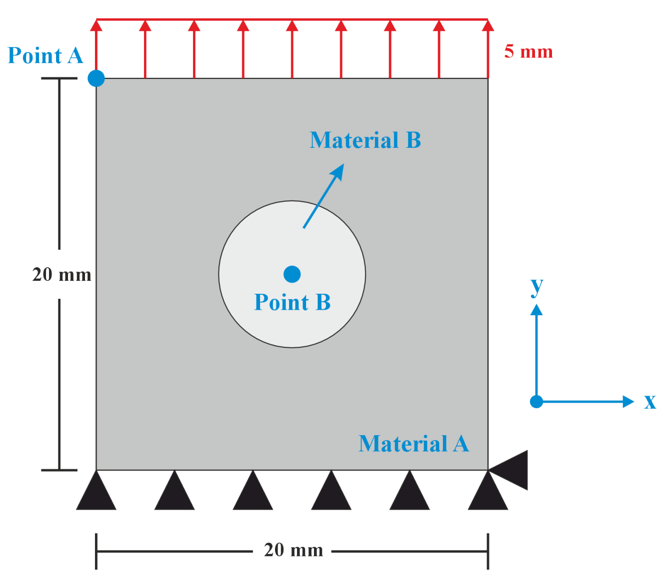

Figure 2: Baseline plate geometry with defined material B. Boundary conditions are the same as in Task 1.

Material Properties

The material properties for the analysis are provided from the task definition, which there are some parameters that need to be calculated based on the author's Immatrikulation Nummer.

Table 1: Elastic and plastic properties of material A.

E (GPa)	ν	σ_{ya} (MPa)	σ_f (MPa)	ϵ_f^f
240	0.3	360	550	0.4

Table 2: Elastic properties of material B.

E_1 (GPa)	E_2 (GPa)	E_3 (GPa)	ν_{12}	ν_{13}	ν_{23}	G_{12} (MPa)	G_{13} (MPa)	G_{23} (MPa)
210	220	230	0.3	0.31	0.32	80	84	87

Table 3: Plastic properties of material B.

R_{11}	R_{22}	R_{33}	R_{12}	R_{13}	R_{23}	σ_{yb}	σ_f (MPa)	ϵ_p^f
1	1.2	1.25	0.8	0.85	0.95	280	450	0.5

Results and Discussion

Q1: Mechanical Response Analysis for Baseline Plate

(a) XXX...

(b) XXX...

(c) XXX...

Q2: Mechanical Response Analysis for Plate with Circular Inclusion

XXX...

Q3: Local Stress Field Distribution and Analysis

XXX...

Q4: Comparison Analysis at Point A

XXX...

Q5: Comparison Analysis at Point B

XXX...

Extra Task

To calculate $\frac{\partial f}{\partial \sigma}$ with Voigt notation tensor $\sigma = (\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6)$ from yield function $f(\sigma) = \sigma_{eq} - \sigma_y$, where σ_{eq} can be described as follows:

$$\sigma_{eq} = \sqrt{\frac{1}{2}\left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + 6(\sigma_4^2 + \sigma_5^2 + \sigma_6^2)\right)} \tag{1}$$

Hence, we can simplify the prescribed equation to:

$$A = \frac{1}{2}\left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + 6(\sigma_4^2 + \sigma_5^2 + \sigma_6^2)\right) \tag{2}$$

We can substitute the equation above into the yield function $f(\sigma)$ and applying the chain rule in the derivative, we obtain:

$$f(\sigma) = \sqrt{A} - \sigma_y \rightarrow \frac{\partial f}{\partial \sigma} = \frac{\partial f}{\partial A} \cdot \frac{\partial A}{\partial \sigma} = \frac{1}{2\sqrt{A}} \cdot \frac{\partial A}{\partial \sigma} \tag{3}$$

Since we defined A as a substituent of the equivalent stress σ_{eq}, we can revert the equation as in the origin form:

$$\frac{\partial f}{\partial \sigma} = \frac{1}{\sigma_{eq}} \cdot \frac{\partial A}{\partial \sigma} \tag{4}$$

Now we need to expand the tensor differential with respect to each stress component in the Voigt notation. Therefore, we can do the differential operation for each stress component and assemble them to a Voigt notation as following sequence:

$$\frac{\partial A}{\partial \sigma_1} = H_1(\sigma_1 - \sigma_2) + H_3(\sigma_1 - \sigma_3) \ ;$$

$$\frac{\partial A}{\partial \sigma_2} = H_2(\sigma_2 - \sigma_3) + H_1(\sigma_2 - \sigma_1) \ ;$$

$$\frac{\partial A}{\partial \sigma_3} = H_2(\sigma_3 - \sigma_2) + H_3(\sigma_3 - \sigma_1) \ ;$$

$$\frac{\partial A}{\partial \sigma_4} = 6H_4\sigma_4 \ ;$$

$$\frac{\partial A}{\partial \sigma_5} = 6H_5\sigma_5 \ ;$$

$$\frac{\partial A}{\partial \sigma_6} = 6H_6\sigma_6 \ ;$$

To summarize the above equations, we can write them in a matrix form as follows:

$$\frac{\partial f}{\partial \sigma} = \frac{1}{\sigma_{eq}} \begin{bmatrix} H_1(\sigma_1 - \sigma_2) + H_3(\sigma_1 - \sigma_3) \\ H_2(\sigma_2 - \sigma_3) + H_1(\sigma_2 - \sigma_1) \\ H_2(\sigma_3 - \sigma_2) + H_3(\sigma_3 - \sigma_1) \\ 6H_4\sigma_4 \\ 6H_5\sigma_5 \\ 6H_6\sigma_6 \end{bmatrix} \tag{5}$$

Conclusion

(Summarize your findings and state key takeaways from the analysis)