Master Thesis

Numerical Study of Modified Newtonian Dynamics

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And He gave you from all you asked of Him. And if you should count the favor of Allah , you could not enumerate them. Indeed, mankind is [generally] most unjust and ungrateful.

(QS. Ibrahim : (14:34))

Dedicated to :

My family who pray me in their continuos prayers.

above every knower there is a higher Knower.

(QS. Yusuf : 76)

Abstract

In this work we simulate the particle motion as a astronomical object which is called galaxy. The particle we set as naturally condition in galaxy scale. Then we use two governing equation to provide this work. The first we implement by using Newtonian dynamics (ND) and another equation using Modified Newtonian dynamics (MOND). By comparing that two equations we get the information how successful from each equation can do the simulation. We consider also two numerical method which are Euler method and symplectic method. Based on all these works we can conclude that MOND $\cdots$.

Key words: *MOND, Newtonian dynamic, Euler method, symplectic method*

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Chapter 1

Introduction

Where are we actually located? Some of us might answer the location of each house. It's the same if we observe from an astrophysical point of view. We are placed on earth, in the Milky Way galaxy with special coordinates. The Milky Way is actually just one of the names of galaxies that can be observed for astronomical observations. There are so many galaxies in our universe, and we still don't really know the information. This is a great mystery in our universe as an astrophysicist.

One interesting issue about galaxies in addition to the mission to find the latest astronomical objects is information about the content and dynamics of the galaxy. The contents of galaxies are usually studied in particle physics. But about galaxy dynamics we can observe them as students of mathematics. Therefore, the main problem that will be given more explanation in the work of this thesis, we do dynamic objects called by galaxies into mathematical simulations.

Basically, we try to consider all the physical conditions that give responsibility for this case. We might combine the two methods between astrophysical theory and mathematical models as best we can to get the physical meaning and also the illustrations. For astrophysical methods we consider Modified Newtonian's Dynamic (MOND) theory as an alternative way to explain the dynamics of galaxies. In addition, MOND actually doesn't really consider galactic content, which means dark matter. So that would be a good way for us to go through this section so that it only functions in mathematical simulations.

1.1 Theory of Galaxy Dynamics

An astronomical object called a galaxy, is a large ensemble of stars and other material that orbits about a shared center and its constituents are united by mutual gravitational interactions. Galaxies come in various global forms and internal morphology. In this section we will be given an overview of galaxies as an overview to go through the main focus of making simulated galaxies as the focus of this research paper..

1.1.1 General Review of Galaxy

Galaxies are star-bound systems of gravity, remnants of stars, interstellar gas, dust, and dark matter [1]. The galaxy always moves dynamically. So, galaxy dynamics are one of the observational objects in astrophysics that we can understand past, present and future predictions of galaxies.

We know that each galaxy has differences from one another. It's easy to identify the type of galaxy we are looking at. We are familiar with galaxies: ellipses, spiral galaxies and disks [2]. One of the best known examples is our Milky Way galaxy.

In our galaxy, the Milky Way, has many kinds of matter, gas and dust in it. Observations provide information that our galaxy has about 1.5 trillion solar mas. Around 200 billion stars, including 4 million solar masses from black holes at the center of galaxies.

Other information about our Milky way is the galactic diameter around 10^5 light year. In the other scale sometimes galactic diameter introduced in 5 Kpc. In addition, the speed of the star to orbit the center of the galaxy has about ~ 230 km/sec. The rotation of each stars have a proportional to distance from center $v \propto r$.

The figure (1.1) shows that our Milky Way galaxy has a complex galaxy

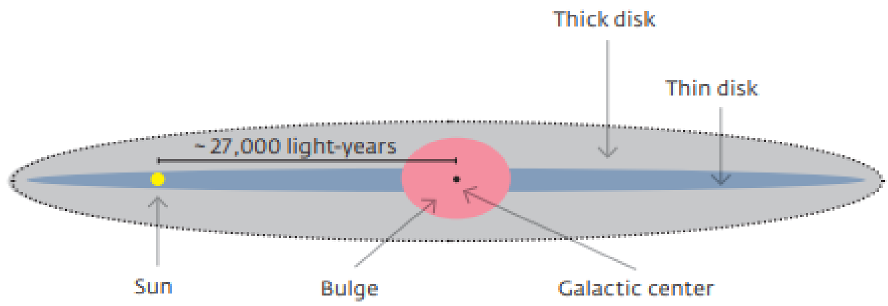

Figure 1.1: Part of Milky way galaxy [3]

structure. Each component has a different behavior. As in the center of the galaxy, it remains in the life of the galaxy, which can one day destroy all matter as an active black hole. Then galactic discs as a medium many stars that make a circular motion. Halo galaxies have the task of guarding globular galaxies. Galaxy content won't come out because of the Halo effect.

The sun as the center of the solar system, the place where we are, has a distance from the center of about ~ 8.4 Kpc. The elegance of astronomical objects will be studied by making numerical simulations. We don't consider DM, so we can ignore that effect by modifying Newton's dynamics. Next we will give a brief discussion as background of this research.

1.2 Point of View MOND

The biggest mystery in cosmology and astrophysics is missing information about dark matter and dark energy. Two of them are inevitable when we study in large-scale physics, such as galaxies, the universe and larger scales. For this reason, a scientist named Milgrom proposed his hypothesis. He said that to solve the DM problem in large-scale physics we need to modify basic Newtonian equations.

Furthermore, by considering this motivation, some researchers can obtain good data when compared with observing data. As in the figure below, MOND predictions get similar data from galaxies, NGC 2403 observations The graph

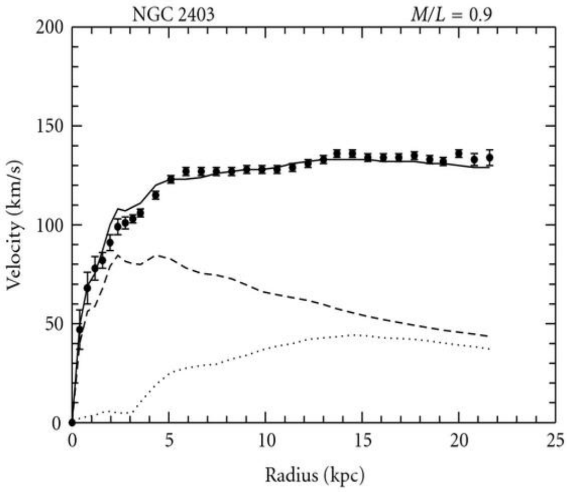

Figure 1.2: Graphic observationally data with show by point and MOND the solid line. Dash-line for Keplerian prediction, dot-line for visible matter other method.[4]

in fig(1.2) shown how successful MOND can predct velocity of the content galaxy. Meanwhile we can see that dash-line from the figure has been resulted from Newtonian dynamic as Keplerian prediction.

Basically all of the information above is a big motivation why we did this research. We want to make a simulated galaxy spin, and make observations about the results obtained. Then, as a big picture of what we actually do and get it, here is written the composition of this thesis. In Chapter 2 we give an introduction to Newton's dynamics as a theoretical basis that provides an explanation of particle motion. Then we also give a brief summary of the scaling of galaxies. This is very important because galaxy objects have large-scale entities where they can be troublesome in the program. In Chapter 3 Modified Newtonian Dynamics (MOND) is displayed to be the main topic of this thesis. Furthermore, the numerical scheme will be displayed in Chapter 4 including two methods, namely the Euler method and the symplectic method. Finally, we show numerical results in Chapter 5 and really provide a brief discussion based on the results obtained.

Chapter 2

Newtonian Dynamics

2.1 Introduction

Newtonian dynamics is a mathematical model that provide to predict the motions of various object which exist in the world around us. The implementation of this model has been familiar in the physical case, since for small scale interaction until large scale. In this chapter we briefly explanation about Newtonian dynamics as a fundamental equation that we intent to modify. We also show the galaxy scaling as a important thing in the program. Then we show a conservation energy theorem based on the Newtonian equation.

2.2 Governing Equation of Newtonian Dynamics

2.2.1 Acceleration in Newtonian Dynamics

Newtonian dynamics is a one of fundamental equation which is responsible for dynamical of object. For many cases this equation as well enough to explain it. The basic method for this equation is Ordinary Differential Equation (ODE), where we have some information of the object and also we have the force which work as responsible it. We show the formula of ND like this

$$m_i a_i(t) = F_i(X(t)), \qquad (i = 1, 2, , \ldots, N), \tag{2.1}$$

where $X(t) = \{x_i(t)\}$. The object $X(t)$ is indicated as many as index i we consider until N. Then we have m as mass of objects and a as acceleration of each object. To make easy to understand, we will say the object as "particle" continuously. Hopefully it will be more cleared.

Then, as we know that force for the particles are given by gravitational force. So we should know the formula of gravity force, and we show as follow

$$F_i(X(t)) = Gm_i \sum_{j=1, j\neq i}^{N} \frac{m_j}{r_{ij}^3(t)} x_{ij}(t). \tag{2.2}$$

We set for condition $m > 0$ and define $r_{ij}(t) := |x_j(t) - x_i(t)|$, $x_{ij}(t) := x_j(t) - x_i(t)$. If we do substitution (2.1) into the last equation (2.2), so we get the acceleration of particle

$$\begin{aligned} m_i a_i(t) &= Gm_i \sum_{j=1, j\neq i}^{N} \frac{m_j}{r_{ij}^3(t)} x_{ij}(t) \\ a_i(t) &= G \sum_{j=1, j\neq i}^{N} \frac{m_j}{r_{ij}^3(t)} x_{ij}(t) \end{aligned} \tag{2.3}$$

That is the acceleration for ND cases. Start by considering this formula, we can easily understood that the velocity of particle isn't constant. This is really important in numerical cases, where we can simulate it based on the equation.

In the galaxy simulation we necessary to consider the interactions of each particles. Because we work for many particle, so the collision between each particles as a something that we should give an intention more. In astrophysics case, especially when explain about collisions between each particles, we sensible for using a softening factor. This value preserve the gravity force from infinite condition [5]. Consider ϵ as a softening factor in $\epsilon > 0$. So, we rewrite the last equation to get this expression

$$a_i(t) \;=\; G \sum_{j=1, j\neq i}^{N} \frac{m_j}{(r_{ij}^{\epsilon})^3} x_{ij}(t) \tag{2.4}$$

where $r_{ij}^{\epsilon} := (r_{ij}^2(t) + \epsilon^2)^{1/2}$. This equation that we do simulation for ND cases in the program.

2.2.2 Velocity of 2 objects in Newtonian Dynamics

We consider for small cases to check how successful the program its work. Two objects have a motion which depend on Newton rule's

$$ma = -G\frac{Mm}{|x|^3}x, \tag{2.5}$$

the left side is about particle motion. Then the right side is the force which obtained from interaction of two particles. In the equation (2.5) M and m are indicated by mass of particles. Generally M usually use for another object that bigger than m.

Then for two particle cases we can define the position as mathematically like this

$$x(t) = (x_1, x_2, 0) \tag{2.6}$$

with x_1 and x_2 is position of each particles.

Therefore the distance r is obtained by calculating the particles position. We calculate for $r = \sqrt{x_1^2 + x_2^2}$, then we can write the acceleration of each particles like this

$$\begin{aligned} \ddot{x}(t) &= -GM\frac{x_1}{r^3} \\ \ddot{x}(t) &= -GM\frac{x_2}{r^3} \end{aligned} \tag{2.7}$$

where we choose the position of each particle is a function of radial motion.

We set the condition

$$
\begin{aligned}
x_1(t) &= r_0\cos(\omega t)\\
x_2(t) &= r_0\sin(\omega t).
\end{aligned}
\tag{2.8}
$$

Start from this condition we need to give an information that ω as angular speed and r_0 is radius of orbit. So, the last expression can be used to reconstruct eq. (2.5) and get the new expression

$$
\begin{aligned}
-\omega^2 r_0\cos(\omega t) &= -GM\frac{1}{r_0^3}r_0\cos(\omega t)\\
\omega^2 r_0^3 &= GM
\end{aligned}
\tag{2.9}
$$

also we define $v_N := |\dot{x}| = r_0\omega$ as speed of motion from Newton dynamics, and then we can make simple equation be like

$$
v_N^2 r_0 = GM \tag{2.10}
$$

2.3 Conservation Energy

Theorem 1. *$E_k(t) - E_p(t) = Const$*

Proof. $E_k(t) :=$ Total kinetic energy $= \sum_{i=1}^{N}\frac{1}{2}m|v_i(t)|^2$.

$$
\begin{aligned}
\frac{d}{dt}E_k(t) &= m\frac{d}{dt}E_k(t) = m\frac{d}{dt}\left(\sum_{i=1}^{N}\frac{1}{2}m|v_i(t)|^2\right) \qquad (2.11)\\
&= m\sum_{i=1}^{N} v_i(t)\cdot\dot{v}_i(t)\\
&= m\sum_{i=1}^{N} \dot{x}_i(t)\cdot f_i(X(t))\\
&= \sum_{i=1}^{N} \dot{x}_i(t)\cdot F_i(X(t))
\end{aligned}
$$

where $F_i(X(t))$ is potential force the system. Sometimes we called as gradient of potential energy.

$E_p(t) :=$ Potential energy $= \sum_{1\le i<j\le N} \frac{m_i m_j G}{|x_j(t)-x_i(t)|}$

$$\begin{aligned}
\nabla_{x_l}\left(\sum_{1\le i<j\le N} \frac{m_i m_j G}{|x_j(t)-x_i(t)|}\right) &= \nabla_{x_l}\left[\sum_{1\le i<j\le N} \frac{m_j m_l G}{|x_j(t)-x_l(t)|} + \right. \\
&\quad \text{terms independent of} x_l] \\
&= \sum_{j=1, j\ne l}^{N} \frac{m_j m_l G}{|x_j(t)-x_i(t)|^3}(x_j - x_l) \\
&= F_l(X)
\end{aligned} \tag{2.12}$$

So, we have a relation :

$$\frac{d}{dt}\left(E_k(t) - E_p(t)\right) = 0, \tag{2.13}$$

$E_k(t) - E_p(t) \equiv const.$ ■

2.4 Two Symmetry Particles for ND

We investigate the simulation by considering for two symmetry particles cases. It is mean that two of particle have opposite direction in order to orbit the circle. We make a notation llike this

$$\text{ND}\begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = f_i(X) \end{cases} \tag{2.14}$$

if $N = 2$, $x_1 = -x_2$. (of course we will work for N as many particles).

$$\begin{cases} \dot{x} = v(t) \\ \dot{v} = -\frac{mG}{4}\frac{x}{|x|^3} \end{cases} \tag{2.15}$$

Rotation solution

- $|x(t)| = R$, $x(t) = R\begin{pmatrix} \cos\omega t \\ \sin\omega t \end{pmatrix}$

- $v(t) = \dot{x}(t) = R\omega \begin{pmatrix} -\sin \omega t \\ \cos \omega t \end{pmatrix}$

- $\dot{v}(t) = -R\omega^2 \begin{pmatrix} \cos \omega t \\ \sin \omega t \end{pmatrix} = -\omega^2 x(t)$

where $\omega^2 = \frac{mG}{4R^3} \Longrightarrow \omega = \sqrt{\frac{mG}{4R^3}}$ as an angular velocity. $\omega = \mathcal{O}\left(\frac{1}{R^{2/3}}\right)$ as $R \longrightarrow \infty$
$|v(t)| = R\omega = \sqrt{\frac{mG}{4R}}$ as a speed. $|v(t)| = \mathcal{O}\left(\frac{1}{R^{1/2}}\right)$ where $R \longrightarrow \infty$.

2.5 Many Symmetry Particles System of Newtonian dynamics

This section we will show for many particle cases in same radius of rotation with considering N step. So, we define condition like this, $x_j \in R^2$. Then we can generally describe as follows

$$x_j(t) = \begin{pmatrix} \cos\left(\omega t + \frac{2\pi j}{N}\right) \\ \sin\left(\omega t + \frac{2\pi j}{N}\right) \end{pmatrix} \tag{2.16}$$

to illustrate the rotation we can show like this figure

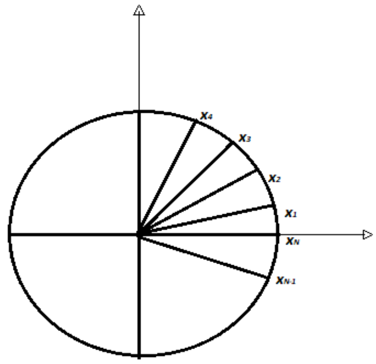

Figure 2.1: Step of rotation particles

identical with previous explanation we have acceleration of that particles as

$$\ddot{x}_j(t) \quad = \quad -\omega^2 x_j(t) \tag{2.17}$$

Then, we have general expression of acceleration as

$$f_j(t) = \frac{MG}{N} \sum_{\substack{l=1 \\ l \neq j}}^{N} \frac{x - l(t)x_j(t)}{|x_l(t) - x_j(t)|^3} \tag{2.18}$$

$$\text{for} \begin{cases} j = N; \\ t = 0 \end{cases}$$

Then we continue to calculate equation 2.18

$$
\begin{aligned}
f_N(t) &= \frac{MG}{NR^2}\sum_{l=1}^{N-1}\frac{\begin{pmatrix}\cos\frac{2\pi l}{N}-1\\ \sin\frac{2\pi l}{N}\end{pmatrix}^{3/2}}{\left\{\left(1-\cos\frac{2\pi l}{N}\right)^2+\sin^2\frac{2\pi l}{N}\right\}} \qquad (2.19)\\
&= \frac{MG}{NR^2}\sum_{l=1}^{N-1}\frac{1}{\left(2\sin\frac{\pi l}{N}\right)^3}\begin{pmatrix}-2\sin^2\frac{\pi l}{N}\\ 2\sin\frac{\pi l}{N}\cos\frac{\pi l}{N}\end{pmatrix}\\
&= \frac{MG}{4NR^2}\sum_{l=1}^{N-1}\frac{1}{\sin^2\frac{\pi l}{N}}\begin{pmatrix}-\sin\frac{\pi l}{N}\\ \cos\frac{\pi l}{N}\end{pmatrix}\\
&= \frac{-MG}{4NR^2}\begin{pmatrix}\sum_{l=1}^{N-1}\frac{1}{\sin^2\frac{\pi l}{N}}\\ (?)\end{pmatrix}
\end{aligned}
$$

The last expression on the second part of bracket actually obtained 0. Therefore we will check it to show you with define $A := \sum_{l=1}^{N-1}\frac{\cos\frac{\pi l}{N}}{\sin^2\frac{\pi l}{N}}$. If we consider the relation $l = N - j$, so $j + l = N$ with

$$N = 1, 2, \cdots, N-1$$

and

$$j = N-1, N-2, \cdots, 1.$$

So we get the relation between j and l is equal to

$$\frac{\pi l}{N} = \frac{\pi(N-j)}{N} = \left(\pi = \frac{\pi j}{N}\right),$$

for simplicity we define $\theta = \frac{\pi l}{N}$. Let's continue to calculate as

$$\begin{aligned} A &= \frac{\cos \pi - \theta}{\sin^2 \pi - \theta} \\ A &= \frac{-\cos \theta}{\sin^2 \theta} \\ A &= -A \\ 2A &= 0 \\ A &= 0. \end{aligned}$$

So, we can show the second part as obtained 0. Again we continue the main calculation 2.20,

$$f_N(t) = -\frac{-MG}{4NR^2} \begin{pmatrix} a_N \\ 0 \end{pmatrix}. \tag{2.20}$$

where $a_N := \frac{1}{N} \sum_{l=1}^{N-1} \frac{1}{\sin^2 \frac{\pi l}{N}} > 0; \lim_{n \to \infty} a_N = \infty$. Since $\ddot{x}_j(t) = f_j(t)$, where in many particles we have,

$$\begin{aligned} -\omega^2 \begin{pmatrix} R \\ 0 \end{pmatrix} &= -\frac{MG}{R^2} \begin{pmatrix} a_N \\ 0 \end{pmatrix} \qquad (2.21) \\ \omega^2 R &= \frac{a_N MG}{R^2} \\ \omega^2 R^2 &= \frac{a_N MG}{4} \end{aligned}$$

if we choose $N = 2$ for 2 particles case, we have a_N as

$$\begin{aligned} a_N &= \frac{1}{2} \frac{1}{\sin \frac{\pi}{2}} = \frac{1}{2} \qquad (2.22) \\ \omega^2 R^3 &= \frac{MG}{8} \\ &= \frac{MG}{4} \qquad (2.23) \end{aligned}$$

This result same as ω for $N = 2$ particles. So this part is general form for many particle that we consider for our simulations.

2.6 Galaxy scaling for Simulation

This section we show about galaxy scale of the simulation. It is so required before we work in the code program, we should fix the parameter that we want to simulate. So, in this table we will be shown some of parameter and unity in galaxy scale

The table 2.1 above still not uniform about their dimension. We need changes

Table 2.1: Scale of galaxy

Variable & constant	Physical Meaning	Unity	Dimension
G	Gravity Constant	13.65×10^{-11}	[Kpc^3,$M_\odot^{-1}$,galyear^{-2}]
m	Mass of particle	1	[$M_\odot$]
t	time scale	1	[galyear]
R	Radius of Milky way	5	[Kpc]

into the SI rules. It's mean that we will use re-scaling the unity into one scale which uniform. Let's we start by this calculation

$$\tilde{t} = \frac{t}{C_1} \tag{2.24}$$

where $\tilde{t}$ is indicated by time after re-scaling and we bring out $C_1 = 10^{15}$ as value of galaxy year. Afterward we do the same for R. Perhaps We can do as a while for initialize the position of particles. So we type in x for this bellow

$$\tilde{x} = \frac{x}{C_2}. \tag{2.25}$$

It's same notation for $\tilde{t}$ is indicated by variable t after re-scaling. Therefore we choose $C_2 = 10^{20}$ in this part referring about galaxy radius scale.

As a result of we succeed to re-scaling variables x and t. We can use it to get velocity v in order we need it. By using the velocity formulation we can write as follows

$$\tilde{v} = \frac{d\tilde{x}}{d\tilde{t}} = \frac{C_2}{C_1}\frac{dx}{dt} = \frac{C_2}{C_1}v \tag{2.26}$$

So, we get v as a new expression which is in normal scale (we call the reverse of galaxy scale as normal scale henceforth).

We still have a mass scale in Sun unity. But the most important thing that we considering mas for conditioning N many particles. So, we need to show the N probability to get a re-scaling mass

$$m = \frac{\text{Total Mass of Galaxy}}{N} \tag{2.27}$$

this case we will consider various N for the future. For simple case we fix $N = 100\text{-}1000$ in our simulation. So we get m for the formulation

$$m = \frac{10^{11}[M_\odot]}{N} = \frac{2 \times 10^{41}[kg]}{N} = \frac{1}{N}C_3 \tag{2.28}$$

this fact perhaps can be considered more depend on our N to set a value of m. So we re-scale m as follows

$$\tilde{m} = \frac{m}{C_3} \tag{2.29}$$

for $C_3 = 1.9 \times 10^{30}$ refers by mass of sun.

Next we will find out the scaling of a as acceleration of particles. We use the formulation a as a derivative of v. Where in equation(2.26) we have the new scaling of v. So the calculation we can check by this formula

$$\tilde{a}_i = \frac{d\tilde{v}}{d\tilde{t}_i} = \frac{d^2\tilde{x}_i}{d\tilde{t}^2} \tag{2.30}$$

Since we have a Newtonian dynamics for calculating the acceleration a, we can use it for scaling the Gravity constant G. Therefore we will give the calculation like this bellow,

$$
\begin{aligned}
\frac{d\tilde{v}}{d\tilde{t}} &= \frac{C_2^2}{C_1}\frac{dv_i}{dt} \\
&= \frac{C_2^2}{C_1} f_i(X(t)) = \frac{C_2^2}{C_1} mG \sum_{j=1, j\neq i}^{N} \frac{x_j - x_i}{|x_j - x_i|}^3 \\
&= \frac{C_2^2}{C_1} mG \sum_{j=1, j\neq i}^{N} \frac{x_j - x_i}{|x_j - x_i|}^3 \qquad (2.31)
\end{aligned}
$$

So we can use this result for scaling G into $\tilde{G}$ as follows

$$\tilde{m}\tilde{G} = \frac{C_2^2}{C_1}$$

and we can use the scaling for m that we gain before

$$\tilde{G} = \frac{C_2^2 C_3}{C1^2} G.[\mathrm{Kpc}^2 \mathrm{M}_\odot^{-1} \mathrm{gy}^{-2}]$$

Then for uniformly the scale of galaxy object, we remake the table(2.1) again include another converting

Table 2.2: Re-scaling of galaxy

Variable & constant	Physical Meaning	Unity	Dimension
$\tilde{G}$	Gravity constant	$6.67 \times 10^{-11} \left(\frac{C_2^2 C_3}{C1^2}\right)$	$[\mathrm{Kpc}^2 \mathrm{M}_\odot^{-1} \mathrm{g}^{-2}]$
$\tilde{m}$	Mass of particle	$1\ (\frac{10^{11}}{N})$	$[M_\odot]$
$\tilde{t}$	time scale	$1\ (\tilde{t}C_1)$	[gyear]
$\tilde{R}$	Radius of Milky way	$2.9(\tilde{x}C_2)$	[Kpc]

Finally we get the uniform dimension like on the table(2.2) that we can do for further step with give an initialization of this simulation.

Chapter 3

Modified Newtonian dynamic

3.1 Introduction of MOND

Physics of the universe is one of elegant science that since have a many mystery inside them. So many kind of physicist work on there, to find all of mystery in our universe. One of the big mystery is about strange particle in our life. The strange matter we usually called as dark matter (DM). The word 'dark' is indicating what actual information about it still mystery.

Naturally, gravitational attraction of each cosmic objects is evidently higher than we expected in General Relativity or Newtonian dynamics from the distribution of the observable matter[6].

As far we know, the amount of DM in the Universe is much larger than that of the observable matter. There is evidence that DM cannot not arbitrary particle which formed by made of any known kind of particles.

These particles are not expected by the Standard model of particle physics. Dark matter has a great explanation of cosmic dynamic which the result about information cosmic large scale still covered as observational data, like (Planck Collaborations, Einstein et.al). Here we necessary to inform that the explanation about particle physics will not be given deeper, just for preliminary of our study case. So, the reader can be found this explanation in physical reference.

Also we still have the other way to solve the DM problem, that is to assume that we are able to detect most of the matter in the Universe. Then we need to modify some of the standard laws of physics. These laws were derived from laboratory experiments and the Solar system observations. But on the galactic scales, many quantities (acceleration, angular momentum, mean density,etc) take values different from those in these experiments by many orders of magnitude.

Caused these conditions we choose the second way to modified from established equation. Newtonian second law or usually called Newtonian dynamic we modified s.t. which we called Modified Newtonian dynamics. From this modified we want to show some of differentiate of this approximation beside we also need it to solving the rotational simulation of the galaxy.

3.2 Governing Equation of MOND

In physics, MOND (Modified Newtonian Dynamics) has actually been another way to describe dynamical of all about interaction can't explain by Newton's law, as well. Milgrom, the name who has propose equation, give adding the small correction in the order of accelerating of the object. He propose the equation caused to answer ND which can't give predict well from galaxy observation.

Basically, from the big reason why MOND should be proposed in physics large scale, we will try to show the formulating of MOND equation. It is mean that we should give the basic modified of ND. Therefore we show like bellow

$$F = m\mu\left(\frac{|a|}{\alpha_0}\right)a \tag{3.1}$$

we introduced $\alpha_0 > 0$ as critical acceleration ($\alpha_0 \sim 10^{-8}cm/s^2 \approx 1.2 \times 10^{-10}km/s^2$). Then $\mu(\cdot)$ as function of interpolating this equation. We defined $s := \frac{|a|}{\alpha_0}$ for simplify our formula. Furthermore, we choose $\mu(s)$ for all possible ($0 \leq s \leq \infty$), it is indicated a non-decreasing function with

$$\mu(0) = 0$$

and also

$$\lim_{s \to \infty} \mu(s) = 1$$

. Specifically for our case we best fixed the value of parameter $\mu(s)$ as follow this equation

$$\mu(s) = \frac{s}{\sqrt{1+s^2}} \tag{3.2}$$

as we know that s we defined as $s := \frac{|a|}{\alpha_0}$.

Furthermore, start from this information and equation (3.1) that we give proposition like followed this

Proposition 3.2.1.

We suppose that $\mu(s)$ is given by (3.2), then (3.1) is equivalent to

$$F\varphi\left(q\right) = ma \tag{3.3}$$

where $q := \frac{|F|}{2m\alpha_0}$ and $\varphi(q) := \sqrt{\frac{1+\sqrt{1+\frac{1}{q^2}}}{2}}$

Proof. for simplicity we define $|F| = G\frac{Mm}{r^3}x_i$, so the last expression can be rewrite as followed

$$\begin{aligned} |F| &= m\mu\left(s\right)\alpha_0 s \\ |F| &= m\alpha_0\frac{s^2}{\sqrt{1+s^2}} \\ |F|^2(1+s^2) &= m^2\alpha_0^2 s^4 \\ m^2\alpha_0^2 s^4 - |F|^2 s^2 - |F|^2 &= 0 \end{aligned}$$

so we can get value of s as root of this expression

$$s^2\alpha_0^2 = \frac{|F|^2 \pm \sqrt{|F|^4 + 4\alpha_0^2 m^2 |F|^2}}{2m^2} \tag{3.4}$$

by definition $|a| = \alpha_0 s$, so we can make the expression

$$\begin{aligned} |a| &= \sqrt{\frac{|F|^2 + |F|\sqrt{|F|^2 + 4m^2\alpha_0^2}}{2m^2}} \\ |a| &= \frac{|F|}{m}\sqrt{\frac{1 + \sqrt{1 + \frac{4m^2\alpha_0^2}{IF|^2}}}{2}} \end{aligned}$$

and we set $q := \frac{|F|}{2m\alpha_0}$, so we get a new expression from the last equation

$$\begin{aligned} |a| &= \frac{|F|}{m}\sqrt{\frac{1 + \sqrt{1 + \frac{1}{q^2}}}{2}} \\ m|a| &= |F|\sqrt{\frac{1 + \sqrt{1 + \frac{1}{q^2}}}{2}} \end{aligned}$$

for each side we times with $\frac{a}{|a|}$, so

$$m|a|\frac{a}{|a|} = \frac{a}{|a|}|F|\sqrt{\frac{1 + \sqrt{1 + \frac{1}{q^2}}}{2}}$$

and we also have the relation of any vector that $\frac{a}{|a|} \equiv \frac{F}{|F|}$, and the expression can be rewrite as

$$\begin{aligned} m|a|\frac{a}{|a|} &= \frac{F}{|F|}|F|\sqrt{\frac{1 + \sqrt{1 + \frac{1}{q^2}}}{2}} \\ ma &= F\sqrt{\frac{1 + \sqrt{1 + \frac{1}{q^2}}}{2}} \qquad (3.5) \end{aligned}$$

and for simplicity we define $\varphi(q) \equiv \sqrt{\frac{1+\sqrt{1+\frac{1}{q^2}}}{2}}$. And then it's make a result of acceleration that we obtain as followed by this equation

$$\begin{aligned} ma &= F\varphi(q) \qquad (3.6) \\ a &= \tfrac{F}{m}\varphi(q) \end{aligned}$$

as usual for our simulation, we define $f_i := a_i$. Something that have a little bit different than natural equation of Newton dynamics. It is possibility the value that we make a comparrison as computational between te natural Newton equation. ■

3.2.1 Two Symmetry Particles for MOND

We have mathematically ND in two symmetry particles cases, as follows **MOND**
Then we give for MOND case, as follows

$$\text{MOND} \begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = \alpha_0 \varphi^{-1} \left(\frac{|f_i(X(t))|}{m\alpha_0} \right) \frac{f_i}{|f_i|} \end{cases} \tag{3.7}$$

Remember that $f_i = \frac{F_i}{m}$. Then, we suppose $N = 2$ and $x_i(t) = -x_2(t)$. (We denote $x(t) := x_i(t), v(t) := v_i(t), r(t) = r_{1,2}(t) = 2|x(t)|$).
for this case we have force in this form

$$f_1(x) = MG\frac{x_2 - x_1}{r_{12}^3} = MG\frac{(-2x(t))}{(2|x(t)|)^3} = -\frac{MG}{4}\frac{x}{|x|^3},$$

and $|f_1(x)| = \frac{MG}{4}\frac{1}{|x|^2}$ So we have this ralation

$$f_2(x) = -f_1(x).$$

Hence we can rewrite eq(3.7) as

$$\text{MOND} \begin{cases} \dot{x} = v \\ \dot{v} = \alpha_0 \varphi^{-1} \left(\frac{MG}{4\alpha_0|x|^2} \right) \left(-\frac{x}{|x|} \right) \end{cases} \tag{3.8}$$

Rotation solution

We use same condition like in ND method as

- $|x(t)| = R$, $x(t) = R \begin{pmatrix} \cos\omega t \\ \sin\omega t \end{pmatrix}$

- $v(t) = \dot{x}(t) = R\omega \begin{pmatrix} -\sin\omega t \\ \cos\omega t \end{pmatrix}$

- $\dot{v}(t) = -R\omega^2 \begin{pmatrix} \cos\omega t \\ \sin\omega t \end{pmatrix} = -\omega^2 x(t)$

where for MOND, we have the dynamical equation like 3.1. We can rewrite with substitution the acceleration

$$\begin{aligned} \frac{F}{m} &= \mu\left(\frac{|a|}{a}\right) a && (3.9) \\ GM\frac{1}{r_{ij}^3} R \begin{pmatrix} \cos\omega t \\ \sin\omega t \end{pmatrix} &= -\mu(s)\omega^2 x(t) && (3.10) \\ GM &= \mu(s)\omega^2 r_{ij}^3 && (3.11) \end{aligned}$$

for r_{ij} actually is radius. So another expression we can use this notation,

$$\mu(s)\omega^2 R^3 = GM$$

remember that $s := \frac{|a|}{\alpha_0}$ and $\mu(s) = \frac{s}{\sqrt{1+s^2}}$. So we get this notation

$$\begin{aligned} \frac{\frac{\omega^2 R}{\alpha_0}}{\sqrt{1+\frac{\omega^2 R}{\alpha_0}}}\omega^2 R^3 &= GM && (3.12) \\ \frac{\omega^2 R}{\sqrt{\alpha_0 + \omega^4 R^2}}\omega^2 R^3 &= GM && (3.13) \\ \omega^4 R^4 &= GM\sqrt{\alpha_0^2 + \omega^4 R^2} \\ \omega^8 R^4 &= G^2 M^2 \left(\alpha_0^2 + \omega^4 R^2\right) \end{aligned}$$

we give $X = \omega^4$ for make a simple calculation to find roots

$$\begin{aligned} X &= \frac{G^2M^2R^2 + \sqrt{G^4R^4M^4 + 4R^8\alpha_0^2G^2M^2}}{2R^{16}} \qquad (3.14) \\ &= \frac{G^2M^2 + \sqrt{G^4R^4M^4 + 4R^8\alpha_0^2G^2M^2}}{2R^{14}} \approx \frac{G^2M^2}{R^{14}} \quad \text{as} R \to \infty. \end{aligned}$$

Thenn we should be back into

$$\omega^2 \approx GMR^{-14} \equiv \omega \approx \sqrt{GM}R^{-\frac{7}{2}}$$

. Then for speed we have

$$|v| = \omega R \approx \sqrt{GM}R^{-\frac{5}{2}}$$

also as $R \to \infty$.

It is for $|v(t)| \implies \mathcal{O}(1)$ as $R \longrightarrow \infty$. Then this is the plot graph based on the ω, $|v(t)|$ and R value for ND and MOND we get the graph for comparing

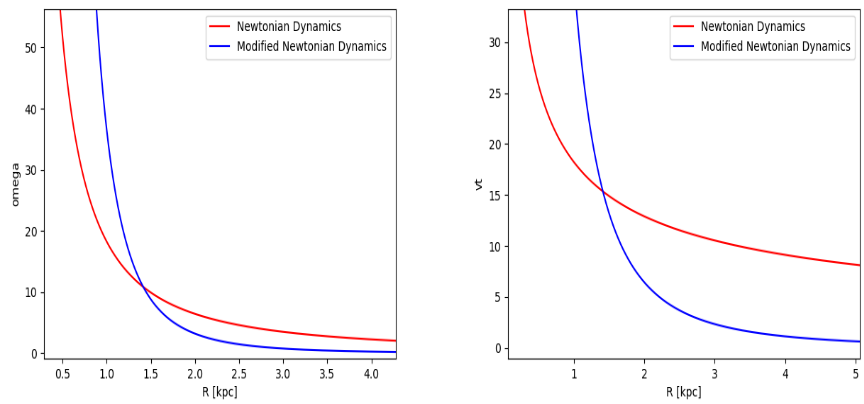

Figure 3.1: Plot graphic ω and v vs R

some dynamical parameters such as ω,$|v|$ depend on radius R. Based on the graph, for MOND dynamics we get more constantly than ND. If we analyze with 1.2, for a while we can give preconclude that accurate enough with data observation.

Chapter 4

Numerical Scheme

4.1 Briefly Numerical Analysis and It's Mathematical Notations

Newtonian dynamic and its modified, basically is problem in ordinary differential case. Sometimes we can calculate by manually. But in order to get much calculation, we need work hard very complicated to solve it. So we will solve this problem by numerical calculation.

So many kind of numerical method to solve ordinary differential equation (ODE), but for easiest way we can use by Euler method. This method maybe is ot really an efficient method, but many of the ideas involved in the numerical solution of differential equations are introduced most simply with it.

Before we discuss more about the numerical solution in this work. It's important to know the mathematical notation that we are going to use it in simulation. So, this part we will give an mathematical overview for this system. Let's start by giving notation for the particles. We write the particle as

$$X = \{x_i\}_{i=1}^{\mathrm{N}} \quad \subset \mathbb{R}^{\mathrm{d}\times\mathrm{N}} \tag{4.1}$$

where X is particle for i as index of each particles as much as N. Then we set for dynamically system, $f_i(X) := \frac{F_i(X)}{m_i}$, and for indicating the step we show

$$\text{ND} \begin{cases} \ddot{x}(t) = f_i(X(t)) & (0 \leq t \leq T), \\ \dot{x}(t) = v_i(t) & (0 \leq t \leq T), \\ x_i(0) = x_i^0 \ \text{; given} & (i = 1, \dots \text{N}) \\ v_i(0) = v_i^0 \ \text{; given} & (i = 1, \dots \text{N}) \end{cases} \tag{4.2}$$

By these notation, we will use it into the program.

4.2 Discretization and The Numerical Scheme

After we fix it the mathematical notation, we also need to attend the time stepping. It is mean that our simulation is depending the function of time, so should fix it. We choose $\Delta t > 0, K := \left[\frac{T}{\Delta t}\right]$, and we consider for our case

$$x_i^k = \begin{pmatrix} x_{\text{i1}}^{\text{k}} \\ x_{\text{i2}}^{\text{k}} \end{pmatrix} \sim x_i(k\Delta t) \tag{4.3}$$

$$v_i = \begin{pmatrix} v_{\text{i1}}^{\text{k}} \\ v_{\text{i2}}^{\text{k}} \end{pmatrix} \sim v_i(k\Delta t) \tag{4.4}$$

where we give notation $(k = 0, 1, \dots, \text{K}, i = 1, \dots, \text{N})$.
Time stepping k start from initially, which mean 0, and will be end for K time. It will be implemented into i-th particle. In our code programe we have this conditions

$$\text{X}^{\text{old}} = (x_i^{\text{old}})_{\text{i=1}}^{\text{N}} \,; \text{V}^{\text{old}} = (v_i^{\text{old}})_{\text{i=1}}^{\text{N}} \tag{4.5}$$

$$\text{X}^{\text{new}} = (x_i^{\text{new}})_{\text{i=1}}^{\text{N}} \,; \text{V}^{\text{new}} = (v_i^{\text{new}})_{\text{i=1}}^{\text{N}} \,. \tag{4.6}$$

So this mean that we don't store the previous data. We overwrite the data by new update data of position x and velocity v.

4.2.1 Euler Scheme

As we said before, the numerical method in order to calculate the differential problem is used Euler method. Therefore we should know how actually this method can work. So we will show the numerical scheme for Euler method in this notation

$$\begin{cases} \frac{x_i^{k+1}-x_i^k}{\Delta t} = v_i^k \\ \frac{v_i^{k+1}-v_i^k}{\Delta t} = f_i(x^k) \\ x_i^0 \quad ; v_i^0 \quad ; \text{given.} \end{cases} \tag{4.7}$$

This scheme will be implemented into the program and we discussing more about the result will be comparing by observation data.

4.2.2 Symplectic scheme

Another scheme that we used in this simulation is symplectic scheme. Generally this method it's better than Euler method. The main reason is about the symplectic always give new velocity update for each step position. So, the result might be closely than the exact.

Then, this is the symplectic scheme which used in the program

$$\begin{cases} \frac{v_i^{k+1}-v_i^k}{\Delta t} = f_i(x^k) \\ \frac{x_i^{k+1}-x_i^k}{\Delta t} = v_i^{(k+1)} \\ x_i^0 \quad ; v_i^0 \quad ; \text{given.} \end{cases} \tag{4.8}$$

Based two numerical method we will show how more accurate between Euler and symplectic method.

4.2.3 Initialization of particles

After we give an uniform scale for our case, the next step is make a condition that we consider as initial position. By this table will be shown some of the initialization By using the table initialization above we can do the simulation

Initialization	Physical meaning	Unity	Value
m	Mass of particle	1	$M_{\odot}$
R	Radius	2.9	Kpc
ω	angular velocity	Random value$(0, \pi)$	1 galyear
$x_i^j\ (i, j)^{\dagger}$	position of particles	Random uniform	Kpc
$v_i^j\ (i, j)$	velocity	(-$v\omega \sin(\omega t), v\omega \cos(\omega t))$	m/s^2
t_0	time initialization	0	$gyear$

for this system.

Chapter 5

Numerical Result & Discussion

In this work we consider for $N = 10$ particles. The first simulation we give $\Delta t = 0.001$ for K as a total of iteration is $K = 5000$. In this case we construct by $K = \frac{t}{\Delta t}$ where $T = 5$. For this ω we set as a radius velocity of the maximum position in random number between $(0, \pi)$.

By using the initialization that we give previous section in the table.(2.2), we do the simulations by two methods. We consider to use Euler and symplectic method as a numerical solver this case ,

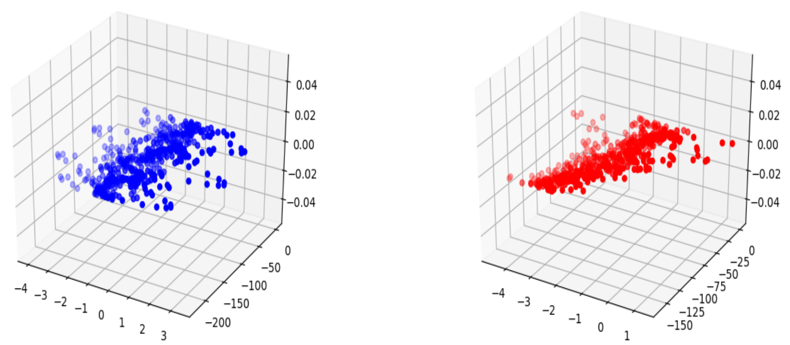

Figure 5.1: Graph from Euler (blue) & symplectic method (red)

Figure (5.1) obtained as Euler method for blue scatter and symplectic for red scatter. By those figures we can say that our program has success work. But

It's should be analyze more about the gravity and dynamically of particle. We can also check by comparing with observation that will be more precisely to conclude this simulation.

Basically, currently we still only show for Newtonian dynamics simulation. We should necessary fix for MOND simulation, beside we show the comparing with observation data result. Then, afterward we can really conclude which one the best method to do simulation if we compare with all of considerations.

Appendix A

Program codes

1. Euler and symplectic

```
import matplotlib.pyplot as plt
from numpy import sin,cos,pi,sqrt,exp,floor,zeros,copy
from numpy.random import normal
from numpy.linalg import norm
from random import uniform
from time import time
from mpl_toolkits.mplot3d import Axes3D

start = time()
def euler_mond(x,v):
    x_tmp = copy(x)
    for i in range(n_particles):
        x[i] += v[i]*dt
        for j in range(n_particles):
            if(i!=j):
                v[i] += (accel_mond(x_tmp[i],x_tmp[j]))*dt

def euler_newton(x,v):
    x_tmp = copy(x)
    for i in range(n_particles):
        x[i] += v[i]*dt
        for j in range(n_particles):
            if(i!=j):
                v[i] += G*f(x_tmp[i],x_tmp[j])*dt

def symplectic_newton(x,v):
    x_tmp = copy(x)
    for i in range(n_particles):
        x[i] += v[i]*dt
        for j in range(n_particles):
```

```
            if(i!=j):
                v[i] += f(x_tmp[i],x_tmp[j])*dt
    for i in range(n_particles):
        x[i] += v[i]*dt

def symplectic_mond(x,v):
    x_tmp = copy(x)
    for i in range(n_particles):
        x[i] += v[i]*dt
        for j in range(n_particles):
            if(i!=j):
                v[i] += (accel_mond(x_tmp[i],x_tmp[j]))*dt
    for i in range(n_particles):
        x[i] += v[i]*dt

def get_init_coordinates_edit():
    x = zeros((n_particles,d))
    i = 0
    while(i<n_particles):
        x1 = normal(-R,R)
        x2 = normal(-R,R)
        if(abs(x1**2+x2**2)<R**2):
            x[i] = ([x1,x2])
            i+=1
        else:
            i=i
    return x

def get_init_velocities():
    v = zeros((n_particles,d))
    for i in range(n_particles):
        v[i] = ([-v_max*omega*sin(omega*t),v_max*omega*cos(omega*t)])
    return v

def f(xi,xj):
    rij = xj-xi
    return (G*m*(rij))/(norm(rij)+epsilon)**3

def accel_mond(xi,xj):
    rij = xj-xi
    varphi = sqrt((m*G/(4*a_0))*(1/norm(rij)**2))
    return a_0*varphi*(f(xi,xj)/norm(f(xi,xj)))

def mu(s):
    return s/sqrt(1+s**2)
```

```
#Global parameter
v_max = 4.56
n_particles = 10 #particles
d = 2 #dimension
m = 10e11/n_particles #[MO]
M_total = m*n_particles
R = 2.9 #[kpc]
G = 13.34*10e-11 #[kpc^3 MO^-1 gy^-2]
omega = sqrt((G*m)/(4*R**3)) #velocities
epsilon = 1.7
T = 5
dt = 0.001
t = 0. #step
N = int(floor(T/dt))
scale = 7.0
a_0 = 1e-10
#initial condition
#x,v = init_two()
x = get_init_coordinates_edit()
v = get_init_velocities()

fig = plt.figure()
ax = fig.gca(projection='3d')
#main loop
for k in range(N):
    t = dt*k
    euler_mond(x,v)
    #print(kinetic_energy())
    #plt.plot(xe[:,0],xe[:,1], 'b.')
    #plt.xlim(right=scale,left=-scale)
    #plt.ylim(top=scale,bottom=-scale)
    #plt.axes(aspect='equal')
    if(k%100==0):
        #plt.plot(x[:,0],x[:,1], 'b.')
        ax.scatter(x[:,0], x[:,1], zs=0, zdir='z',c='c')
#filename='./figures/plot.png'
#plt.savefig(filename)
print("Time for running ", N, "iteration :", time()-start, "seconds")
plt.show()
```

2. Plot Two Particles

```
import matplotlib.pyplot as plt
import numpy as np
from time import time

start = time()
```

```
#Global parameter
a_0 = 10e-1
m = 10e11 #[MO]
R = np.arange(0.5,2.9,0.001) #[kpc]
G = 13.34*10e-11 #[kpc^3 MO^-1 gy^-2]
#Newton Dynamic
omega = np.sqrt((G*m)/(4*R**3)) #velocities
vt = omega*R
plt.xlabel("R [kpc]")
plt.ylabel("|v|")
plt.plot(R,vt, 'r-', label='ND')
#MoND
omega = np.sqrt((a_0*G*m)/(4))/R #velocities
vt = omega*R
plt.plot(R,vt, 'b-',label='MOND')
plt.legend()
#plt.title("Total Energy of Symplectic Newton Dynamics")
#filename='./figures/plot.png'
#plt.savefig(filename)
print("Time for running ", time()-start, "seconds")
plt.show()
```

Bibliography

[1] Hupp, E.; Roy, S.; Watzke, M. (August 12, 2006). "NASA Finds Direct Proof of Dark Matter". NASA. Retrieved April 17, 2007.

[2] Bilek, Michal . 2016. Galaxy interactions: dark matter vs. Modified Newtonian dynamics (MOND).,arXiv:1601.01240v1 [astro-ph.GA] 6 Jan 2016.

[3] BÜHRKE, Thomas. *Archaeology of the Milky Way.* PHYSICS & ASTRONOMY Galaxy. Article in The Astronomical magazine. Max Planck Research· January 2016.

[4] Sanders, R.H,. Modified Newtonian Dynamics: A Falsification of Cold Dark Matter., Hindawi Publishing Corporation Advances in Astronomy Volume 2009, Article ID 752439, 9 pages doi:10.1155/2009/752439.

[5] Dyer, Charles and Peter 1993 Softening in N-body simulation of collisionless systems ApJ. 409 67–60.

[6] Jungwiert, Bruno. 2015, Galaxy interactions: dark matter vs. Modified Newtonian dynamics (MOND), arXiv:1601.01240v1 [astro-ph.GA] 6 Jan 2016.