Problem set 15,16

Graded

Student

Total Points

93 / 100 pts

Question 1

Exercise 7.1.2a

4 / 4 pts

✔ **– 0 pts** Correct

– 4 pts no answer

– 4 pts illegible

– 4 pts wrong problem

– 2 pts incorrect domain

– 2 pts incorrect co-domain

– 1 pt supplied the function g equals the domain / co-domain

– 1 pt did not specify elements defining the domain / co-domain

Question 2

Exercise 7.1.2b

6 / 6 pts

✔ **– 0 pts** Correct

– 6 pts no answer

– 6 pts illegible

– 6 pts wrong problem

– 2 pts incorrect g(1)

– 2 pts incorrect g(3)

– 2 pts incorrect g(5)

Question 3

Exercise 7.1.2c

2 / 2 pts

✔ **− 0 pts** Correct

− 2 pts no answer

− 2 pts illegible

− 2 pts wrong problem

− 2 pts incorrect

− 1 pt supplied function equates a value

Question 4

Exercise 7.1.2d

2 / 2 pts

✔ **− 0 pts** Correct

− 2 pts no answer

− 2 pts illegible

− 2 pts wrong problem

− 1 pt incorrect a

− 1 pt incorrect b

− 0.5 pts implied unique inverse

Question 5

Exercise 7.1.2e

2 / 2 pts

✔ **− 0 pts** Correct

− 2 pts no answer

− 2 pts illegible

− 2 pts wrong problem

− 1 pt incorrect a

− 1 pt incorrect b

− 0.5 pts incorrectly equated pre-images to images

Question 6

Exercise 7.1.2f

2 / 2 pts

✔ **– 0 pts** Correct

– 2 pts no answer

– 2 pts illegible

– 2 pts wrong problem

– 1 pt incorrect delimiters () {}

– 1 pt incorrect punctuation (commas)

– 1 pt incorrect values

Question 7

Exercise 7.1.9d

2 / 2 pts

✔ **– 0 pts** Correct

– 2 pts no answer

– 2 pts illegible

– 2 pts wrong problem

Question 8

Exercise 7.1.9e

2 / 2 pts

✔ **– 0 pts** Correct

– 2 pts no answer

– 2 pts illegible

– 2 pts wrong problem

Question 9

Exercise 7.1.9f

2 / 2 pts

✔ **– 0 pts** Correct

– 2 pts no answer

– 2 pts illegible

– 2 pts wrong problem

Question 10

Exercise 7.1.16

5 / 5 pts

✔ **– 0 pts** Correct

– 5 pts no answer

– 5 pts illegible

– 5 pts wrong problem

– 2 pts failue to use set difference definition

– 2 pts weak / vague reasoning

Question 11

Exercise 7.1.29b

3 / 3 pts

✔ **– 0 pts** Correct

– 3 pts no answer

– 3 pts illegible

– 3 pts wrong problem

– 3 pts incorrect

Question 12

Exercise 7.1.39b

5 / 5 pts

✔ **– 0 pts** Correct

– 5 pts no answer

– 5 pts wrong problem

– 1 pt incorrect $g(A)$

– 1 pt incorrect $g(X)$

– 1 pt incorrect $g^{-1}C)$

– 1 pt incorrect $g^{-1}(D)$

– 1 pt incorrect $g^{-1}(Y)$

Question 13

Exercise 7.2.6b

8 / 8 pts

✔ **− 0 pts** Correct

− 8 pts no answer

− 8 pts illegible

− 8 pts wrong problem

− 4 pts incorrect one-to-one

− 2 pts failure to use one-to-one definition

− 2 pts weak / vague reasoning one-to-one

− 4 pts incorrect onto

− 2 pts failure to use onto definition

− 2 pts weak / vague reasoning onto

Question 14

Exercise 7.2.8a

8 / 8 pts

✔ **− 0 pts** Correct

− 8 pts no answer

− 8 pts illegible

− 8 pts wrong problem

− 4 pts incorrect one-to-one

− 2 pts failure to use one-to-one definition

− 2 pts weak / vague reasoning one-to-one

− 4 pts incorrect onto

− 2 pts failure to use onto definition

− 2 pts weak / vague reasoning onto

Question 15

Exercise 7.2.18

5 / 5 pts

✔ **– 0 pts** Correct

– 5 pts no answer

– 5 pts illegible

– 5 pts wrong problem

– 1 pt failure to comment on x≠1

– 1 pt minor algebraic error

– 1 pt incomplete

– 2 pts weak / vague reasoning

Question 16

Exercise 7.2.22b

5 / 5 pts

✔ **– 0 pts** Correct

– 5 pts no answer

– 5 pts illegible

– 5 pts wrong problem

– 5 pts incorrect yes/no

– 2 pts did not show how to reach a positive number

– 2 pts did not show how to reach a negative number

– 1 pt did not show how to reach 0

Question 17

Exercise 7.2.24a

4 / 4 pts

✔ **– 0 pts** Correct

– 4 pts no answer

– 4 pts illegible

– 4 pts wrong problem

– 4 pts incorrect

– 3 pts incorrect counterexample

– 2 pts vague/weak explanation

Question 18

Exercise 7.3.5

2 / 2 pts

✔ **– 0 pts** Correct

– 2 pts no answer / incorrect

– 2 pts illegible

– 2 pts wrong problem

Question 19

Exercise 7.3.7

3 / 3 pts

✔ **– 0 pts** Correct

– 3 pts no answer

– 3 pts illegible

– 3 pts wrong problem

– 1 pt incorrect on (1)

– 1 pt incorrect on (2)

– 1 pt incorrect on (3)

Question 20

Exercise 7.3.11

6 / 6 pts

✔ **– 0 pts** Correct

– 3 pts failure to prove 1 identity

– 3 pts did not reduce both compositions completely to x

– 1.5 pts did not reduce 1 composition completely to x

– 6 pts no answer

– 6 pts illegible

– 6 pts wrong problem

– 3 pts Need to show these compositions give the identity for every x except 1.

Question 21

Exercise 7.3.17

6 / 6 pts

✔ **– 0 pts** Correct

– 2 pts weak / vague explanation

– 4 pts incorrect counterexample

– 6 pts incorrect yes/no

– 6 pts no answer

– 6 pts illegible

– 6 pts wrong problem

Question 22

Exercise 7.3.24

6 / 6 pts

✔ **– 0 pts** Correct

– 6 pts no answer

– 6 pts illegible

– 6 pts wrong problem

– 1 pt incorrect $g \circ f$

– 1 pt incorrect $(g \circ f)^{-1}$

– 1 pt incorrect g^{-1}

– 1 pt incorrect f^{-1}

– 1 pt incorrect $f^{-1} \circ g^{-1}$

– 1 pt incorrect relationship

Question 23

Exercise 7.4.9

0 / 4 pts

– 0 pts Correct

– 4 pts no answer

– 4 pts illegible

✔ **– 4 pts** wrong problem

– 2 pts mapping is not shown to be onto

– 2 pts mapping is not shown to be one-to-one

– 1 pt arithmetic error / weak reasoning

Question 24

Exercise 7.4.12

3 / 3 pts

✔ **– 0 pts** Correct

– 3 pts no answer

– 3 pts illegible

– 3 pts wrong problem

– 3 pts did not use a mapping

– 1.5 pts mapping is not shown to be onto

– 1.5 pts mapping is not shown to be one-to-one

– 2 pts arithmetic error / weak reasoning

Question 25

Exercise 7.4.20

0 / 3 pts

– 0 pts Correct

– 3 pts no answer

– 3 pts illegible

– 3 pts wrong problem

✔ **– 3 pts** both incorrect examples

– 1.5 pts only 1 correct example

Remember, for a function f from the integers to the integers, we can only produce an integer when given an integer input.

When f(x) = sqrt(x), f(2) = sqrt(2), which is not an integer.

Similarly, when f(x) = e^x, f(1) = e, which is also not an integer.

Remember that by definition of function, we need to be able to map any element of our domain to some element of our co-domain. However, in both of your examples, we have some integer input which outputs an non-integer. Hence, we do not have a function from the integers to the integers.

I also worked with the following students (provide EMLPIDs only)

EMPLID	EMPLID	EMPLID
EMPLID	EMPLID	EMPLID
EMPLID	EMPLID	EMPLID
EMPLID	EMPLID	EMPLID

My answers came in part or in full from the following sources

Put your answer in each indicated box. Answers must be handwritten, legible and use correct notation.
Study the answers in Appendix A to similar problems so you know what your approach should be.
Larger boxes indicate that you are expected to provide substantial detail.

1. Exercise 7.1.2a domain co-domain

$\{1, 3, 5\}$

$\{a, b, c, d\}$

2. Exercise 7.1.2b $g(1)$ $g(3)$ $g(5)$

b

b

b

3. Exercise 7.1.2c

$\{b\}$

4. Exercise 7.1.2d for part 1 for part 2

no

yes

5. Exercise 7.1.2e for part 1 for part 2

inverse of b $= \{1, 3, 5\}$

inverse of c $= \emptyset$

6. Exercise 7.1.2f

$\{(1,b), (3,b), (5,b)\}$

7. Exercise 7.1.9d

6

$s(5) = 1 + 5 = 6$

8. Exercise 7.1.9e

39

$s(18) = 1 + 2 + 3 + 6 + 9 + 18 = 39$

9. Exercise 7.1.9f

32

$s(21) = 1 + 3 + 7 + 21 = 32$

10. Exercise 7.1.16

$(F-G)(x) = F(x) - G(x)$ By substitution

$= -(G(x) - F(x))$ By factoring

Assume $(F-G) = (G-F)$. Then $(F-G)(x) = G(x) - F(x)$.
However, $(F-G)(x) = -(G(x) - F(x))$. Since our assumption led to a contradiction, $(F-G) \neq (G-F)$

11. Exercise 7.1.29b

$H(00110, 10111)$

$$\begin{array}{r} 00110 \\ 10111 \\ \hline \end{array}$$

$1+1=2$

$\boxed{2}$

12. Exercise 7.1.39b

$g(A)$

$\{a\}$

$g(X)$

$\{a, d\}$

$g^{-1}(C)$

$\{1, 2, 3\}$

$g^{-1}(D)$

$\emptyset$

$g^{-1}(Y)$

$\{1, 2, 3, 4\} = X$

13. Exercise 7.2.6b

one-to-one

g is one-to-one because $\forall x \in X$, there exists exactly one $y \in Y$ such that $g(x) = y$

onto

g is onto because $\forall y \in Y, \exists x \in X$ such that $g(x) = y$.

14. Exercise 7.2.8a one-to-one

H is not one-to-one because $H(b)=y$ and $H(c)=y$ and this violates the definition of a one-to-one function

onto

H is not onto because x and z are both elements of Y that have no inverse image in the domain of H

15. Exercise 7.2.18

Theorem: $f(x) = \frac{x+1}{x-1}$ for all $x \neq 1$ is a one to one function

Suppose $f(x_1) = f(x_2)$

We must show $x_1 = x_2$

By substitution, $f(x_1) = \frac{x_1+1}{x_1-1}$

$f(x_2) = \frac{x_2+1}{x_2-1}$

Since $f(x_1) = f(x_2)$, $\frac{x_1+1}{x_1-1} = \frac{x_2+1}{x_2-1}$

By cross multiplying, we get $(x_1+1)(x_2-1) = (x_2+1)(x_1-1)$

By factoring, we get $(x_1)(x_2) + (x_2) - (x_1) \cancel{-1} = (x_1)(x_2) - (x_2) + (x_1) \cancel{-1}$

$(x_1)\cancel{(x_2)} + (x_2) - (x_1) = (x_1)\cancel{(x_2)} - (x_2) + (x_1)$

$(x_2) - (x_1) = (x_1) - (x_2)$

$2x_2 = 2x_1$

$x_2 = x_1$

Since $x_1 = x_2$, $f(x_1) = f(x_2)$

and $f(x) = \frac{x+1}{x-1}$ for all $x \neq 1$ is a one to one function

QED

16. Exercise 7.2.22b

Theorem: Let S be the set of all strings of 0's and 1's and define $D: S \to \mathbb{Z}$ for all $s \in S$

$D(s)$ = the number of 1's minus the number of 0's

We must show D is onto.

Proof:

Let a be any particular integer

We must show $D(n) = a$

Consider the following string of 0's and 1's

$$\text{"}\underbrace{1\,1\,1\,1\ldots}_{\substack{n \\ \downarrow \\ (a+1)\text{ ones}}}\underbrace{0}_{\text{one zero}}\text{"}$$

Since we have $(a+1)$ 1's and one 0,

$D(s) = a+1 - 1 = a$

Since $D(s) = a$, $\exists s \in S \ni D(s) = a$

Since we used a generic particular, we know that $\forall x \in \mathbb{Z}, \exists s \in S$ such that $D(s) = x$

By definition of onto, $D(s)$ is onto

QED

17. Exercise 7.2.24a

N is not one-to-one

Counterexample!

s_1 = "aabb"

s_2 = "aab"

$N(s_1) = 2$

$N(s_2) = 2$

Since $s_1 \neq s_2$ but $N(s_1) = N(s_2)$,

$N(s)$ is not one-to-one

QED

18. Exercise 7.3.5

$(f \circ f)(x) = x$

$H(0) = 6(0) = 0$
$K(0) = 0$

$H(1) = 6(1) = 6$
$K(6) = 6 \bmod 4 = 2$

$H(2) = 6(2) = 12$
$K(12) = 12 \bmod 4 = 0$

19. Exercise 7.3.7

$(K \circ H)(0)$	$(K \circ H)(1)$	$(K \circ H)(2)$
0	2	0

20. Exercise 7.3.11

Check that both compositions of H and H^{-1} give the identity function

$$(H \circ H^{-1})(x) = \frac{\left(\frac{x+1}{x-1}\right) + 1}{\left(\frac{x+1}{x-1}\right) - 1} = \frac{(x+1) + (x-1)}{(x+1) - (x-1)}$$

$$= \frac{2x}{2} = x = I_R(x)$$

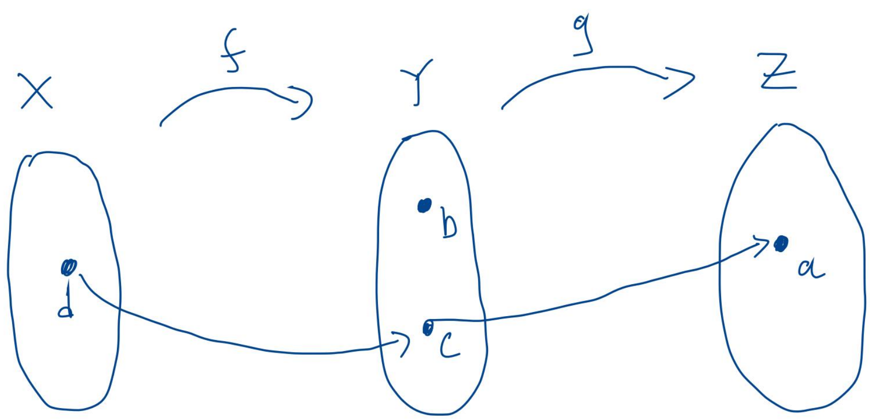
$$(H^{-1} \circ H)(x) = \frac{\left(\frac{x+1}{x-1}\right) + 1}{\left(\frac{x+1}{x-1}\right) - 1} = \frac{(x+1) + (x-1)}{(x+1) - (x-1)} = \frac{2x}{2} = x$$

$$= I_R(x)$$

Hence, for all x in $\mathbb{R}$, $(H \circ H^{-1})(x)$ and $(H^{-1} \circ H)(x)$ both give $I_R(x)$

21. Exercise 7.3.17

Counterexample: $X = \{d\}$ $Y = \{b, c\}$ $Z = \{a\}$



$(g \circ f)$ is onto because every element in Z has an element in Y that maps to it.

f is not onto because b is an element of Y and there exists no element in X that maps to b.

$g(x) = -x$
$-g^{-1}(x) = x$
$g^{-1}(x) = -x$

22. Exercise 7.3.24

$(g \circ f)$	$(g \circ f)^{-1}$	g^{-1}
$-(x+3)$	$-x-3$	$-x$

$f(x) = x+3$
$x = f^{-1}(x)+3$
$f^{-1}(x) = x-3$

$f^{-1}(x) = x-3$
$g^{-1}(x) = -x$
$f^{-1}(g^{-1}(x)) = f^{-1}(-x) = -x-3$
$= -(x+3)$

f^{-1}	$f^{-1} \circ g^{-1}$
$x-3$	$-(x+3)$

How $(g \circ f)^{-1}$ and $f^{-1} \circ g^{-1}$ are related

$(g \circ f)^{-1} = -x-3 = -(x+3)$
$f^{-1} \circ g^{-1} = -(x+3)$

$(g \circ f)^{-1}$ is equal to $f^{-1} \circ g^{-1}$

23. Exercise 7.4.9

Consider $\mathbb{Z}^{+}$ and $\mathbb{Z}^{nonneg}$
We must show that $|\mathbb{Z}^{+}| = |\mathbb{Z}^{nonneg}|$
Define $\mathbb{Z}^{+} \to \mathbb{Z}^{nonneg}$ by the rule $f(x) = x-1$
We must show that $f(x)$ has a one-to-one correspondence
Suppose $f(x_1) = f(x_2)$
By substitution, $x_1 - 1 = x_2 - 1$
$+1 \quad +1$
$x_1 = x_2$
Since $x_1 = x_2$, $f(x)$ has a one to one correspondence
By definition of equal cardinality, since $f(x)$ has a one to one correspondence, $|\mathbb{Z}^{+}| = |\mathbb{Z}|$
QED

24. Exercise 7.4.12

Theorem: $S = \{x \in \mathbb{R} \mid 0 < x < 1\}$ $W = \{x \in \mathbb{R} \mid a < x < b\}$

Prove that S and W have the same cardinality

Proof:

Let a, b any real numbers such that $a < b$

Define $S \to W$ by the rule $f(x) = x + a$ for all real numbers in S.

We must show that f(x) has a one to one correspondence.

Suppose $f(x_1) = f(x_2)$

By substitution, $x_1 + a = x_2 + a$

$-a \quad -a$

$x_1 = x_2$

Since $x_1 = x_2$, f(x) has a one to one correspondence.

Since there is a one to one correspondence between S and W, by definition of equal cardinality, set S has the same cardinality as set W

QED

25. Exercise 7.4.20

Example 1:

$f(x) = \sqrt{x}$

onto because all integers in $(-\infty, 0)$ have no inverse image

Example 2:

$f(x) = e^x$

onto because all integers in $(-\infty, 0]$ have no solution