Problem set 3,4

● Graded

Student

Total Points

92 / 100 pts

Question 1

Exercise 2.1.35

5 / 5 pts

✔ **– 0 pts** Correct

– 5 pts no answer / incorrect template / incorrect

– 5 pts wrong problem

– 5 pts illegible

– 2.5 pts only "and" or $\wedge$ is acceptable here

– 2 pts not $\leq$ is $>$

– 2 pts not $>$ is $\leq$

Question 2

Exercise 2.2.8

8 / 8 pts

✔ **– 0 pts** Correct

– 8 pts no answer / incorrect template

– 8 pts wrong problem

– 8 pts illegible

– 1 pt 1 incorrect entry

– 2 pts 2 incorrect entries

– 3 pts 3 or 4 incorrect entries

– 6 pts 5 or more incorrect entries

– 4 pts precedence order (p. 46 text) makes this $\sim (p \vee q) \rightarrow r$

Question 3

Exercise 2.2.15

8 / 8 pts

✔ **– 0 pts** Correct

– 8 pts no answer / incorrect template

– 8 pts wrong problem

– 8 pts illegible

– 1 pt 1 incorrect entriy / steps

– 2 pts 2 incorrect entries

– 3 pts 3 or 4 incorrect entries /steps

– 5 pts 5 or more incorrect entries / steps

– 3 pts no/incorrect statement about logical equivalence

Question 4

Exercise 2.2.20g

9 / 9 pts

✔ **– 0 pts** Correct

– 2 pts small logical error

– 7 pts implication must be negated $\sim (p \rightarrow q) \equiv p \wedge \sim q$

– 9 pts no answer / incorrect template

– 9 pts wrong problem

– 9 pts illegible

– 5 pts multiple logical errors

Question 5

Exercise 2.2.27

9 / 9 pts

✔ **– 0 pts** Correct

– 9 pts no answer / incorrect template

– 9 pts wrong problem

– 9 pts illegible

– 2 pts 1 incorrect entriy / step

– 4 pts 2 incorrect entries /steps

– 7 pts 3 or more incorrect entries / steps

– 2 pts no indication about why logically equivalent

Question 6

Exercise 2.2.46

8 / 8 pts

✔ **– 0 pts** Correct

– 8 pts no answer / incorrect template

– 8 pts wrong problem

– 8 pts illegible

– 2 pts d is incorrect

– 3 pts e is incorrect

– 3 pts f is incorrect

– 0 pts Click here to replace this description.

Question 7

Exercise 2.3.9

9 / 9 pts

✔ **– 0 pts** Correct

– 9 pts no answer / incorrect template

– 9 pts wrong problem

– 9 pts illegible

– 1 pt which are premises?

– 1 pt which is conclusion?

– 2 pts no sentence about reason for validity

– 1 pt imprecise reason about validity

– 3 pts 1 or 2 incorrect table entries

– 6 pts 3 or more incorrect table entries

Question 8

Exercise 2.3.11

9 / 9 pts

✔ **– 0 pts** Correct

– 3 pts claimed argument is valid

– 1 pt imprecise reason about validity

– 2 pts incorrect/ no sentence about $reason$ for validity

– 9 pts no answer / incorrect template

– 9 pts wrong problem

– 9 pts illegible

– 1 pt which are premises?

– 1 pt which is conclusion?

– 3 pts 1 or 2 missing/incorrect table entries

– 6 pts 3 or more incorrect table entries

– 3 pts Incorrect set up for truth table

Question 9

Exercise 2.3.15

8 / 9 pts

– 0 pts Correct

– 9 pts no answer / incorrect template

– 9 pts wrong problem

– 9 pts illegible

✔ **– 1 pt** which are premises?

– 1 pt which is conclusion?

– 2 pts no sentence about $reason$ for validity

– 1 pt imprecise / unclear reason about validity

– 3 pts 1 or 2 incorrect / empty table entries

– 6 pts 3 or more incorrect / empty table entries

Only one premise which is q

Question 10

Exercise 2.3.28

9 / 9 pts

✔ **– 0 pts** Correct

– 9 pts no answer / incorrect template

– 9 pts wrong problem

– 9 pts illegible

– 2 pts no definition of symbols in argument

– 3 pts incorrect argument form

– 4 pts you were asked to name the argument form that guarantees its validity

– 6 pts Incorrect logic form. Expected logic form:

$p \to q$
q
$\therefore p$

Question 11

Exercise 2.3.32

2 / 9 pts

– **0 pts** Correct

– **9 pts** no answer / incorrect template

– **9 pts** wrong problem

– **9 pts** illegible

– **2 pts** no definition of symbols in argument

✔ – **3 pts** incorrect argument

– **4 pts** incorrect argument form

✔ – **4 pts** you were asked to name the argument form that guarantees its validity

Your argument doesn't match what's being said in the question.

Let's go through it line by line:
"If I get a Christmas bonus, then I'll buy a stereo" becomes "if p then r" [p -> r]
"If I sell my motorcycle, I'll buy a stereo" becomes "if q then r" [q -> r]
"∴ If I get a Christmas bonus or I sell my motorcycle, then I'll buy a stereo" becomes "if p or q, then r" [(p v q) -> r]
So to the argument would be:
p -> r
q -> r
∴(p v q) -> r

Then, if we look at Table 2.3.1 in the textbook, we'll find that the argument form you provided (p->q, r->q, therefore (p v r) ->q) is seems similar to Proof by Division into Cases.

If we examine a little bit further, you will find that your argument form and division into cases end up saying the same thing.

In this question, our argument form in English would boil down to so: given two initial statements (call them p, q), both implying another statement (r), if either of those p or q true, then we can conclude r.

With Division into Cases, the same applies. Given p implies r and q implies r, if p or q end up being true, r can be concluded.

Thus, the rule of inference justifying the validity of your argument form, would be division into cases.

Question 12

Exercise 2.3.42

8 / 8 pts

✔ **– 0 pts** Correct

– 3 pts no premises

– 8 pts no answer / incorrect template / no correct steps

– 8 pts wrong problem

– 8 pts illegible

– 7 pts only 1 or 2 correct steps

– 6 pts only 2 correct steps

– 5 pts only 3 correct steps

– 3 pts only 5 correct steps

– 1 pt almost! missing 1 correct step

Put your answer in each indicated box. Answers must be handwritten, legible and use correct notation.
Be sure to study the answers in Appendix A to similar problems so you know what your approach should be.
Larger boxes indicate that you are expected to provide substantial detail.

1. Exercise 2.1.35

let $p = (x \leq -1)$
$q = (x > 1)$

$\sim(p \vee q) \equiv \sim p \wedge \sim q$ De Morgan's Law for real numbers

$= |x > -1 \text{ and } x \leq 1|$

2. Exercise 2.2.8

p	q	r	$\sim p$	$\sim p \vee q$	$(\sim p \vee q) \rightarrow r$
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	T	T
F	F	F	T	T	F

3. Exercise 2.2.15

p	q	r	$(q \to r)$	$p \to (q \to r)$	$(p \to q)$	$(p \to q) \to r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	T	F
F	T	F	F	T	T	T
F	F	T	T	T	T	F
F	F	F	T	T	T	T

$p \to (q \to r) \not\equiv (p \to q) \to r$

since their truth columns are not the same for all values of p, q, r

4. Exercise 2.2.20g

→ Mentioned in class

let p = "n is divisible by 6"
let q = "n is divisible by 7"
let r = "n is divisible by 3"

Statement
$p \to (q \wedge r)$

Find
$\sim(p \to (q \wedge r))$

$p \to q \equiv \sim p \vee q$

$p \to (q \wedge r) \equiv \sim p \vee (q \wedge r)$

$\sim(p \to (q \wedge r)) \equiv \sim(\sim p \vee (q \wedge r))$

$\sim(\sim p \vee (q \wedge r))$

$p \wedge \sim(q \wedge r)$ De Morgan's Law and double negation law

$p \wedge \sim q \vee \sim r$ De Morgan's Law

n is divisible by 6 and not divisible by 7 or not divisible by 3

converse inverse

5. Exercise 2.2.27

p	q	$p \to q$	$q \to p$	$\sim p$	$\sim q$	$\sim p \to \sim q$
T	T	T	T	F	F	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

$q \to p \equiv \sim p \to \sim q$

6. Exercise 2.2.46 (answer yes or not)

(c)	(d)	(e)	(f)
No	No	Yes	No
$q \to p$	$\sim p \to \sim q$		

p = "Compound X is boiling"

q = "temperature must be at least 150°C"

Given:

$p \to q$

7. Exercise 2.3.9

Premises — Conclusion

p	q	r	$p \wedge q$	$\sim r$	$\sim q$	$(p \wedge q) \to \sim r$	$p \vee \sim q$	$\sim q \to p$	$\sim r$
T	T	T	T	F	F	F	T	T	F
T	T	F	T	T	F	T	T	T	T
T	F	T	F	F	T	T	T	T	F
T	F	F	F	T	T	T	T	T	T
F	T	T	F	F	F	T	F	T	F
F	T	F	F	T	F	T	F	T	T
F	F	T	F	F	T	T	T	F	F
F	F	F	F	T	T	T	T	F	T

Critical rows

The argument form is invalid because there exists a critical row (row #3) where all the premises are true and the conclusion is false. A valid argument should always have a true conclusion if the premises are true.

8. Exercise 2.3.11

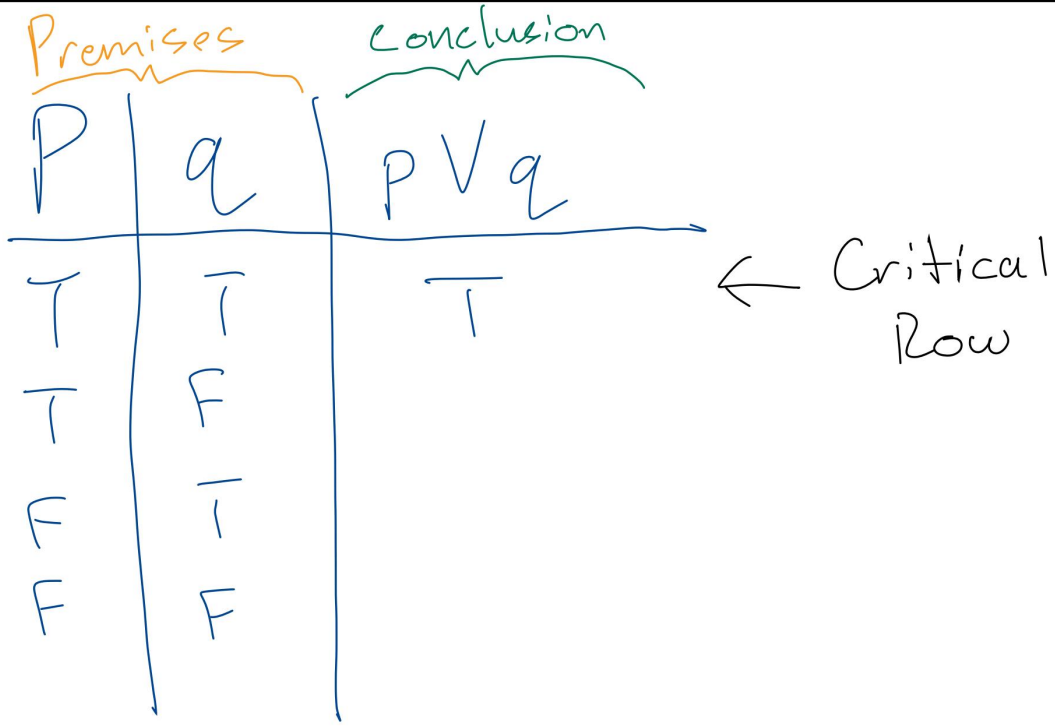
premises conclusion

p	q	r	~p	~q	~r	qVr	p→(qVr)	~qV~r	~pV~r
T	T	T	F	F	F	T	T	F	
T	T	F	F	F	T	T	T	T	T
T	F	T	F	T	F	T	T	T	F
T	F	F	F	T	T	F	F	T	
F	T	T	T	F	F	T	T	F	
F	T	F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	T	T	T
F	F	F	T	T	T	F	T	T	T

Critical Rows

The argument form is invalid because there exists a possibility (as shown by row #3 of the truth table) that all the premises are true but the conclusion is false. A valid argument must always have a true conclusion if all of the premises are true

9. Exercise 2.3.15



The logical form of the argument is valid because for all cases in which the premises are true, the conclusion is also true.

10. Exercise 2.3.28

Argument

p = "# of rationals equals # of irrationals"

q = "set of irrationals is infinite"

① $p \to q$ premise

② q premise

③ $\therefore p$ conclusion

Explanation

Converse Error

11. Exercise 2.3.32

Argument

p = christmas bonus

q = buy stereo

r = sell motorcycle

① $p \to q$

② $r \to q$

③ $\sim p \vee q$

④ $\sim r \vee q$

⑤ $(\sim p \vee q) \wedge (\sim r \vee q)$

⑥ $q \vee (\sim p \wedge \sim r)$

⑦ $(\sim p \wedge \sim r) \vee q$

⑧ $\sim(\sim p \wedge \sim r) \to q$

⑨ $(\sim(\sim p) \vee \sim(\sim r)) \to q$

⑩ $(p \vee r) \to q$

Explanation

① premise

② premise

③ definition of conditional on ①

④ definition of conditional on ②

⑤ Conjunction on ③ and ④

⑥ distributive law on ⑤

⑦ Commutative law on ⑥

⑧ Definition of conditional on ⑦

⑨ De Morgan's Law on ⑧

⑩ Double negation law on ⑨

12. Exercise 2.3.42 (The statements are *not* in order. You have to reorder them to write the full proof. Be sure to look at similar problems for this one!)

(1) $p \vee q$	premise	Prove t
(2) $q \to r$	premise	
(3) $p \wedge s \to t$	premise	
(4) $\sim r$	premise	
(5) $\sim q \to u \wedge s$	premise	
(6) $\sim q$	modus tollens on (2) and (4)	
(7) $u \wedge s$	modus ponens on (5) and (6)	
(8) s	Conjunctive simplification on (7)	
(9) p	elimination on (1) and (6)	
(10) $p \wedge s$	conjunction on (9) and (8)	
(11) t	modus ponens on (3) and (10)	