Problem set 21,22

Graded

Student

Total Points

91.6 / 100 pts

Question 1

Exercise 9.5.4

3.5 / 3.5 pts

✔ **– 0 pts** Correct

– 3.5 pts no answer

– 3.5 pts illegible

– 3.5 pts wrong problem

– 3.5 pts incorrect

– 1 pt factorials required

Question 2

Exercise 9.5.7a

3.5 / 3.5 pts

✔ **– 0 pts** Correct

– 3.5 pts no answer

– 3.5 pts illegible

– 3.5 pts wrong problem

– 3 pts this is a combination not a permutation

– 0.1 pts factorials required

– 3.5 pts incorrect

Question 3

Exercise 9.5.7b(i)

3.5 / 3.5 pts

✔ **– 0 pts** Correct

– 3.5 pts no answer

– 3.5 pts illegible

– 3.5 pts wrong problem

– 3.5 pts Incorrect

– 3 pts this is a combination not a permutation

– 2 pts multiply, don't add

– 0.1 pts factorials required

Question 4

Exercise 9.5.7b(ii)

3.5 / 3.5 pts

✔ **– 0 pts** Correct

– 3.5 pts no answer

– 3.5 pts illegible

– 3.5 pts wrong problem

– 3 pts this is a combination not a permutation

– 3 pts this is either a complement (subtract) or the sum of multiple terms

– 2 pts missed multiple terms

– 1 pt missed one term

– 0.1 pts factorials required

– 3.5 pts incorrect

Question 5

Exercise 9.5.7b(iii)

3.5 / 3.5 pts

✔ **– 0 pts** Correct

– 3.5 pts no answer

– 3.5 pts illegible

– 3.5 pts wrong problem

– 3 pts this is a combination not a permutation

– 2 pts missed more than 1 term (should be 4)

– 1 pt missed one term (should be 4)

– 0.1 pts factorials required

– 3.5 pts Incorrect

Question 6

Exercise 9.5.7c

3.5 / 3.5 pts

✔ **– 0 pts** Correct

– 3.5 pts no answer

– 3.5 pts illegible

– 3.5 pts wrong problem

– 1 pt missed one term

– 0.1 pts factorials required

– 3.5 pts Incorrect

Question 7

Exercise 9.5.7d

3.5 / 3.5 pts

✔ **– 0 pts** Correct

– 3.5 pts no answer

– 3.5 pts illegible

– 3.5 pts wrong problem

– 3 pts this is a combination not a permutation

– 2 pts treat this as a 2-term sum

– 1 pt missed one term

– 0.1 pts factorials required

Question 8

Exercise 9.5.14a

3.4 / 3.5 pts

- **– 0 pts** Correct
- **– 3.5 pts** incorrect
- **– 3.5 pts** incorrect template
- **– 3.5 pts** no answer
- **– 3.5 pts** illegible
- **– 3.5 pts** wrong problem
- **– 1 pt** incorrect factorial representation / evaluation
- ✔ **– 0.1 pts** factorials required

Question 9

Exercise 9.5.14b

3.4 / 3.5 pts

- **– 0 pts** Correct
- **– 3.5 pts** no answer
- **– 3.5 pts** incorrect template
- **– 3.5 pts** illegible
- **– 3.5 pts** wrong problem
- **– 3 pts** this is the sum of 4 combinations
- **– 3 pts** incorrect terms
- ✔ **– 0.1 pts** factorials required
- **– 0.1 pts** arithmetic error

Question 10

Exercise 9.5.14c

3.9 / 4 pts

- **– 0 pts** Correct
- **– 4 pts** incorrect template
- **– 4 pts** no answer
- **– 4 pts** illegible
- **– 4 pts** wrong problem
- **– 4 pts** this is the sum of 16 combinations or use the complement
- **– 2.5 pts** missed multiple terms
- **– 1 pt** missed 1 term
- ✔ **– 0.1 pts** factorials required
- **– 0.1 pts** arithmetic error

Question 11

Exercise 9.5.14d

3.9 / 4 pts

- **– 0 pts** Correct
- **– 4 pts** wrong template
- **– 4 pts** no answer
- **– 4 pts** illegible
- **– 4 pts** wrong problem
- **– 2.5 pts** this is the sum of 2 combinations
- **– 1.5 pts** missing/incorrect 1 term
- ✔ **– 0.1 pts** factorials required

Question 12

Exercise 9.5.26b

4 / 4 pts

- ✔ **– 0 pts** Correct
- **– 4 pts** no answer
- **– 4 pts** illegible
- **– 4 pts** wrong problem
- **– 4 pts** incorrect ((impossible)

Question 13

Exercise 9,5,26e

0 / 4 pts

- **– 0 pts** Correct
- **– 4 pts** no answer
- **– 4 pts** illegible
- **– 4 pts** wrong problem
- ✔ **– 4 pts** incorrect (impossible)

Question 14

Exercise 9.5.30

4 / 4 pts

- ✔ **– 0 pts** Correct
- **– 4 pts** no answer
- **– 4 pts** illegible
- **– 4 pts** wrong problem
- **– 4 pts** incorrect (there is double counting here by ordering the suit selection)

Question 15

Exercise 9.6.4a

3.5 / 3.5 pts

- ✔ **– 0 pts** Correct
- **– 3.5 pts** no answer / incorrect
- **– 3.5 pts** illegible
- **– 3.5 pts** wrong problem
- **– 3.5 pts** this is a $V(n,r)$ problem
- **– 0.1 pts** factorials required

Question 16

Exercise 9.6.4b

3.5 / 3.5 pts

- ✔ **– 0 pts** Correct
- **– 3.5 pts** no answer/incorrect :(
- **– 3.5 pts** illegible
- **– 3.5 pts** wrong problem
- **– 3.5 pts** this is a $V(n,r)$ problem
- **– 0.1 pts** factorials required

Question 17

Exercise 9.6.4c

3.5 / 3.5 pts

✔ **– 0 pts** Correct

– 3.5 pts no answer / incorrect

– 3.5 pts illegible

– 3.5 pts wrong problem

– 3.5 pts this is a $V(n, r)$ problem

– 0.1 pts factorials required

– 0.5 pts there are 4 cases to be added, missed 1

– 1 pt there are 4 cases to be added, missed 2

– 2 pts there are 4 cases to be added

Question 18

Exercise 9.6.18a

3.5 / 3.5 pts

✔ **– 0 pts** Correct

– 3.5 pts no answer

– 3.5 pts illegible

– 3.5 pts wrong problem

– 3.5 pts this is a $V(n, r)$ problem

– 0.1 pts factorials required

Question 19

Exercise 9.5.14b

3.5 / 3.5 pts

- ✔ **– 0 pts** Correct
- **– 3.5 pts** Incorrect
- **– 3.5 pts** no answer
- **– 3.5 pts** illegible
- **– 3.5 pts** wrong problem
- **– 3.5 pts** this is a $V(n, r)$ problem
- **– 0.1 pts** factorials required
- **– 0.5 pts** there are 16 cases (0 through 15) to be added, missed 1
- **– 1 pt** there are 16 cases (0 through 15) to be added, missed several
- **– 3 pts** there are 16 cases (o through 15) to be added

Question 20

Exercise 9.6.14c

0 / 4 pts

- **– 0 pts** Correct
- ✔ **– 4 pts** incorrect
- **– 4 pts** no answer
- **– 4 pts** illegible
- **– 4 pts** wrong problem
- **– 4 pts** this is a $V(n, r)$ problem
- **– 0.1 pts** factorials required
- **– 0.5 pts** there are 21 cases (0 through 20) to be added, missed 1
- **– 1 pt** there are 21 cases (0 through 20) to be added, missed several
- **– 3 pts** there are 16 cases (o through 15) to be added

Question 21

Exercise 9.6.14d

4 / 4 pts

✔ **– 0 pts** Correct

– 4 pts Incorrect

– 4 pts no answer

– 4 pts illegible

– 4 pts wrong problem

– 4 pts this is a $V(n, r)$ problem

– 0.1 pts factorials required

– 1 pt this is also an includion/exclusion problem -- you needed to add prior problems' answers but also subtract to avoid double counting

– 0.5 pts incorrect index on intersection

Question 22

Exercise 9.7.11

3.5 / 3.5 pts

✔ **– 0 pts** Correct

– 3.5 pts no answer

– 3.5 pts illegible

– 3.5 pts wrong problem

– 3.5 pts incorrect

Question 23

Exercise 9.7.15

7 / 7 pts

✔ **– 0 pts** Correct

– 4 pts this should be a proof by induction, look at the answer to 9.7.13 in Appendix A

– 7 pts no answer

– 7 pts illegible

– 7 pts wrong problem

– 7 pts incorrect

– 2 pts good start but you need to use Pascal's formula, look at the answer to 9.7.13 in Appendix A

Question 24

Exercise 9.7.24

4 / 4 pts

✔ **– 0 pts** Correct

– 4 pts no answer

– 4 pts illegible

– 4 pts wrong problem

– 4 pts incorrect

– 0.1 pts factorials required

– 1 pt terms like u^2 must be raised to a power

– 1 pt coefficients like -3 must be raised to a power

– 1 pt missed a term

– 2 pts missed 2 terms

Question 25

Exercise 9.7.34

3.5 / 3.5 pts

✔ **– 0 pts** Correct

– 3.5 pts incorrect

– 3.5 pts no answer

– 3.5 pts illegible

– 3.5 pts wrong problem

– 0.1 pts factorials required

– 1.5 pts forgot to count from 0

Question 26

Exercise 9.7.44

5 / 5 pts

✔ **– 0 pts** Correct

– 5 pts no answer

– 5 pts illegible

– 5 pts wrong problem

– 5 pts incorrect approach -- use the binomial formula

Put your answer in each indicated box. Answers must be handwritten, legible and use correct notation.
Study the answers in Appendix A to similar problems so you know what your approach should be.
Larger boxes indicate that you are expected to provide substantial detail.
UNLESS OTHERWISE INSTRUCTED: do not use P(n,r), C(n,r) or V(n,r) notation as a final answer, do not reduce fractions, and do not expand factorials.

Students learning counting techniques often ask, "How do I know what to multiply and what to add? When do I use the multiplication rule and when do I use the addition rule?" Unfortunately, these questions have no easy answers. You need to imagine, as vividly as possible, the objects you are to count. You might even start to make an actual list of the items you are trying to count to get a sense for how to obtain them in a systematic way. You should then construct a model that would allow you to continue counting the objects one by one if you had enough time. If you can imagine the elements to be counted as being obtained through a multistep process (in which each step is performed in a fixed number of ways regardless of how preceding steps were performed), then you can use the multiplication rule. The total number of elements will be the product of the number of ways to perform each step. If, however, you can imagine the set of elements to be counted as being broken up into disjoint subsets, then you can use the addition rule. The total number of elements in the set will be the sum of the number of elements in each subset.

One of the most common mistakes students make is to count certain possibilities more than once.

— Discrete Structures, Susanna Epps, fourth edition, p.577

1. Exercise 9.5.4

$P(8,3) = \binom{8}{3} 3!$

2. Exercise 9.5.7a

$\binom{13}{7}$

3. Exercise 9.5.7b(i)

$\binom{7}{4} \cdot \binom{6}{3}$

4. Exercise 9.5.7b(ii)

$\binom{13}{7} - \binom{7}{7}$

5. Exercise 9.5.7b(iii)

$\binom{7}{1}\binom{6}{6} + \binom{7}{2}\binom{6}{5} + \binom{7}{3}\binom{6}{4}$

6. Exercise 9.5.7c

$\binom{11}{6} + \binom{11}{6} + \binom{11}{7}$

7. Exercise 9.5.7d

$\binom{11}{5} + \binom{11}{7}$

8. Exercise 9.5.14a

$\binom{16}{7}$

9. Exercise 9.5.14b

$\binom{16}{13} + \binom{16}{14} + \binom{16}{15} + \binom{16}{16}$

10. Exercise 9.5.14c

$2^{16} - \binom{16}{0}$

11. Exercise 9.5.14d

$\binom{16}{0} + \binom{16}{1}$

12. Exercise 9.5.26b

0

13. Exercise 9.5.26e

$3^4 - 2^4\binom{3}{1} + (1)^4\binom{3}{2}$

14. Exercise 9.5.30

Error in step 1: Since you are choosing a pair, it should be $\binom{13}{2}$

15. Exercise 9.6.4a

$V(30, 8) = C(30-1+8, 30)$

16. Exercise 9.6.4b

$V(26, 8) = C(26+8-1, 26)$

17. Exercise 9.6.4c

$C(30-1+8, 30) - C(26+8-1, 26)$

18. Exercise 9.6.18a

$V(30, 4) = C(30+4-1, 30)$

19. Exercise 9.6.18b

16 or more quarters $\Rightarrow C(14+4-1, 14)$

$C(30+4-1, 30) - C(14+4-1, 14)$

20. Exercise 9.6.18c

20 or more dimes $\Rightarrow C(10+4-1, 10)$

$C(30+4-1, 30) - C(10+4-1, 10)$

21. Exercise 9.6.18d

$C(30+4-1, 30) - C(14+4-1, 14) - C(10+4-1, 10)$

22. Exercise 9.7.11

$n=8$: 1 8 28 56 70 56 28 8 1

$n=9$: 1 9 36 84 126 126 84 36 9 1

23. Exercise 9.7.15

Theorem: Let r be a fixed nonnegative integer. For all integers n with $n \geq r$, $\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$

Proof:

Let $P(n)$ be the property $\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$

Basis:

$P(r)$: $\sum_{i=r}^{r} \binom{i}{r} = \binom{r+1}{r+1}$

$\downarrow$

$\binom{r}{r} = 1 \quad \binom{r+1}{r+1} = 1$

Since the left side equals the right side, our basis step is true.

Inductive Step:

Assume for some k where $k \geq r$, $P(k)$ is true

We must show $P(k+1)$ is also true

$P(k+1)$: $\sum_{i=r}^{k+1} \binom{i}{r} = \binom{k+2}{r+1}$

$\binom{k+1}{r} + \sum_{i=r}^{k} \binom{i}{r}$

$\binom{k+1}{r} + \binom{k+1}{r+1}$ Pascal's Formula

$\binom{k+2}{r+1} = \binom{k+2}{r+1}$

$\therefore P(k+1)$ is true

Since we have proved the basis step and the inductive step, our theorem is true via mathematical induction

QED

24. Exercise 9.7.24

$$\binom{4}{0}(u^2)^4 + \binom{4}{1}(u^2)^3(-3v)^1 + \binom{4}{2}(u^2)^2(-3v)^2 + \binom{4}{3}(u^2)^1(-3v)^3 + \binom{4}{4}(-3v)^4$$

25. Exercise 9.7.34

coefficient: $(-3)^5(2)^9\binom{14}{5}$ term: (x^9y^{10})

26. Exercise 9.7.44

$$\sum_{i=0}^{m}\binom{m}{i}4^i = \sum_{i=0}^{m}\binom{m}{i}(1)^{m-i}(4)^i$$

$$= (1+4)^n = (5)^n$$