CSCI 15000§001, Final Exam May 18, 2023$_{P}$

[redacted]

Directions:

Put your name and EMPLID in the boxes above. Once you are allowed to open the exam, put your name on the top of EVERY page. Write your answers in the space provided. Only the parts of the answers that you write down will be graded. Support all your answers. You may not use your text book or your notes. You may not use anyone else's text book, notes, or work. You may not use a calculator, computer, cell phone, etc. You may not enlist the aid of any persons, spirits or other beings, or any denizens of this or any other planes, living, dead or otherwise.

Have your ID card available. Someone will be by to compare your face to your picture and your name on the card to your name on the exam.

Gradescope will be used to assist us in grading this exam. Because of this, your exam pages will be separated. It is important to write your name on EVERY page of this exam.

If we are unable to read your answer, the answer is wrong. Be sure to write legibly. Tiny writing, faint writing, and poor writing can adversely affect your grade. Prof. Schweitzer's eyes are the arbiter of "legible", and his eyes are older than yours.

There are eight questions with point values as marked.

Do Not Open This Exam Until Instructed To Do So

If you earn a D in the class and would rather have an F, put an X in this box. This will not affect your grade if you earn a C or better. If you have already elected to take a P/NC you probably don't want to do this. $\boxed{}$

1. (15) Use the truth table below to determine if

$$q \to r, q \to p, (p \wedge r) \to q \vdash q$$

is a valid argument form. Be sure to use the "standard (in this class)" order of values for the variables (many points will be lost if you don't, and part of the question is "do you know this order"). Be sure to indicate why this is or is not a valid argument form.

p	q	r	$q \to r$ ~~$p \to r$~~	$q \to p$ ~~$p \to q$~~	$(p \wedge r) \to q$ ~~$(q \wedge r) \to \neg p$~~	Q ~~p~~	
T	T	T	T	T	T	T	X
T	T	F	F	T	T	T	
T	F	T	T	T	F	F	
T	F	F	T	T	T	F	X
F	T	T	T	F	T	T	
F	T	F	F	F	T	T	
F	F	T	T	T	T	F	X
F	F	F	T	T	T	F	X

Why is this or is this not a valid argument form?

This statement is not valid as some of the critical rows conclude Q to be false, thus this is not a valid argument.

2. (15)

   (a) Prove that for all integers a, if a^3 is divisible by 2, then a is divisible by 2.

[redacted]

   (b) Prove that $\sqrt[3]{2}$ is irrational.

[redacted]

3. (10) Kleebob is a card game played with a deck consisting of six suits each with 13 denominations. Like a poker deck, the denominations are A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, and K. The six suits are □ (boxes), ♣ (clubs), ♢ (diamonds), ♡ (hearts), ♠ (spades), and △ (triangles). Boxes and triangles are green, clubs and spades are black, and diamonds and hearts are red. Every player in a game is dealt 6 cards. These 6 cards are called a "hand", and the order they are dealt in does not matter.

   Hands are "named" in a style similar to poker. For example "a pair" has exactly two cards of the same denomination and no other cards that match in denomination. "Two pair" are two cards of one denomination, two cards of another denomination, and two cards that are unrelated to each other or to either pair. "Three of a kind" is three cards of the same denomination and three cards unrelated to those or each other.

   Leave your answers to the questions below as combinatorial expressions. When appropriate, for example, $\binom{4}{2}$ is better than 6. Use the empty space for you work, but be sure your answer is in the box.

   (a) How many hands have exactly one pair?

   [redacted]

   (b) How many hands have 5 of a kind?

   [redacted]

   (c) How many hands have exactly one pair and one three of a kind?

   [redacted]

   (d) How many hands have only green cards?

   [redacted]

   (e) How many hands have a 5 of a kind and a pair?

   [redacted]

4. (10) Consider the graph $G =< V, E >$ with $V = \{v_1, v_2, \ldots v_8\}$ and $E = \{e_1 = (v_1, v_5), e_2 = (v_5, v_6), e_3 = (v_1, v_6), e_4 = (v_2, v_6), e_5 = (v_2, v_3), e_6 = (v_3, v_4), e_7 = (v_3, v_7), e_8 = (v_2, v_3), e_9 = (v_6, v_6), e_{10} = (v_6, v_7), e_{11} = (v_7, v_8), e_{12} = (v_4, v_8)\}$.

   (a) On the diagram below (which indicates the vertices v_1 through v_8) draw G.

   [redacted]

   (b) Is there an Eulerian Circuit? [redacted]

   (c) If so, starting at v_1, list the vertices in the order you traverse them. If not, briefly state why not.

   [redacted]

   (d) Is this graph "simple"? Why or why not? (Your answer should show that you know the definition of "simple")

   [redacted]

5. (15) Let the sequence $b_1, b_2, b_3, \ldots$ be defined by $b_k = 3^k - 2^k$. Use strong induction to show that $b_n = 5 \cdot b_{n-1} - 6 \cdot b_{n-2}$ for $n \in \mathbb{Z}, n \geq 3$. Be sure to identify the proposition you are proving and to indicate what you are doing as follows:

   (a) What is "$P(k)$"?

   [redacted]

   (b) Basis:

   [redacted]

   (c) Induction:

   [redacted]

6. (10)

   (a) Define "$a|b$": $a|b$ if and only if

   [redacted]

   (b) To prove "If $a|b$ then $a|2b$" by contradiction, you would start out by assuming

   [redacted]

   and conclude your proof by showing

   [redacted]

   (c) To prove "If $a|b$ then $a|2b$" by contrapositive, you would start out by assuming

   [redacted]

   and conclude your proof by showing

   [redacted]

   (d) To prove "If $a|b$ then $a|2b$" by a direct proof, you would start out by assuming

   [redacted]

   and conclude your proof by showing

   [redacted]

7. (15)Prove the following by induction. Be sure to clearly identify what "$P(k)$" is and what you are doing.

$$\forall n \in \mathbb{N}^{\geq 1} \quad \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

(a) What is "$P(k)$"?

[redacted]

(b) In terms of $P(k)$ what do you need to prove in the Basis step?

[redacted]

(c) Prove the Basis st[redacted]

(d) In terms of $P(k)$, what do you need to prove for the Induction step?

[redacted]

(e) Prove the induction step:

[redacted]

8. (10)

   (a) What does it mean for a function $f : A \to B$ to be (that is *define*) "onto" (a.k.a. "surjective").

   [redacted]

   (b) What does it mean for a function $f : A \to B$ to be (that is *define*) "one to one" (a.k.a. "injective").

   [redacted]

   (c) If A and B are sets, and $f : A \to B$ is both one-to-one and onto, what does that tell you about A and B?

   [redacted]

   (d) Using your definition, prove or disprove that $g : \mathbb{Z} \to \mathbb{Z}$ given by $g(n) = 3n + 4$ is one to one.

   [redacted]

   (e) Using your definition, prove or disprove that g (as above) is onto.

   [redacted]