Problem Set 9.10

● Graded

Student

Total Points

95 / 100 pts

Question 1

1. Exercise 5.1.4

5 / 5 pts

✔ **– 0 pts** Correct

– 5 pts no answer

– 5 pts wrong problem

– 5 pts illegible

– 1 pt 1 incorrect term

– 2.5 pts 2 incorrect terms

– 5 pts 3 ior more ncorrect terms

Question 2

2 Exercise 5.1.15

3 / 5 pts

– 0 pts Correct

– 5 pts no answer

– 5 pts wrong problem

– 5 pts illegible

– 2 pts no power of -1

✔ **– 2 pts** incorrect from value

– 2 pts incorrect to value

– 2 pts no Sigma

– 2 pts incorrect term representation

Question 3

3. Exercise 5.1.24 (just show the computation)

5 / 5 pts

✔ **− 0 pts** Correct

− 5 pts no answer

− 5 pts wrong problem

− 5 pts illegible

− 4 pts only a 1

Question 4

4, Exercise 5.1.38 (just show the computation)

5 / 5 pts

✔ **− 0 pts** Correct

− 5 pts no answer

− 5 pts wrong problem

− 5 pts illegible

− 2 pts missing/incorrect from value

− 2 pts missing/incorrect to value

− 2 pts missing/incorrect + an extra term

− 2 pts incorrect term representation

Question 5

5. Exercise 5.1.41

5 / 5 pts

✔ **− 0 pts** Correct

− 5 pts no answer

− 5 pts wrong problem / incorrect

− 5 pts illegible

− 2 pts incorrect from value

− 2 pts incorrect to value

− 2 pts no Sigma

− 2 pts incorrect term representation

Question 6

6. Exercise 5.1.45

5 / 5 pts

✔ **– 0 pts** Correct

– 5 pts no answer

– 5 pts wrong problem

– 5 pts illegible

– 3 pts no Pi

– 2 pts incorrect from value

– 2 pts incorrect to value

– 2 pts incorrect term representation

Question 7

7. Exercise 5.1.50

5 / 5 pts

✔ **– 0 pts** Correct

– 5 pts no answer

– 5 pts wrong problem

– 5 pts illegible

– 3 pts no Sigma

– 2 pts incorrect from value

– 2 pts incorrect to value

– 2 pts incorrect term representation

Question 8

8. Exercise 5.1.67

5 / 5 pts

✔ **– 0 pts** Correct

– 5 pts no answer

– 5 pts wrong problem

– 5 pts illegible

– 3 pts severe algebraic error

– 1.5 pts algebraic error

Question 9

9. Exercise 5.2.4a

3 / 3 pts

- ✔ **– 0 pts** Correct
- **– 3 pts** no answer
- **– 3 pts** wrong problem
- **– 3 pts** illegible
- **– 1.5 pts** incorrect P(2)
- **– 1.5 pts** answer should include "yes" or "true"

Question 10

10. Exercise 5.2.4b

3 / 3 pts

- ✔ **– 0 pts** Correct
- **– 3 pts** no answer
- **– 3 pts** wrong problem
- **– 3 pts** illegible
- **– 3 pts** incorrect formula
- **– 1.5 pts** incomplete formula

Question 11

11. Exercise 5.2.4c

3 / 3 pts

- ✔ **– 0 pts** Correct
- **– 3 pts** no answer
- **– 3 pts** wrong problem
- **– 3 pts** illegible
- **– 3 pts** incorrect formula
- **– 1.5 pts** incomplete formula

Question 12

12. Exercise 5.2.4d

0 / 3 pts

– 0 pts Correct

– 3 pts no answer

– 3 pts wrong problem

– 3 pts illegible

✔ **– 3 pts** this should be an if-then statement

Question 13

13. Exercise 5.2.7

6 / 6 pts

✔ **– 0 pts** Correct

– 6 pts no answer

– 6 pts wrong problem

– 6 pts illegible

– 4.5 pts no proof

– 3 pts incorrect skeleton

– 1 pt incorrect P(1) statement

– 1 pt incorrect P(k) statement

– 1 pt incorrect P(k+1) statement

– 3 pts multiple prrof steps missing / no reasons

– 1 pt 1 proof step missing / no reason

Question 14

14. Exercise 5.2.14

6 / 6 pts

✔ **– 0 pts** Correct

– 6 pts no answer

– 6 pts wrong problem

– 6 pts illegible

– 4.5 pts no proof

– 3 pts incorrect skeleton

– 1 pt incorrect P(0) statement

– 1 pt incorrect P(k) statement

– 1 pt incorrect P(k+1) statement

– 3 pts multiple proof steps missing / no reasons

– 1 pt 1 proof step missing / no reason

Question 15

15. Exercise 5.2.23

5 / 5 pts

✔ **– 0 pts** Correct

– 5 pts no answer

– 5 pts incorrect template

– 5 pts wrong problem

– 5 pts illegible

– 1.5 pts incorrect formula

– 1.5 pts incorrect match to formula

– 1.5 pts incorrect lower bound

Question 16

16. Exercise 5.2.27

5 / 5 pts

✔ **– 0 pts** Correct

– 5 pts no answer

– 5 pts wrong problem

– 5 pts illegible

– 2 pts incorrect value for a

– 2 pts incorrect value for r

– 1.5 pts inocrrect formula

– 1.5 pts incorrect match to formula

– 2 pts incorrect value for n (number of terms)

– 2 pts incorrect simplification

Question 17

17. Exercise 5.3.7a

2.5 / 2.5 pts

✔ **– 0 pts** Correct

– 2.5 pts no answer

– 2.5 pts wrong problem / incorrect

– 2.5 pts illegible

– 1.5 pts incorrect P(2)

– 1.5 pts answer should include "yes" or "true"

Question 18

18. Exercise 5.3.7b

2.5 / 2.5 pts

✔ **– 0 pts** Correct

– 2.5 pts no answer

– 2.5 pts wrong problem / incorrect

– 2.5 pts illegible

– 0.5 pts incorrect formula

– 1.5 pts incomplete formula

Question 19

19. Exercise 5.3.7c

2.5 / 2.5 pts

✔ **− 0 pts** Correct

− 2.5 pts no answer

− 2.5 pts wrong problem / incorrect

− 2.5 pts illegible

− 3 pts incorrect statement

− 1.5 pts incomplete statement

Question 20

20. Exercise 5.3.7d

2.5 / 2.5 pts

✔ **− 0 pts** Correct

− 2.5 pts no answer

− 2.5 pts wrong problem

− 2.5 pts illegible

− 3 pts this should be an in-then statement

Question 21

21. Exercise 5.3.9

8 / 8 pts

✔ **− 0 pts** Correct

− 8 pts no answer

− 8 pts wrong problem

− 8 pts incorrect template

− 8 pts illegible

− 4.5 pts no proof

− 3 pts incorrect skeleton

− 1 pt incorrect P(0) statement

− 1 pt incorrect P(k) statement

− 1 pt incorrect P(k+1) statement

− 3 pts multiple proof steps missing / incorrect / no reasons

− 1 pt 1 proof step missing / incorrect / no reason

Question 22

22. Exercise 5.3.17

8 / 8 pts

✔ **− 0 pts** Correct

− 8 pts no answer

− 8 pts wrong problem

− 8 pts illegible

− 4.5 pts no proof

− 3 pts incorrect skeleton

− 1 pt incorrect P(1) statement

− 1 pt incorrect P(k) statement

− 1 pt incorrect P(k+1) statement

− 3 pts multiple proof steps missing/incorrect / no reasons

− 1 pt 1 proof step missing / no reason

− 1 pt 2-3 minor errors

I also worked with the following students (provide EMLPIDs only)

EMPLID	EMPLID	EMPLID
EMPLID	EMPLID	EMPLID
EMPLID	EMPLID	EMPLID
EMPLID	EMPLID	EMPLID

My answers came in part or in full from the following sources

Put your answer in each indicated box. Answers must be handwritten, legible and use correct notation.
Study the answers in Appendix A to similar problems so you know what your approach should be.
Larger boxes indicate that you are expected to provide substantial detail.

1. Exercise 5.1.4

$d_m = 1 + \left(\frac{1}{2}\right)^m$
$m \geq 0$

$(\underbrace{2, 1.5, 1.25, 1.125}_{\text{First 4 terms}}, \ldots)$

2. Exercise 5.1.15

$a_n = (-1)^{n-1} \left(\frac{n-1}{n}\right)$ with a_1 being first term of the sequence

3. Exercise 5.1.24 (Just show the computation)

$$\sum_{j=0}^{0} (j+1)(2^j) = (0+1)(2^0) = 1$$

4. Exercise 5.1.38 (Just show the computation)

$$\sum_{k=1}^{m+1} k^2 = \sum_{k=1}^{m} k^2 + (m+1)^2$$

5. Exercise 5.1.41

$$\sum_{k=1}^{m} \frac{k}{k+1} + \frac{(m+1)}{(m+2)} = \sum_{k=1}^{m+1} \frac{k}{k+1}$$

6. Exercise 5.1.45

$$\prod_{n=1}^{3} (n+1)^2 - 1$$

7. Exercise 5.1.50

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{n}{(n+1)!} = \sum_{k=1}^{n} \frac{k}{(k+1)!}$$

8. Exercise 5.1.67

$$\frac{n!}{(n-2)!} = \frac{(n)\cdot(n-1)\cdot\cancel{(n-2)!}}{\cancel{(n-2)!}} = \boxed{(n)(n-1)}$$

9. Exercise 5.2.4a

$$\sum_{i=1}^{n-1}(i)(i+1) = \frac{n(n-1)(n+1)}{3}$$

$$P(2): \sum_{i=1}^{2-1}(i)(i+1) = \frac{2(2-1)(2+1)}{3}$$

$$(1)(1+1) \quad 2 = 2$$

Since both sides of the equation are equivalent, $P(2)$ is true

10. Exercise 5.2.4b

$$P(k): \sum_{i=1}^{k-1} i(i+1) = \frac{k(k-1)(k+1)}{3}$$

11. Exercise 5.2.4c

$$P(k+1): \sum_{i=1}^{k} i(i+1) = \frac{(k+1)(k)(k+2)}{3}$$

12. Exercise 5.2.4d

For the $P(k+1)$ formula, we must show that the left side of the equation is equal to the right side

$$\sum_{i=1}^{k} i(i+1) = \sum_{i=1}^{k-1} i(i+1) + k(k+1) = \frac{k(k-1)(k+1)}{3} + k(k+1) = \frac{(k)(k+1)(k-1+3)}{3}$$

$$= \frac{(k)(k+1)(k+2)}{3}$$

13. Exercise 5.2.7

Theorem: Let P(n) be $\sum_{i=1}^{n}(5i-4) = \frac{n(5n-3)}{2}$

Proof by Mathematical Induction:

We must show that P(n) is true for all $n \geq 1$

Basis:

$$P(1): \sum_{i=1}^{1}(5(1)-4) = \frac{(1)(5(1)-3)}{2}$$

$$(5-4) = \frac{2}{2}$$

$$(1) = 1$$

Since both the left and right side of the equation is 1, P(1) is true

Inductive Step: Assume for some that $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

By Substitution, $P(k+1): \sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

We must show P(k+1) is true.

By definition of Σ, the left side of the P(k+1) equation is $\sum_{i=1}^{k} i + (k+1) = \frac{k(k+1)}{2} + (k+1)$

$= \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+2)(k+1)}{2}$ which is the right side of the equation for P(k+1)

Since we have proved the basis step and the inductive step, the theorem is true

QED

14. Exercise 5.2.14

Theorem: Let $P(n)$ be $\sum_{i=1}^{n+1} i \cdot 2^i = n \cdot 2^{n+2} + 2$

Proof by Mathematical Induction:

We must show that $P(n)$ is true for all integers $n \geq 0$

Basis:

$$P(0): \sum_{i=1}^{1} i \cdot 2^i = 0 \cdot 2^{0+2} + 2$$

$$1 \cdot 2^1 = 0 + 2$$

$$2 = 2$$

Since both sides of $P(0)$ is 2, $P(0)$ is true.

Inductive Step: Assume for some k $P(k)$: $\sum_{i=1}^{k+1} i \cdot 2^i = (k)(2)^{k+2} + 2$

By substitution, $P(k+1)$: $\sum_{i=1}^{k+2} i \cdot 2^i = (k+1)(2)^{k+3} + 2$

We must show $P(k+1)$ is true

By definition of $\sum$, $P(k+1)$: $\sum_{i=1}^{k+1} i \cdot 2^i + (k+2) \cdot 2^{k+2}$

$$= (k)(2)^{k+2} + 2 + (k+2)(2)^{k+2}$$

$$= (k)(2)^{k+2} + (k+2)(2)^{k+2} + 2$$

$$= (k+k+2)(2)^{k+2} + 2 = (2k+2)(2)^{k+2} + 2$$

$$= (k+1)(2)(2)^{k+2} + 2$$

$$= (k+1)(2)^{k+3} + 2$$

Since, $(k+1)(2)^{k+3} + 2$ is the right side of $P(k+1)$, we have proved the basis step and the inductive step so the theorem is true.

QED

15. Exercise 5.2.23

$\frac{(n)(n+1)}{2}$ $\quad 7+8+9+10+\ldots+600$ is $\sum_{i=7}^{600} n$

$\frac{(600)(601)}{2} = \sum_{i=1}^{600} n = 1+2+3+4+5+6+\sum_{i=7}^{600} = 21 + \sum_{i=7}^{600}$

$\frac{360600}{2} = 180300$ $\quad 180300 - 21 = \sum_{i=7}^{600} = \boxed{180,279}$

16. Exercise 5.2.27

$5^3+5^4+5^5+\ldots+5^k = 5^3(1+5+5^2+5^3+\ldots+5^{k-3}$

$(5)^3\sum_{i=0}^{k-3}5^i = (125)\left(\frac{5^{k-2}-1}{5-1}\right) = \boxed{\left(\frac{125}{4}\right)(5^{k-2}-1)}$

17. Exercise 5.3.7a

$P(2)$: $2^2 < (2+1)!$

$4 < (3)!$

$4 < 6$

Since the inequality $4<6$ is true, $P(2)$ is also true

18. Exercise 5.3.7b

$P(k)$: $2^k < (k+1)!$

19. Exercise 5.3.7c

$P(k+1)$: $2^{k+1} < (k+2)!$

20. Exercise 5.3.7d

We must show $2^{k+1} < (k+2)!$ is true in the inductive step which would show that $P(k+1)$ is also true which would prove our entire theorem

$2^k < (k+1)!$

$2^{k+1} < (k)(k+1)!$

$(k+2)! = (k+2)(k+1)!$

$(k)(k+1)! < (k+2)(k+1)!$

$\therefore 2^{k+1} < (k+2)(k+1)!$

$\therefore P(k+1)$ is true

21. Exercise 5.3.9

Theorem: Let $P(n)$ be $6 \mid 7^n - 1 \ \forall n \in \mathbb{N}, n \geq 0$

Proof by Mathematical Induction:

We must show P_n is true for all $n \geq 0$

Basis:

$P(0)$: $6 \mid 7^n - 1 = 6 \mid 7^0 - 1 = 6 \mid (1-1) = 6 \mid 0$

Since $6 \mid 0$ is true, $P(0)$ is also true

Inductive Step:

Assume for some k that $P(k)$ is true, that is, $6 \mid 7^k - 1$

By Substitution, $P(k+1)$ equals $6 \mid 7^{k+1} - 1$

Since $6 \mid 7^k - 1$, $\exists m \in \mathbb{Z} \ni 7^k - 1 = 6m$

If we multiply both sides by 7, we get

$7(7^k - 1) = 7^{k+1} - 7 = 42m$

If we add 6 to both sides, we get $7^{k+1} - 1 = 6(7m+1)$

Since addition is closed under multiplication and subtraction, $7m+1$ is an integer.

By definition of divisibility, $7^{k+1} - 1$ is divisible by 6 so $P(k+1)$ is true

Since we have proved the basis step and the inductive step, the theorem is true

QED

22. Exercise 5.3.17

Theorem: Let $P(n)$ be $1+3n \leq 4^n$ $\forall n \in \mathbb{N} \ni n>0$

Proof by Mathematical Induction:

We must show $P(n)$ is true for all $n \geq 0$

Basis:

$P(0)$: $1+3(0) \leq 4^{(0)}$

$1+0 \leq 1$

$1 \leq 1$

Since $1 \leq 1$ is true, $P(0)$ is true

Inductive Step:

Assume for some k that $P(k)$ is true

By substitution, $P(K)$: $1+3k \leq 4^k$

We must show $P(k+1)$ is true

By substitution, $P(k+1)$: $1+3(k+1) \leq 4^{k+1}$

$3k+4 \leq 4(4^k)$

On the right side, multiplying the inductive hypothesis by 4, we get that $4+12k \leq (4)(4^k)$

Since $3k+4 < 12k+4 \leq (4)(4)^k$ by transitivity of $<$ and $\leq$, $3k+4 \leq (4)(4)^k$

Thus, $P(k+1)$ is true

Since we have proved both the basis and the inductive step, the theorem is true

QED