Problem set 11,12

Graded

Student

Total Points

100 / 100 pts

Question 1

Exercise 5.4.2

10 / 10 pts

✔ **− 0 pts** Correct

- **− 10 pts** no answer
- **− 10 pts** illegible
- **− 10 pts** wrong problem
- **− 7 pts** no proof
- **− 3 pts** incorrect skeleton
- **− 1.5 pts** incorrect P(b) statement
- **− 1.5 pts** incorrect P(k) statement
- **− 1.5 pts** incorrect P(k+1) statement
- **− 1 pt** 1 proof step missing / no reason/ mistake
- **− 3 pts** multiple proof steps missing /incorrect/ no reasons

Question 2

Exercise 5.4.7

10 / 10 pts

✔ **− 0 pts** Correct

− 10 pts no answer

− 10 pts illegible

− 10 pts wrong problem

− 7 pts no proof

− 3 pts incorrect skeleton

− 1.5 pts incorrect P(b) statement

− 1.5 pts incorrect P(k) statement

− 1.5 pts incorrect P(k+1) statement

− 1 pt 1 proof step missing / no reason

− 3 pts multiple prrof steps missing / no reasons

Question 3

Exercise 5.6.2

8 / 8 pts

✔ **− 0 pts** Correct

− 8 pts no answer

− 8 pts illegible

− 8 pts wrong problem

− 2 pts incorrect first term

− 2 pts incorrect second term

− 2 pts incorrect third term

− 2 pts incorrect fourth term

Question 4

Exercise 5.6.6

8 / 8 pts

- ✔ **– 0 pts** Correct
- **– 8 pts** no answer
- **– 8 pts** illegible
- **– 8 pts** wrong problem
- **– 2 pts** incorrect first term
- **– 2 pts** incorrect second term
- **– 2 pts** incorrect third term
- **– 2 pts** incorrect fourth term

Question 5

Exercise 5.6.8

8 / 8 pts

- ✔ **– 0 pts** Correct
- **– 8 pts** no answer
- **– 8 pts** illegible
- **– 8 pts** wrong problem
- **– 2 pts** incorrect first term
- **– 2 pts** incorrect second term
- **– 2 pts** incorrect third term
- **– 2 pts** incorrect fourth term

Question 6

Exercise 5.6.12

10 / 10 pts

- ✔ **– 0 pts** Correct
- **– 10 pts** no answer
- **– 10 pts** illegible
- **– 10 pts** wrong problem
- **– 2 pts** incorrect k expression
- **– 3 pts** incorrect k-1 expression
- **– 3 pts** incorrect substitution for k+1 expression
- **– 2 pts** algebraic errors

Question 7

Exercise 5.6.14

10 / 10 pts

- ✔ **– 0 pts** Correct
- **– 10 pts** incorrect template
- **– 10 pts** no answer
- **– 10 pts** illegible
- **– 10 pts** wrong problem
- **– 2 pts** incorrect k-1 expression
- **– 2 pts** incorrect k-2 expression
- **– 4 pts** incorrect substitution for k expression
- **– 2 pts** algebraic errors / missing

Question 8

Exercise 5.6.38a

8 / 8 pts

- ✔ **– 0 pts** Correct
- **– 8 pts** no answer
- **– 8 pts** illegible
- **– 8 pts** wrong problem
- **– 8 pts** incorrect formula
- **– 4 pts** partially correct formula

Question 9

Exercise 5.6.38b

8 / 8 pts

- ✔ **– 0 pts** Correct
- **– 8 pts** no answer / incorrect
- **– 8 pts** illegible
- **– 8 pts** wrong problem
- **– 4 pts** correctly applied incorrect formula
- **– 4 pts** incorrectly applied correct formula

Question 10

Exercise 5.6.38c

8 / 8 pts

✔ **– 0 pts** Correct

- **– 8 pts** no answer
- **– 8 pts** illegible
- **– 8 pts** wrong problem
- **– 4 pts** correctly applied incorrect formula
- **– 4 pts** incorrectly applied correct formula

Question 11

Exercise 2.5.3

3 / 3 pts

✔ **– 0 pts** Correct

- **– 3 pts** no answer
- **– 3 pts** illegible
- **– 3 pts** wrong problem
- **– 3 pts** incorrect

Question 12

Exercise 2.5.9

3 / 3 pts

✔ **– 0 pts** Correct

- **– 3 pts** no answer
- **– 3 pts** illegible
- **– 3 pts** wrong problem
- **– 3 pts** incorrect

Question 13

Exercise 5.6.25b

3 / 3 pts

✔ **– 0 pts** Correct

- **– 3 pts** no answer
- **– 3 pts** illegible
- **– 3 pts** wrong problem
- **– 3 pts** incorrect

Question 14

Exercise 5.6.25c

3 / 3 pts

✔ **− 0 pts** Correct

− 3 pts incorrect template

− 3 pts no answer

− 3 pts illegible

− 3 pts wrong problem

− 3 pts incorrect

Put your answer in each indicated box. Answers must be handwritten, legible and use correct notation.
Study the answers in Appendix A to similar problems so you know what your approach should be.
Larger boxes indicate that you are expected to provide substantial detail.

1. Exercise 5.4.2

Theorem: Prove b_n is divisible by 4 for all integers ≥ 1

$b_1 = 4 \quad b_2 = 12 \quad b_k = b_{k-1} + b_{k-2}$ for all $k \geq 3$

Let $P(n)$: b_n is divisible by 4

Proof by mathematical induction:

We must show $P(n)$ is true for all integers $n \geq 1$

Basis:

$4 = (4)(1)$ and $12 = (4)(3)$ so by definition of divisibility, $P(1)$ is true and $P(2)$ is true

$P(3)$: $b_3 = b_2 + b_1 = 12 + 4 = 16$

$16 = (4)(4)$ so by definition of divisibility, 16 is divisible by 4 so $P(3)$ is true

Inductive Step:

Assume for some k that $P(1), \ldots, P(k)$ are all true

We must show $P(k+1)$ is true

By substitution, $P(k+1)$: $b_{k+1} = b_{k-1} + b_k$

Since $P(k-1)$ and $P(k)$ are both true, b_{k-1} and b_k are both divisible by 4

By definition of divisibility, $\exists a, b \in \mathbb{Z} \ni \quad b_{k-1} = 4a$ and $b_k = 4b$

By substitution, $b_{k+1} = 4a + 4b = 4(a+b)$

Since $\mathbb{Z}$ is closed under addition, $a+b$ is also an integer

By definition of divisibility, since $(a+b)$ is an integer, b_{k+1} is divisible by 4

Since we have proved the basis and the inductive step, our theorem is true

QED

2. Exercise 5.4.7

Theorem:

$g_1 = 3$, $g_2 = 5$, $g_k = 3g_{k-1} - 2g_{k-2}$ for $k \geq 3$

$g_n = 2^n + 1$ for all integers $n \geq 1$

Proof by mathematical induction:

Let $P(n): g_n = 2^n + 1$

We must show $P(n)$ is true for all integers $n \geq 1$

Basis:

$P(1) = 2^1 + 1 = 3 = g_1$

$P(2) = 2^2 + 1 = 5 = g_2$

$P(3) = g_3 = 3(5) - 2(3) = 9 = 2^3 + 1$

Since we have shown that $P(1), P(2), P(3)$ are all true, we have proved our basis.

Inductive step:

Assume that for some k, $P(1), \ldots, P(k)$ is true

We must show $P(k+1)$ is true

By substitution, $P(k): g_k = 2^k + 1$

$P(k-1): g_{k-1} = 2^{k-1} + 1$

By substitution, $P(k+1): g_{k+1} = 3g_k - 2g_{k-1} = 3(2^k + 1) - 2(2^{k-1} + 1)$

$3(2^k + 1) - 2(2^{k-1} + 1) = (3)(2^k) + (3) - 2^k - 2 = 2(2)^k + 1$

$= 2^{k+1} + 1$

Since $P(k+1)$ is true, we have proved our inductive step

Since both the basis and the inductive step are true, our theorem is true by strong mathematical induction

QED

For problems that ask to "show that," see the answers to similar problems In Appendix A to understand what "show that" means.

3. Exercise 5.6.2

$b_k = b_{k-1} + 3k$ for $k \geq 2$

$b_1 = 1$

$b_2 = 1 + 3(2) = 7$

$b_3 = 7 + 3(3) = 16$

$b_4 = 16 + 3(4) = 28$

$b_5 = 28 + 3(5) = 43$

4. Exercise 5.6.6

$t_k = t_{k-1} + 2t_{k-2}$ for all integers $k \geq 2$

$t_0 = -1$

$t_1 = 2$

$t_2 = 2 + 2(-1) = 0$

$t_3 = 0 + 2(2) = 4$

$t_4 = 4 + 2(0) = 4$

$t_5 = 4 + 2(4) = 12$

5. Exercise 5.6.8

$v_k = v_{k-1} + v_{k-2} + 1$ for all integers $k \geq 3$

$v_1 = 1$ $v_2 = 3$

$v_3 = 3 + 1 + 1 = 5$

$v_4 = 5 + 3 + 1 = 9$

$v_5 = 9 + 5 + 1 = 15$

$v_6 = 15 + 9 + 1 = 25$

6. Exercise 5.6.12

$s_n = \frac{(-1)^n}{n!}$ $\quad s_k = \frac{-s_{k-1}}{k}$

Substitute k and $k-1$ into s_n to get $s_k = \frac{(-1)^k}{k!}$ and $s_{k-1} = \frac{(-1)^{k-1}}{(k-1)!}$

$$s_k = -\frac{s_{k-1}}{k}$$

$$= -\frac{\left(\frac{(-1)^{k-1}}{(k-1)!}\right)}{k}$$ substitute s_{k-1}

$$= \frac{-(-1)^{k-1}}{k(k-1)!}$$ simplify with algebra

$$= \frac{(-1)^k}{k!}$$ simplify further with algebra

7. Exercise 5.6.14

$d_n = 3^n - 2^n$ $\quad d_k = 5d_{k-1} - 6d_{k-2}$

Substitute $k-1$ and $k-2$ into d_n to get

$d_{k-1} = 3^{k-1} - 2^{k-1}$

$d_{k-2} = 3^{k-2} - 2^{k-2}$

By substitution $d_k = 5(3^{k-1} - 2^{k-1}) - 6(3^{k-2} - 2^{k-2})$

By factoring $d_k = (5)(3^{k-1}) - (5)(2^{k-1}) - 6(3^{k-2}) + 6(2^{k-2})$

By using algebra $d_k = (5)(3^{k-1}) - (5)(2^{k-1}) - 2 \cdot 3(3^{k-2}) + 3 \cdot 2(2^{k-2})$

By using exponent properties $d_k = (5)(3^{k-1}) - (5)(2^{k-1}) - 2(3^{k-1}) + 3(2^{k-1})$

$d_k = (3)(3^{k-1}) - (2)(2^{k-1})$

$d_k = (3)^k - (2)^k$

8. Exercise 5.6.38a

$$S_n = S_{n-1}\left(1 + \frac{.03}{12}\right)$$

9. Exercise 5.6.38b

$S_0 = \$10{,}000$

$$S_{12} = 10{,}000\left(1 + \frac{.03}{12}\right)^{12} \approx \boxed{\$10{,}304.16}$$

10. Exercise 5.6.38c

$$\frac{\$10{,}304.16 - \$10{,}000.00}{\$10{,}000} = \frac{\$304.16}{\$10{,}000} \approx 0.0304$$

$$\boxed{\approx 3.04\%}$$

11. Exercise 2.5.3

100011111

12. Exercise 2.5.9

54

11011 0

$2^1 + 2^2 + 2^4 + 2^5$

$2+4 + 16 + 32 = 54$

13. Exercise 5.6.25b

$F_{k+2} = F_{k+1} + F_k$

14. Exercise 5.6.25c

$F_{k+3} = F_{k+2} + F_{k+1}$