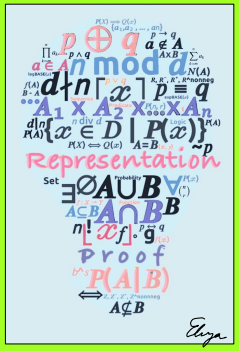
Discrete Structures



Lecture 4: Applications and arguments

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HUNTER

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Last time

- ★ **Every equivalence change must have a reason**
- ★ **Every step in a proof must have a reason**
- DeMorgan's laws: $\sim(p \wedge q) \equiv (\sim p \vee \sim q)$ $\sim(p \vee q) \equiv (\sim p \wedge \sim q)$
- Commutative laws: $p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$
- Associative laws: $(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Distributive laws: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
  $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- Identity laws: $p \vee \boldsymbol{c} \equiv p$ $p \wedge \boldsymbol{t} \equiv p$
- Negation laws: $p \vee \sim p \equiv \boldsymbol{t}$ $p \wedge \sim p \equiv \boldsymbol{c}$
- Idempotent laws: $p \vee p \equiv p$ $p \wedge p \equiv p$
- Universal bound laws: $p \vee \boldsymbol{t} \equiv \boldsymbol{t}$ $p \wedge \boldsymbol{c} \equiv \boldsymbol{c}$
- Absorption laws: $p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$
- How to prove logical equivalence with a truth table or with laws

We saw how logic simplified propositions and justified proof steps

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Today's outline

- Applications of propositional laws
- Introduction to argumentation
- Some valid argument forms

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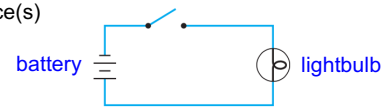
Propositional logic for circuit design (1)

- Current flows through wire indicated (blue lines)
- When it reaches a closed switch it continues, but an open switch stops it

- Switching circuit is a path defined by wiring, power source, switches, and device(s)



[Claude Shannon, 1928]

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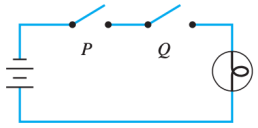
Propositional logic for circuit design (2)

- When switches are in series, all must be closed for switching circuit to function



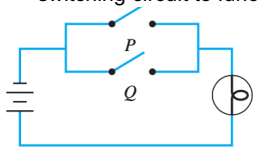
Switches "in series"

Switches		Light Bulb
P	*Q*	State
closed	closed	on
closed	open	off
open	closed	off
open	open	off

Do these tables look familiar?

conjunction!

- When switches are in parallel, at least one must be closed for switching circuit to function



Switches "in parallel"

Switches		Light Bulb
P	*Q*	State
closed	closed	on
closed	open	on
open	closed	on
open	open	off

disjunction!

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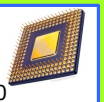
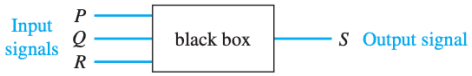
Modern computer hardware



- To reason about circuit design, electrical engineers write 1 and 0 (bits) instead of "on" and "off" or T and F
- In 1971, modern computers began to replace wiring with digital logic circuits
- Switches replaced with devices that respond to signals that are discrete rather than continuous,
- CPU design is still based on reasoning with logic
- Combinations of signals can be transformed by black boxes that are best described by input/output tables



Input			Output
P	*Q*	*R*	*S*
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	1
0	0	0	0

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Other applications of propositional logic

- Consistent system specifications

  The error message is stored in the buffer or it is retransmitted.
  The error message is not stored in the buffer.
  If the error message is stored in the buffer, then it is retransmitted.

p: The error message is stored in the buffer
q: The error message is retransmitted
Specifications in propositional logic: $p \vee q$, $\sim p$, $p \rightarrow q$
What truth values for p and q would make this consistent?

- Boolean based web page search
- Solving logic puzzles

An island has two kinds of people: knights (who always tell the truth) and knaves (who always lie). Person A tells you "B is a knight" and person B tells you they are of different types. Is each of them a knight or a knave?

Any questions?

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- Applications of propositional laws
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Necessary and sufficient

- Necessary condition is required for something to be true $\sim p \rightarrow \sim q$
  p is necessary for q
  To matriculate at Hunter you must have a high school diploma
  $\sim diploma \rightarrow \sim matriculation$
- Sufficient condition permits something to be true $p \rightarrow q$
  p is sufficient for q
  To buy cigarettes it is enough to be 18 $be18 \rightarrow buycigarettes$
- Necessary and sufficient condition is both $(p \rightarrow q) \wedge (\sim p \rightarrow \sim q)$
  You must be 21 to buy alcohol in NYC and that is the only condition
  $(be21 \rightarrow buyalcohol) \wedge (\sim be21 \rightarrow \sim buyalcohol)$

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Argument

Parker is a turkey or annoying
Parker is not a turkey
∴ Parker is annoying

- Argument = sequence of propositions intended to show that an assertion is true
- Conclusion = final statement or statement form
  Parker is annoying
- Premises = all but last statement in an argument
  Parker is a turkey or annoying
  Parker is not a turkey
- ∴ is read “therefore” and should come just before the conclusion

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Validity

- The form of an argument is different from its content
- Argument form = sequence of proposition forms

p = Parker is a turkey or annoying
q = Parker is not a turkey
∴ Parker is annoying

$p \vee q$ Juan is home or at work
$\sim p$ Juan is not home
$\therefore q$ ∴ Juan is at work

p and q are propositional variables in this argument
$p \vee q$, $\sim p$, q are propositional forms in this argument

- Valid argument form when any propositions are substituted for its propositional variables and the premises are all true, then the conclusion is also true
- Valid argument is one with a valid form

Validity ≠ truth

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How to prove an argument form is valid

- Build a truth table for propositional variables, premises and conclusion
- Identify all critical rows (those where all premises are true)
- Confirm conclusion is true in all critical rows (ignore the others)

$p \vee (q \vee r)$
$\sim r$
$\therefore p \vee q$

p	q	r	$q \vee r$	$\sim r$	$p \vee (q \vee r)$	$p \vee q$
T	T	T	T	F	T	
T	T	F	T	T	T	T
T	F	T	T	F	T	
T	F	F	F	T	T	T
F	T	T	T	F	T	
F	T	F	T	T	T	T
F	F	T	T	F	F	
F	F	F	F	T	F	

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Today's outline

- ✓ Applications of propositional laws
- ✓ Introduction to argumentation
- • Some valid argument forms

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Modus ponens and modus tollens

p = Parker is a turkey
q = Parker is annoying

modus ponens

$p \rightarrow q$ $p \rightarrow q$ = If Parker is a turkey then Parker is annoying
p p = Parker is a turkey
$\therefore q$

What can you conclude?

modus tollens

$p \rightarrow q$ $p \rightarrow q$ = If Parker is a turkey then Parker is annoying
$\sim q$ $\sim q$ = Parker is not annoying
$\therefore \sim p$

What can you conclude?

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Disjunctive addition and disjunctive syllogism

p = Parker is a turkey
q = Parker is annoying

disjunctive addition

p — p = Parker is a turkey
$\therefore p \vee q$ — What can you conclude?

disjunctive syllogism (aka elimination)

$p \vee q$ — $p \vee q$ = Parker is a turkey or annoying
$\sim p$ — $\sim p$ = Parker is not a turkey
$\therefore q$ — What can you conclude?

$p \vee q$ — $p \vee q$ = Parker is a turkey or annoying
$\sim q$ — $\sim q$ =Parker is not annoying
$\therefore p$ — What can you conclude?

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Conjunction and conjunctive simplification

p = Parker is a turkey
q = Parker is annoying

Conjunction

p — p = Parker is a turkey
q — q = Parker is annoying
$\therefore p \wedge q$ — What can you conclude?

Conjunctive simplification (aka specialization)

$p \wedge q$ — $p \wedge q$ = Parker is a turkey and Parker is annoying
$\therefore p$ — What can you conclude?

$p \wedge q$ — What else can you conclude?
$\therefore q$

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Find the mistake in the program

There is an undeclared variable or there is a syntax error in lines 1-5.
If there is a syntax error in the lines 1-5, then there is a missing semicolon or a variable name is misspelled.
There is not a missing semicolon.
There is not a misspelled variable name.
Prove there is an undeclared variable.

1 $u \vee s$	premise
2 $s \rightarrow (m \vee v)$	premise
3 $\sim m$	premise
4 $\sim v$	premise
5 $\sim m \wedge \sim v$	conjunction of 3 and 4
6 $\sim(m \vee v) \rightarrow \sim s$	contrapositive of 2
7 $(\sim m \wedge \sim v) \rightarrow \sim s$	DeMorgan on 6
8 $\sim s$	modus ponens on 5 and 7
9 u	disjunctive syllogism on 1 and 8

How many variables?
What are they?
u = undeclared variable
s = syntax error
m = semicolon missing
v = variable name
How many premises?
What are they?

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Prove we will be home by sunset

It is not sunny this afternoon and it is colder than yesterday.
We will go swimming if it is sunny.
If we do not go swimming then we will take a canoe trip.
If we take a canoe trip then we will be home by sunset

1 $\sim p \wedge q$	premise
2 $r \rightarrow p$	premise
3 $\sim r \rightarrow s$	premise
4 $s \rightarrow w$	premise
5 $\sim p$	conjunctive simplification of 1
6 $\sim r$	modus tollens on 2 and 5
7 s	modus ponens on 3 and 6
8 w	modus ponens on 7 and 4

How many variables?
What are they?
p = sunny
q = colder
r = swim
s = canoe
w = home by sunset
How many premises?
What are they?

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Hypothetical syllogism

p = Parker is a turkey
q = Parker is annoying
r = Parker is smart

Hypothetical syllogism (aka transitivity of $\rightarrow$)

$p \rightarrow q$ $\quad p \rightarrow q$ = If Parker is a turkey then Parker is annoying
$q \rightarrow r$ $\quad q \rightarrow r$ = If Parker is is annoying then Parker is smart
$\therefore p \rightarrow r$ What can you conclude?

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Prove the last sentence

If you send me an email, then I will finish writing the program.
If you do not send me an email, then I will go to sleep early
If I go to sleep early, then I will wake up feeling refreshed.
If I do not finish writing the program, then I will wake up feeling refreshed.

How many variables?
What are they?
p = email
q = finish
r = early
s = refreshed
How many premises?
What are they?

1	$p \rightarrow q$	premise
2	$\sim p \rightarrow r$	premise
3	$r \rightarrow s$	premise
	Show $\sim q \rightarrow s$	
4	$\sim q \rightarrow \sim p$	contrapositive of 1
5	$\sim q \rightarrow r$	hypothetical syllogism on 4 and 2
6	$\sim q \rightarrow s$	hypothetical syllogism on 5 and 3

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Propositional proof by cases

p = Parker is a turkey
q = Parker is annoying
r = Parker is smart

proof by cases
$p \vee q$
$p \rightarrow r$
$q \rightarrow r$
$\therefore r$

$p \vee q$ = Either Parker is a turkey or Parker is annoying
$p \rightarrow r$ = If Parker is a turkey then Parker is smart
$q \rightarrow r$ =If Parker is annoying then Parker is smart

Case 1: p
Case 2: q
What can you conclude?

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Proof methods (so far)

Truth table

Sequence of statements with reasons

Valid argument forms (modus ponens, modus tollens,…)

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What you should know

- **Validity ≠ truth**
- What an argument form is and how to prove one is valid
- How to use each of the many argument forms covered here

Next up: *Proofs*

Time to finish up that Opening sheet!

Problem set 3,4 is due on Thursday, September 14 at 11PM

Any questions?