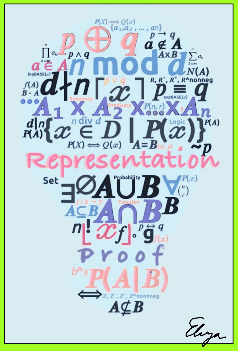
Discrete Structures



Lecture 2: Introduction to logic

Susan L. Epstein

HUNTER

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Last time

- What you can expect in CSCI 150
- ★ **This is a course about Truth, spelled P-R-O-O-F**
- Notation for sets
- Cartesian products
- Delimiter conventions and cardinality
- How to define and use relations and functions

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Today's outline

- Introduction to logical reasoning
- Logical equivalence

Why logic?

- Natural language is one used for human communication
  Japanese Spanish Arabic Urdu
- Can convert most natural language sentences into propositional logic
- Logic provides symbolic representations for ideas
- Logic has many ways to say the same thing
- Logic also has clear tests for whether 2 statements *do* say the same thing
- Given individual statements about the world and information about whether or not they are true, a computer can manipulate them to reason about the world

Truth

- Mathematics is a set of languages
  logic number theory combinatorics probability
- Each language has its own vocabulary
  conjunct conditional universal tautology
  - Its own set of symbols
    $\vee$ $\exists$ $\Leftrightarrow$ ~
  - Its own ways to express ideas
    truth table sentence grammar
  - Its own axioms, ideas that it accepts as true to begin with
    truth falsity contradiction
- Mathematicians look for new truths in a language
- Proof method = how to argue for a new truth

- We focus on truth because it allows us to trust the source
  interest on a loan trajectory for a rocket diagnosis of a disease

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Truth matters

Do not invent mathematical notation or language

- Common frame of reference
- Build from truth to more truth
- Mathematical truth provides a substantial body of knowledge
- Logic provides symbolic representations for ideas
- A computer can manipulate those forms to reason about the world
- In this course, there are only 2 truth values: T = true and F = false
  - Bit 1 represents T = true
  - Bit 0 represents F = false

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Propositional logic

- Proposition = sentence that declares a fact
- Proposition is **either true or false** but not both
  - George Washington was president of the United States.
  - 2 + 2 = 4
  - 2 + 2 = 7
  - Queen Elizabeth was president of the United States
- Some sentences are not propositions
  - What time is it? $3x - 4y$
  - Read the syllabus carefully. Look out!
- Propositional variable = letter that represents a proposition
  - $p, q, r, s, \ldots$
- Given some propositions, logic builds new propositions with logical operators

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Truth table

Truth table specifies every possible combination of truth and falsity for all propositional variables in those propositions

p	proposition about p
T	$1, 0$
F	

p	q	proposition about p, q
T	T	
T	F	$11, 10, 01, 00$
F	T	
F	F	

p	q	r	proposition about p, q, r
T	T	T	$111, 110, 101, 100, 011, 010, 001, 000$
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Required **sequence of values** is like counting **down** in binary

How many lines in a truth table for n variables?

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Logical operator: negation

- $\sim p$ denotes the negation of proposition p (aka $\neg p$)

p	$\sim p$
T	F
F	T

- Read $\sim p$ as "not p" or "it is false that p" or "it is not the case that p"

p: Jane's preferred OS is Linux
$\sim p$: Jane's preferred OS is not Linux
Jane's preferred OS isn't Linux
Linux is not Jane's preferred OS
Linux isn't Jane's preferred OS

p: My hard drive has at least 3TB of memory
$\sim p$: My hard drive does not have at least 3TB of memory
My hard drive has less than 3TB of memory

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iff = if and only if

Logical operator: conjunction

- $p \wedge q$ denotes the conjunction of p and q
- $p \wedge q$ is true **iff** p is true and q is true (p and q are both true)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- Read $p \wedge q$ as "p and q"

p: I am hungry q: I am wide awake
$p \wedge q$: I am hungry and I am wide awake
I am hungry and wide awake
I am both hungry and wide awake

- Translation hint: "but" is really "and" as in "I like you but I love football"

I am hungry but wide awake

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Logical operator: disjunction

- $p \lor q$ denotes the disjunction of p and q (aka inclusive or)
- $p \lor q$ is true **iff** at least one of p and q is true

p	q	$p \lor q$
T	T	T
T	F	T
F	T	T
F	F	F

- $p \lor q$ is false only if both p and q are false
- Read $p \lor q$ as "p or q"

$x \leq 3$ $(x < 3) \lor (x = 3)$

p: I am hungry q: I am wide awake

$p \lor q$: Either I am hungry or I am wide awake
I am hungry or I am wide awake
I am hungry or wide awake

Exclusive or

- $p \oplus q$ denotes the exclusive or of p and q
- Read $p \oplus q$ as "exactly 1 of p and q is true"

p	q	$p \lor q$	$p \oplus q$
T	T	T	F
T	F	T	T
F	T	T	T
F	F	F	F

p: I am hungry q: I am wide awake

$p \oplus q$: I am hungry or I am wide awake but not both
Exactly 1 of "I am hungry" and "I am wide awake" is true

Tautology

- Tautology = proposition that is always true for any combination of truth values of its component propositions
- Denote tautology as t

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

Either Jim is alive or Jim is not alive

Jim is alive or Jim is not alive

Jim is alive or not alive

- Test for tautology: truth table column entries are all T

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Contradiction

- Contradiction = proposition that is always false for any combination of truth values of its component propositions
- Denote contradiction by c

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

Jim is both alive and not alive

Jim is alive and not alive

Jim is alive and Jim is not alive

- Test for contradiction: truth table column entries are all F

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Example

Evaluate by truth table $((p \wedge q) \vee (p \wedge \sim q)) \wedge \sim p$

p	q	$p \wedge q$	$\sim q$	$p \wedge \sim q$	$(p \wedge q) \vee (p \wedge \sim q)$	$\sim p$	$((p \wedge q) \vee (p \wedge \sim q)) \wedge \sim p$

Example

Evaluate by truth table $((p \wedge q) \vee (p \wedge \sim q)) \wedge \sim p$

p	q	$p \wedge q$	$\sim q$	$p \wedge \sim q$	$(p \wedge q) \vee (p \wedge \sim q)$	$\sim p$	$((p \wedge q) \vee (p \wedge \sim q)) \wedge \sim p$
T	T						
T	F						
F	T						
F	F						

Example

Evaluate by truth table $((p \wedge q) \vee (p \wedge {\sim}q)) \wedge {\sim}p$

p	q	$p \wedge q$	${\sim}q$	$p \wedge {\sim}q$	$(p \wedge q) \vee (p \wedge {\sim}q)$	${\sim}p$	$((p \wedge q) \vee (p \wedge {\sim}q)) \wedge {\sim}p$
T	T	T					
T	F	F					
F	T	F					
F	F	F					

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Example

Evaluate by truth table $((p \wedge q) \vee (p \wedge {\sim}q)) \wedge {\sim}p$

p	q	$p \wedge q$	${\sim}q$	$p \wedge {\sim}q$	$(p \wedge q) \vee (p \wedge {\sim}q)$	${\sim}p$	$((p \wedge q) \vee (p \wedge {\sim}q)) \wedge {\sim}p$
T	T	T	F				
T	F	F	T				
F	T	F	F				
F	F	F	T				

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Example

Evaluate by truth table $((p \wedge q) \vee (p \wedge \sim q)) \wedge \sim p$

p	q	$p \wedge q$	$\sim q$	$p \wedge \sim q$	$(p \wedge q) \vee (p \wedge \sim q)$	$\sim p$	$((p \wedge q) \vee (p \wedge \sim q)) \wedge \sim p$
T	T	T	F	F	T		
T	F	F	T	T	T		
F	T	F	F	F	F		
F	F	F	T	F	F		

Example

Evaluate by truth table $((p \wedge q) \vee (p \wedge \sim q)) \wedge \sim p$

p	q	$p \wedge q$	$\sim q$	$p \wedge \sim q$	$(p \wedge q) \vee (p \wedge \sim q)$	$\sim p$	$((p \wedge q) \vee (p \wedge \sim q)) \wedge \sim p$
T	T	T	F	F	T		
T	F	F	T	T	T		
F	T	F	F	F	F		
F	F	F	T	F	F		

Example

Evaluate by truth table $((p \wedge q) \vee (p \wedge \sim q)) \wedge \sim p$

p	q	$p \wedge q$	$\sim q$	$p \wedge \sim q$	$(p \wedge q) \vee (p \wedge \sim q)$	$\sim p$	$((p \wedge q) \vee (p \wedge \sim q)) \wedge \sim p$
T	T	T	F	F	T	F	
T	F	F	T	T	T	F	
F	T	F	F	F	F	T	
F	F	F	T	F	F	T	

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Example

Evaluate by truth table $((p \wedge q) \vee (p \wedge \sim q)) \wedge \sim p$

p	q	$p \wedge q$	$\sim q$	$p \wedge \sim q$	$(p \wedge q) \vee (p \wedge \sim q)$	$\sim p$	$((p \wedge q) \vee (p \wedge \sim q)) \wedge \sim p$
T	T	T	F	F	T	F	F
T	F	F	T	T	T	F	F
F	T	F	F	F	F	T	F
F	F	F	T	F	F	T	F

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Comments

- Translate from English to logic carefully

  but nor neither either
- Hint: "neither A nor B" is "not A and not B" as in "I am neither hungry nor tired"
- Translate from logic to English rigidly and then refine it

  p: "I am hungry" q: "I am tired"

  $p \oplus q$ "Exactly one of ' I am hungry' and 'I am tired' is true"

  "I am either hungry or tired but not both"
- If you understand a logical operator you should not have to memorize its truth table

Any questions?

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Code and logic

- Negation as !, NOT
- Conjunction as AND for logic variables
- Disjunction as OR for bitwise operations

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Today's outline

- ✓ Introduction to logical reasoning
- Logical equivalence

Definition of logical equivalence

- Logically equivalent (aka equivalent) propositions have the same truth values for all truth values of their components
- Denoted as $p \equiv q$

p	$\sim p$	$\sim\sim p$
T	F	T
F	T	F

- Because p and $\sim\sim p$ satisfy this definition of logical equivalend, we have the

double negation law: $p \equiv \sim\sim p$

- Logical equivalence allows one logical statement to be substituted for another
- Test for logical equivalence: build a truth table and test the 2 columns

Quick quiz

Are $\sim(p \wedge q)$ and $\sim p \wedge \sim q$ logically equivalent?

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge \sim q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

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What you should know

★ **Do not invent mathematical notation or language**

- What a proposition is
- Definitions of negation, conjunction, disjunction, and exclusive or
- What conjuncts, disjuncts, tautologies, and contradictions are in formal logic
- How to build and use a truth table

Next up: *Laws and logical equivalence*

Time to finish up that Opening sheet!

Problem set 1,2 is due on Friday, September 8 at 11PM

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