(5) What are the following:

- Proposition
- Propositional Form
- Argument
- Argument Form
- Valid Argument Form

(10) Some truth table questions and proving valid argument form. I can’t remember it so here is an example of equal difficulty:

$$
\begin{array}{l}
p \land (q \lor \neg r) \\
\neg p \to (q \land r) \\
\hline
\therefore q \land r
\end{array}
$$

Prove whether this is a valid argument form using a truth table (beware: these tests don’t always give valid argument forms so don’t assume they are valid → hint from me :^))

(5) Define the Method of Exhaustion. What must be true about domain A for the method to be any useful?

(10) Prove or disprove the following: If x and y are even, then $2|xy$.

(10)

a. What does $a|b$ mean (rigorous definition)?
b. Prove or disprove the following: If $a|bc$, then $a|b$ or $a|c$.

(10)

a. Prove $\sqrt{7}$ is irrational.
b. Show why this proof would not work on $\sqrt{9}$. "$\sqrt{9}$ is rational" is not sufficient proof. You must show what step of your proof in part a would not work with $\sqrt{7}$.

(10) A combinatorics question. It was essentially the following problem but with different numbers (the method of solving is exactly the same).

(10) Kleebob is a card game played with a deck consisting of six suits each with 13 denominations. Like a poker deck, the denominations are A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, and K. The six suits are □ (boxes), ♣ (clubs), ◇ (diamonds), ♡ (hearts), ♠ (spades), and △ (triangles). Boxes and triangles are green, clubs and spades are black, and diamonds and hearts are red. Every player in a game is dealt 6 cards. These 6 cards are called a "hand", and the order they are dealt in does not matter.

Hands are "named" in a style similar to poker. For example "a pair" has exactly two cards of the same denomination and no other cards that match in denomination. "Two pair" are two cards of one denomination, two cards of another denomination, and two cards that are unrelated to each other or to either pair. "Three of a kind" is three cards of the same denomination and three cards unrelated to those or each other.

Leave your answers to the questions below as combinatorial expressions. When appropriate, for example, $\binom{4}{2}$ is better than 6. Use the empty space for you work, but be sure your answer is in the box.

a. How many hands have exactly one pair?

b. How many hands have 5 of a kind?

c. How many hands have exactly one pair and one three of a kind?

d. How many hands have only green cards?

e. How many hands have 5 of a kind and a pair?

(10) Prove some summation = linear equation proof by induction. I can’t remember the exact question so here is one of equal difficulty:

Prove $\displaystyle\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1} \quad \forall n \in \mathbb{Z}^+$

(10)

a. Prove that a Completed Graph has [n(n-1)] / 2 vertices through mathematical induction.

b. Show that a simple graph has less than or equal to [n(n-1)] / 2 vertices.

(10) A test has eight questions and five topics. The topics are Induction, Strong Induction, Functions, Graph Theory, and Set Theory.

a. Must there be at least two questions on one topic?
b. Must there be at least two questions on Graph Theory?
c. There are 3 questions on Induction, 2 on Strong Induction, 1 on Functions, 2 on Graph Theory, and 2 on Set Theory. But of course, $3+2+1+2+2 = 10$ which is not $= 8$. How is this possible?

(10) We know that n1+n2+n3+...+nk-1+nk is equal to (n1+n2)+n3+...+(nk-1+nk) and likewise (n1+(n2+n3))+...+(nk-1)+nk. Essentially, the sum of k numbers does not change no matter the order in which you add them. Prove that for any number of even integers, the sum will be even no matter which order you add them by strong induction.