Question 2

Truth Table (note column headers were corrected) **15** / 15 pts

2.1 **p,q,r in right form** **2** / 2 pts

✔ **+ 2 pts** Correct: In canonical order. From the top: TTT
TTF
TFT
TFF
FTT
FTF
FFT
FFF

+ 1 pt Anything that gets all 8 values BUT getting this wrong DOES effect other grades.

+ 0 pts incorrect

2.2 **q -> r** **2** / 2 pts

✔ **+ 2 pts** Correct: Note heading was corrected in class. T
F
T
T
T
F
T
T

+ 0 pts incorrect. Note that error in order of part 1 WILL result in 0 here.

2.3 **q -> p** **2** / 2 pts

✔ **+ 2 pts** Correct: Note heading corrected in class. T
T
T
T
F
F
T
T

+ 0 pts incorrect. Note that error in order of part 1 WILL result in 0 here.

2.4 **(p^r) -> q** **2** / 2 pts

✔ **+ 2 pts** Correct: Note heading corrected in class. T
T
F
T
T
T
T
T. No harm in writing in values of $p \wedge r$

+ 0 pts incorrect. Note that error in order of part 1 WILL result in 0 here.

2.5 **p** **2** / 2 pts

✔ **+ 2 pts** Correct: Heading corrected in class. T
T
F
F
T
T
F
F

+ 0 pts incorrect. Note that error in order of part 1 WILL result in 0 here.

2.6 **why valid or not** **5** / 5 pts

✔ **+ 5 pts** Correct: No, this is not valid because line 4 has all hypotheses true but the conclusion false.

+ 3 pts Incorrect answer that clearly indicates knowledge of definition.

+ 2 pts "No" only, no good reason

+ 0 pts incorrect

Question 3

cube root of 2 is irrational **10** / 15 pts

3.1 **divisable by 2** **7** / 7 pts

✔ **+ 7 pts** Correct: Proof by contrapositive, I'll show that $\forall a \in \mathbb{Z}$ if $2 \not| a$ then $2 \not| a^3$. So let a_0 be an integer not divisible by 2. Then $a_0 = 2k + 1$ for some $k \in \mathbb{Z}$. So $a^3 = (2k+1)^3 = 8k^3 + 12k^2 + 6k + 1$ which has remainder of 1 after dividing by 2, so is not divisible by 2.

+ 5 pts close but flawed

+ 3 pts For a setup ("I'm assuming this to show that") that would work only

+ 1 pt Indicates knowledge of what | means (or odd/even)

+ 0 pts incorrect

– 2 pts using $a|2$ or $\frac{a}{2}$ when $2|a$ is intended

3.2 **cube root 2** **3** / 8 pts

+ 8 pts Correct: A proof by contradiction, I'll assume $\sqrt[3]{2}$ is rational and derive a contradiction: If it is rational, then $\sqrt[3]{2} = p/q$ where $p, q \in \mathbb{Z}, q \neq 0, p, q$ in lowest terms. Then $2 = p^3/q^3$ so $2q^3 = p^3$ so $2|p^3$ and by above problem, $2|p$. So $p = 2k$ and so $2q^3 = (2k)^3 = 8k^3$ and $q^3 = 4k^3$. So q^3 and thus q are both divisible by 2. But this has both p and q divisible by 2, contradicting "p and q in lowest terms.

+ 3 pts Any setup ("I'm assuming this to prove that") that would work

✔ **+ 1 pt** assuming "rational" without stating why

✔ **+ 2 pts** Getting definition of Rational. Must include for $p/q; p, q \in \mathbb{Z}, q \neq 0$

+ 2 pts successfully showing both p and q are even

+ 0 pts incorrect

+ 1 pt only one shown even

1 why?

2 why not? This is what you are to prove!

Question 4

Kleebob **0** / 10 pts

4.1 **one pair** **0** / 2 pts

+ 2 pts Correct: $\binom{13}{1}\binom{6}{2}\binom{12}{4}\binom{6}{1}^4$ or equivalent

+ 1 pt Picking the pair: $\binom{13}{1}\binom{6}{2}$

+ 1 pt Picking the 4 non-pair: $\binom{12}{4}\binom{6}{1}^4$ (or picking them first)

+ 0.5 pts Partially correct

✔ **+ 0 pts** incorrect

4.2 **5 of a kind** **0** / 2 pts

+ 2 pts Correct: $\binom{13}{1}\binom{6}{5}\binom{12}{1}\binom{6}{1}$ or equivalent like $\binom{13}{1}\binom{6}{5}72$

+ 1 pt Pick five of a kind: $\binom{13}{1}\binom{6}{5}$ or equivalent like $13 * 6$

+ 0.5 pts Partially Correct

✔ **+ 0 pts** incorrect

4.3 **pair and 3 of a kind** **0** / 2 pts

+ 2 pts Correct $\binom{13}{1}\binom{6}{2}\binom{12}{1}\binom{6}{3}\binom{11}{1}\binom{6}{1}$

+ 1 pt Picking the pair: $\binom{13}{1}\binom{6}{2}$ (or pick 3k first so pair is $\binom{12}{1}\binom{6}{2}$)

+ 1 pt Pick 3 of a kind: $\binom{12}{1}\binom{6}{3}$ (or pick them first with $\binom{13}{1}\binom{6}{3}$)

+ 0.5 pts Partially Correct

✔ **+ 0 pts** incorrect

4.4 **green cards** **0** / 2 pts

+ 2 pts Correct: $\binom{26}{6}$, (there are 26 green cards, choose any 6 of them)

✔ **+ 0 pts** incorrect

4.5 **5 of a kind and a pair** **0** / 2 pts

+ 2 pts Correct: 0 or "none" or similar

✔ **+ 0 pts** incorrect

Question 5

Graph **6** / 10 pts

5.1 **The graph** **4** / 4 pts

✔ **+ 4 pts** Correct: But GradeScope won't let me draw the answer. Must have loop on v_6 and parallel edges between v_2 and v_3

+ 3 pts missing loop on v_6 only

+ 3 pts missing parallel edges only

+ 3 pts missing one edge or one incorrect edge only

+ 2 pts missing or incorrect any two edges

+ 0 pts missing 3 or more edges or 3 or more incorrect edges

5.2 **Eulerian circuit?** **0** / 2 pts

+ 2 pts Correct: "No"

✔ **+ 0 pts** incorrect

5.3 **Why E.Circ?** **0** / 2 pts

+ 2 pts Correct: At least one vertex (here, v_2 and v_7) has odd degree ("an odd number of edges on the vertex" is also OK)

✔ **+ 0 pts** incorrect

5.4 **simple** **2** / 2 pts

✔ **+ 2 pts** Correct: No, it is not simple, because it has a loop on v_6 or because it has parallel edges between v_2 and v_3

+ 0 pts incorrect

Question 6

Strong Induction **13** / 15 pts

6.1 **P(k)** **4** / 4 pts

✔ **+ 4 pts** Correct: $b_k = 3^k - 2^k = 5b_{k-1} - 6b_{k-2}$

+ 0 pts incorrect

6.2 **Basis** **5** / 5 pts

✔ **+ 5 pts** Correct: Knowing $b_1 = 3^1 - 2^1 = 1$ and $b_2 = 3^2 - 2^2 = 5$ we find $b_3 = 3^3 - 2^3 = 27 - 8 = 19$ and $5b_2 - 6b_1 = 5 * 5 - 6 * 1 = 25 - 6 = 19$ so they are the same.

+ 0 pts incorrect

– 1 pt incorrect math

– 2 pts incomplete solution

6.3 **Induction** **4** / 6 pts

✔ **+ 2 pts** Clearly stating what they will prove by induction (set up)

✔ **+ 2 pts** Starting induction, applying inductive hypothesis

+ 6 pts Correct: Assuming $b_i = 3^i - 2^i = 5b_{i-1} - 6b_{i-2}, \forall i < k$ to show $b_k = 3^k - 2^k = 5b_{k-1} - 6b_{k-2}$. $5b_{k-1} - 6b_{k-2} = 5(3^{k-1} - 2^{k-2}) - 6(3^{k-2} - 2^{k-2})$ (by induction) $= 5 * 3^{k-1} - 5 * 2^{k-1} - 6 * 3^{k-2} + 2^{k-2} = 5 * 3^{k-1} - 6 * 3^{k-2} - 5 * 2^{k-1} + 6 * 2^{k-2} = 3^{k-2}(5 * 3 - 6) - 2^{k-2}(5 * 2 - 6) = 3^{k-2} * 9 - 2^{k-4} * 4 = 3^k - 2^k$

+ 2 pts doing the arithmetic

+ 0 pts incorrect

Question 7

Defs **6** / 10 pts

7.1 **a | b** **0** / 2 pts

+ 2 pts Correct: there is an $i \in \mathbb{Z}$ with $b = a * i$

✔ **+ 0 pts** incorrect

7.2 **contradiciton assume** **0** / 2 pts

+ 2 pts Correct: "The negation of $a|b \rightarrow a|2b$" or "$a|b$ and $a \not| 2b$"

✔ **+ 0 pts** incorrect

7.3 **contradiction prove** **1** / 1 pt

✔ **+ 1 pt** Correct: Any contradiction.

+ 0 pts incorrect

7.4 **contrapositive assume** **1** / 1 pt

✔ **+ 1 pt** Correct: $a \not| 2b$ or "a does not divide $2b$"

+ 0 pts incorrect

7.5 **contrapositive conclude** **1** / 1 pt

✔ **+ 1 pt** Correct: "$a \not| b$" or "a does not divide b"

+ 0 pts incorrect

7.6 **direct assume** **2** / 2 pts

✔ **+ 2 pts** Correct: "$a|b$" or "a divides b"

+ 0 pts incorrect

7.7 **direct prove** **1** / 1 pt

✔ **+ 1 pt** Correct: "$a|2b$" or "a divides $2b$"

+ 0 pts Incorrect

Question 8

induction **10** / 15 pts

8.1 **P(k)** **0** / 3 pts

+ 3 pts Correct: $P(k)$ is the statement $\sum_{i=1}^{k} \frac{1}{i*(i+1)} = \frac{k}{k+1}$ OR is the statement $\frac{1}{1*2} + \cdots + \frac{1}{k*(k+1)} = \frac{k}{k+1}$. Note limit of sum etc must be k

+ 1 pt One "side" of $P(k)$, ie "$\sum_{i=1}^{k} \frac{1}{i(i+1)}$" OR "$\frac{k}{k+1}$"

✔ **+ 0 pts** incorrect

+ 1.5 pts 2 answers of which 1 is correct

8.2 **Basis** **3** / 3 pts

✔ **+ 3 pts** Correct: "$P(1)$"

+ 0 pts incorrect

+ 1.5 pts language instead of symbols

+ 1 pt contains P(1) but wrong

+ 2.5 pts formula instead of notation

8.3 **Prove basis** **3** / 3 pts

✔ **+ 3 pts** Correct: $\sum_{i=1}^{1} \frac{1}{i(i+1)} = \frac{1}{1*2} = \frac{1}{2}$. $\frac{1}{1+1} = \frac{1}{2}$. They are the same.

+ 0 pts incorrect

+ 0.5 pts ½ in there somehow

+ 2 pts proved P(2) and sometimes more also

+ 1.5 pts extra erroneous material

+ 2 pts inadequate separation of LHS and RHS

8.4 **Induction** **3** / 3 pts

✔ **+ 3 pts** Correct: $P(k) \rightarrow P(k+1)$ or $P(k-1) \rightarrow P(k)$ or equivalent

+ 0 pts incorrect

+ 2 pts extra erroneous material

+ 1.5 pts no assumption of P(k)

8.5 **Prove induction** **1** / 3 pts

+ **3 pts** Correct: Assuming $P(k)$ I will show $P(k+1)$. $\sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \sum_{i=1}^{k} \frac{1}{i(i+1)} + \frac{1}{(k+1)(k+2)}$. By induction, $= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$ which is $= \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{k^2+2k+1}{(k+1)(k+2)} = \frac{(k+1)(k+1)}{(k+1)(k+2)} = \frac{k+1}{k+2}$

+ **0 pts** no answer

+ **0 pts** did not use full statement of P(k)

+ **1.5 pts** final steps incorrect or missing

+ **0 pts** multiple errors

+ **2.5 pts** used n not k

+ **2.3 pts** last step required factoring

✔ + **1 pt** used n not k AND final steps incorrect

Question 9

Functions **6** / 10 pts

9.1 **onto** **2** / 2 pts

✔ + **2 pts** Correct: $\forall b \in B \exists a \in A$ with $f(a) = b$ OR "every $b \in B$ comes from some $a \in A$ under f" OR equivalent

+ **0 pts** incorrect

9.2 **one to one** **0** / 2 pts

+ **2 pts** Correct: Either "if $f(x_1) = f(x_2)$ then $x_1 = x_2$" OR "$x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$" OR same with $\rightarrow$s. Note NO POINTS for getting the definition "backwards", eg "$x_1 = x_2 \rightarrow f(x_1) = f(x_2)$"

✔ + **0 pts** incorrect

9.3 **bijection?** **0** / 2 pts

+ **2 pts** Correct: $|A| = |B|$ OR "A and B are the same size" or "...same cardinality"

✔ + **0 pts** incorrect

9.4 **Show one-one** **2** / 2 pts

✔ + **2 pts** Correct: If $g(n_1) = g(n_2)$ then $3n_1 + 4 = 3n_2 + 4$ so $3n_1 = 3n_2$ and so $n_1 = n_2$

+ **0 pts** incorrect

9.5 **Show onto** **2** / 2 pts

✔ + **2 pts** Correct: g is not onto. For example, $6 \in \mathbb{Z}$ but $6 = 3n + 4$ implies $n = 2/3 \notin \mathbb{Z}$

+ **1 pt** knowing "not onto"

+ **0 pts** incorrect