Dynamic M-Player Constrained Game for Collision Avoidance and Trajectory Planning in Serial Manipulators

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Abstract

The navigation of multiple robots in shared workspaces is a significant challenge in robotics. This work introduces a new approach that combines a game-theoretic framework with optimization-based collision avoidance, specifically using Augmented Lagrangian Games (ALGames). The main goal is to minimize individual costs for a group of serial robots while adhering to polyhedral constraints and achieving desired trajectory goals.

Our method focuses on finding the open-loop Nash equilibrium for trajectory optimization by solving the problem through a Newton solver integrated with a line search algorithm, effectively addressing smooth but non-convex constraints. Each robot independently optimizes its trajectory, considering the strategies of other robots to prevent collisions and ensure smooth operation within the shared environment.

To demonstrate the concept, we conducted a case study simulation involving basic two-degree-of-freedom robots. The simulation results show that our approach successfully controls multiple robots, ensuring collision-free navigation and efficient trajectory planning. This framework holds promise for real-world applications where multiple robots must operate concurrently in dynamic and constrained environments.

1 Introduction

In the field of robotics, navigating multiple serial manipulators in shared workspaces is a challenging task. As articulated machines become more common in industries such as manufacturing and healthcare, ensuring efficient and collision-free operation is increasingly important. Traditional methods, though effective in some scenarios, often struggle to coordinate multiple manipulators in dynamic environments. This article introduces a new approach that combines game theory with optimization-based collision avoidance, using Augmented Lagrangian Games (ALGames) to address this challenge.

Serial manipulators, consisting of interconnected segments and joints, are highly versatile and dexterous. However, their operation in shared workspaces presents unique challenges, such as

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interference, collision risks, and coordination complexity. Classical methods frequently fail to effectively manage these issues, resulting in suboptimal performance and increased safety hazards Poudel et al. [2023]. Centralized planning approaches suffer from scalability issues and computational bottlenecks, while decentralized strategies often lack the necessary coordination for collision-free navigation Antonyshyn et al. [2023], Khatib [1986]. Heuristic-based methods can produce suboptimal trajectories and reactive responses, compromising efficiency and safety Mohanan and Salgoankar [2018], LaValle and Kuffner Jr [2001].

In response, we propose a game-theoretic approach to serial manipulator navigation. By modeling the strategic interactions between multiple manipulators as a non-cooperative dynamic game, we aim to find an open-loop Nash equilibrium where each manipulator independently optimizes its trajectory while considering the actions of others. Augmented Lagrangian Games (ALGames) provide a powerful framework for balancing individual objectives with collective goals such as collision avoidance and trajectory optimization.

1.1 Related Work

Cleac'h et al. [2019] introduced ALGames, a numerical solver for constrained dynamic games, which serves as the backbone of our approach. Zhang et al. [2020] addressed an optimization-based collision avoidance problem with smooth but non-convex constraints, providing methodologies for easier calculation of minima within a game theory setting.

Schulman et al. [2013] focused on locally optimal, collision-free trajectories involving stationary objects, offering insights into efficient path-planning methodologies. Bergen [1999] and Ong and Gilbert [1997] developed algorithms for finding minimum distances and penetration between convex sets, laying the groundwork for proximity-based collision avoidance strategies. Liniger and Lygeros [2019], Fisac et al. [2019], and Cococcioni et al. [2021] explored game theoretical approaches to dynamical collision avoidance and path planning, providing adaptable principles for our project.

While each of these studies informs our work, Cleac'h et al. [2019] and Zhang et al. [2020] offer key components for our approach. Our project integrates these methodologies, extending them by incorporating optimization-based collision avoidance within a game theory framework to address dynamic obstacles in shared workspaces.

In game theoretical settings involving collision avoidance, common techniques like big-M notation Liniger and Lygeros [2019], Antonyshyn et al. [2023] and the signed distance approach Ong and Gilbert [1997] are often employed due to their simplicity and ease of integration. However, these methods have limitations. Big-M notation often oversimplifies the manipulator's representation due to its large length-to-width ratio, and both signed distance and big-M notation constraints are non-smooth, presenting challenges in solver implementation.

Acknowledging these limitations, our approach deviates from conventional methods. We aim to develop a more accurate and efficient solution for handling polyhedral constraints in collision avoidance scenarios. While our approach shares similarities with Zhang et al. [2020], we extend upon it by managing collisions between two controlled objects, rather than only considering static obstacles.

2 Problem Setup

Let $\mathbb{E}(x_k)$ represent the occupied space by the controlled object $\mathbb{O} = \{x : Ax \leq b\}$ at time k.

$$\mathbb{E}(x_k) = R(x)\mathbb{O} + t(x) \tag{1}$$

Here, $R(x)$ denotes the rotational matrix, and $t(x)$ denotes the translation vector. Then, combining these equations, the constrained finite horizon optimal control problem for M players can be written as follows.

$$
\begin{aligned}
\min_{X,U} \quad & \sum_{k=0}^{N}(x_k - x_{ref})^T Q^\nu (x_k - x_{ref}) + (u_k^\nu)^T R_k^\nu (u_k^\nu) \\
& s.t. \quad x_{k+1} = x_k + u\Delta t \\
& \mathbb{E}^\nu(x_k) \cap \mathbb{E}^{\nu^-}(x_k) = \emptyset \\
& \forall \nu \in M
\end{aligned}
\tag{2}
$$

Where Q^ν and R^ν denote the state and input cost matrices for player ν and Δt denotes the time step from the discretization.

Lemma 1. *Let the following inequalities describe the shapes of the objects under consideration.*

$$
\mathbb{H} = \{x : Hx \leq h\} \quad \mathbb{F} = \{y : Fy \leq f\} \tag{3}
$$

Suppose that the controlled objects of shapes $\mathbb{H}$ and $\mathbb{F}$, denoted as $\mathbb{E}_1(x)$ and $\mathbb{E}_2(x)$ respectively, are given as in (1)*. Let d_{min} be the desired safety margin. Then, $dist(\mathbb{E}_1, \mathbb{E}_2)$ is equivalent to the following optimization problem with optimal value of the objective being equivalent to the distance between the sets.*

$$
\begin{aligned}
\max_{\gamma,\phi} & -(HR(x^1)t(x^1) + h)^T\gamma - (FR(x^2)t(x^2) + f)^T\phi \\
& s.t. \quad R(x^2)^T H^T \gamma + R(x^1)F^T\phi = 0 \\
& ||F\phi|| \leq 1 \\
& \gamma \geq 0, \quad \phi \geq 0
\end{aligned}
\tag{4}
$$

Proof. Here, we will not present the full proof due to space constraints. Interested readers can refer to the full proof in [Boyd and Vandenberghe, 2004, Section 8.2]. ☐

We can obtain the following dynamic game formulation by incorporating these constraints into (2).

$$
\begin{aligned}
\min_{X,U,\gamma,\phi} \quad & \sum_{k=0}^{N}(x_k - x_{ref})^T Q^\nu (x_k - x_{ref}) + (u_k^\nu)^T R_k^\nu (u_k^\nu) \\
& s.t. \quad x_{k+1} = x_k + u\Delta t \\
& (HR(x_k^\nu)t(x_k^\nu) + h)^T\gamma + (FR(x_k^{-\nu})t(x_k^{-\nu}) + f)^T\phi < -d_{min} \\
& R(x_2)^T H^T\gamma + R(x_1)F^T\phi = 0, \quad ||F\phi|| \leq 1 \\
& \gamma \geq 0, \quad \phi \geq 0 \\
& \forall \nu \in M
\end{aligned}
\tag{5}
$$

3 Analysis

In this section, we present the analysis and methodologies to approach the problem described in the previous section.

3.1 ALGames

The ALGames algorithm aims to solve Generalized Nash Equilibrium Problems (GNEPs) in discretized trajectory optimization. It utilizes an augmented Lagrangian formulation, coupled with Newton's method for root-finding, to iteratively converge towards equilibrium solutions for multi-player dynamic games.

In the context of discretized trajectory optimization with N time steps, we consider M players, each optimizing their control inputs. The control input of player ν at time step k are denoted by u_k^ν. Player

ν optimizes their control sequence $U^\nu = [(u_1^\nu)^T \dots (u_{N-1}^\nu)^T]^T \in \mathbb{R}^{\bar{m}_\nu}$, where $\bar{m}_\nu = m_\nu(N-1)$. The combined strategies of all players except ν are $U^{-\nu}$.

The state trajectory $X = [(x_2)^T \dots (x_N)^T]^T \in \mathbb{R}^{\bar{n}}$, with $\bar{n} = n(N-1)$, follows the system dynamics: $x_{k+1} = f(x_k, u_k) = f(x_k, u_k^1, \dots, u_k^M)$, subject to the dynamics constraints: $D(X, U) = 0$.

Each player minimizes their cost function $J^\nu(X, U^\nu) : \mathbb{R}^{\bar{n}+\bar{m}_\nu} \to \mathbb{R}$, subject to dynamics and inequality and equality constraints represented in concatenated form $C = [C_{ci}, C_{ce}], C_{ci} \in R^{n_{ci}}, C_{ce} \in R^{n_{ce}}$:

$$\begin{aligned} \min_{X, U^\nu} \quad & J^\nu(X, U^\nu), \\ \text{s.t.} \quad & D(X, U) = 0, \\ & C(X, U) \leq 0. \end{aligned} \tag{6}$$

To solve this Generalized Nash Equilibrium Problem (GNEP), ALGames introduces Lagrange multipliers μ_ν for dynamics constraints and λ for inequality constraints, along with penalty weights ρ. The augmented Lagrangian for player ν is:

$$L^\nu(X, U) = J^\nu + \mu_\nu^T D + \lambda^T C + \frac{1}{2} C^T I_\rho C \tag{7}$$

where I_ρ is a diagonal matrix:

$$I_{\rho,kk} = \begin{cases} 0 & \text{if } C_k(X, U) < 0 \text{ and } \lambda_k = 0, \text{ for } k \leq n_{ci} \\ \rho_k & \text{otherwise.} \end{cases} \tag{8}$$

At optimality, the gradient of the augmented Lagrangian of the player ν concerning the states and its inputs is zero:

$$\nabla_{y^\nu} L^\nu(X, U, \mu^\nu) = G^\nu(X, U, \mu^\nu) = 0 \tag{9}$$

where $y^\nu = [X, U^\nu]$. It's important to note that this equality constraint preserves coupling between players since the gradient G^ν depends on the other players' strategies $U^{-\nu}$.

To find the generalized Nash equilibrium, we form the G vector by concatenating the augmented Lagrangian gradients of all players and the dynamics constraints:

$$G(X, U, \mu) = \left[(\nabla_{y^1} L^1)^T, \dots, (\nabla_{y^m} L^M)^T, D^T\right]^T \tag{10}$$

where $\mu = [\mu^1, \dots, \mu^M]^T$. At a generalized Nash equilibrium, all the players are acting optimally and dynamics constraints are respected. Then, we solve the following root-finding problem:

$$\begin{aligned} \min_{y} \quad & 0 \\ \text{s.t.} \quad & G(X, U, \mu) = 0 \\ & D(X, U) = 0 \end{aligned} \tag{11}$$

where $y = [X_1, U_1, \mu_1, \dots X_N, U_N, \mu_N]^T$.

Using Newton's method for the root-finding problem above, we compute the search direction δy in the primal-dual space by inverting the Jacobian H of G concerning the overall state vector y: $\delta y = -H^{-1}G$, and apply a backtracking line-search to ensure convergence. Newton's method iteratively updates the solution by moving in the direction that nullifies the gradient, achieving rapid local convergence to the equilibrium point.

3.2 Combining ALGames with OBCA

To solve the problem we formalized at (5), we propose to combine the two previously explained approaches. We will be extending state vectors of the robots y^ν by adding the dual variables γ and ϕ respectively. The overall state vector is also extended likewise.

The kinodynamic constraints only include the state transitions and can be formulated as follows:

$$D_N(X_N, X_{N-1}, U_N) = X_N - X_{N-1} - (\Delta t)U_N = 0 \tag{12}$$

For the constraints, we are considering the ones defined at (5).

In nested optimization problems with Lagrange multipliers (λ), it's vital to understand that these multipliers, linked to constraints, cannot be updated independently. When solving the inner optimization problem, all λ's for constraints should be updated together at each timestep to maintain consistency and ensure all constraints are satisfied simultaneously. This collective update prevents the risk of nullifying satisfied constraints and fosters convergence towards a solution where all constraints are met concurrently, enhancing the optimization process's effectiveness and constraint satisfaction.

4 Results

In this section, we present the results of our simulations, designed to evaluate the effectiveness of our approach to collision avoidance in a game theoretical setting.

4.1 Chosen Examples and Motivation

For our simulations, we selected scenarios involving two basic two-degree-of-freedom (2-DOF) serial manipulators operating within a shared workspace. In the simulation, manipulators are placed on opposite sides of a rectangular workspace and we aim to move the manipulators without collision.

The 2-DOF manipulators were selected because they represent a simple yet non-trivial case, allowing us to clearly illustrate the effectiveness of our collision avoidance strategy while keeping the computational complexity manageable.

These scenarios help us address the core question posed in the introduction: Can a game-theoretic approach with optimization-based collision avoidance effectively manage the trajectories of multiple serial manipulators in shared workspaces? By focusing on 2-DOF manipulators, we ensure that our results are both illustrative and reproducible.

4.2 Simulation Setup and Parameters

We set up our simulations by defining the workspace, initial positions, and target positions for the manipulators. The polyhedral constraints were set to represent the physical boundaries of the manipulators and their immediate environment. The parameters of the simulation can be found in Table 1.

Table 1: Parameters for the Workspace and Dynamics

Parameter Name	**Value**
Workspace Dimensions	(5m x 2m)
Link Lengths	60 cm
Link Widths	10 cm
Time Horizon	2 s
Time Step	0.2 s
Max Joint Velocity	1.5 rad/s

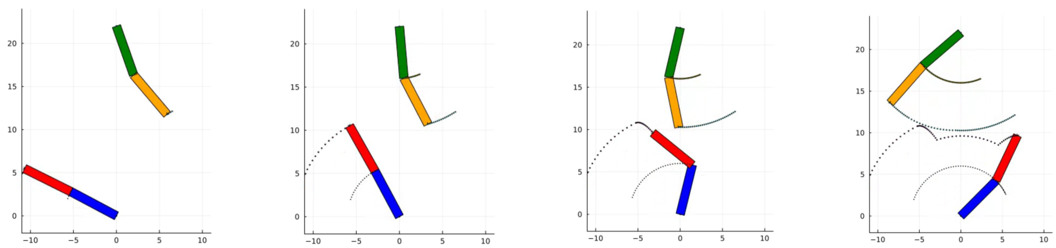
4.3 Results and Analysis

The simulation results indicate that our approach successfully manages the trajectories of the two manipulators, ensuring collision-free operation. A sample trajectory can be seen in Figure 1. Key findings from our simulations include:

- The manipulators were able to reach their target positions without any collisions, demonstrating the effectiveness of our collision avoidance strategy.

- The trajectories were smooth and efficient, with the manipulators making optimal movements to avoid collisions while minimizing their individual costs.
- Our approach handled dynamic interactions between the manipulators well, adjusting their paths in real-time to prevent collisions.

These conclusions support our analysis by showing that a game-theoretic approach with optimization-based collision avoidance can effectively manage the trajectories of multiple serial manipulators. The simulations provided new insights into the practical implementation of our method, revealing that the incorporation of polyhedral constraints and real-time optimization can significantly enhance the safety and efficiency of robotic operations in shared workspaces.



A - Initial Positions | B - Before the Collision | C - Collision Avoidance | D - Final Positions

Figure 1: Trajectories of the dynamic system at different stages

5 Conclusion

Our simulation results demonstrate the effectiveness of our game-theoretic approach with optimization-based collision avoidance for coordinating multiple robotic arms in shared workspaces. The method successfully guided the robotic arms to their target positions without collisions, showcasing the robustness of our approach in managing interactions among multiple robots in confined spaces.

Despite the positive outcomes, there are still some important unresolved questions that need further exploration. Firstly, because of the smooth but non-convex constraints mentioned in Section 2, the results are sensitive to the initial conditions and must be chosen carefully. Additionally, in the augmented Lagrangian form of the nested optimization problem, constraints add extra costs, which can shift the minimum of the objective. However, this creates a tradeoff in satisfying the constraints, as some constraints may be very small and can be ignored to satisfy others. Therefore, it's worth considering other solvers such as interior point methods for greater precision.

Future investigations should delve into evaluating the performance and computational feasibility of our approach in scenarios involving more robots and higher degrees of freedom to ascertain its scalability. Furthermore, additional studies are imperative to adapt and validate our approach in practical, real-world settings. Moreover, exploring adaptive strategies capable of dynamically modifying the game-theoretic model based on real-time feedback could augment the robustness and efficiency of our approach.

In conclusion, our study presents a promising approach to the challenge of collision avoidance in shared workspaces utilizing game theory and optimization techniques. While our initial findings are encouraging, addressing the aforementioned open questions will be pivotal in advancing this method toward practical, real-world applications.

6 Partnership work and resources

All tasks were completed through collaboration, with regular group meetings aiding progress. For all project-related files, please refer to our GitHub repository at `https://github.com/alifuatsahin/ManipulatorGameTheoreticPlanner`.

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