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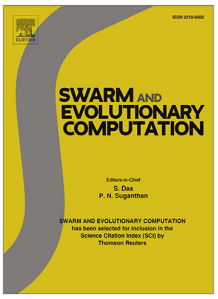

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Modeling and Designing a Robotic Swarm: a Quantum Computing Approach

Maria Mannone[1,2,*], Valeria Seidita[1], Antonio Chella[1,3]

Abstract

Nature is a neverending source of inspiration for technology. Quantum physics suggests applications toward quantum computing. Swarms' self-organization leads to robotic swarm developments. Here, apply quantum computing is applied to swarm robotics. We model local interactions with a quantum circuit, testing it on simulators and quantum computers. To relate local with global behavior, we develop a block matrix-based model. Diagonal sub-matrices contain information on single robots; off-diagonal sub-matrices are the pairwise interaction terms. Comparing different swarms means comparing different block matrices. Choosing initial values and computation rules for off-diagonal blocks (with a particular logic gate), model different behaviors can be modeled. To show the global-behavior emergence, we propose a specific pairwise-interaction logic gate, embedding the corresponding quantum circuit in an ant-foraging-inspired algorithm. To implement a first application, we choose the foraging-ant behavior for its clarity and importance in nature, running experiments with toy swarms (3 and 10 robots). We consider ants' individual and collective back-and-forth movements between the nest and the food source, analyzing the effect of entanglement. Our research can help shed light on quantum potentialities for swarms. The implications of our findings and results concern the future development of

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a decision-making system, based on the advantages of swarms and quantum computing. While an ant-foraging scenario is chosen as an example of application, our study is not focused on optimization. We present a new methodology, open to non-optimal solutions. Future developments can concern improvements toward optimization.

Keywords: quantum computing, swarm robotics, search & rescue, logic gates
2010 MSC: 03G12, 81P68, 15A99, 93C85

1. Introduction

Antoni Gaudí, architect and saint, considered Nature as his supreme teacher and source of inspiration. Science and beauty in nature can also inspire the development of refined and autonomous human artifacts such as robots. Robotics tries to catch structures and mechanisms, including examples of self-organized collective behavior in nature, results of the so-called *swarm intelligence* [1].

The twirling elegance of flocking birds and schooling fish, the architectural ingenuity of termites [2], locusts grouping [3], the organization of ants are the input for mathematical modeling [4, 5] and robotic developments [6, 7]. Some characteristics of natural swarms are caught in human-made artificial swarms, such as self-organization, scalability, and decentralization. Computational techniques mimicking behavioral and social patterns in nature can help solve complex tasks [8].

Another source of natural inspiration is given by quantum computing, an extension of computer science derived from the principles of quantum physics [9], of growing importance for artificial intelligence [10]. The main reason is the extension of quantum computational efficiency to enhance a robotic set-up.

In this article, we develop a mathematical description of the swarm with a model of nested matrices, and pairwise interactions represented by reversible logic gates. In particular, we consider an ant-foraging scenario, for its impor-

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tance in nature and recent robotic applications.[4]

In particular, we try to connect local and global behavior of a swarm, letting the global behavior emerge from simple local pairwise interactions. Tthese local interactions are modeled via a quantum circuit. Then, to test the idea, the circuit in a nest-food-nest ant scenario is exploited.[5] The algorithm can lead to a novel decision-making system based on quantum computing. The novelty of our work is two-fold: first, we define general matrices which can be adapted to whatever swarm with whichever interaction laws; second, we shape the interaction terms starting from probability-based quantum concepts. We develop and extend dimensionally the ideas *in nuce* in [11]. The key technical difficulty to overcome is the definition from scratch of a formalism describing both local and global aspects of the robotic swarm, including a quantum-based approach to connect the world of robots with the flourishing field of quantum computing.

The article is organized as follows. A brief literature survey is presented in Section 2, providing the motivations for our research in Section 3. Then, all information to reproduce our research are presented in Section 4. Our theoretical approach is presented in Section 5, with nested matrices (subsection 5.1) and quantum computing (subsection 5.2). We briefly describe our results in Section 6, presenting our case study, with toy swarms of 3 and 10 robots on the plane simulating the case of nest-food-nest ant path. We also present a comparison between our method and two optimization approaches, the PSO and NL-SHADE-RSP algorithms. Section 7 contains a discussion of strengths and limitations of our strategy and possible developments of this research. In Section 8, our findings are summarized. The Appendix contains the search and rescue pseudocodes, the quantum codes for pairwise interactions, and an example of the nested matrices for a 3-robot swarm.

[4]See the ant-inspired codes for NASA: `https://www.nasa.gov/feature/students-develop-robotic-code-in-first-swarmathon-challenge`

[5]There are the following steps: starting from the nest, random reshuffle, quantum gate information exchange, and final points achievement.

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2. Literature survey

2.1. Swarms of robots

Swarms of robots are an example of artificial swarm intelligence. A robotic swarm is constituted by multiple autonomous and simple robots, collaborating to achieve a task that is impossible for single units. Each individual robot in a swarm is a simple element that can perform only a few simple tasks (in traditional swarms, which are biologically inspired, there is only one task). Moreover, the single element does not know the global goal of the swarm but it contributes to its achievement thanks to self-adaptation, self-regulation, communication, and cooperation with the other elements of the swarm [12, 13, 14]. The swarm behavior is in fact an *emerging* effect. If the study of nature can be the input for robotic applications, it can also constitute its target. In fact, robots can be a benchmark to model and investigate complex phenomena, such as morphogenesis in multi-cellular creatures [15]. Medical applications include miniature robotic swarms to deliver medications inside the human body [16].

In robotic swarms, as well as in natural swarms, individual behavior is governed by simple rules allowing a few simple actions. The individual element of the robot must interact with the environment and with its conspecifics to allow complex behavior to emerge. Communication and interaction with other elements of the swarm are key to coordinate and collectively perform a complex goal. Natural examples include heavy prey transporting, foraging, or massive and complex structures building.

Intuitively, in swarm robotics a large number of simple embodied agents and their actions are designed to let a complex collective behavior emerge. Robots have to interact between them and with their environment [17].

In robotic swarms, one can distinguish between a micro-level, with individual behaviors, controls, and pairwise interactions, a macro-level with the global swarm action [18]. Properties of a swarm include scalability, robustness (the lack of an individual does not affect the global behavior), self-regulation [13, 12] (no external commands are required), and self-awareness [12] (each individual

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knows its own position, speed, and action) [18].

As described by [19], examples of self-organized approaches for swarms of robots concern spatial organization, navigation, decision making, and miscellaneous task. Spatial organization are aggregation, pattern formation, self-assembly, object clustering and assembling. Navigation tasks include collective exploration, coordinated motion, collective transport, collective localization. Decision-making tasks are about consensus, task allocation, collective fault detection, collective perception, synchronization, and group-size regulation. Finally, miscellaneous tasks concern self-healing, self-reproduction, human-swarm interaction. Research on swarm robotics range from the development of cyber-swarms [20] and swarm specialization into specific tasks [21].

Concerning the software improvements, there are several optimization approaches to robotic swarms. A notorious example is the application of particle swarm optimization (PSO) [22] to refine search and rescue robotic missions [23, 24], where each particle represents a robot, also in maritime scenarios [25]. Another source of natural inspiration for swarm robotics is evolution. Evolutionary algorithms, and in particular the successful genetic algorithm (GA), are based on candidate solutions of a problem, and a problem-dependent objective function [26]. Also differential evolution-based algorithms [27] can be considered. The applications of these concepts to robotics leads to evolutionary robotics [28]. Evolutionary approaches are used to optimize problem-solving techniques. Other approaches are the aforementioned ant-colony optimization (ACO) [29, 27], inspired by ants' collective behavior, and particle swarm optimization (PSO), suggested by birds' collective flight [22]. According to [30, 31], the philosophy of PSO lead to several other bio-inspired optimization methods, such as GA and ACO. In [32, 33], evolutionary and genetic processes are also exploited. However, our research does not focus on optimization. We rather investigate if quantum computing can be applied to model simple, local pairwise robot interactions, and if a global behavior of swarm can emerge from them. Nevertheless, an example of PSO is adapted to our scenario, quantitatively comparing its outcomes with ours (Table 10), and then of NL-SHADE-RSP

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algorithm [34]. A basic example of PSO is considered for its conceptual importance. Further comparisons, with the aid of machine learning applications, will be considered for future research, including more recent developments of PSO itself [35], and enhancements of ant-inspired algorithms with evolutionary approaches [36, 37]. However, while a great part of recent research on swarm robotics deals with optimization techniques for specific goals and scenarios [20], there is another side of the research to be explored, that is, the definition of a more general approach. As noticed by Nedja et al. [38], several solutions are too problem-specific, and thus a general methodology which could be adapted to different problems, algorithms, and devices is yet to be found. In this regard, the matrix-based methodology that is proposed in Section 5 might help fill this gap. Our approach tries to model the emerging swarm behavior from local interactions. Indeed, global-local connections and swarm-emergence from simple rules are not trivial. Analogies with physics have been made, concerning the micro/macroscopic behavior descriptions through Langevin and Fokker-Planck equations, respectively [18]. However, the increasing complexity of robotic swarms requires a more complex and general treatment [39]. For all these reasons, robotic swarms are amongst the most challenging topics in robotics [40].

In a robotic swarm, the individual does not have a complex objective in its own. Instead, it acts to reach a higher-level objective. Therefore, one can talk of force multiplication. The individual in fact feels the environment and its peers, and communicates with them, while acting individually and autonomously.

A standard approach to engineering a robot swarm is unthinkable. A swarm is a complex system that is more than the sum of its components. Thus, to describe a swarm we should remind that it is a complex system whose parts interact with each other and their environment [41]. In addition, swarms are characterized by consensus formation [42] in networks with neighbor-dependent synergy and observer effect.[6] Therefore, a top-down engineering approach is

[6] In our approach, we consider information exchange between all elements of the swarm, as

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not feasible. A design methodology suitable for a swarm must analyze the system and implement the model at the micro-level: the goal of each element, its capabilities, and the exchange of messages. On the other hand, at the macro level, one should analyze and implement the "how" producing the behavior needed for the swarm's goal achievement. From a modeling point of view, the single individual task requires an interaction with the environment, while the interaction with other elements of the swarm requires a peer communication. *Collective behavior* is thus a key word in this research. Finally, concerning the hardware side, there are examples of research and commercially-available robotic swarms suitable for terrestrial [43, 44, 45], aerial [46], aquatic [47, 48], and outer space [49] motion.

2.2. Quantum computing and its application to biology

To model tasks and communications, we choose to exploit *quantum computing* [9, 50]. Quantum computing is a branch of computer science based on the principles of quantum mechanics. In a nutshell, it is an approach to computer science based on quantum probability amplitudes and reversible gates. Reversible gates are used in analogy with invertible operators ruling quantum mechanics.

In classic computer science, the units of memory are 0 and 1, that is, the possible values of the so-called *bit*. However, in quantum computing there is the *qubit*, the quantum bit, that can assume all values between 0 and 1. This is a consequence of the principle of state superposition in quantum mechanics. In addition, to measure the value of a qubit and store it in a classic bit, it is necessary to perform a measurement operation which is a *destructive* one: the wavefunction, representing the state, collapses to one of its values. All further measures will then give the same result. Destructive measure occurs in quantum

all robots were the neighbors of each other. Consensus is implicit in reaching the robot with the highest "reward," a concept that is explained later. In the case of a larger swarm where one can consider subsets of neighbors, the consensus formation can deal with the comparison of max rewards in each subset.

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physics, and it was the first operation measure where the observer influenced the observed system.

In general, quantum computing enhances speed and efficiency of classical algorithms, and nowadays quantum computers and simulators can be accessed remotely (e.g., IBM, Amazon, and so on). The main cost of these gains is the access itself to quantum computers or simulators, and thus, the need for an internet access. One of the major motivations for the use of quantum computing is its power of calculation.

The advantages of quantum speed-up come with some drawbacks: it is the case of *decoherence*, that is, a loss of information of the quantum system into the environments for effect of its interaction with it. The collapse of the wave-function is necessary to perform measurements, but such a phenomenon should be controlled. Constructors of quantum computers have to take into account the risk of errors given to decoherence. Decoherence does not appear in simulations because there are no interaction with the environment—unless they are modeled in purpose.

Starting from initial insights in physics [51] and computer sciences [52], quantum computing has recently been applied in robotics [53, 54] and artificial intelligence [10]. Quantum particle swarm behavior inspires algorithm improvements [55] and swarm optimization [56, 19]. Examples of quantum-inspired algorithms include quantum harmonic oscillator algorithm as an heuristic optimization algorithm [57]. The quantum paradigm helped enhance techniques to solve multiobjective large-scale optimization problems [58]. Other applications involve improvements to quantum-inspired evolutionary algorithms [59], to solve the positioning-antenna problems in networks [60], enhancement of grey levels in images [61], face multi-objective large-scale problems [62]. In particular, Cao et al. considered position uncertainty, that is, as a quantum reference, to formalize their approach [62]. In fact, based on quantum mechanics postulates, quantum computing is essentially probabilistic [63]. Probability has already been used for threshold-based robotic swarm behavior [64]. One of the most singular phenomena of quantum mechanics is entanglement [65]. Two entangled particles

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are parts of the same system, in the sense that a measure on one of them affects the other one. With quantum computing, one can build up circuits to make entangled states [50]. The entanglement has an essential role for pure-state (not mixed) quantum algorithms, while it does not necessarily lead in general to computational speed-up [66]. The idea of entanglement has been proposed to entirely model a swarm of robots, as a theoretical simplification to group and connect robots' behavior [67], or to enable a different communication strategy between two complex robots [68]. However, the application of the quantum paradigm to swarm autonomous devices is, to the best of our knowledge, still a largely unexplored field [69, 11].

In a recent study, simulated underwater swarm localization made use of fuzzy logic [70]. Because quantum logic can be seen as a particular example of fuzzy logic [71], we choose the quantum paradigm to investigate and model the swarm behavior. Also, a quantum algorithm has been developed for the path planning of a single robot [72].

Pioneering applications of quantum computing in biology deal with simulation technologies [73]: imaging, spectroscopy, microscopy, molecule dynamics [74], and protein structure prediction [75]. Quantum neural networks are considered key tools for neuroscience: from genes, to molecules, to cells, to neural structures, up to the human behavior—seen as an emerging effect [76]. In Table 1, a synthetic overview of the literature framework is proposed.

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Table 1: A synthetic overview of the literature framework.

area	topic	authors	year	ref.
quantum mechanics	basics	Feynman et al.	1965	[77]
	entanglement	Einstein et al.	1935	[78]
		Bell	1964	[65]
		Greenberger et al.	1989	[79]
	probabilities	Hemmo and Shenker	2020	[63]
quantum computing	general	Stolze and Suter	2004	[50]
	entanglement and speed-up	Josza and Linden	2003	[66]
	for AI	Wichert	2020	[10]
	AI, learning	Kwak et al.	2008	[80]
	for robots	Dong et al.	2006	[52]
		Benioff	1998	[51]
		Dong et al.	2008	[81]
		Zhu et al.	2010	[6]
		Lamata et al.	2021	[68]
		Atchade-Adelomou et al.	2021	[54]
		Ivancevic	2016	[67]
		Chella et al.	2022	[72]
	for swarms of robots	Koukam et al.	2021	[69]
		Mannone et al.	2022	[11]
swarms of robots	general	Hamann	2018	[18]
	overview	Shranz et al.	2020	[19]
	terrestrial (ants)	Berman et al.	2011	[7]
	terrestrial (kilobots)	Rubenstein et al.	2014	[43]
	terrestrial (e-pucks)	Alkilabi et al.	2017	[44]
	terrestrial	Groß et al.	2006	[45]
	aerial	Oung	2013	[46]
	aquatic	Schmickl et al.	2015	[48]
	outer space	Kang	2018	[49]
	general	Zambonelli et al.	2011	[12]
	response probability	Wu et al.	2011	[64]
	hormone-inspired	Shen et al.	2020	[13]
	foraging	Pitonakova et al.	2020	[14]
	general	Sahin	2004	[17]
	micro, health	Dong and Sitti	2020	[16]
	future	Dorigo et al.	2020	[39]
	fuzzy, underwater	Sabra and Fung	2020	[70]
	consensus formation	Mañas-Álvarez et al.	2023	[42]
natural swarms	general	Eberhart	2001	[1]
	flocking birds	Hemelrijk and Hildenbrandt	2012	[4]
	termites	Noirot	2000	[2]
	foraging ants	Plowes et al.	2013	[82]

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3. Motivations

We described above the motivations for quantum computing. In this Section, we enunciate the details of our research questions motivating our work.

Our first research question is: how can one connect local and global behavior to model the swarm behavior emergence from local behavior of the single and pairs of robots?

The second question is: is it possible to model a local decision system through quantum computing, to be the core part of a whole swarm approach?

Swarms of animals make use of a mixture of instinct, individual intelligence, environment and peer observations for individual decision-making. Individual decisions are the pieces in the puzzle of the emerging swarm intelligence.

Robotic swarms rely on simple individual behavior and decision-making which can be modeled first and coded then. The local behavior, with single-agent decisions and pairwise interactions, needs to be connected with the global behavior, that is, the emerging swarm behavior. We need a theoretical framework to connect the local with the global behavior, and a logic model to build up decision-making structures.

Our quantum circuit models pairwise interactions, and, to be tested, it is included inside an original algorithm (codes 1, 2) for the robotic swarm. For our case study, we consider the biological inspiration of ants moving between the nest and the food location. Group food retrieval in *Aphaenogaster cockerelli*, an ant species, is in fact the inspiration source for swarm robotic studies [7]. We focus on ant colonies because of their simplicity of modeling, importance as a classic biological model, and easy generalizability. While our research may appear as an optimization study, we are developing some initial elements for a decisional system. In fact, even though we will be considering and comparing trajectories of robots, we are not developing a system to reach a target with the fastest trajectory. We are not looking at the best possible solution of a problem, but instead, at applications of quantum computing. The quantum enhancement of an algorithm usually yields faster solutions than the classical

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counterparts. In our research, we are open to sub-optimal solutions. Thus, we preferred to not analyze our results with statistical approaches, but rather with qualitative observations. Nevertheless, some quantitative comparisons (between our method and the results of a particle swarm optimization adapted to our scenario) are proposed in Table 10.

Quantum computing is used here to model probabilistic information sent by robots and probabilistic trajectory outcomes. Here, the quantum paradigm involves:

- definitions of rewards (as probability amplitudes of 'target yes' and 'target no' as 0/1) and positions (left/right as 0/1 along x, up/down as 0/1 along y);
- use of quantum circuits, with quantum logic gates, for robots' probabilistic decision-making;
- definition of an entangled GHZ state.

The Greenberger–Horne–Zeilinger state (GHZ state for short) is an entangled state which, for three qubits, takes the form: $|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ [79]. We will consider a GHZ for five qubits. In an enhanced and shortened version of our code discussed later in the article, we omit the GHZ state, because convergence is faster.

A closely-related research has been developed by Koukam and others [69]. The authors, focusing on agents, considered a quantum circuit, relating robot's perceptions with robot's actions, picking up an action from a list, as activating an item. The idea has been implemented with IBM quantum simulators. In our study, we also build up quantum circuits, embedding them into a theoretical and nested approach to swarms, to model a search and rescue behavior. Our research can inherit expertise in agent systems and multi-robots, adding the condition of swarm as a relationship between local and global behavior. In [69], the authors used entangled W states, that is, states of the form $|\psi_W\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$. In our study, we choose entangled GHZ state, because

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we focus on space positions elements and we are interested in bringing together all robots in a position (indicated by qubit 0) or in an opposite one (indicated by 1).

We aim to model the connection between local and global behavior through nested matrices. A block matrix represents the swarm, while each sub-matrix represents a single robot's behavior or a pairwise interaction term. Changing the structure of matrices, it is possible to model swarm features. We present the general theoretical idea and an experiment with a toy 3- and 10-robot swarm implementation, confirming our expectations. In addition, even though our approach is open to suboptimal solutions, we present a comparison between our results and the results obtained with two optimization approaches, the PSO and NL-SHADE-RSP algorithms, assessing the degree of suboptimality, and verifying the validity of our proposal.

4. Experimental

In our research, we propose a matrix-based model of a robotic swarm (the details are provided in Section 5). We focus on off-diagonal sub-matrices, representing interaction terms. We design a quantum-based local-interaction approach, modeling relationships between input information (position and reward as food proximity) and behavioral outcome. We implement a simulation of toy 3- and 10-robot swarms inspired by ant foraging in nature. The robots search for the food and then get back to the nest. The food location and the nest are targets to reach. Regarding communication strategies, broadcast communication [14] are considered here. In the Appendix, all the necessary information to replicate our experiment are provided.

Our proposed quantum circuit is included in two Python codes, created in Jupyter Notebook environment and corresponding to Algorithm 1 and Algorithm 2, accessible online, jointly with screenshots and a video simulation. Qiskit QASM simulator is called locally from the Notebook. An alternative code could involve the loop repetition of the solely quantum gate, until the

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target is reached. The additional materials can be found in the GitHub folders `https://github.com/medusamedusa/3-robot` and `https://github.com/medusamedusa/10_little_ants`.

5. Theory and Calculation

In this section, we describe the theoretical background of our method, with nested matrices (subsection 5.1) and quantum computing for decision-making in search and rescue (subsection 5.2). Then, in Section 6, a toy model with a robotic swarm is implemented.

5.1. The overall matrix

In our former research [11], we presented a category-theoretic framework to connect specific, existing swarms of robots with their typologies, going upward in abstraction toward main classes of swarms (for underwater, flying, walking robots), up to conceptual "ideal" swarms. Thus, we can make vertical comparisons, between swarms of different ontology, and horizontal comparisons, between swarms of the same level of reality [11]. A comparison between different swarms becomes in this way a comparison between block matrices.

In our approach, a swarm can be described by a block matrix at each time point, where diagonal sub-matrices represent individual information of each robot, and off-diagonal sub-matrices contain information on pairwise interaction. Equation (1) shows the matrix S_n. The dimensions of the matrix are computed as $(nm) \times (nm)$, where n is the number of robots and m is the size of the matrix blocks, depending upon the degree of freedom and detail of information exchange. Thus, if $n = 3$ and $m = 4$ (as the (4×4) blocks in Paragraph 5.2),

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S_3 has dimensions 12×12 (see Figure 1 in the separated file of the Appendix).

$$S_n^{(t)} = \begin{pmatrix} R_1 & R_1 * R_2 & ... & R_1 * R_{n-1} & R_1 * R_n \\ R_2 * R_1 & R_2 & ... & R_2 * R_{n-1} & R_2 * R_n \\ & & . & & \\ & & . & & \\ & & . & & \\ R_{n-1} * R_1 & R_{n-1} * R_2 & ... & R_{n-1} & R_{n-1} \\ R_n * R_1 & R_n * R_2 & ... & R_n * R_{n-1} & R_n \end{pmatrix} \quad (1)$$

The case with $n = 2$ robots is described in [11]. In the case of a 3-robot swarm, the matrix S_3 is constituted by three single-robot terms (diagonal sub-matrices) and six pairwise interaction terms (off-diagonal sub-matrices).

$$S_3^{(t)} = \begin{pmatrix} R_1 & (R_1 * R_2) & (R_1 * R_3) \\ (R_2 * R_1) & R_2 & (R_2 * R_3) \\ (R_3 * R_1) & (R_3 * R_2) & R_3 \end{pmatrix} \quad (2)$$

In eq. 2, off-diagonal sub-matrices are not symmetric: e.g., R_1 might be sending a message to R_2, while R_2 might not be sending any message in turn. Even if the two robots are sending messages to each other, their content will be different, so the terms are different. In the case study of Section 6, we consider broadcast communication: each robot sends a message to all the other robots. Inside the matrix of eq. (2), one can define the structure of single-robot terms (eq. 4) and pairwise interaction terms (eq. 5). In each single-robot sub-matrix (eq. 4), diagonal blocks indicate robot's own 'perception' and off-diagonal blocks correspond to robot's communication tools.

A specific form that can be taken by diagonal and off-diagonal sub-matrices is proposed at the end of Section 5.2. Let us now introduce the quantum formalism needed to develop our modeling.

5.2. The quantum machinery

We formalize pairwise robotic interaction terms as quantum gates. To this aim, we quantize robots positions and target positions. That is, for each robot,

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we model the position along the x-axis, the position along y, and the reward as quantum superposition of $|0\rangle$ and $|1\rangle$. In our study, we measure the individual reward in terms of target proximity.

In the case of one-dimensional movement, the x-position of the i-th robot is given by:

$$|q_0\rangle = \alpha_i^x|0\rangle + \beta_i^x|1\rangle,$$

where α_i^x is the probability amplitude for outcome $|0\rangle$ along x (left) and β_i^x is the amplitude for outcome $|1\rangle$ (right), and the reward is given by:

$$|q_1\rangle = \gamma_i|0\rangle + \delta_i|1\rangle,$$

where γ_i is the probability amplitude to obtain $|0\rangle$, *failure*, and δ_i to obtain $|1\rangle$, *success*. In the case of a 2-robot toy swarm and motion along one dimension only, one can thus define a reversible logic gate as the one presented in Table 2. If a robot had a successful reward (1) in position 1 at time 0, the other robot reaches it at time 1. Otherwise, the second robot explores around position 0. The so-obtained decision system can be described through a reversible gate. Actual configurations are quantum superposition of states. For a detailed account of states and some simulations, see [11]. The code to implement this gate can be found in the Appendix.

While considering a motion along the plane, we have:

$$|q_0\rangle = \alpha_i^x|0\rangle + \beta_i^x|1\rangle, \quad |q_1\rangle = \alpha_i^y|0\rangle + \beta_i^y|1\rangle, \quad |q_2\rangle = \gamma_i|0\rangle + \delta_i|1\rangle, \qquad (3)$$

where $|q_0\rangle$, $|q_1\rangle$ are the positions along x and y, respectively, and $|q_2\rangle$ is the reward. In particular, α_i^x is the probability amplitude for outcome $|0\rangle$ along x (left), β_i^x for $|1\rangle$ along x (right), α_i^y for $|0\rangle$ along y (up), and β_i^y for $|1\rangle$ along y (down). The pairwise interaction of two robots on the x-y plane can be described through Table 3, implemented through the circuit of Figure 1 obtained with IBM Quantum Composer. On the plane, in case of failure (reward 0) of robot 1, then robot 2 has more than one option, leading to outcome indeterminacy—and the gate is no longer reversible.

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Table 2: Truth tables (reversible equivalents of XNOR gates), representing the interaction between robot 1, R_1 (q_0: position, q_1: reward) and robot 2, R_2 (q_2: position, q_3: reward). At time t_0, R_1 sends to R_2 a message with its position and reward. According to this information, R_2 can choose to reach the first robot or not at t_1. The reward of R_1 is copied in the output to guarantee the same number of inputs and outputs, making the gate reversible. After having sent the message, R_1 stops and waits for the motion of R_2. Once the R_2 reaches the new position at t_2, it can send to R_1, in turn, its position and obtained reward. And, similarly, R_1 can decide to reach R_2 or not at t_2. The first table represents the situation $t_0 \rightarrow t_1$, and the second table, $t_1 \rightarrow t_2$.

q_0	q_1	q_0	q_2
0	0	0	1
0	1	0	0
1	0	1	0
1	1	1	1

q_2	q_3	q_2	q_0
0	0	0	1
0	1	0	0
1	0	1	0
1	1	1	1

Table 3: The idea of Table 2 is now extended to two dimensions in space, with the truth table for two robots R_i, R_j on the plane, no longer reversible because of the indeterminacy on x, y in the case of 0 reward. In this application, we do not consider any waiting time. All robots are exchanging information and moving only if its reward is lower than their own.

q_0	q_1	q_2	q_4	q_3	q_2
x-pos	y-pos	reward	y-pos	x-pos	reward
R_i	R_i	R_i	R_j	R_j	R_i
0	0	0	0/1	0/1	0
0	0	1	0	0	1
0	1	0	0/1	0/1	0
0	1	1	1	0	1
1	1	1	1	1	1
1	0	0	0/1	0/1	0
1	1	0	0/1	0/1	0
1	0	1	0	1	1

In a 3-robot swarm, such an indeterminacy is solved because each robot is receiving information from two robots, not only one, and it can let only the most successful robot (reward = 1 or just higher than the other) enter the table. However, if the most successful robot presents a H (Hadamard) gate for

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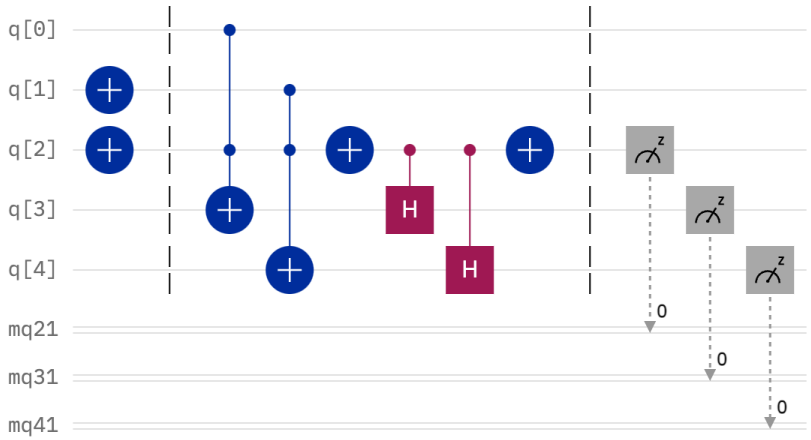

Figure 1: Quantum circuit realizing the truth table of Table 3. The circuit is made of NOT, Toffoli, and Hadamard gates. The white "plus" inside a blue circle indicates the NOT gate; the same symbol connected with two other smaller blue circles indicates the Toffoli gate. The symbol containing the red square with the white H indicates the Hadamard gate. The gray boxes with the Z letter characterize the measurement operation. These are standard symbols used in quantum computing, to indicate logic gates. Each line indicate a qubit ($q[0], ..., q[4]$) and a classical bit ($mq21$, $mq31$, $mq41$), where the results of the measurements are stored. At the end of the circuit, there are measurements for each qubit. As an example, the initial configuration with $|q_0\rangle = 0$, $|q_1\rangle = 1$, $|q_2\rangle = 1$ is shown.

the reward state, then one can also get a superposition of possibilities as the output (Tables 4, 5, 6).

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Table 4: Theoretical expectations for inputs and outputs of the proposed quantum circuit for a selection of eigenstates and state superpositions. The input is given by the values of (x, y)-position and reward values of robot R_i, that is, the message sent by R_i to R_j, the second robot. The output is thus the expected $(x, y)-$position to be reached by R_j. The reward of the first robot is just copied into the output, to have the same number of inputs and outputs, necessary for reversible gates in quantum computing. In fact, in the output, the reward is the one of R_i. The reward of R_j will be computed once the second robot actually will have reached the expected position.

	R_i			(exp.) R_j		
label	x	y	rew.	y	x	(rew.)
A	$R_y(1.9)$	1	1	1	$R_y(1.9)$	1
B	$R_y(1.9)$	0	1	0	$R_y(1.9)$	1
C	0	0	1	0	0	1
D	0	0	0	H	H	0
E	1	1	0	H	H	0
F	1	0	1	0	1	1
G	H	1	1	1	H	1
H	1	1	H	H (more 1)	H (1)	H
I	H	H	H	H	H	H
L	1	1	$R_y(1.9)$	1	1	$R_y(1.9)$
M	0	1	1	1	0	1
N	1	1	1	1	1	1

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Table 5: Comparison and agreement between theoretical expectations (Table 4) and measured outcomes, obtained with the QASM simulator. Empty cells indicate 0 measurements. Screenshots of measurements A_1, ..., N_1 can be retrieved at `https://github.com/medusamedusa/3-robot/blob/main/QASM_simulator.zip`. Here and in Table 6, the states indicate x, y, and reward, respectively.

	states								agr.
label	000	001	010	011	100	101	110	111	
A_1						345		679	✓
B_1		345		653					✓
C_1		1024							✓
D_1	245		265		254		260		✓
E_1	254		232		258		280		✓
F_1				1024					✓
G_1						540		484	✓
H_1	152		124		146		128	474	✓
I_1	247		259		270		248		✓
L_1	87		84		100		75	678	✓
M_1						1024			✓
N_1								1024	✓

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Table 6: Comparison between the outcomes obtained with three IBM quantum computers (C), located in Bogotà (B), Lima (L), and Manila (M), for states A, ..., N and agreement with respect to theoretical expectations (Table 5). Some relevant results in agreement with the expectations are highlighted in bold. Screenshots of A_2, ..., N_2 can be retrieved at `https://github.com/medusamedusa/`, in the zip files *quantum_computer_Bogota.zip*, *quantum_computer_Lima.zip*, and *quantum_computer_Manila.zip*, respectively.

		states								agr.
label	C	000	001	010	011	100	101	110	111	
A_2	B	177	197	99	136	129	**139**	74	82	∼
	L	144	163	108	117	152	**160**	63	**117**	∼
	M	88	78	83	119	70	**234**	88	**264**	✓
B_2	B	212	**181**	127	**146**	120	94	73	71	∼
	L	240	**196**	136	156	126	75	53	42	∼
	M	99	**283**	86	**302**	66	67	44	77	✓
C_2	B	144	**251**	88	128	111	132	78	92	✓
	L	199	**318**	100	130	122	168	65	22	✓
	M	114	**384**	70	96	69	129	84	78	✓
D_2	B	**368**	59	**141**	21	**294**	25	**106**	10	✓
	M	**248**	106	**174**	44	**225**	33	**173**	21	✓
	L	**260**	44	**250**	33	**208**	33	**179**	17	✓
E_2	B	**203**	66	**125**	59	**256**	90	**154**	71	✓
	L	304	51	218	41	216	26	131	37	✓
	M	259	41	184	34	229	51	174	52	✓
F_2	B	98	234	106	**209**	77	115	74	111	∼
	L	165	257	111	**230**	106	49	74	32	∼
	M	69	324	72	**280**	62	91	49	77	∼
G_2	B	135	206	71	125	138	**188**	74	87	∼
	L	120	137	106	73	109	**222**	114	**143**	✓
	M	55	177	45	129	75	**293**	59	**191**	✓
H_2	B	161	109	141	55	202	113	152	91	✓
	L	185	61	204	72	156	64	205	77	✓
	M	184	77	162	112	113	95	113	168	✓
I_2	B	198	140	137	121	155	89	110	74	✓
	L	231	104	143	94	216	62	106	68	∼
	M	188	128	128	83	190	109	112	86	✓
L_2	B	224	85	192	100	122	93	119	89	x
	L	142	156	101	154	138	124	108	101	x
	M	128	107	148	123	102	110	77	**229**	✓
M_2	B	134	115	108	**192**	160	68	119	128	x
	L	146	173	127	199	149	118	106	86	x
	M	68	198	65	101	81	**339**	76	96	✓
N_2	B	147	154	112	253	58	108	65	127	x
	L	150	190	99	139	103	130	111	102	x
	M	67	98	81	304	78	104	76	**216**	∼

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Table 4 presents, for a selection of states, input values and expected values. Table 5 shows the agreement between these expectations and the output obtained with the QASM simulator. Table 6 shows the comparison between the outcomes obtained through three different IBM quantum computers, presenting a larger quantum noise.

Once our quantum circuit is defined and tested on simulators and quantum computers, it can be embedded into a code for a robotic swarm, to verify the impact of quantum-modeled local interactions on the swarm global behavior. Thus, we can now more precisely shape our matrices. In fact, in practical implementations with robots, we can either use the circuit of Figure 1 in a loop until robots reach the target, or we can embed the circuit in a more complex code with several steps. Each step corresponds to a shot of the matrix, with broadcast information and consequent action of each robot, activating the sub-matrices corresponding to pairwise interaction with the most successful robot. This strategy has been chosen for the case study in Section 6. In the Appendix, the matrices for each step of a run are presented.

$$R_i(t) = \begin{pmatrix} x_i(t) \text{ (Where I am)} & \dot{x}_i(t) \text{ (Where I go)} & send\,(A) \text{ on} & send\,(B) \text{ on} \\ y_i(t) \text{ (Where I am)} & \dot{y}_i(t) \text{ (Where I go)} & send\,(C) \text{ on} & send\,(D) \text{ off} \\ receive\,(A) \text{ on} & receive\,(B) \text{ on} & type \text{ e-puck} & wheels\ 2 \\ receive\,(D) \text{ on} & receive\,(D) \text{ off} & camera \text{ on} & motor\ 1 \end{pmatrix} \tag{4}$$

Let us focus on a 3-robot toy swarm moving on the plane, analyzing in more detail the structure of matrices. In (4), positions x, y are considered as the quantum superposition of states, see Section 5.2 for details. Letters A, ..., D indicate different communication channels.

In each pairwise-interaction sub-matrix (eq. 5), the first diagonal block contains the information, sent by the i-th robot (R_i), about its (x,y)-position probability amplitudes and reward. The second diagonal block contains the possible behavioral response of the j-th robot (R_j), according to the reward of R_i. If the reward is high, R_j follows R_i; otherwise, the position of the R_j robot

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at the subsequent time interval is assumed as a quantum superposition of the possible outcome of x- and y-positions.

$$(R_i * R_j)(t) = \begin{pmatrix} \alpha_i^x(t) & \beta_i^x(t) & \gamma_i(t) & 0 \\ \alpha_i^y(t) & \beta_i^y(t) & 0 & \delta_i(t) \\ 0 & 0 & \alpha_j^x(t+1) & \beta_j^x(t+1) \\ 0 & 0 & \alpha_j^y(t+1) & \beta_j^y(t+1) \end{pmatrix} \tag{5}$$

In principle, one should evaluate six interaction terms. However, we can compute only three of them, halving computational times. In fact, the R_j gets signals from R_i and R_k, comparing this information with its own information on position and reward. R_j chooses which robot to follow (or, more precisely, which robot should enter the quantum circuit to decide the position) according to its highest reward. R_j remains stationary if it already has the highest reward. In the presented pseudocodes (see the Appendix) and in the corresponding Jupyter/Python codes we created, it has been necessary to compute just one interaction term, having as inputs positions' and rewards' probability amplitudes of the robot with the highest reward, and as output, positions probability amplitudes assigned to the other two robots. Small fluctuations are added to avoid a superposition of these two robots. An example of the matrix output, when the proposed quantum circuit is included in our ant-inspired code for three robots, is proposed at the end of the Appendix.

5.3. Pseudocodes

Before moving to the results, let us describe the pseudocodes used in our method. We present here our original and improved algorithms, Algorithms 1 and 2, respectively.

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Algorithm 1 original quantum-gate driven search

1: Class Target and instances (T_1: Food, T_2: Nest)
2: Robots R_1 R_2, R_3 as classes, values as attributes
3: **if** All robots have a high but not max reward: **then**
4: little random variation of R_1
5: **end if**
6: **Inputs**: initial robots' positions
7: Reward evaluation Robots i, j, k (as Euclidean distance from the target)
8: **if** $R_{1,2,3}$ have a reward lower than a threshold: **then**
9: Re-initialize randomly their positions and rewards
10: **else** Do not re-evaluate positions and rewards
11: **end if**
12: **for** $i, j, k = 1, 2, 3$ **do**
13: R_i gets information from robots j, k
14: Evaluation of the highest probability of 'Yes' as reward
15: **if** If the i-th robot had already the highest reward: **then**
16: the other robots reach it
17: **end if**
18: **end for**
19: Evaluation of the highest probability of 'Yes' as reward
20: **if** If the i-th robot has the highest reward: **then**
21: it does not enter the circuit of Figure 1 for probabilistic decision-making
22: it does not move
23: **else if** robot j has the highest reward: **then**
24: x-y positions and reward of R_j at time t are the circuit inputs
25: circuit output: most likely positions at $t + 1$ for both R_i and R_k
26: the 2 most frequent configurations: arrays; occurrences: weights
27: **end if**
28: update positions, reward, graph
29: repeat lines 3, 8, 15
30: **if** $R_{1,2,3}$ have a reward difference lower than a threshold: **then**
31: lock robots in a GHZ. 1(0): higher-reward robot position +(-) fluctuation; measure of the qubits
32: **end if**
33: overall fluctuation or overall flip
34: **Outputs**: final robots' positions and final rewards
35: repeat from Line 3 to Line 34 to get back to the nest, calculating rewards with respect to the nest (T_2).

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Algorithm 2 enhanced quantum-gate driven search

```
choose the number of robots
initialize robots' positions as state superpositions
if all rewards are below a certain threshold then
    for each robot do
        randomly reshuffle positions
        if a position hits the obstacle then
            reshuffle the position for that robot
        end if
    end for
end if
find the robot with the highest reward and let it enter the circuit
find the new suggested position through the circuit
for all robots do
    evaluate the new rewards
end for
```

Regarding time complexity, conditional statements have a complexity of $O(n \log n)$, and time loops of $O(n)$; because our algorithms contain nested *for* and *if*, the complexity is of $O(n)$. Space complexity is also $O(n)$, because of the sorting function: *What is the most successful robot?*. The sorting function adopted for our examples is classical. The quantum part regards the computation of the logic gate $R_i \rightarrow R_j$. In next research, the sorting operation could be made quantum as well, using the Grover search, with $O(\sqrt{n})$.

6. Results: A case study

At the end of Section 5, the pseudocodes used for our method have been presented. In this Section, we analyze the effectiveness of our quantum circuit inside the algorithm, and then we compare it with other search methods. First, setup and conducted experiments based on your model are presented (subsection 6.1); then, a quantitative comparison between our method and the PSO variant

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(subsection 6.2) and the NL-SHADE-RSP with midpoint algorithm (subsection 6.3) are discussed.

6.1. Setup and experiments with our model

Before presenting the implementation results, we recall that robot positions are defined here as quantum state superpositions: for R_i, there is a probability amplitude α_i^x to stay in the point 0 of the x-axis, β_i^x to stay in the point 1 of the x-axis, and corresponding α_i^y, β_i^y for the y-axis, respectively. If $(\beta_i^x, \beta_i^y) = (0.99, 0.99)$ is almost sure to find the robot in (1,1): visually, wave functions have a peak in (1,1). So, one can reasonably (within quantum indeterminacy) localize R_i in $(1, 1)$. If $(\beta_i^x, \beta_i^y) = (0.5, 0.5)$, however, there is the same probability amplitude to find the robot in any point of the xy-plane. With a little abuse of notation, in this situation we indicate the robot in $(0.5, 0.5)$, that is, considering the positions as the peaks of the wavefunctions describing the states: half-way between the two extremes of the $[0, 1]$ segment along x and y. This is why β_i^x, β_i^y are considered to build our (approximate) visual representations. The target is treated the same way, making possible the comparisons between target's and robots' positions.

The reward is evaluated as the distance from the target. It is the measure of δ, the amplitude probability to get 1, that is, 'success.' This is a conceptual simplification: the robots cannot truly know their distance from the target. They can assess their approximate positions and distances interpreting their vision (if we are considering robot vision, or smell for ants), as 'yes,' 'no,' 'maybe.' In principle, robots can rotate to improve their camera vision, and they should also avoid eventual obstacles in their path. In future research, one can model robot vision, leaving the evaluation of rewards to the 'precision' of target proximity assessment. E.g., if only the 30% of target is visible on a camera, then that robot will have, as a reward, 30% or 'yes' and 70% of 'no.' The overall code would remain the same, except for the reward's evaluation.

Let us now describe an implementation of our method for a small-sized

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robotic swarm. We start with the comparison of convergence efficiency[7] of our decision-making system for swarms of different sizes. In Table 7, we consider the enhanced and shortened code, allowing the customization of the number of elements of the swarm. Once can notice that the scalability starts being visible for $N \geq 5$. A more robust convergence is presented for 10 robots. Trials performed with swarms of 15 units, not reported here, show results similar to the ones obtained for 10 robots. In analogy with the biological model of foraging ants [82], in this analysis we consider the motion from the nest to the source of food (the target). In the table, the closer the average reward to 1, the more successful the swarm.

Table 7: We present a comparison between the rewards for swarms of different sizes, with $N = 2, 3, 5, 10$ robots, respectively. For these simulations, we used the second (and shorter) algorithm, and we placed the nest (start) in (0.2, 0.9), and the food (target) in (0.9, 0.2).

		average reward			
N	trial	t_0	t_1	t_2	t_3
2	1	0.249	0.689	0.682	0.760
	2	0.217	0.637	0.777	0.781
	3	0.245	0.534	0.698	0.733
3	1	0.244	0.694	0.779	0.801
	2	0.256	0.546	0.585	0.673
	3	0.271	0.756	0.836	0.920
5	1	0.253	0.778	0.798	0.892
	2	0.245	0.637	0.831	0.899
	3	0.241	0.685	0.774	0.856
10	1	0.256	0.596	0.684	0.821
	2	0.251	0.661	0.843	0.878
	3	0.255	0.601	0.833	0.924

[7]While talking about our algorithms, we use the term "efficiency" to indicate the reduction of code lines and the improved accuracy in target reaching, shown by the second code.

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Let us now observe in detail the performance with the longer algorithm, complete with GHZ, for a 3-robot swarm. Following the biologic example of ants, we consider a motion back-and-forth between the nest (T_2) and the food location (T_1), evaluating robots' behavior in terms of their rewards (Table 8). When the convergence is not optimal, additional run of the code allow to reach a more precise convergence. In the majority of our tests, the robots converge to the targets, showing a concordance with the model. In the lower part of Table 8, the food position is changed.

Table 8: Top: Comparative table with the results of eight runs of the first code (longer), between two targets: T_2, the food source, and T_1, the nest, in analogy with the Ant Lines model. Numbers in bold highlight the most relevant contributions of the gate to improve rewards. The arrival position of the first path is the starting point for the second path. The last column shows the convergence, which is successful if the final rewards are ≥ 0.8, and approximate if ≥ 0.6, < 0.8. Bottom: Robots' behavior in terms of reward time points for changing food locations. Details of the gate states outcomes and populations can be found at `https://github.com/medusamedusa/3-robot`; files test(number)_Jan_29.pdf (table above) and trial(number)_different_food_Jan_29.pdf (table below).

test	path	initial rewards	rewards before circuit	rewards after circuit	rewards before GHZ	final rewards	
1	$T_2 \to T_1$	(0.3, 0.34, 0.29)	(0.68, 0.6, 0.57)	(0.68, 0.8, 0.53)	(0.8, 0.8, 0.82)	(0.85, 0.85, 0.85)	✓
	$\to T_2$	(0.15, 0.15, 0.15)	(**0.58**, **0.56**, **0.38**)	(**0.58**, **0.78**, **0.38**)	(0.78, 0.86, 0.81)	(0.92, 0.92, 0.92)	✓
2	$T_2 \to T_1$	(0.3, 0.34, 0.29)	(0.78, 0.84, 0.81)	(0.7, 0.84, 0.69)	(0.87, 0.84, 0.53)	(0.81, 0.81, 0.81)	✓
	$\to T_1$	(0.25, 0.25, 0.25)	(0.9, 0.93, 0.86)	(0.68, 0.93, 0.86)	(0.93, 0.93, 0.89)	(0.9, 0.9, 0.9)	✓
3	$T_2 \to T_1$	(0.3, 0.34, 0.29)	(**0.76**, **0.55**, **0.41**)	(**0.76**, **0.6**, **0.6**)	(0.7, 0.7, 0.7)	(0.67, 0.67, 0.67)	∼
	$\to T_1$	(0.33, 0.33, 0.33)	(0.86, 0.8, 0.83)	(0.86, 0.78, 0.83)	(0.78, 0.76, 0.83)	(0.83, 0.83, 0.83)	✓
4	$T_2 \to T_1$	(0.3, 0.34, 0.29)	(**0.78**, **0.56**, **0.25**)	(**0.78**, **0.78**, **0.78**)	(0.78, 0.78, 0.78)	(0.67, 0.67, 0.67)	∼
	$\to T_1$	(0.41, 0.41, 0.41)	(**0.41**, **0.41**, **0.41**)	(**0.7**, **0.7**, **0.41**)	(0.7, 0.7, 0.7)	(0.77, 0.77, 0.77)	∼
5	$T_2 \to T_1$	(0.3, 0.34, 0.29)	(0.79, 0.4, 0.65)	(0.79, 0.52, 0.67)	(0.79, 0.71, 0.71)	(0.77, 0.77, 0.77)	∼
	$\to T_1$	(0.53, 0.53, 0.53)	(0.53, 0.53, 0.53)	(0.46, 0.46, 0.53)	(0.89, 0.89, 0.93)	(0.85, 0.85, 0.85)	✓
6	$T_2 \to T_1$	(0.3, 0.34, 0.29)	(**0.68**, **0.21**, **0.79**)	(**0.8**, **0.8**, **0.79**)	(0.78, 0.8, 0.6)	(0.77, 0.77, 0.77)	∼
	$\to T_1$	(0.29, 0.29, 0.29)	(0.35, 0.42, 0.53)	(0.46, 0.46, 0.53)	(0.7, 0.7, 0.72)	(0.72, 0.72, 0.72)	∼
7	$T_2 \to T_1$	(0.3, 0.34, 0.29)	(0.92, 0.9, 0.9)	(0.92, 0.9, 0.9)	(0.9, 0.69, 0.98)	(0.9, 0.9, 0.9)	✓
	$\to T_1$	(0.25, 0.25, 0.25)	(0.63 0.57 0.66)	(0.46, 0.66, 0.66)	(0.75, 0.75, 0.66)	(0.66, 0.66, 0.66)	∼
8	$T_2 \to T_1$	(0.3, 0.34, 0.29)	(0.64, 0.62, 0.59)	(0.64, 0.77, 0.54)	(0.77, 0.77, 0.6)	(0.81, 0.81, 0.81)	✓
	$\to T_1$	(0.38, 0.38, 0.38)	(0.86, 0.93, 0.87)	(0.59, 0.93, 0.87)	(0.93, 0.96, 0.91)	(0.92, 0.92, 0.92)	✓
food position (T_1')	path	initial rewards	rewards before circuit	rewards after circuit	rewards before GHZ	final rewards	
(0.2, 0.9)	$T_2 \to T_1'$	(0.6, 0.6, 0.64)	(**0.6**, **0.6**, **0.64**)	(**0.86**, **0.86**, **0.64**)	(0.81, 0.86, 0.82)	(0.81, 0.81, 0.81)	✓
	$\to T_2'$	(0.65, 0.65, 0.65)	(0.65, 0.65, 0.65)	(0.68, 0.64, 0.75)	(0.74, 0.84, 0.82)	(0.92, 0.92, 0.92)	✓
(0.9, 0.9)	$T_2 \to T_1'$	(0.19, 0.23, 0.2)	(0.6, 0.6, 0.64)	(0.2, 0.23, 0.6)	(0.98, 0.94, 0.98)	(0.93, 0.93, 0.93)	✓
	$\to T_2$	(0.65, 0.65, 0.65)	(0.86, 0.88, 0.91)	(0.86, 0.88, 0.91)	(0.74, 0.84, 0.82)	(0.91, 0.91, 0.91)	✓
(0.5, 0.0)	$T_2 \to T_1'$	(0.42, 0.44, 0.38)	(0.5, 0.44, 0.5)	(0.5, 0.5, 0.5)	(0.55, 0.55, 0.5)	(0.71, 0.71, 0.71)	∼
	$\to T_2$	(0.69, 0.69, 0.69)	(0.69, 0.69, 0.69)	(0.78, 0.7, 0.69)	(0.76, 0.76, 0.76)	(0.81, 0.81, 0.81)	✓

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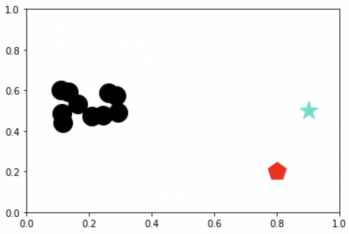
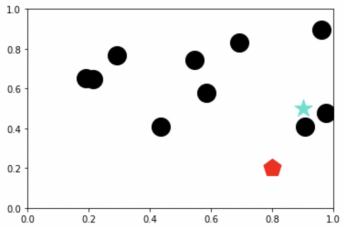
From our tests, one can see that the proposed gate works better when the robots are in exploration phase, that is, in sight of the target but not very close to it. In a real application, one can argue that, if robots are all already close enough to the target, they are not required to enter the gate. In the majority of cases, the gate actually improves the robot reward statuses. Fluctuations are justified by the probabilistic nature of this approach. Our toy-swarm converges to the target (final rewards ≥ 0.8) in 14 out of 19 paths, and it reaches some closeness (final reward ≥ 0.6) in all paths.

Here, we propose the entanglement (GHZ state) to let robots act as a whole system in the latest steps of the search process. In other studies, the entanglement condition is imposed right at the beginning [67]. However, here we avoid any strong hypothesis in the first steps of the process, preferring instead to focus on the proposed circuit action.

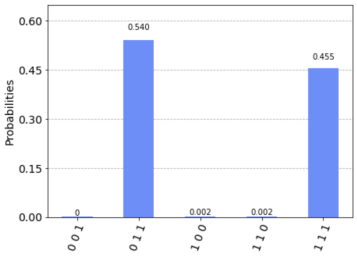
Figure 2 shows the path simulation obtained with the shorter code and 10 robots. One can notice that, to reach the target, a smaller number of passages is required, and the convergence is high (the average reward is 0.925). Considering as metrics the number of direction changes across the different methods, we can qualitatively assess that, even in its original version, our method requires a smaller number of turns for the robots. Moreover, in the shorter version of the code, there is only one change of direction, after the computation of the gate. The number of direction changes for the bounded round walk and the NetLogo simulation is visibly higher than 1. To make the code more general, we included a known obstacle (indicated by the red pentagon), that is avoided by the swarm. If, after the random generation of positions (step 1), the obstacle position would be hit by a robot, a new cycle of random-number generation is activated.



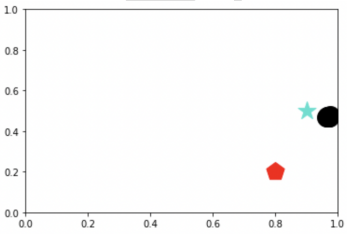
(a) Test A, initial configuration, in analogy with the ant-lines example



(b) Second step with the scattering



(c) Results of the gate computation



(d) Final configuration, after the gate

Figure 2: Same setup of Figure 3, improved code (the second and shorter one) and results of the test for 10 robots. The red pentagon indicates an obstacle which is avoided. The complete output of this test is in the PDF *testA.pdf*, available in the shared Git folder `https://github.com/medusamedusa/10_little_ants`.

6.2. Comparison with a PSO variant

In this subsection, we first present a qualitative comparison with another ant-colony model, coded in NetLogo; then, we focus on a PSO variant. Our method is applied to a 3-robot swarm for the first case, and to a 10-robot swarm in the second case. Figure 3 also shows a qualitative visual comparison between paths back-and-forth in Test 1, a similar setup with Ant Lines in NetLogo and a Python-made 3-objects bounded random walk. A video simulation for Test 1 is available at `https://github.com/medusamedusa/3-robot`.

Let us now present a comparison between particle swarm optimization (PSO) approach [83] and our proposed method. See the Appendix for the code references. Our results are shown in Table 9 and Figure 4. For both approaches, we consider a 1×1 square. However, PSO is conceptually different from our

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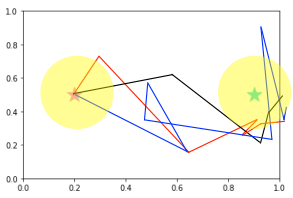
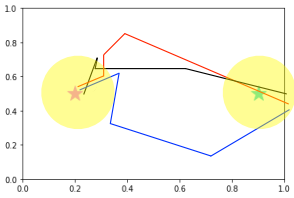
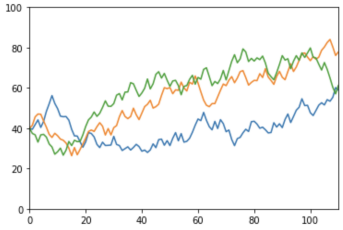


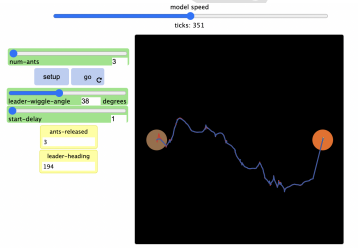
(a) Test 1, path $T_2 \to T_1$



(b) Test 1, path $T_1 \to T_2$



(c) Bounded random walk from left to right



(d) 3-ant example with NetLogo from nest to food

Figure 3: Visual comparison between paths for our setup, ant lines, and bounded random walks. For the simulations of our method, we used here the first (longer) code. In (a) and (b): the red star indicates the center of the nest, and the turquoise star indicates the food source. The yellow disks highlight the range of convergence. As one can see in Figure 2, with the enhanced and shortened code, the convergence is significantly improved, presenting a smaller number of steps. Computational details of the bounded random walks and the 3-ant example with NetLogo are provided in the Appendix.

approach. Our method is not an optimization technique. For our PSO example, there is a surface living in three dimensions. The equivalent of the target is the minimum of the surface. We define an object function, that is, $f(x,y) = (x-0.9)^2 + (y-0.5)^2$, having its minimum in $(0.9, 0.5)$, the same point where the considered target is. In Table 10, the considered target is $(0.8, 0.9)$, and the objective function is changed accordingly. Running the simulation, we estimate the precision of PSO particles in making it to the target. Starting with the PSO with the same initial conditions of our approach (all robots in a small cluster centered in a point of the plane), obtained with

$$X = \text{np.random.rand}(2,\ \text{n\_particles}) * 0.1 + 0.2,$$

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$$Y = \text{np.random.rand}(2,\ \text{n\_particles}) * 0.1 + 0.2,$$

(X, V in the code), the convergence is 10 times less precise than for our method. Starting with the PSO with all-scattered robots, that is, with

$$X = \text{np.random.rand}(2,\ \text{n\_particles}) * 0.9,$$

$$Y = \text{np.random.rand}(2,\ \text{n\_particles}) * 0.01,$$

then the target is more precisely reached. In both tests with PSO, 49 iterations are considered. However, with our method, only four passages are needed. To quantitatively compare our results against the PSO ones, we compute distances (Euclidean, Manhattan, cosine dissimilarity) between the swarm barycenter and the target coordinates. Results are presented in Table 10. We consider PSO taking into account the constraints of our scenario. When the initial parameters are the same as the ones we used, the results obtained with our approach appear as more precise. In fact, the distance values between the barycenter of the swarm and the target are smaller, see Table 10. If the PSO initial condition is the scattered search rather than all particles in a "nest," then in one case PSO outperforms our method (test f), and in another one, our method outperforms PSO (test n). However, these are only initial considerations. In future research, one can exploit machine learning to run further tests, and statistical techniques to analyze the results, also taking into account recent developments of PSO [35], and enhancements of ant-colony approaches with the genetic algorithm (GA) [36, 37]. More details about future improvements, also inspired by GA-derived and differential evolution-based competition-winner algorithms [34, 84], are discussed in Section 7.

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Table 9: Comparison between our method (second code) and a PSO test (see Figure 4). The PDF with the complete outputs can be found in the folder `https://github.com/medusamedusa/10_little_ants/tree/main/test_PSO`.

	nest (starting points)	food (target)	error
expected	(0.6, 0.6)	(0.9, 0.5)	0.
our method	centered in (0.6, 0.6)	(0.89, 0.65/0.66)	0.01
PSO - test 1	centered in (0.6, 0.6)	(0.79..., 0.59,...)	0.1
PSO - test 2	scattered	(0.90, 0.49)	0., 0.01

Table 10: Distance comparison between the position of the robotic-swarm barycenter and the target coordinates, at the end of the search process. In tests (a)-(e) and (g)-(m), the starting point of robots are centered in the nest at $(0.6, 0.6)$, while in tests (f) and (n) they are scattered through the arena (that is, with an initial condition that differs from ours). The pairs of tests (d),(e) and (l),(m) lead to the same numerical results, respectively. The complete outputs can be found in the folder `https://github.com/medusamedusa/10_little_ants/tree/main/test_distance_comparison_PSO`.

				barycenter/target distance		
test	method	start	target	Euclidean	Manhattan	Cosine
a	our	centered	(0.9, 0.5)	0.047	0.066	0.001
b	our	centered	”	0.124	0.131	0.006
c	our	centered	”	0.019	0.025	$2.95 * 10^{-6}$
d	PSO	centered	”	0.137	0.193	0.010
e	PSO	centered	”	0.137	0.193	0.010
f	PSO	scattered	”	$7.65 * 10^{-5}$	$7.70 * 10^{-5}$	$2.90 * 10^{-9}$
g	our	centered	(0.8, 0.9)	0.0196	0.025	0.0001
h	our	centered	”	0.071	0.099	0.0018
i	our	centered	”	0.126	0.155	0.0014
l	PSO	centered	”	0.418	0.506	0.057
m	PSO	centered	”	0.418	0.506	0.057
n	PSO	scattered	”	0.386	0.462	0.0497

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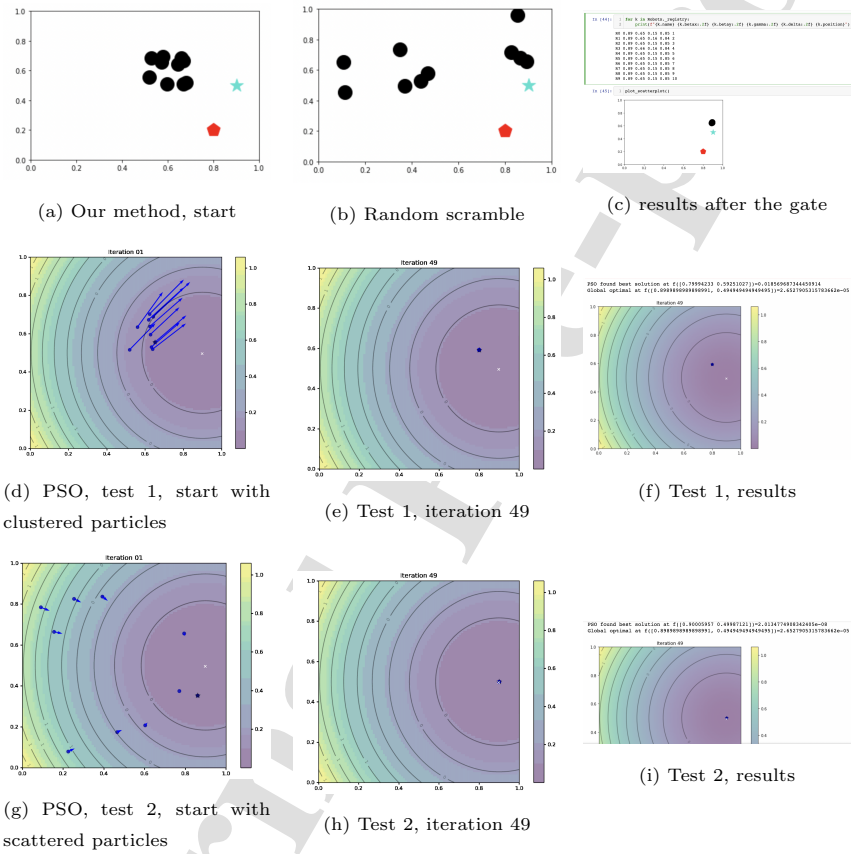

(a) Our method, start

(b) Random scramble

(c) results after the gate

(d) PSO, test 1, start with clustered particles

(e) Test 1, iteration 49

(f) Test 1, results

(g) PSO, test 2, start with scattered particles

(h) Test 2, iteration 49

(i) Test 2, results

Figure 4: A comparison between our method (a-c), applied to 10 robots, for a target in (0.9, 0.5), clustered robots around (0.6, 0.6), and an obstacle, with an example of PSO (d-i), where the target is represented by the minimum of an objective function.

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6.3. Comparison against an NL-SHADE-RSP algorithm

Finally, we performed a comparison between the precision of the results achieved with our method, and with a winner of a CEC-winner in 2022. In particular, we focused on a differential evolution-based algorithm, NL-SHADE-RSP with midpoint [34]. During the competition, the code has been used to optimize objective functions. Here, we wrote two new objective functions, having global minima in correspondence of two target locations of our scenario, respectively. In [34], the conditions of our scenario are reproduced, with a population of ten units initially starting in a disk centered on $(0.6, 0.6)$ with a radius of 0.1. The exploration space is a square of side $[0, 1]$. A total of 25 runs for each one of the two objective functions have been completed. Table 11 shows the data of first evaluations of the two functions in ten different runs for each objective function. The objective function is evaluated 300 or 400 times to reach the target with an error of 10^{-9}. After one or three evaluations, the results have an error of the same order of magnitude of the results attained with our methodology: see test a-c and test g-h from Table 10. After four evaluations, the error is of the same order of magnitude, or one order less. The advantage shown by our methodology (which leads to suboptimal solutions) is the simplicity of passages and the precision corresponding to the first evaluations of an advanced and way more complex optimization algorithm. It should be precised that the method NL-SHADE-RSP [34] has not been devised for small functions, and that our methodology is not an optimization technique. Thus, such a comparison gives an idea of the orders of magnitude of the error a target is reached with, in our 4-step code, and according to the first evaluations of a recent and efficient algorithm.

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Table 11: Outputs of a recent version of NL-SHADE-RSP with midpoint [34], tested on two different objective functions (f. obj. 1 and f. obj. 2), having as minima the targets in our scenario: $(0.9, 0.5)$ and $(0.8, 0.9)$, respectively. Population is constituted by 10 units, as in our test of Table 10. The complete data can be retrieved at `https://github.com/medusamedusa/10_little_ants/tree/main/SHADE`.

		distance from the target	
run	no. of evaluations	f. obj. 1	f. obj. 2
1	1	0.0564498	0.129196
	4	0.00699566	0.121177
	50	0.00428091	0.0590199
2	1	0.052212	0.0804435
	4	0.052212	0.0804435
	50	0.00898058	0.0434829
3	1	0.0368419	0.0804435
	4	0.0135726	0.0804435
	50	0.000914063	0.0434829
4	1	0.0135726	0.126022
	4	0.0135726	0.0887293
	50	0.000914063	0.026067
5	1	0.0961737	0.218962
	4	0.0785412	0.109809
	50	0.035633	0.0689708
6	1	0.158216	0.167935
	4	0.0593486	0.0561922
	50	0.0136765	0.0326968
7	1	0.0320644	0.170921
	4	0.0320644	0.147435
	50	0.000156165	0.0736671
8	1	0.0583604	0.0661496
	4	0.0583604	0.0661496
	50	0.0140189	0.0661496
9	1	0.0961737	0.218962
	4	0.0785412	0.109809
	50	0.035633	0.0689708
10	1	0.0583604	0.0804435
	4	0.0583604	0.0804435
	50	0.0140189	0.0434829

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7. Discussion

Addressing the problematics of a formal description of robotic swarms, we developed a matrix-based approach distinguishing between single robots' information and pairwise interactions. Then, we focused on interactions, developing a quantum-based decision model to connect information exchange and local decisions with the overall swarm behavior. We devised a strategy and implemented it for 3-robot and 10-robot toy swarms. The proposed strategy could constitute a new software design paradigm, with communications included in the feedback loop. The core idea of our study can in the future be enriched making the whole system scalable, finding patterns of behavior, adding one more spatial dimension, and introducing learning.

While we worked with three and ten robots only, our algorithm can be extended to a generic number of robots. The structure of the matrix would remain the same. Scalability is thus possible. From 3 to N robots, one would have no longer $3! = 6$ but $N!$ interaction sub-matrices. However, given the described patterns of behavior, with only the more successful robot entering the circuit, also in the case of N robots, the evaluation of only one interaction term could just be required. At time t, all robots broadcast information about their reward. The position of the only robot with the highest reward enters the decision system of each robot, allowing them to move accordingly at $t + 1$. Other steps of reshuffle (see the Pseudocode in Materials and Methods) allow robots to improve their exploration also in cases of low reward for all of them.

Here, the reward has been evaluated as the distance from the target. In the future, this information might be recovered through camera observations, sound recognition, or even odor recognition, as it happens for ants with pheromones. In any case, the main structure of the algorithm would remain unchanged.

Our study involved robots in the plane. This is an extension of our first approach with movements along a line [11]. One can extend the present study by adding one more spatial dimension, as sketched in [85]. The circuit does only require one more qubit. This change does not alter the core idea. A

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tridimensional motion would make the modeling of swarm robotic motion in the air or underwater possible. In our study, we first considered the ideal model of motion in an obstacle-less two-dimensional space, with battery-less robots. Then, in our enhanced code, we included a known obstacle. The passage from simulation to real robots would of course require some battery information. The precision has been greatly improved through the enhanced and shortened code. It includes the nest→food path, allowing robots to quickly reach the target without the need for the GHZ passage. We also quantitatively measured the different precision of target reaching achieved with our method and with an application of particle swarm optimization. While we are not considering an optimization approach here, we set up the parameters of a PSO example and of a NL-SHADE-RSP algorithm [34] to match the characteristics of our scenario. When we have the same starting parameters, the results obtained with our approach appear as being more precise. Refined approaches of machine learning will be exploited to investigate in more detail the advantages of our code with respect to existing classic approaches, and ultimately to refine our method. We considered a basic example of PSO for its conceptual importance, being it at the base of recent and valuable developments [35]. Regarding the quantum improvement of classical ant-foraging approaches, we qualitatively considered the number of steps and direction changes in the NetLogo application and in our simulation. Our robots reach the target with less changes of directions, that is, with different steps. However, this article presents the methodology. More detailed evaluations of performance time will be considered in future research.

In our current research, we considered sub-optimal solutions. However, in our case, the focus was on the methodology definition, rather than on the search of an optimal solution. In addition, our methodology is general. It is specialized into a specific task according to the choices made for the logic gate. Thus, the comparison we have run are merely examples of the results we can obtain with respect to other methods.

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7.1. *Hints toward future research*

In next steps of the research, one can take into account the progressive refinement of its applications. We can find out which logic gate or which degree of parameters' detail may yield the more precise results. And, only in that case, a precise comparison with optimization approaches can make sense. At that point, we will dig into the most recent and effective approaches of PSO [35], GA + ant-colony [36], competition-winners of GA-based and differential evolution-based algorithms [34, 84] (CEC 2022 Bound Constrained Single Objective Numerical Optimization benchmark problems). These comparisons would, in turn, help us refine the logic gates to obtain the best results. We have already considered NL-SHADE-RSP with midpoint [34] for a first comparison, commenting the results of Table 11 with respect to the outputs of our methodology shown in Table 10. We noticed that the error in target-reaching with our 4-step method, leading to suboptimal solutions, are of the same order of magnitude of the first objective-function evaluations in [34]. In general, the idea of genetic selection could correspond, for us, to the attention devoted to the most successful robots. However, the other robots would not die or be substituted; instead, they would just follow the more successful ones. As an example of how these connections could be developed, let us consider the key ideas of the CEC-winner, differential evolution-based algorithm proposed by Biedrzycki and co-authors [34], called NL-SHADE-RSP. It is a GA-derived algorithm where the k-means is used to split into half large populations, evaluating their midpoint. In our case, we could run experiments with a large number of robots, and split the population into two parts, distinguishing between the robots closer and less close to the target. The bound constraint can be given, in our case, by the size of the arena (as we already did for PSO). The objective could be a function having a minimum or so in a neighborhood of the target (similarly to what we did for PSO and NL-SHADE-RSP). The step of population size diminution can lead, in our case, to the shift of attention toward only those robots that are really close to the target. In the following step, one would just give all other robots the command to reach their most successful peers. From such a comparison, we could refine

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our search-and-rescue applications, getting to the target with a limited number of passages and with a greater precision. We could also, quantitatively, find out how to tune the parameters within our matrices to get scenarios and results as close as possible to NL-SHADE-RSPv[34] and other recent works [84].

As another future development, learning can be added by defining arrays of "probable target locations." This can easily be done for T_2, with robots storing their initial position if, as in our examples, they start from the nest. It can also be done for food position (T_1): once a robot obtains a reward ≥ 0.8, it can store its (probable) position within an array, and communicate this information to the other robots. If, during subsequent exploration steps, the reward of a robot gets higher than the stored information, the memory array is updated. Otherwise, if rewards are constant or diminish, at least one robot can get back to the stored position. The entire algorithm can be performed as a loop, and the search can be stopped once the stored position corresponds to a reward of around 0.9. Thus, one would have a loop while the rewards are ≤ 0.9.

The introduction of learning would connect our research with Quantum Reinforcement Learning studies, with discrete successive states and decisions [81, 80]. Our research may thus constitute a key connection between quantum computing, swarm robotics, swarm modeling, and future developments in this research area. As mentioned above, we found that the proposed gate works better when the robots are in exploration phase, rather than already close to the target. The enhancement of the code eliminated this issue.

8. Conclusions

In this article, we proposed a quantum approach to swarm robotics. Our strategy is a two-step modeling technique, where we first model the individual behavior, and then we tune the swarm to observe the emerging behavior.

Our approach to the swarm is based on nested matrices. Diagonal sub-matrices represent information (position, speed, activated sensors) of each individual robot. Off-diagonal sub-matrices are the pairwise interaction terms,

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containing the information about sent and received messages, and suggestions of behavior according to the received signals. A core element of our approach is the use of an original quantum circuit. To test it and measure the effect of quantum noise, we compared theoretical expectations of our circuit with measurements obtained through IBM QASM simulator and three real IBM quantum computers. To test our approach on a swarm of robots, our circuit was then included in a Jupyter Notebook original code, written in Python. This idea can lead to a quantum-based decision-making system.

We simulated the behavior of a 3-robot swarm and then of a 10-robot swarm as case studies. We considered a search and rescue mission, inspired by the foraging ant behavior, for its simplicity and importance in nature. We obtained an overall success in target finding (food retrieval and return to the nest), observing the circuit effect on swarm behavior. We compared our method with other case studies coded in NetLogo, with bounded random walks, and with an optimization approach of particle swarm, trying to recreate conditions similar to the ones in our research. We also compared our results against two optimization algorithms, the particle swarm optimization (PSO) and the NL-SHADE-RSP with midpoint algorithms.

In our code (second version), a known obstacle to be avoided is also included. The size of the swarm can be modified by suitably setting the N number of robots. The study has been developed for a 2-dimensional scenario, but it can be easily generalized to a 3-dimensional scenario, as sketched in [85]. In our study, we chosen to focus on a simplified scenario to validate the approach. We deliberately ignored unknown obstacles, battery information, information storage, and techniques for target detection. Regarding the last point, in our current simulation, we considered the Euclidean distance as a measure of target proximity. However, in a realistic scenario, we should consider visual feedback, or infrared detection, or sonar-collected information to name but a few. A limitation on the feasibility of our approach is the access to quantum simulators and the risk of decoherence, that are common issues for all quantum applications.

Swarm robotics is one of the countless technologies inspired by nature, with

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its swarms and self-organizational structures. Another natural source of inspiration for computer science is quantum physics. Our research aims to merge nature-inspired swarms and quantum computing. Our work constitutes one of the first steps toward the adaptation of the quantum paradigm to swarm robotics development. The resources of quantum computing have started being explored in full only in the latest years, and the extension of their computational power to artificial intelligence and robotic development is yet to come.

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Author contributions

M.M., V.S., and A.C. designed the experiments; M.M. conducted the experiments; M.M. and V.S. wrote the paper; M. M. revised the manuscript according to the reviewers' and editor's comments.

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Declaration of interests

The authors declare no competing interests.

Data availability

All codes and data are available at `https://github.com/medusamedusa/3-robot` and `https://github.com/medusamedusa/10_little_ants`. These are public repositories, copyright M.M.

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Author Statement

Maria Mannone, Valeria Seidita, and Antonio Chella: conceptualization, methodology

Maria Mannone: software, validation

Maria Mannone, Valeria Seidita: writing-original draft presentation

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Valeria Seidita, Antonio Chella: supervision

Maria Mannone, Valeria Seidita, and Antonio Chella: reviewing and editing

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Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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