Exercise Report-2

For

INF 5620/9620 -Numerical Methods for the Partial Differential Equation

Hao, Zhao

Department of Geoscience & Department of Informatics, University of Oslo

September 28, 2015

(1)Exercise a:

Based on the 1-D wave equation with variable velocity coefficient

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial \mathrm{x}}\left(q(x)\frac{\partial u}{\partial \mathrm{x}}\right) + f(x,t)$$

When the q(x) = 1 + (x – L/2)^4, and the exact solution: cos(pi*x/L)*cos(W*t), we can derive the initial condition :I(x),V(x), and the corresponding source term f(x,t) for simulation. These initial conditions listed in the program.

Based on the Neumann condition $\frac{\partial \mathrm{y}}{\partial \mathrm{x}} = 0$, at x =0 and X= Nx*dx, we can derive the follow discretized scheme for simulation.

$$u_i^{n+1} = -u_i^{n-1} + 2u_i^n + 0.5 * C^2\big((q_{i+1} + q_i)(u_{i+1}^n - u_i^n) - (q_i + q_{i-1})(u_i^n - u_{i-1}^n)\big) + dt^2 * f$$

Plus the variation on the edge X=0 and X=L, the following scheme are used for the first step calculation, and the rest calculating:

$$u_0^1 = dt * V + u_0^0 + 0.5 * C^2((q_1 + q_0)(u_1^0 - u_0^0)) + 0.5 * dt^2 * f$$

$$u_{Nx}^1 = dt * V + u_{Nx}^0 + 0.5 * C^2((q_{Nx} + q_{Nx-1})(u_{Nx}^0 - u_{Nx-1}^0)) + 0.5 * dt^2 * f$$

$$u_0^{n+1} = -u_0^{n-1} + 2 * u_0^n + C^2((q_1 + q_0)(u_1^0 - u_0^0)) + dt^2 * f$$

$$u_{Nx}^{n+1} = -u_{Nx}^{n-1} + 2 * u_{Nx}^n + C^2((q_{Nx} + q_{Nx-1})(u_{Nx}^0 - u_{Nx-1}^0)) + dt^2 * f$$

Based on the above scheme, the program is implemented and executed.

N.B, in the codding, when handling the boundary condition, the 'method_1' is the approach mentioned above (course text equation 55), and "method_2" is approach from course text equation 52.

(2)Exercise b:

When the q(x) = 1 + cos(pi*x/L), and the exact solution: cos(pi*x/L)*cos(W*t), we can derive the initial condition :I(x),V(x), and the corresponding source term f(x,t) for simulation. These initial conditions listed in the program, and the identical scheme as the 'exercise-a' is used for the computation.

(3)Exercise c:

When the q(x) = 1 + cos(pi*x/L), and the exact solution: cos(pi*x/L)*cos(W*t), we can derive the initial condition :I(x),V(x), and the corresponding source term f(x,t) for simulation.

These initial conditions listed in the program, but the discretized scheme is slightly changed for handling the boundary condition based on the assumption: $u_i - u_{i-1} = 0, at\ i = N_x$ and $u_{i+1} - u_i = 0, at\ i = 0$, the calculation of the boundary at i=0 and i=Nx is modified as below:

$$u_0^1 = u_1^1 = dt * V + u_1^0 + 0.5 * C^2((q_2 + q_1)(u_2^0 - u_1^0)) + 0.5 * dt^2 * f$$

$$u_{Nx}^1 = u_{Nx-1}^1 = dt * V + u_{Nx-1}^0 + 0.5 * C^2((q_{Nx-1} + q_{Nx-2})(u_{Nx-1}^0 - u_{Nx-2}^0)) + 0.5 * dt^2 * f$$

$$u_0^{n+1} = u_1^{n+1} = -u_1^{n-1} + 2 * u_1^n + C^2((q_2 + q_1)(u_2^0 - u_1^0)) + dt^2 * f$$

$$u_{Nx}^{n+1} = u_{Nx-1}^{n+1} = -u_{Nx-1}^{n-1} + 2 * u_{Nx-1}^n + C^2((q_{Nx-1} + q_{Nx-2})(u_{Nx-1}^0 - u_{Nx-2}^0)) + dt^2 * f$$

Based on the above scheme, the program is implemented as "method_3" and executed.

(4)Exercise d:

According to the context, the discretized scheme can be rewritten with below expression by replacing the physical boundary to Xi+1/2 at i = Nx, and Xi-1/2 at I = 0:

$$[D_t D_t u]_i^n = \frac{1}{dx} * \left([qD_x u]_{i+\frac{1}{2}}^n - [qD_x u]_{i-\frac{1}{2}}^n\right) + [f]_i^n$$

Thus we can derive the discretized scheme for the boundary calculation as follows:

$$u_0^1 = dt * V + u_0^0 + 0.5 * C^2(0.5 * (q_1 + q_0)(u_1^0 - u_0^0)) + 0.5 * dt^2 * f$$

$$u_{Nx}^1 = dt * V + u_{Nx}^0 + 0.5 * C^2((-0.5 * (q_{Nx} + q_{Nx-1})(u_{Nx}^0 - u_{Nx-1}^0)) + 0.5 * dt^2 * f$$

$$u_0^{n+1} = -u_0^{n-1} + 2 * u_0^n + C^2(0.5 * (q_1 + q_0)(u_1^0 - u_0^0)) + dt^2 * f$$

$$u_{Nx}^{n+1} = -u_{Nx}^{n-1} + 2 * u_{Nx}^n + C^2(-0.5 * (q_{Nx} + q_{Nx-1})(u_{Nx}^0 - u_{Nx-1}^0)) + dt^2 * f$$

Based on the above scheme, the program is implemented as "method_4" and executed.

Remarks:

The convergence calculation (the divergence rate analysis) is included in each exercise function of the program and plotted in the terminal, please have a check.

Many thanks for your review for my report☺