Cluster and Dimension Reduction

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Task 1: analyze the data unempstates.csv. The objective of the analysis is to group states together if they have similar trends in unemployment rate.

1.Use PCA to reduce the dimension of unemployment-rate information.

Requirement: Generate a screeplot and determine the number of principle components based on this plot. Plot the loadings for first principal component.

Load the data and get the summary.

```
unemp <- read.csv("C:/Users/daisy/OneDrive/Study/DM/week7/unempstates.csv",
    header = FALSE, sep = ",")
unemp[1:3, ]
```

```
##    V1  V2   V3  V4  V5  V6  V7  V8  V9 V10 V11 V12 V13 V14 V15 V16 V17 V18
## 1  AL  AK   AZ  AR  CA  CO  CT  DE  FL  GA  HI  ID  IL  IN  IA  KS  KY  LA
## 2 6.4 7.1 10.5 7.3 9.3 5.8 9.4 7.7  10 8.3 9.9 5.5 6.4 6.9 4.2 4.3 5.7 6.2
## 3 6.3   7 10.3 7.2 9.1 5.7 9.3 7.8 9.8 8.2 9.8 5.4 6.4 6.6 4.2 4.2 5.6 6.2
##   V19 V20  V21 V22 V23 V24 V25 V26 V27 V28 V29  V30 V31  V32 V33 V34 V35
## 1  ME  MD   MA  MI  MN  MS  MO  MT  NE  NV  NH   NJ  NM   NY  NC  ND  OH
## 2 8.8 6.9 11.1  10 6.2   7 5.8 5.8 3.6 9.8 7.2 10.5 8.9 10.2 6.7 3.2 8.3
## 3 8.6 6.7 10.9 9.9   6 6.8 5.8 5.7 3.5 9.5 7.1 10.4 8.8 10.2 6.5 3.3 8.2
##   V36  V37 V38 V39 V40 V41 V42 V43 V44 V45 V46 V47 V48 V49 V50
## 1  OK   OR  PA  RI  SC  SD  TN  TX  UT  VT  VA  WA  WV  WI  WY
## 2 6.4 10.1 8.1 7.8 7.6 3.6 5.9 5.9 6.1 8.8 6.2 8.7 8.3 5.9 4.2
## 3 6.3  9.8 8.1 7.8 7.4 3.5 5.9 5.9 5.9 8.7 6.1 8.7 8.1 5.7 4.1
```

```
dim(unemp)
```

```
## [1] 417  50
```

There are 416 observations in the data set and 50 variables. 50 variables represent 50 states in Unites State and each state is characterized by a feature vector of very large dimension(416),with its components representing the 416 monthly observation.

Get the state names and convert them into column.

```
unemplab <- unemp[1, ]
unemplab <- as.data.frame(t(unemplab))
```

Import the data and set the header as True so we can import the data as numeric.

```
library(matrixStats)
unemp1 <- read.csv("C:/Users/daisy/OneDrive/Study/DM/week7/unempstates.csv",
    header = TRUE, sep = ",")
# Transform the row and column in the original dataset, so that we can get
# the pca for 50 #states
unemp5 <- as.data.frame(t(unemp1))
```

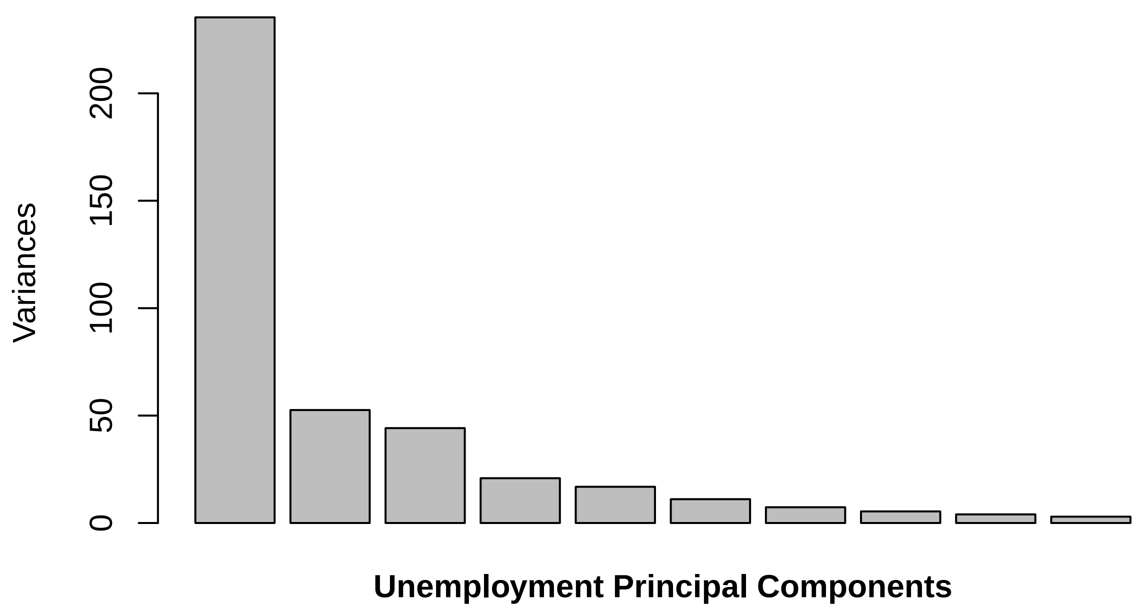
```r
unemp5 <- cbind(unemplab, unemp5)
names(unemp5)[names(unemp5) == "1"] <- "StateName"
# Get the correlation matrix from all variants cor(unemp5[,-1]) Get the pca
# for the unemployment of 50 states.
pcaunemp5 = prcomp(unemp5[, -1], scale = TRUE)
# pcaunemp5 Get the loadings of PCA pcaunemp5$rotation

## use 'predict' to project data onto the loadings
unemp5pc = predict(pcaunemp5)
# unemp5pc
```

1) Generate a screeplot and determine the number of principle components based on this plot.

Plot the variances against the number of the principal component; retain only factors with eigenvalues greater than 1.

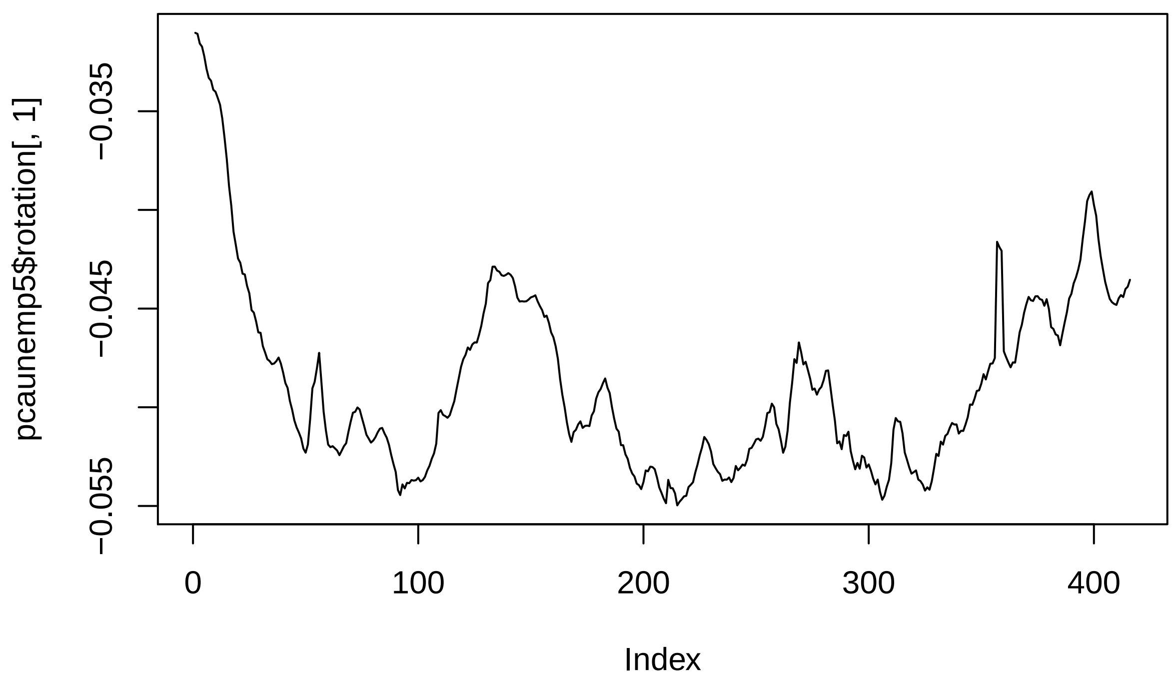
```r
plot(pcaunemp5, main = "")  ## same as screeplot(pcafood)
mtext(side = 1, "Unemployment Principal Components", line = 1, font = 2)
```



The first rectangle's height is almost 225, that means we can choose 225 variables for principle components.

2) Plot the loadings for first principal component.

```r
plot(pcaunemp5$rotation[, 1], type = "l")
```



2. Generate a scatterplot to project states on the first two principal components.

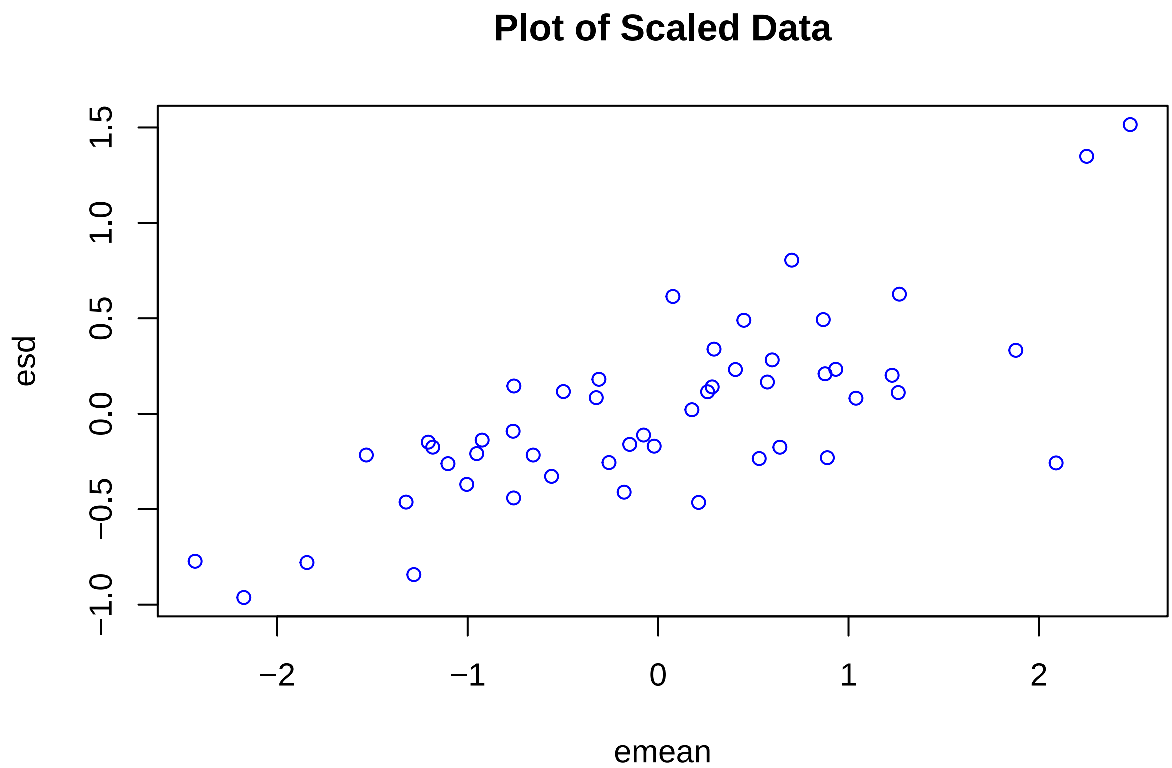
```
# Get the mean value of each state and save them into columns
emean <- colMeans(unemp1)
# Convert the dataframe into matrix and get the standard deviation for each
# column
unemp_matrix <- as.matrix(unemp1)
esd <- colSds(unemp_matrix)
unemp2 <- cbind(unemplab, emean, esd)
unemp2[1:3, ]
```

```
##     1    emean      esd
## V1 AL 6.644952 2.527530
## V2 AK 8.033173 1.464966
## V3 AZ 6.120673 1.743672
```

```
states = unemp2[, 1]
n = length(states)
data = (as.matrix(unemp2[, -1]))
```

Center the data according to the mean and get the scatterplot of the original data.

```
my.scaled.data = apply(data, 2, function(x) (x - mean(x)))
plot(my.scaled.data, cex = 0.9, col = "blue", main = "Plot of Scaled Data")
```



Calculate the covariance matrix.

```
my.cov = cov(my.scaled.data)
my.cov
```

```
##           emean       esd
## emean 1.2308637 0.3930731
## esd   0.3930731 0.2279926
```

Calculate the eigenvectors and eigenvalues of the covariance matrix

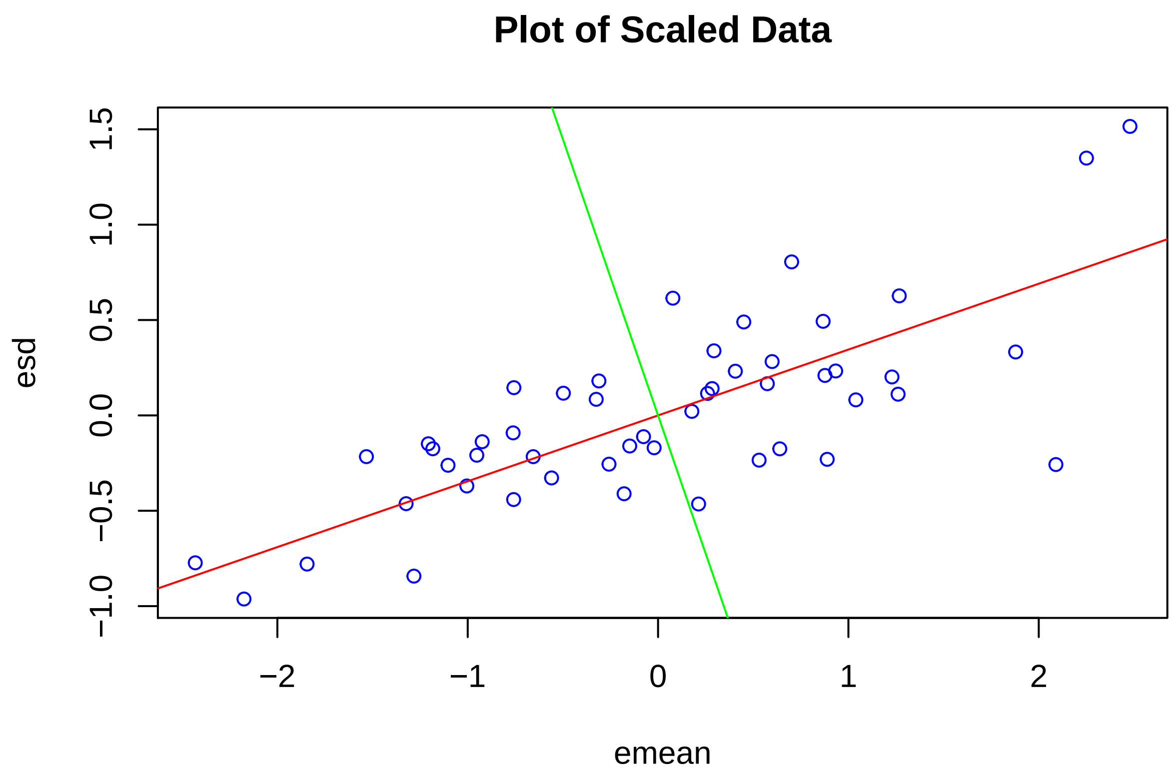
```
my.eigen = eigen(my.cov)
my.eigen
```

```
## $values
## [1] 1.36656552 0.09229076
##
## $vectors
##            [,1]       [,2]
## [1,] -0.9452548  0.3263333
## [2,] -0.3263333 -0.9452548
```

1) Plot the Eigenvectors over the scaled data and load the first and second principle on it.

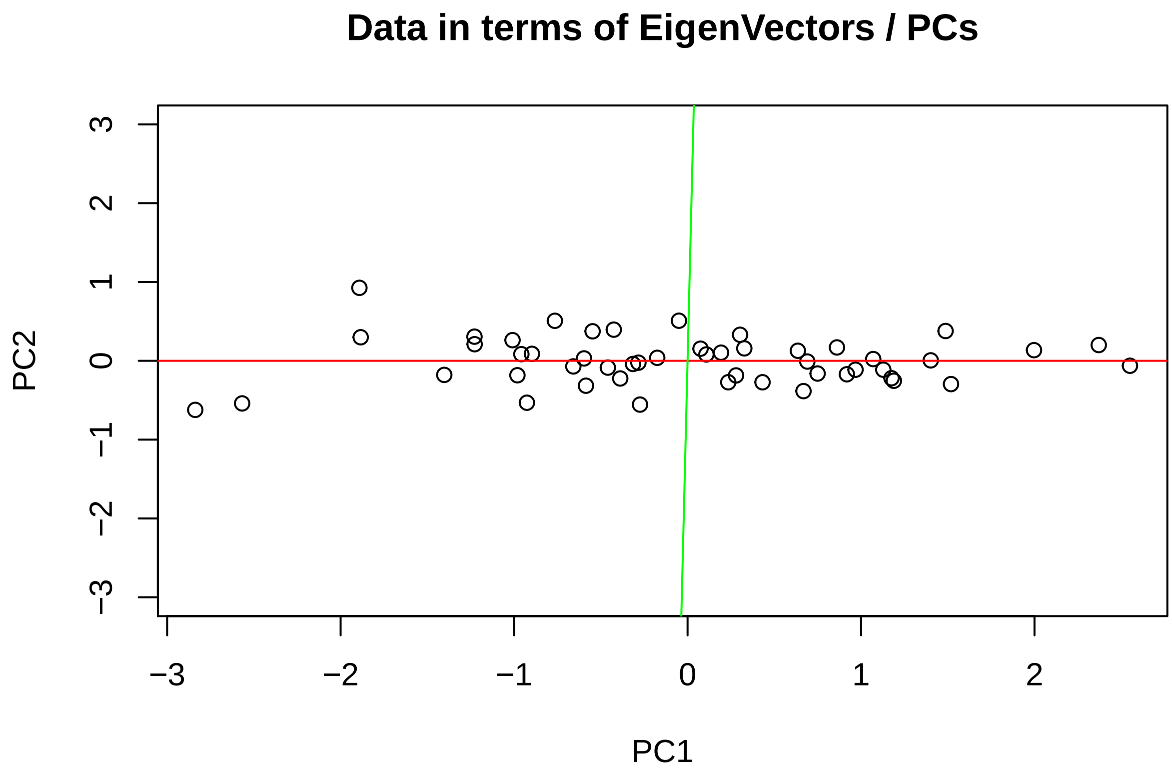
```
plot(my.scaled.data, cex = 0.9, col = "blue", main = "Plot of Scaled Data")
# Plot the loadings for the first principal components
pc1.slope = my.eigen$vectors[2, 1]/my.eigen$vectors[1, 1]
```

```r
abline(0, pc1.slope, col = "red")
# Plot the loadings for the second pricipal components
pc2.slope = my.eigen$vectors[2, 2]/my.eigen$vectors[1, 2]
abline(0, pc2.slope, col = "green")
```



2) Generate a scatterplot to project states on the first two principal components.

```r
## get the P matrix
loadings = my.eigen$vectors
## project data onto the loadings
scores = my.scaled.data %*% loadings
## plot the projected data on the first two PCs
plot(scores, ylim = c(-3, 3), main = "Data in terms of EigenVectors / PCs",
    xlab = "PC1", ylab = "PC2")
abline(0, 0, col = "red")
abline(0, 90, col = "green")
```



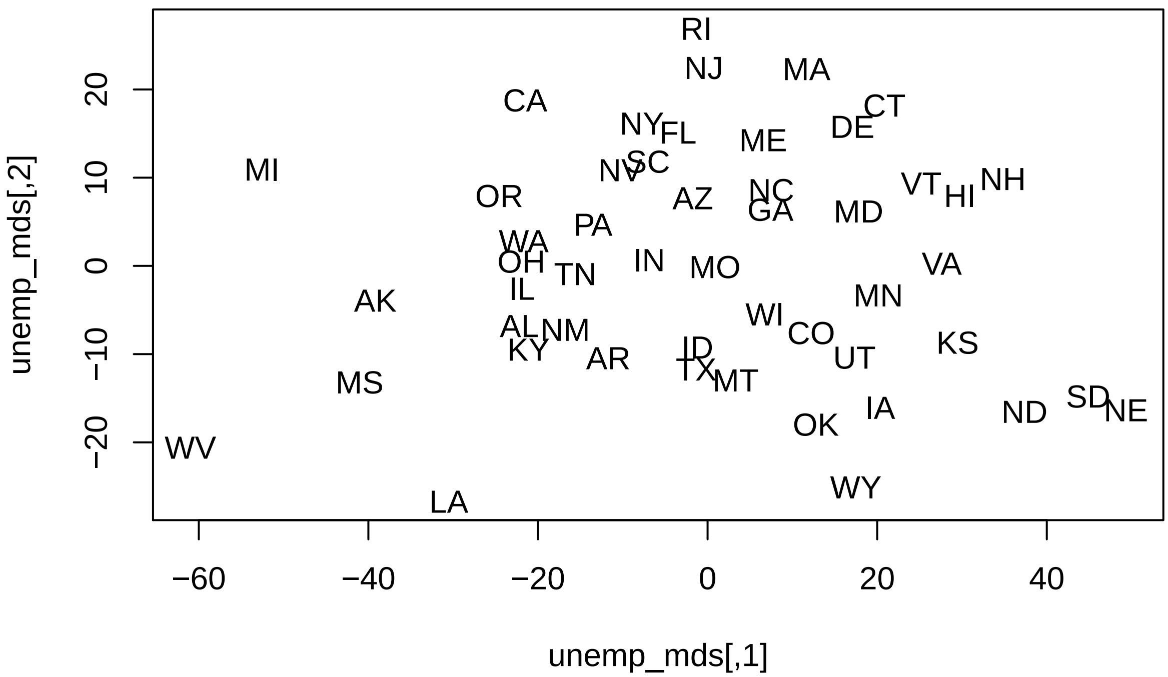
3. Generate an MDS map to plot states on a two-dimensional space.

```
# Define a function to make the MDS graph for dataset
MDS <- function(dataset) {
    ## calculate distance matrix
    unemp_dist = dist(dataset[, -1])
    unemp_dist

    ## visualize clusters
    unemp_mds <- cmdscale(unemp_dist)
    unemp_mds

    plot(unemp_mds, type = "n")
    text(unemp_mds, labels = dataset[, 1])
    return(unemp_mds)
}

# Convert the row and column in the dataset so that we can make MDS using
# State label.
unemp3 <- as.data.frame(t(unemp))
MDS(unemp3)
```




```
##             [,1]         [,2]
## V1  -22.2019162  -6.76831967
## V2  -39.1805355  -3.88282712
## V3   -1.7348762   7.72138063
## V4  -11.7083958 -10.41309760
## V5  -21.5188934  18.81105419
## V6   12.2070091  -7.56321854
## V7   20.8401142  18.22341824
## V8   17.0677004  15.81375928
## V9   -3.4981858  15.16360313
## V10   7.4033873   6.41083521
## V11  29.7500837   7.98479206
## V12  -1.2069633  -9.20200701
## V13 -21.8965063  -2.53877144
## V14  -6.8990654   0.70535026
## V15  20.3347208 -16.03505184
## V16  29.4758727  -8.65266113
## V17 -21.1176436  -9.43865150
## V18 -30.5189107 -26.67578778
## V19   6.5531511  14.29825605
## V20  17.7974513   6.22443094
## V21  11.6616389  22.35949189
## V22 -52.5446525  10.97138162
## V23  20.1242140  -3.29125554
## V24 -41.0407972 -13.15375945
## V25   0.8613684  -0.08142266
```

```
## V26   3.3027452 -12.92786831
## V27  49.3320695 -16.31138941
## V28 -10.2869072  10.89807049
## V29  34.8145052   9.89602566
## V30  -0.4556686  22.45708805
## V31 -16.7930257  -7.19569411
## V32  -7.7478062  16.18180533
## V33   7.4952226   8.63781582
## V34  37.3649459 -16.49203587
## V35 -21.9722912   0.53540658
## V36  12.7645961 -17.94162978
## V37 -24.5764978   7.97535389
## V38 -13.5046932   4.71451380
## V39  -1.3156246  26.92548559
## V40  -7.0470520  11.86187133
## V41  44.8826807 -14.75214565
## V42 -15.6059535  -0.87705863
## V43  -1.3520873 -11.68759443
## V44  17.3150375 -10.38198874
## V45  25.1844868   9.38537400
## V46  27.6114453   0.29861030
## V47 -21.6851730   2.85368831
## V48 -60.9859792 -20.55073560
## V49   6.7641317  -5.44671215
## V50  17.4875224 -25.04717868
```

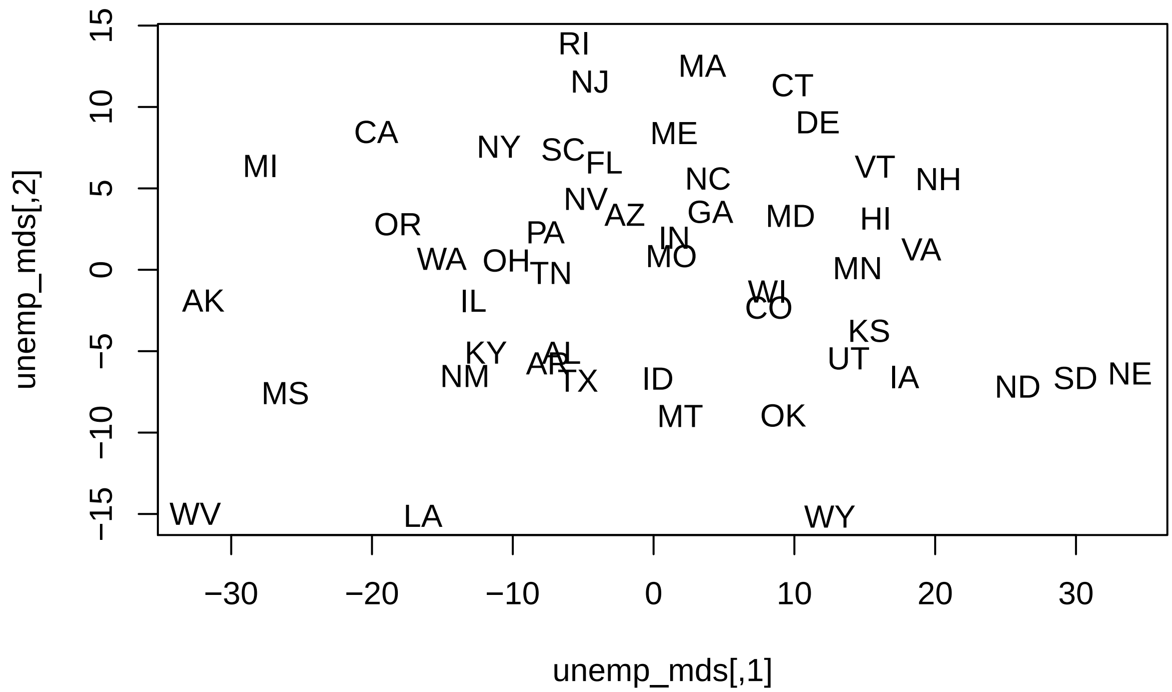
4.Use k-means and hierarchical clustering to group states.

Requirement: Specifically, you will generate 8 MDS maps for the states and color the states based on different clustering methods (k-means, h-clustering with single-link, h-clustering with complete-link, h-clustering with average-link) and different number of clusters (k = 4, k = 8). For each hierarchical clustering method, generate a dendrogram.

1)Clustering preperetion–Standardzation

According to the oringal MSD map we can know, there are some outliers in the dataset and therefore, we need to standardize the dataset first to make all the points much tighter clustered.

```r
library(robustHD)
unemp4 <- as.data.frame(t(unemp1))
unemp4 <- standardize(unemp4)
unemp4 <- cbind(unemplab, unemp4)
names(unemp4)[names(unemp4) == "1"] <- "StateName"
MDS(unemp4)
```




```
##              [,1]        [,2]
## V1   -6.5355811  -5.0678416
## V2  -31.9769380  -1.8617508
## V3   -2.0406812   3.4072423
## V4   -7.4869880  -5.7186257
## V5  -19.7041330   8.4932739
## V6    8.1843777  -2.2958469
## V7    9.8645337  11.3634713
## V8   11.6676032   9.0875711
## V9   -3.5092848   6.6185632
## V10   4.0250607   3.5857061
## V11  15.7813751   3.1753140
## V12   0.2958574  -6.6560207
## V13 -12.8017876  -1.8710739
## V14   1.4605140   1.9982251
## V15  17.7959176  -6.5625014
## V16  15.3045015  -3.7207949
## V17 -11.9098420  -5.0586222
## V18 -16.3816475 -15.0834333
## V19   1.4554736   8.4256526
## V20   9.7184628   3.3384255
## V21   3.4562174  12.5618152
## V22 -27.9253764   6.4070665
## V23  14.4870208   0.1308453
## V24 -26.1567057  -7.5474443
## V25   1.2606443   0.8705779
```

```
## V26   1.8870779  -8.9512119
## V27  33.8349865  -6.3389913
## V28  -4.8196179   4.3826963
## V29  20.2305322   5.5988618
## V30  -4.5195217  11.5705117
## V31 -13.4013028  -6.4948622
## V32 -10.9903394   7.5884656
## V33   3.8622253   5.6296717
## V34  25.8587787  -7.1461755
## V35 -10.4539202   0.5972584
## V36   9.2075217  -8.9205162
## V37 -18.1540669   2.8265885
## V38  -7.6877798   2.3280738
## V39  -5.6492155  13.9367466
## V40  -6.4242696   7.4171784
## V41  29.9625144  -6.6195970
## V42  -7.2949297  -0.1658545
## V43  -5.3740677  -6.7815091
## V44  13.8416624  -5.4299929
## V45  15.7343157   6.3648588
## V46  19.0133426   1.2797923
## V47 -15.0423899   0.7009426
## V48 -32.5519183 -14.9570262
## V49   8.0796064  -1.3057752
## V50  12.5221813 -15.1299288
```

As we can see from the MDS map, there are still some points that are far from the central but most of the points are distributed evenly in the MDS map after standardzation.

2) K-Means Clustering

k=4

```r
set.seed(1)
grpUnemp1 = kmeans(unemp4[, -1], centers = 4, nstart = 10)
# grpUnemp1

## list the cluster assignments
o = order(grpUnemp1$cluster)
data.frame(unemp4$StateName[o], grpUnemp1$cluster[o])
```

```
##     unemp4.StateName.o. grpUnemp1.cluster.o.
## V6                   CO                    1
## V11                  HI                    1
## V15                  IA                    1
## V16                  KS                    1
## V23                  MN                    1
## V27                  NE                    1
## V29                  NH                    1
## V34                  ND                    1
## V36                  OK                    1
## V41                  SD                    1
## V44                  UT                    1
## V45                  VT                    1
```

```
## V46                        VA                        1
## V49                        WI                        1
## V50                        WY                        1
## V3                         AZ                        2
## V7                         CT                        2
## V8                         DE                        2
## V9                         FL                        2
## V10                        GA                        2
## V12                        ID                        2
## V19                        ME                        2
## V20                        MD                        2
## V21                        MA                        2
## V25                        MO                        2
## V26                        MT                        2
## V28                        NV                        2
## V30                        NJ                        2
## V32                        NY                        2
## V33                        NC                        2
## V39                        RI                        2
## V43                        TX                        2
## V1                         AL                        3
## V4                         AR                        3
## V13                        IL                        3
## V14                        IN                        3
## V17                        KY                        3
## V22                        MI                        3
## V35                        OH                        3
## V37                        OR                        3
## V38                        PA                        3
## V40                        SC                        3
## V42                        TN                        3
## V47                        WA                        3
## V2                         AK                        4
## V5                         CA                        4
## V18                        LA                        4
## V24                        MS                        4
## V31                        NM                        4
## V48                        WV                        4
```

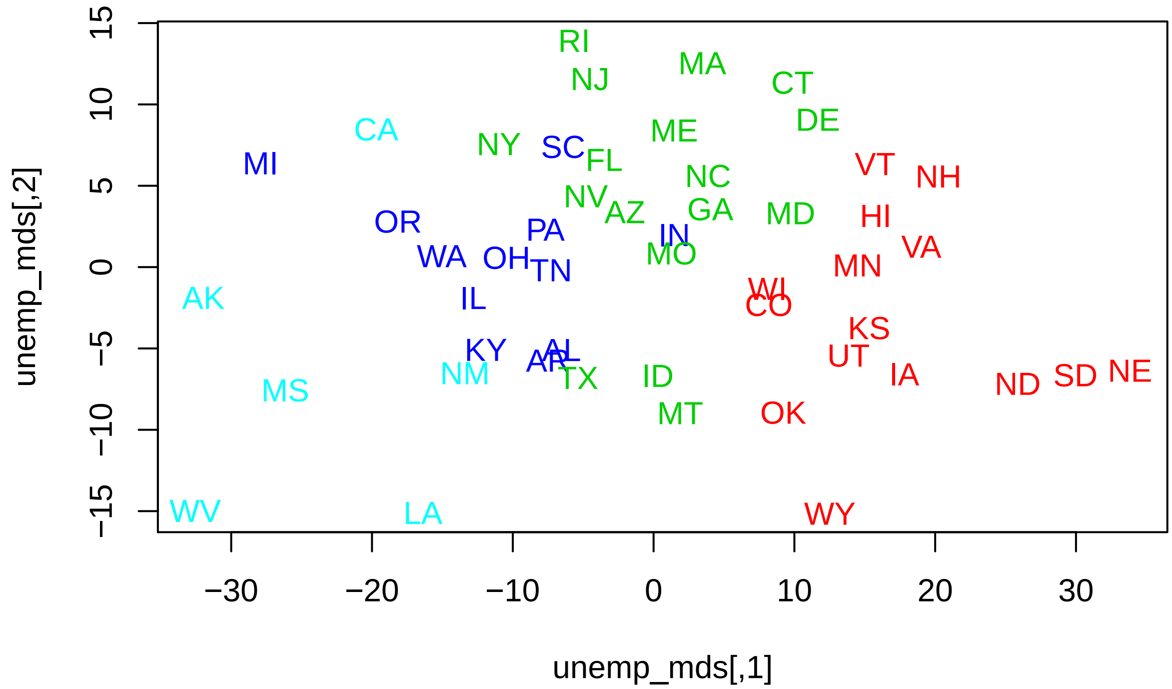
Adjust the MDS function to make MDS map after clustering

```
adMDS <- function(dataset, grpName) {
    ## calculate distance matrix
    unemp_dist = dist(dataset[, -1])
    unemp_dist

    ## visualize clusters
    unemp_mds <- cmdscale(unemp_dist)
    unemp_mds

    plot(unemp_mds, type = "n")
    text(unemp_mds, labels = dataset[, 1], col = grpName$cluster + 1)
    return(unemp_mds)
}
# MDS map for k-means with 4 cluster
```

```r
adMDS(unemp4, grpUnemp1)
```




```
##             [,1]        [,2]
## V1   -6.5355811  -5.0678416
## V2  -31.9769380  -1.8617508
## V3   -2.0406812   3.4072423
## V4   -7.4869880  -5.7186257
## V5  -19.7041330   8.4932739
## V6    8.1843777  -2.2958469
## V7    9.8645337  11.3634713
## V8   11.6676032   9.0875711
## V9   -3.5092848   6.6185632
## V10   4.0250607   3.5857061
## V11  15.7813751   3.1753140
## V12   0.2958574  -6.6560207
## V13 -12.8017876  -1.8710739
## V14   1.4605140   1.9982251
## V15  17.7959176  -6.5625014
## V16  15.3045015  -3.7207949
## V17 -11.9098420  -5.0586222
## V18 -16.3816475 -15.0834333
## V19   1.4554736   8.4256526
## V20   9.7184628   3.3384255
## V21   3.4562174  12.5618152
## V22 -27.9253764   6.4070665
## V23  14.4870208   0.1308453
```

```
## V24 -26.1567057  -7.5474443
## V25   1.2606443   0.8705779
## V26   1.8870779  -8.9512119
## V27  33.8349865  -6.3389913
## V28  -4.8196179   4.3826963
## V29  20.2305322   5.5988618
## V30  -4.5195217  11.5705117
## V31 -13.4013028  -6.4948622
## V32 -10.9903394   7.5884656
## V33   3.8622253   5.6296717
## V34  25.8587787  -7.1461755
## V35 -10.4539202   0.5972584
## V36   9.2075217  -8.9205162
## V37 -18.1540669   2.8265885
## V38  -7.6877798   2.3280738
## V39  -5.6492155  13.9367466
## V40  -6.4242696   7.4171784
## V41  29.9625144  -6.6195970
## V42  -7.2949297  -0.1658545
## V43  -5.3740677  -6.7815091
## V44  13.8416624  -5.4299929
## V45  15.7343157   6.3648588
## V46  19.0133426   1.2797923
## V47 -15.0423899   0.7009426
## V48 -32.5519183 -14.9570262
## V49   8.0796064  -1.3057752
## V50  12.5221813 -15.1299288
```

k=8

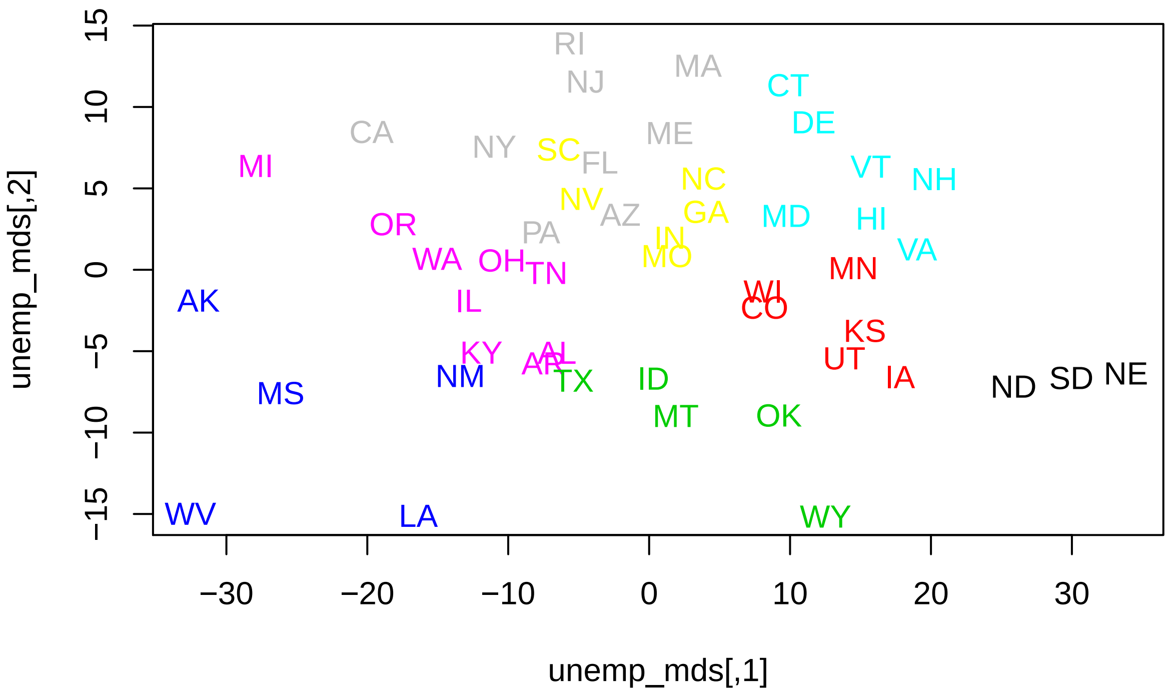
```r
set.seed(1)
grpUnemp2 = kmeans(unemp4[, -1], centers = 8, nstart = 10)
# grpUnemp2

## list the cluster assignments
o = order(grpUnemp2$cluster)
data.frame(unemp4$StateName[o], grpUnemp2$cluster[o])
```

```
##     unemp4.StateName.o. grpUnemp2.cluster.o.
## V6                   CO                    1
## V15                  IA                    1
## V16                  KS                    1
## V23                  MN                    1
## V44                  UT                    1
## V49                  WI                    1
## V12                  ID                    2
## V26                  MT                    2
## V36                  OK                    2
## V43                  TX                    2
## V50                  WY                    2
## V2                   AK                    3
## V18                  LA                    3
## V24                  MS                    3
## V31                  NM                    3
## V48                  WV                    3
```

```
## V7                        CT                        4
## V8                        DE                        4
## V11                       HI                        4
## V20                       MD                        4
## V29                       NH                        4
## V45                       VT                        4
## V46                       VA                        4
## V1                        AL                        5
## V4                        AR                        5
## V13                       IL                        5
## V17                       KY                        5
## V22                       MI                        5
## V35                       OH                        5
## V37                       OR                        5
## V42                       TN                        5
## V47                       WA                        5
## V10                       GA                        6
## V14                       IN                        6
## V25                       MO                        6
## V28                       NV                        6
## V33                       NC                        6
## V40                       SC                        6
## V3                        AZ                        7
## V5                        CA                        7
## V9                        FL                        7
## V19                       ME                        7
## V21                       MA                        7
## V30                       NJ                        7
## V32                       NY                        7
## V38                       PA                        7
## V39                       RI                        7
## V27                       NE                        8
## V34                       ND                        8
## V41                       SD                        8
```

```r
# MDS map for k-means with 8 clusters
adMDS(unemp4, grpUnemp2)
```




```
##             [,1]        [,2]
## V1   -6.5355811  -5.0678416
## V2  -31.9769380  -1.8617508
## V3   -2.0406812   3.4072423
## V4   -7.4869880  -5.7186257
## V5  -19.7041330   8.4932739
## V6    8.1843777  -2.2958469
## V7    9.8645337  11.3634713
## V8   11.6676032   9.0875711
## V9   -3.5092848   6.6185632
## V10   4.0250607   3.5857061
## V11  15.7813751   3.1753140
## V12   0.2958574  -6.6560207
## V13 -12.8017876  -1.8710739
## V14   1.4605140   1.9982251
## V15  17.7959176  -6.5625014
## V16  15.3045015  -3.7207949
## V17 -11.9098420  -5.0586222
## V18 -16.3816475 -15.0834333
## V19   1.4554736   8.4256526
## V20   9.7184628   3.3384255
## V21   3.4562174  12.5618152
## V22 -27.9253764   6.4070665
## V23  14.4870208   0.1308453
## V24 -26.1567057  -7.5474443
## V25   1.2606443   0.8705779
```

```
## V26   1.8870779  -8.9512119
## V27  33.8349865  -6.3389913
## V28  -4.8196179   4.3826963
## V29  20.2305322   5.5988618
## V30  -4.5195217  11.5705117
## V31 -13.4013028  -6.4948622
## V32 -10.9903394   7.5884656
## V33   3.8622253   5.6296717
## V34  25.8587787  -7.1461755
## V35 -10.4539202   0.5972584
## V36   9.2075217  -8.9205162
## V37 -18.1540669   2.8265885
## V38  -7.6877798   2.3280738
## V39  -5.6492155  13.9367466
## V40  -6.4242696   7.4171784
## V41  29.9625144  -6.6195970
## V42  -7.2949297  -0.1658545
## V43  -5.3740677  -6.7815091
## V44  13.8416624  -5.4299929
## V45  15.7343157   6.3648588
## V46  19.0133426   1.2797923
## V47 -15.0423899   0.7009426
## V48 -32.5519183 -14.9570262
## V49   8.0796064  -1.3057752
## V50  12.5221813 -15.1299288
```

3) Hierarchical clustering with single link.

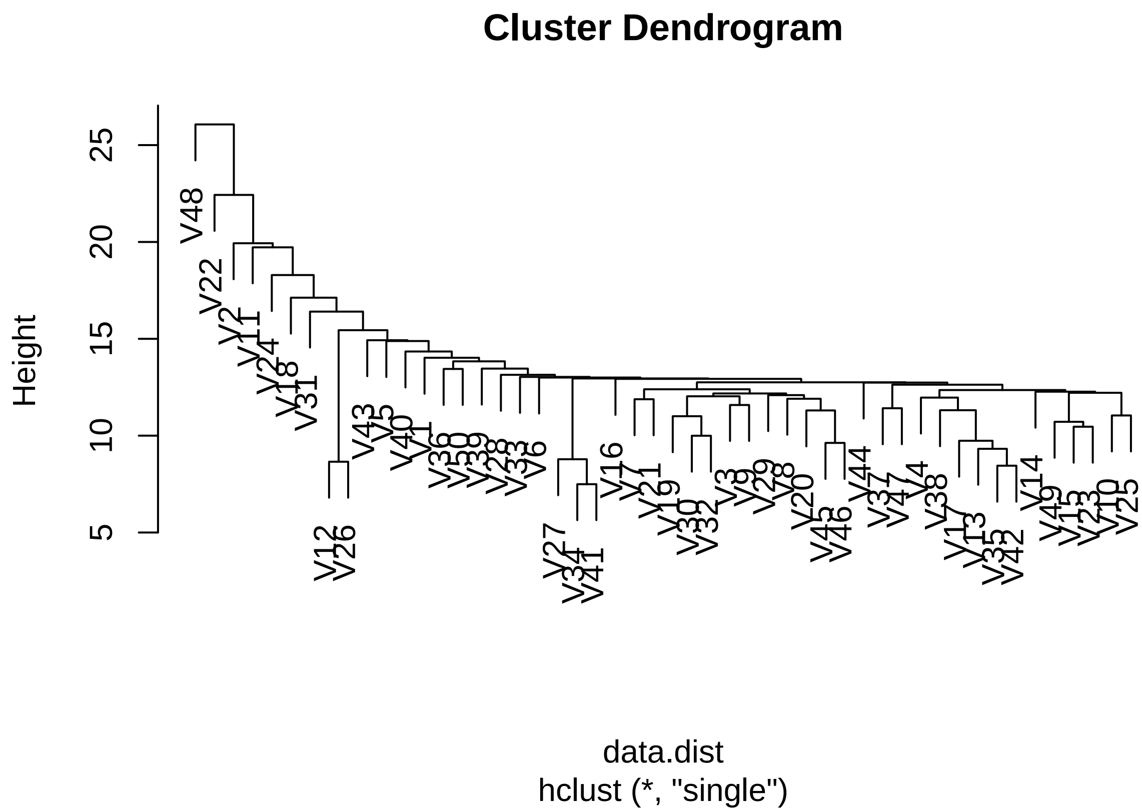
```r
library(cluster)

## use hclust,cutree for hierarchical clustering
data.dist = dist(unemp4[, -1])  ## use dist to obtain distance matrix

# hc_plot function is used for generate MDS map and save the cluster result.
hc_plot <- function(hc_agg, n) {
    hc1 = cutree(hc_agg, k = n)
    hc1 <- as.data.frame(hc1)
    names(hc1)[names(hc1) == "hc1"] <- "cluster"
    adMDS(unemp4, hc1)
    return(hc1)
}
```

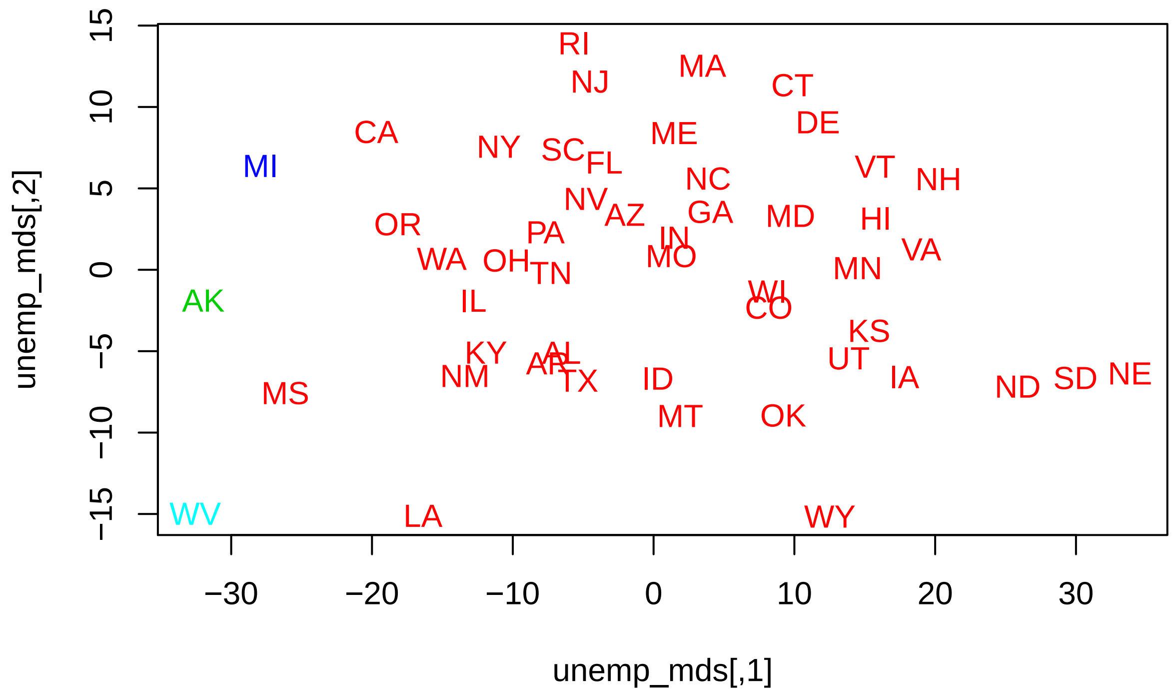
Dendrogram for single method.

```r
hc_s = hclust(data.dist, method = "single")
plot(hc_s)
```



k=4

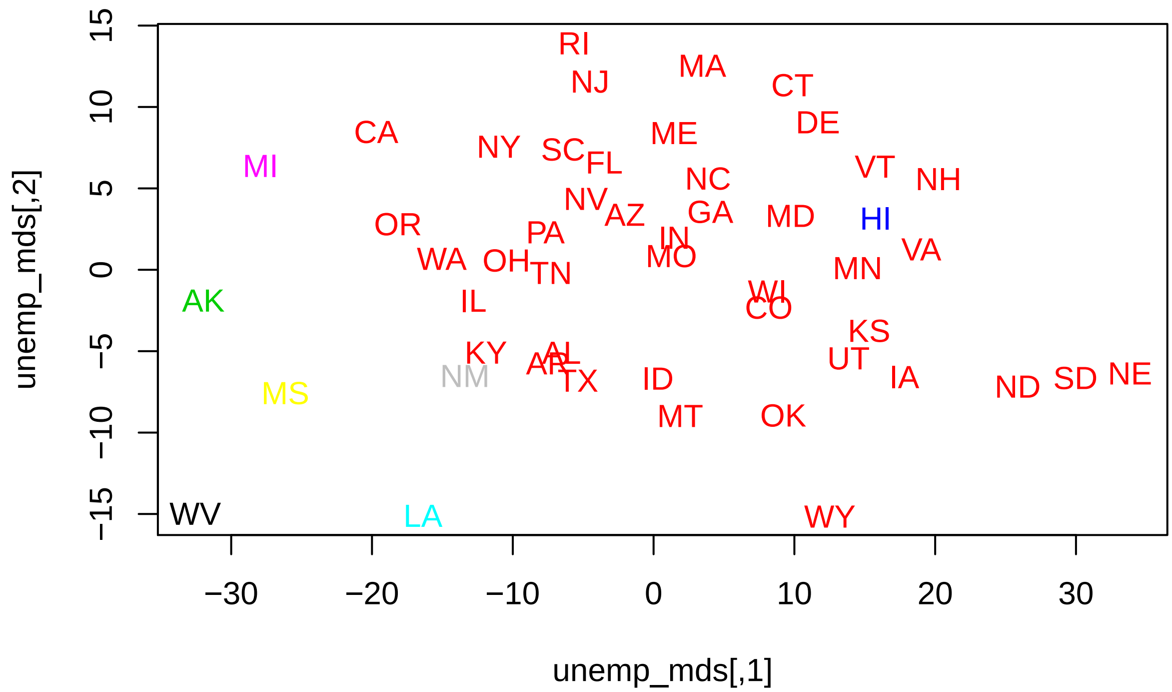
```
grpUnemp3 <- hc_plot(hc_s, 4)
```




```
# grpUnemp3
```

k=8

```
grpUnemp4 <- hc_plot(hc_s, 8)
```

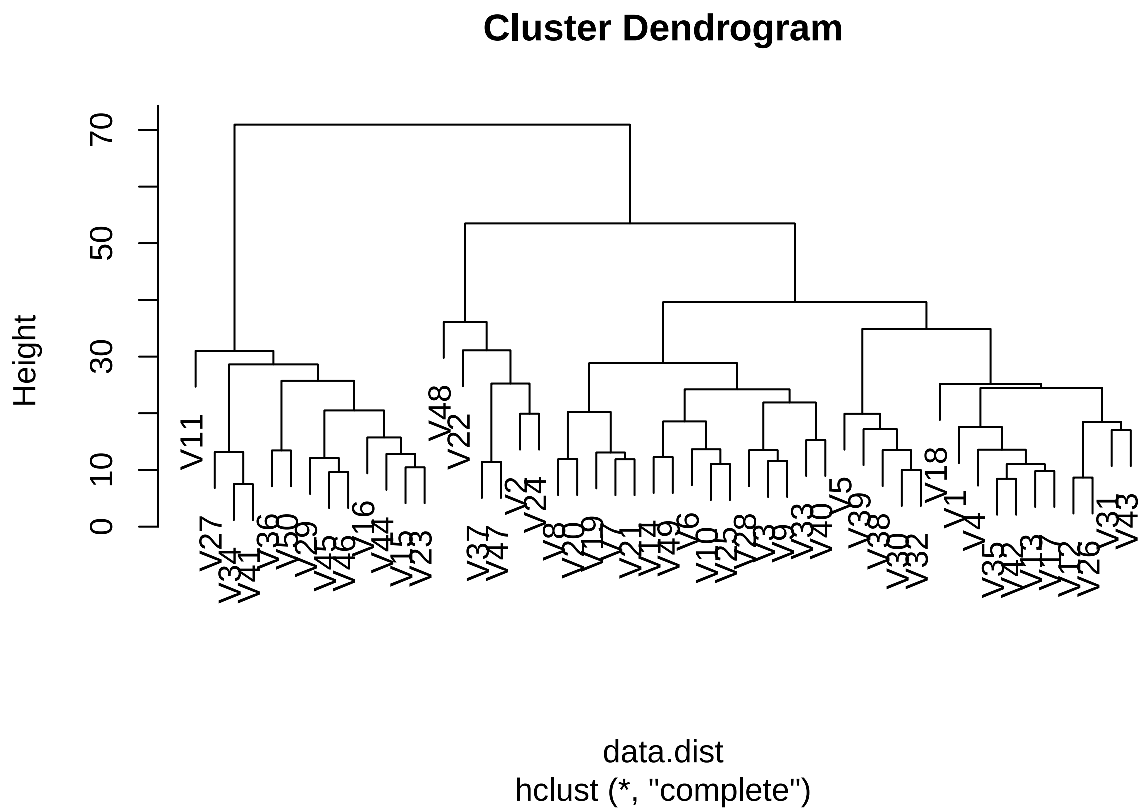



```
# grpUnemp4
```

4) Hierarchical clustering with complete link.

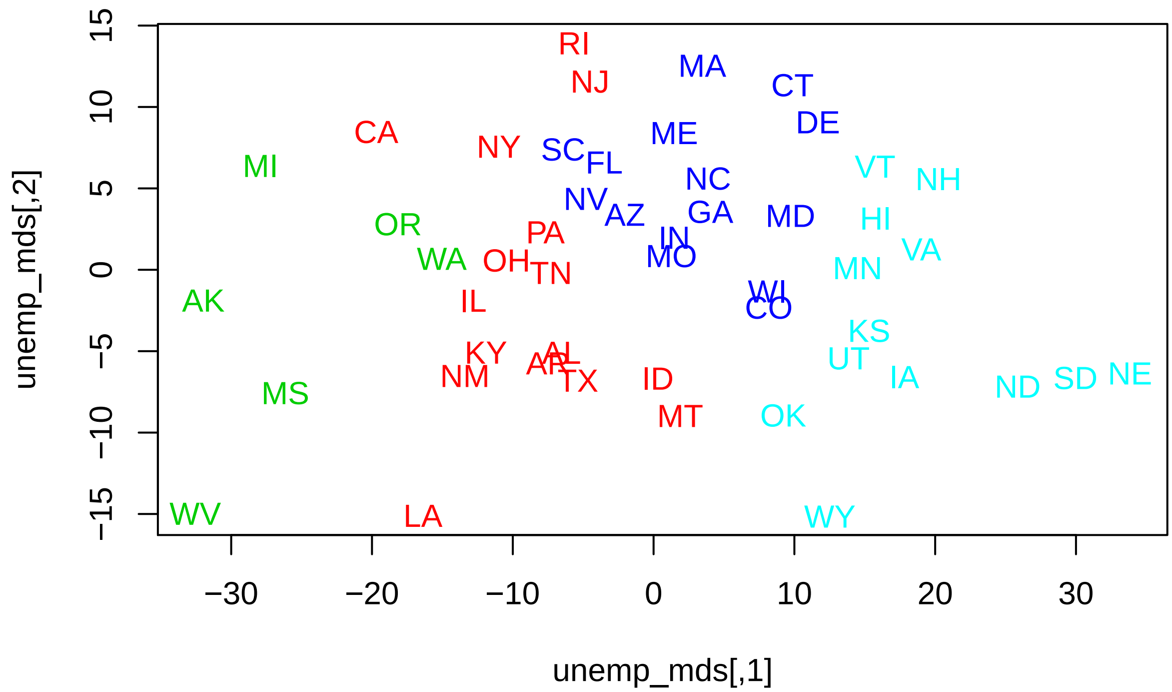
Dendrogram for complete method.

```
hc_c = hclust(data.dist, method = "complete")
plot(hc_c)
```



k=4

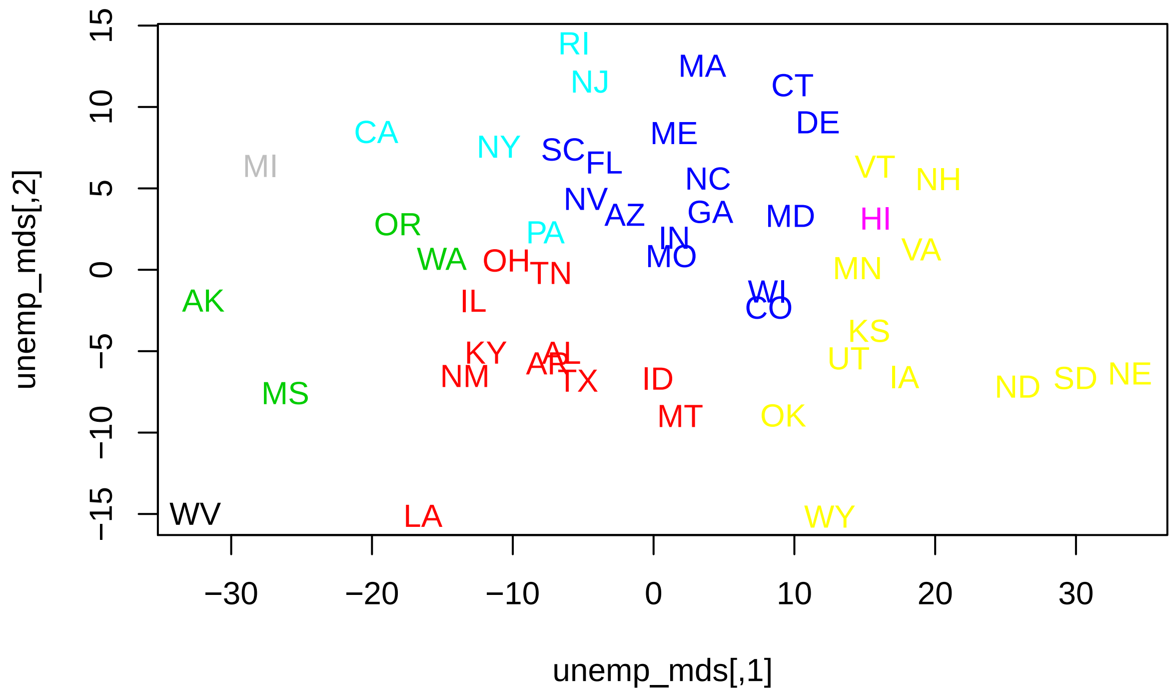
```r
grpUnemp5 <- hc_plot(hc_c, 4)
```




```
# grpUnemp5
```

k=8

```
grpUnemp6 <- hc_plot(hc_c, 8)
```

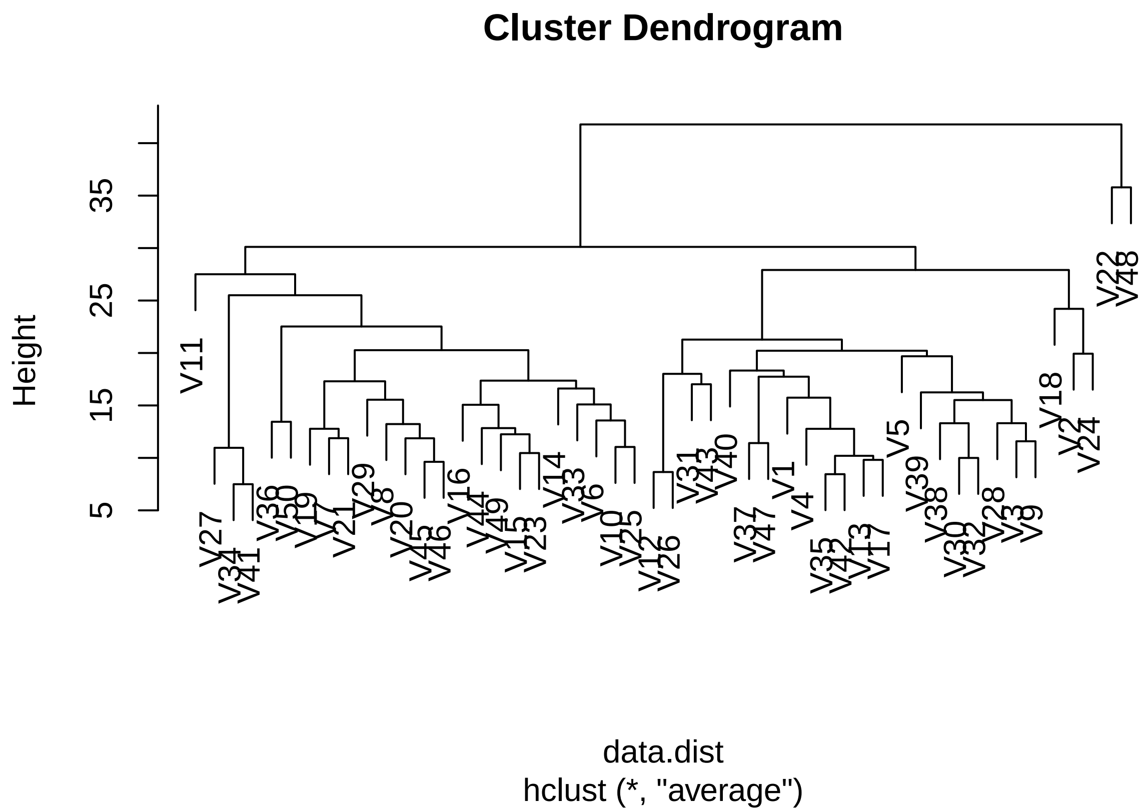



```
# grpUnemp6
```

5) Hierarchical clustering with average link.

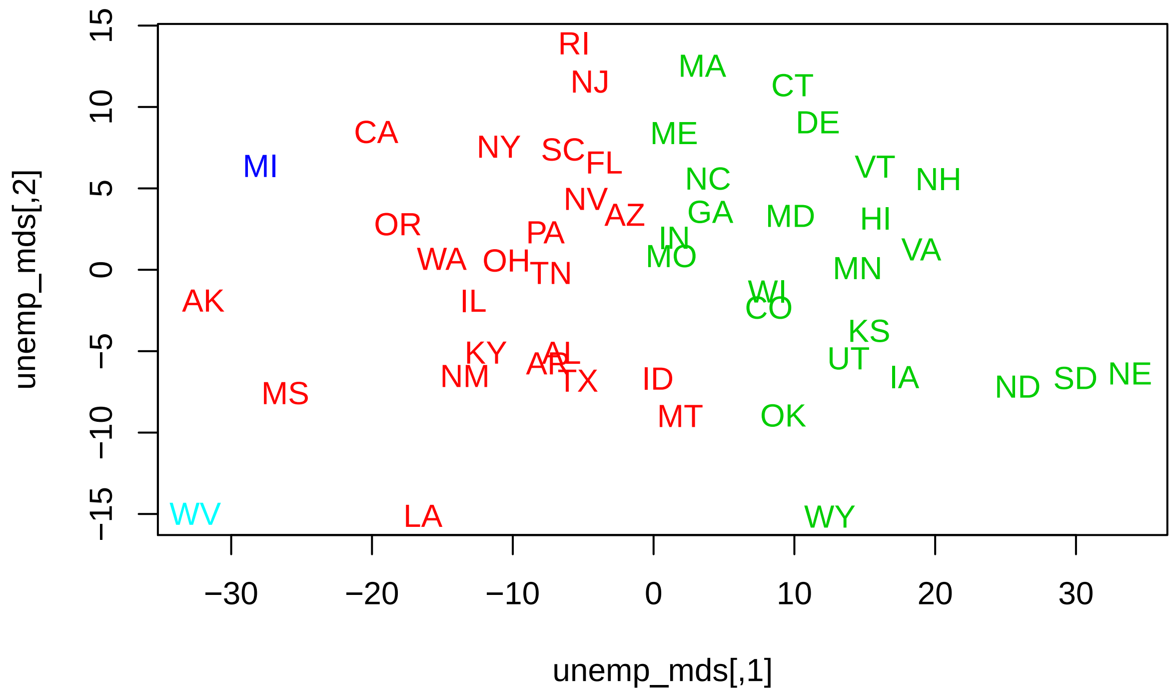
Dendrogram for avarage method.

```
hc_a = hclust(data.dist, method = "average")
plot(hc_a)
```



k=4

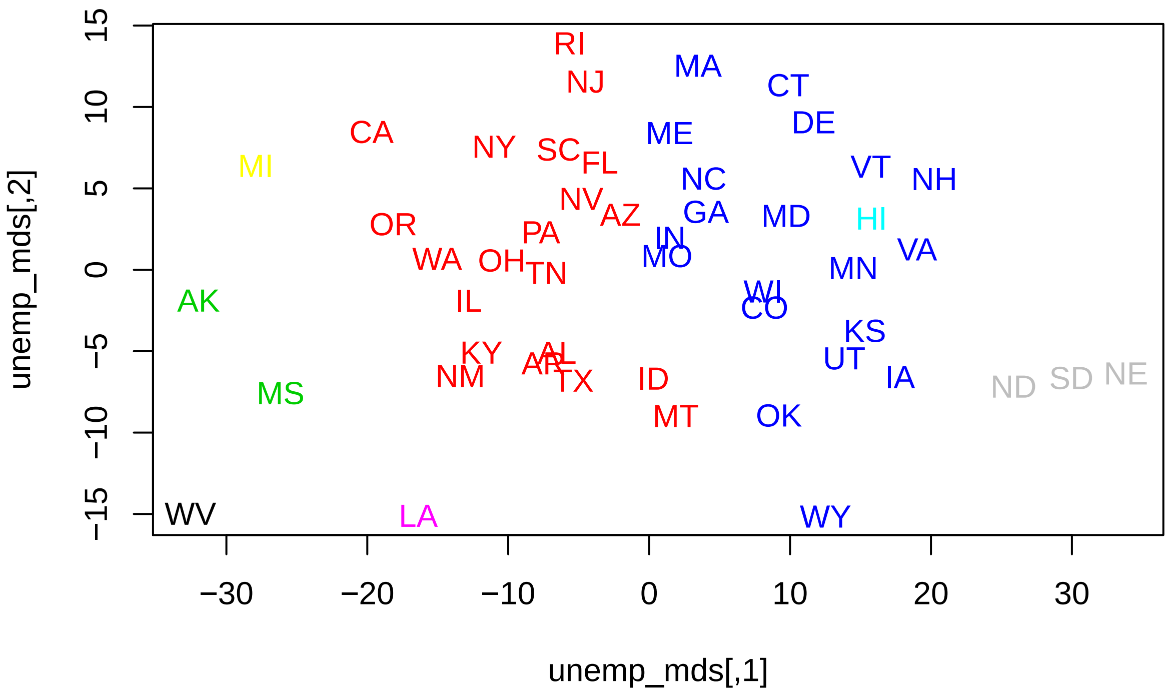
```
grpUnemp7 <- hc_plot(hc_a, 4)
```




```r
# grpUnemp7
```

k=8

```r
grpUnemp8 <- hc_plot(hc_a, 8)
```




```
# grpUnemp8
```

5. Based on your observation, choose two clustering results (from the 8 solutions) that are most meaningful and explain why.

I would like tp choose h-clustering with complete-link with 4 cluster and k-means with 4 cluster to be the most meaningful method. According to the MDS map we can see, in this two methods, all similar objects are clustered together and objects which far away from each other are divided into the different clustered. That means the similarity in each cluster is high and differences between the clusters are large. This indicates a good clustering results. While if we divided the objects into 8 clusters, some of objects are clustered into wrong groups.

Task 2: analyze US Senator Roll Call Data. The objective is to identify and visualize the clustering patterns of senators' voting activities.

1.Create a senator-by-senator distance matrix for the 113th Congress.

Load the packages and data into R

```
library(foreign)
library(ggplot2)
```

```
data.url = "http://www.yurulin.com/class/spring2017_datamining/data/roll_call"
# data.dir = file.path('data', 'roll_call') data.files =
# list.files(data.dir)
data.files = c("sen101kh.dta", "sen102kh.dta", "sen103kh.dta", "sen104kh.dta",
    "sen105kh.dta", "sen106kh.dta", "sen107kh.dta", "sen108kh_7.dta", "sen109kh.dta",
    "sen110kh_2008.dta", "sen111kh.dta", "sen112kh.dta", "sen113kh.dta")
sen113 <- read.dta("C:/Users/daisy/OneDrive/Study/DM/week7/sen113kh.dta")

sen113 <- as.data.frame(sen113)
```

Add all roll call vote data frames to a single list.

```
rollcall.data = lapply(data.files, function(f) {
    read.dta(file.path(data.url, f), convert.factors = FALSE)
})

dim(rollcall.data[[1]])
```

```
## [1] 103 647
```

```
head(rollcall.data[[1]][, 1:12])
```

```
##   cong    id state dist  lstate party eh1 eh2        name V1 V2 V3
## 1  101 99908    99    0 USA       200   0   0 BUSH         1  1  1
## 2  101 14659    41    0 ALABAMA   100   0   1 SHELBY, RIC  1  1  1
## 3  101 14705    41    0 ALABAMA   100   0   1 HEFLIN, HOW  1  1  1
## 4  101 12109    81    0 ALASKA    200   0   1 STEVENS, TH  1  1  1
## 5  101 14907    81    0 ALASKA    200   0   1 MURKOWSKI,   1  1  1
## 6  101 14502    61    0 ARIZONA   100   0   1 DECONCINI,   1  1  1
```

Remove the president data in Sen113kh

```
sen113kh <- na.omit(sen113)
# grpUnemp8
```

The function takes a single data frame of roll call votes and returns a Senator-by-vote matrix.

```
rollcall.simplified <- function(df) {
    no.pres <- subset(df, state < 99)
    ## to group all Yea and Nay types together
    for (i in 10:ncol(no.pres)) {
        no.pres[, i] = ifelse(no.pres[, i] > 6, 0, no.pres[, i])
        no.pres[, i] = ifelse(no.pres[, i] > 0 & no.pres[, i] < 4, 1, no.pres[,
            i])
        no.pres[, i] = ifelse(no.pres[, i] > 1, -1, no.pres[, i])
    }

    return(as.matrix(no.pres[, 10:ncol(no.pres)]))
}

rollcall.simple = lapply(rollcall.data, rollcall.simplified)

sen113_simple = rollcall.simplified(sen113kh)
```

1) Senator-by-senator distance matrix for the 113th Congress.

Multiply the matrix by its transpose to get Senator-to-Senator tranformation and calculate the Euclidan distance between each Senator.

```
rollcall.dist = lapply(rollcall.simple, function(m) dist(m %*% t(m)))

sen113_dist = dist(sen113_simple %*% t(sen113_simple))

sen113_dist
```

```
##              2          3          4          5          6          7
## 3    248.43711
## 4   4124.24902 3993.06536
## 5   7180.62184 7050.87519 3221.72811
## 6   1502.47130 1397.74068 2737.42507 5863.22846
## 7   1446.39103 1332.98162 2791.51446 5913.50243  259.81339
## 8   6841.65579 6710.54282 2862.05556  424.96823 5513.33438 5563.00863
## 9    647.35539  525.13427 3561.49477 6636.13283  996.03363  919.47703
## 10  7478.10792 7348.20788 3539.62682  392.16833 6170.77515 6219.89349
## 11  7509.16154 7378.99953 3536.76787  396.37230 6186.75771 6236.04971
## 12  7398.33495 7268.08799 3429.93877  275.47777 6076.84770 6125.48488
## 13  7611.34975 7481.06543 3633.93877  460.27926 6287.05185 6336.57242
## 14  7756.69414 7627.22263 3792.74887  613.25362 6438.30568 6488.18981
## 15  7710.73278 7581.35601 3749.20285  568.15667 6393.25551 6443.12960
## 16  7617.42575 7487.28402 3642.95718  469.43051 6294.24841 6343.41848
## 17  7604.48295 7474.61022 3633.39029  464.61489 6281.34404 6330.60045
## 18   606.48825  549.15845 3659.43028 6706.64611 1123.32453 1086.31717
## 19  7575.18957 7444.63431 3606.02648  435.81992 6253.44825 6302.49046
## 20  1576.99588 1456.01065 2648.48296 5743.42006  451.11418  454.83733
## 21  1500.52524 1378.32979 2812.90846 5943.93422  471.79869  437.56714
## 22  7767.03740 7636.83370 3802.83421  620.07580 6448.77461 6498.20075
## 23  7491.93393 7362.42236 3550.20873  388.21128 6182.18165 6231.65460
## 24   480.18434  587.97959 4487.35055 7511.38183 1900.00711 1846.72792
## 25   351.49395  442.55960 4344.02947 7383.01537 1743.14486 1687.59918
## 26  7740.55780 7610.51720 3769.27009  596.40674 6418.86960 6468.15012
## 27  1776.15230 1653.79987 2433.49604 5541.11983  563.96720  557.56614
## 28   940.82411  844.27780 3367.55965 6489.70523  728.83469  691.03328
## 29  7274.85230 7144.05466 3276.85489  289.09168 5939.33852 5988.87644
## 30   447.69186  443.69697 3937.56600 7034.34972 1292.49990 1233.06083
## 31  7555.14844 7425.06074 3600.68521  421.69420 6240.57754 6290.05493
## 32   533.77523  431.33630 3688.20932 6736.51423 1148.19423 1091.26761
## 33   315.21263  355.59809 4151.33870 7184.49212 1582.08691 1527.66227
## 34   404.20787  471.53897 4120.56598 7140.60488 1567.31873 1527.93488
## 35   471.77325  430.70059 3889.16212 6985.47135 1249.77678 1188.68709
## 36   529.32032  431.67001 3696.46615 6750.66841 1161.15718 1112.92363
## 37  6847.88778 6716.64648 2896.87124  384.47757 5530.70728 5578.84450
## 38  4477.63822 4347.07660  420.23565 2917.60038 3084.91815 3133.75350
## 39  7379.39022 7249.18582 3395.40366  293.90475 6050.19223 6099.63417
## 40  7572.87686 7442.64019 3609.16389  445.15166 6254.60422 6303.88460
## 41  7771.54534 7641.52053 3807.00578  622.13744 6453.47294 6502.97901
## 42  7662.48935 7533.55149 3717.38120  541.08502 6352.97513 6403.23450
## 43  3592.42884 3478.67877 2044.88655 4073.55999 2692.71294 2732.88675
## 44  4430.79011 4307.20652 1738.55716 3110.47279 3349.10451 3390.03687
## 45  6580.29581 6453.91013 2723.75825  746.10656 5297.83475 5351.20753
```

```
## 46   7698.07080 7567.66998 3732.90155  551.93478 6379.43336 6428.28492
## 47   7614.92160 7484.78844 3653.34819  481.83296 6297.21645 6346.56238
## 48   7652.94198 7522.79124 3674.66012  512.02051 6327.19156 6376.48932
## 49   7733.48848 7603.46875 3765.11540  587.17289 6412.74372 6461.93152
## 50   1418.35680 1278.65359 2775.51040 5858.16507  550.01455  495.99698
## 51   1123.82917 1003.48144 3213.54166 6336.88078  662.44849  582.18811
## 52   7179.63836 7049.71985 3212.54370  185.97580 5856.64793 5905.94836
## 53    944.59092  819.90426 3307.73729 6407.70224  753.47196  657.31956
## 54   5186.36192 5057.73517 1759.47094 2219.11153 3984.17294 4030.59288
## 55   5518.99701 5394.16564 1876.23719 1797.54833 4281.25449 4334.40734
## 56   7314.98879 7185.64611 3341.68535  235.03829 5989.38653 6040.21854
## 57    951.86974  830.71234 3322.59718 6423.23626  764.34286  677.68355
## 58    515.70146  468.60538 3895.97613 6995.20836 1258.21779 1181.09737
## 59    907.99119  809.92407 3323.01881 6385.52660  840.34160  803.93967
## 60   7335.72191 7205.37390 3367.68556  274.31004 6012.56967 6061.80196
## 61   1892.62463 1776.45236 2368.13091 5521.61480  522.53421  578.16866
## 62   7564.80495 7435.12737 3593.52863  423.07210 6243.93898 6294.25603
## 63   7736.18381 7606.48887 3777.64556  598.00167 6420.30171 6469.71908
## 64   3773.26781 3657.19031 1964.30293 3876.38930 2831.90819 2872.56801
## 65   3424.22780 3307.36345 1863.53803 4091.26240 2466.88184 2508.19796
## 66   6260.87310 6135.30366 2483.34613 1103.52345 5001.99080 5056.09345
## 67   7642.85143 7512.64634 3667.98337  489.25862 6319.63559 6369.07913
## 68   7749.75877 7619.72165 3782.64392  593.26470 6430.75322 6480.28734
## 69   7704.84192 7575.08231 3755.03622  570.44982 6393.55042 6443.16801
## 70   7785.52458 7655.45244 3820.16008  635.64849 6467.23395 6516.53558
## 71    765.40120  677.93510 3533.59944 6641.52610  897.42799  835.83013
## 72   7208.40329 7077.73869 3224.89070  221.33233 5878.85278 5927.84978
## 73   7185.66524 7054.69489 3200.52824  224.37692 5855.96166 5905.08679
## 74    960.36035  845.24730 3410.75857 6532.39083  831.32424  746.06367
## 75   7675.15140 7545.78757 3721.62800  541.15340 6361.48450 6411.13204
## 76   1221.19491 1115.91756 3071.30900 6189.84709  532.06485  472.70287
## 77    415.20597  357.97067 3828.74444 6865.63428 1274.89451 1222.02578
## 78    811.58117  765.10718 3650.60488 6616.59973 1274.20485 1260.45904
## 79   7606.23074 7476.95493 3658.62433  486.64155 6295.48902 6346.54355
## 80   7655.04696 7525.74216 3699.15369  521.46524 6340.36497 6391.67185
## 81   7520.67085 7390.62359 3548.92885  383.77076 6198.42383 6247.69501
## 82    681.56291  612.73567 3576.47452 6676.16327  939.27951  904.63750
## 83   7687.13035 7557.75363 3726.35090  554.31850 6369.34714 6419.38938
## 84   7735.33690 7605.93190 3778.14783  602.17688 6419.64103 6469.74760
## 85    254.77441  391.86350 4253.88411 7310.40218 1640.51821 1584.33866
## 86   1615.52778 1494.26872 2645.13365 5780.46261  361.41666  322.79870
## 87    569.38739  515.88855 3843.50257 6948.06894 1199.82332 1130.43045
## 88   7693.23320 7562.79340 3723.48855  541.45452 6372.33513 6421.28118
## 89   1271.11054 1164.70984 2980.95018 6097.91768  410.92578  372.31841
## 90   1500.36396 1380.08732 2748.49122 5869.85741  407.78671  362.78506
## 91    456.98578  413.32675 3797.86677 6886.20810 1159.20749 1111.58311
## 92    409.43620  496.87322 4174.08601 7184.72421 1637.07911 1595.53847
## 93    581.76026  693.46665 4429.43100 7423.72784 1891.94635 1850.16432
## 94   1553.91666 1438.86344 2716.99117 5856.11279  347.69958  373.97326
## 95   7204.42427 7074.65618 3288.26748  324.64596 5905.03319 5954.63248
## 96   7689.36968 7559.25241 3730.77177  545.09265 6373.78836 6423.27292
## 97   7579.04658 7448.60953 3602.22306  442.97291 6253.87376 6302.80826
## 98   7427.72105 7297.19172 3449.83565  313.65905 6100.83421 6149.96366
## 99   7795.83030 7665.83348 3831.39870  646.03405 6477.90846 6527.37551
```

```
## 100 7675.67574 7546.01756 3710.83387  530.94821 6356.58871 6406.35833
## 101 6310.00404 6180.15647 2316.97151  972.47057 4970.73405 5021.86639
## 102 7231.47018 7101.62545 3318.53748  323.17023 5933.09135 5981.21426
## 103  250.41965  339.41567 4081.16503 7147.47648 1447.29679 1398.71191
## 104 7747.34290 7617.75367 3785.84812  601.26700 6430.47572 6479.73557
## 105  275.92028  334.54297 4179.11546 7245.33560 1553.57588 1491.65043
## 106  283.16073  335.48323 4142.42996 7214.36220 1510.32182 1452.60628
##              8          9         10         11         12         13
## 3
## 4
## 5
## 6
## 7
## 8
## 9   6291.24622
## 10   772.73799 6937.00533
## 11   748.31210 6963.55197  201.73002
## 12   623.29768 6851.90404  251.84916  182.22788
## 13   811.12576 7064.57161  291.48413  180.83694  233.00858
## 14   988.33294 7213.52196  315.57566  271.87313  391.52011  222.72180
## 15   944.79627 7167.58906  278.11868  237.17293  352.65564  195.40983
## 16   822.93560 7071.00629  276.55560  169.28674  238.62732   72.97945
## 17   818.15096 7058.95021  272.61145  170.17638  237.31624   99.02525
## 18  6371.42590  529.58569 7001.90260 7034.02239 6924.49926 7137.36009
## 19   792.40835 7029.29556  241.16177  139.57077  207.73300   97.83660
## 20  5390.37828 1016.09498 6052.21183 6069.75972 5956.88308 6168.01175
## 21  5586.14303  937.92804 6255.16491 6266.46144 6154.98497 6365.05326
## 22   990.01818 7222.50545  324.38249  278.69159  396.67619  217.72689
## 23   775.17998 6950.64378  116.24113  199.17580  245.65423  273.74258
## 24  7178.77462 1033.58696 7803.44289 7840.81157 7730.14883 7943.94398
## 25  7047.47714  880.62080 7677.42327 7712.22257 7601.22806 7814.88548
## 26   956.27036 7195.20465  330.74613  247.46717  361.05540  173.45893
## 27  5192.19925 1224.03922 5844.45729 5861.15518 5751.56857 5962.84798
## 28  6137.02974  483.52559 6796.73348 6813.57623 6702.37630 6913.19629
## 29   460.26731 6723.45231  494.35210  398.29010  289.35791  408.71873
## 30  6687.69078  505.77169 7337.10352 7360.72585 7248.84570 7461.30826
## 31   794.38152 7011.71384  168.68906  147.38046  218.51316  173.96264
## 32  6396.79029  340.85334 7034.83262 7065.68234 6954.03221 7167.48680
## 33  6849.83190  725.26271 7478.68016 7513.80316 7402.92517 7616.58152
## 34  6809.26619  798.24746 7432.96791 7469.62616 7360.19959 7573.38227
## 35  6638.67818  435.80615 7288.26029 7312.22736 7200.45165 7412.89208
## 36  6411.75358  410.20361 7047.68260 7079.01017 6968.82960 7181.67773
## 37   247.94556 6301.65819  699.52627  710.91772  585.79945  792.28152
## 38  2551.23186 3911.83448 3237.84434 3223.31894 3116.93567 3317.59567
## 39   598.35441 6831.03103  333.27766  229.92825  163.15637  264.89054
## 40   813.87960 7028.14093  186.41888  108.36512  229.46459  171.06432
## 41   995.94628 7227.21537  329.03343  283.00177  398.10426  220.86648
## 42   929.59830 7122.91794  215.73827  254.20858  354.41642  259.04826
## 43  3814.25969 3175.49398 4315.75150 4394.23691 4295.02910 4503.31034
## 44  2867.64485 3958.39791 3342.44058 3422.71179 3327.11662 3534.22382
## 45   651.09523 6049.99339  970.03557 1048.41118  953.43432 1149.55035
## 46   918.74915 7152.58827  278.51750  215.14414  322.19870  159.56503
## 47   847.63023 7070.84507  209.95476  161.80235  263.93749  156.92036
## 48   858.15908 7106.26949  319.52621  200.14994  275.19629   88.34025
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## 49    950.39045 7188.63248  324.68138  248.83931  353.08639  168.09224
## 50   5507.54719  844.56557 6163.45536 6184.36197 6072.92549 6284.54350
## 51   5981.15089  598.25329 6645.38231 6660.23791 6548.50120 6759.59163
## 52    430.10580 6633.91378  414.55156  386.73505  276.58814  454.68671
## 53   6057.48628  394.46546 6712.27160 6732.85073 6621.41707 6833.28581
## 54   1978.84461 4677.74999 2464.30031 2539.51117 2441.09033 2647.08387
## 55   1550.84364 5000.74684 2052.70845 2120.98067 2016.84109 2223.40707
## 56    547.59931 6769.84165  353.28317  304.15785  212.22394  343.10640
## 57   6071.12000  417.13187 6729.54798 6748.85627 6636.41590 6848.13252
## 58   6646.14896  451.22611 7299.19557 7321.30187 7209.04876 7421.26465
## 59   6044.63597  547.25954 6683.54801 6712.04581 6601.61314 6814.83536
## 60    587.21717 6788.94344  288.02778  232.86262  176.51629  315.70714
## 61   5165.60200 1344.26076 5832.11042 5841.73887 5732.27843 5941.00757
## 62    787.75885 7020.34095  239.34912  161.61374  210.89334  138.48827
## 63    974.90205 7193.14139  285.66064  257.18670  378.14547  226.68260
## 64   3621.79541 3344.51476 4114.82806 4194.64313 4096.60469 4304.43272
## 65   3813.71079 2983.92175 4346.29750 4415.38594 4313.58436 4524.24657
## 66    951.03417 5738.02954 1325.39051 1409.21858 1313.12071 1511.63719
## 67    849.61638 7096.56544  282.13118  182.60066  260.40353   90.65870
## 68    965.69198 7205.17939  315.69764  266.21796  368.35716  187.93616
## 69    955.70759 7163.44694  253.30416  265.70661  367.94972  242.59019
## 70   1007.32070 7240.97148  346.24702  297.82209  409.14667  226.96255
## 71   6291.17930  379.02507 6947.41254 6967.31024 6855.46373 7067.23864
## 72    402.59160 6658.49044  481.27747  416.42887  290.04827  446.67326
## 73    387.54484 6635.48687  492.47030  431.65727  309.82737  470.46998
## 74   6177.94302  473.60004 6839.79006 6855.99088 6744.23087 6955.44427
## 75    919.51781 7133.35321  237.01688  224.92665  331.46795  204.11761
## 76   5834.84867  704.60343 6498.91937 6514.24976 6401.63471 6612.38414
## 77   6528.77783  474.64618 7162.55220 7196.36985 7084.02979 7297.37151
## 78   6290.89548  773.27809 6906.41571 6948.64440 6837.93960 7051.39922
## 79    872.08600 7066.19282  193.64917  236.56500  310.81827  237.55631
## 80    905.75604 7113.88101  234.01709  236.15884  325.56720  211.90800
## 81    734.36095 6974.82552  240.79452  136.45878  167.89878  129.57237
## 82   6330.55701  415.68618 6978.76536 7001.97893 6891.00704 7102.83922
## 83    927.04315 7144.52462  259.02316  221.43622  335.78118  189.52045
## 84    979.82498 7193.46801  287.34822  268.84940  389.27240  238.22258
## 85   6972.68929  782.93486 7606.76791 7639.00471 7528.21705 7741.36390
## 86   5424.96710 1060.11933 6090.65973 6102.49515 5992.05708 6201.79530
## 87   6598.05987  449.86331 7252.72625 7273.92886 7161.63668 7373.69819
## 88    904.13882 7147.31810  295.28122  218.23382  308.77662  129.24396
## 89   5746.15176  754.54688 6405.04395 6422.07965 6310.91451 6522.08011
## 90   5514.93935  935.20319 6178.86527 6193.32326 6082.15472 6292.54376
## 91   6542.45550  400.97756 7187.36127 7213.02634 7102.09138 7314.51003
## 92   6854.62297  844.04088 7476.39913 7514.38520 7404.41179 7617.91658
## 93   7097.17218 1090.26419 7711.70792 7753.09732 7643.82607 7857.57138
## 94   5500.17572 1013.41305 6167.25539 6179.04151 6067.70443 6277.66836
## 95    617.91342 6666.55008  313.10861  413.12468  374.25793  532.25370
## 96    920.30647 7145.39271  250.63519  225.94911  328.50266  178.72045
## 97    780.57223 7031.56384  302.77219  177.15530  215.03256   90.27735
## 98    629.69993 6879.71053  325.13997  219.10956  139.38436  210.75816
## 99   1019.59109 7251.88176  348.35901  305.64522  420.81587  239.08367
## 100   902.77129 7131.24849  257.34801  198.35574  305.31623  152.54180
## 101   615.60458 5759.33416 1303.10207 1285.59480 1170.42855 1364.23385
## 102   625.76433 6692.55990  302.35575  407.17318  354.55183  509.51349
```

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## 103 6808.06081  631.96914 7445.22827 7475.45082 7364.16648 7577.54439
## 104  975.70487 7203.78088  311.49639  267.65836  377.80948  207.77151
## 105 6903.51229  670.22757 7544.62179 7574.20273 7462.02265 7675.28273
## 106 6872.36699  640.31320 7514.13056 7543.23492 7431.16882 7644.25215
##             14         15         16         17         18         19
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## 15    68.89122
## 16   201.39265  169.37827
## 17   212.11553  182.92895   81.43709
## 18  7281.07423 7235.36841 7143.41928 7130.24256
## 19   226.01106  189.59167   89.44272   97.31393 7100.71623
## 20  6320.74632 6275.12916 6175.15239 6163.78350 1213.29469 6133.96878
## 21  6520.14892 6474.83035 6372.45785 6361.76619 1199.14803 6331.97244
## 22    70.29936   91.64060  197.12686  215.85875 7291.97737  221.10857
## 23   305.89541  267.43971  258.22277  248.40089 7015.85333  223.27785
## 24  8086.47259 8040.25304 7949.76339 7936.58239  905.38224 7906.66390
## 25  7958.98838 7912.75818 7820.91152 7807.92988  791.85352 7778.03233
## 26    92.67686   98.42256  150.68510  167.73193 7265.62172  183.43936
## 27  6113.92010 6068.63354 5969.57034 5957.81319 1366.28584 5928.10408
## 28  7065.71412 7020.24458 6920.40902 6908.60268  767.08409 6879.58436
## 29   604.43858  568.21123  429.37280  430.43815 6803.13090  415.52497
## 30  7610.80508 7565.07422 7468.01547 7456.34582  631.67713 7426.84327
## 31   232.11204  192.19261  155.73375  152.63355 7080.28347  131.24405
## 32  7313.99713 7267.72000 7173.81091 7161.65002  437.31225 7131.74488
## 33  7760.01372 7713.77528 7622.38270 7609.37816  600.82527 7579.58653
## 34  7715.01017 7668.84959 7579.02256 7565.86459  538.77082 7536.29040
## 35  7562.62124 7516.84176 7419.51380 7407.85144  608.70190 7378.08491
## 36  7327.04947 7281.42514 7187.87083 7175.69446  322.17697 7145.61509
## 37   950.76969  905.35131  799.38852  791.86236 6374.96706  761.74274
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## 39   437.25507  395.80551  278.58392  274.23530 6905.33345  249.06826
## 40   210.62763  177.76389  153.97727  160.61445 7097.47321  124.64750
## 41    67.56478   95.88013  202.60800  217.32464 7296.24773  226.59656
## 42   154.28221  133.98134  237.06117  236.28373 7185.72439  236.55443
## 43  4614.70790 4569.26920 4504.69311 4488.06261 3124.50876 4458.57197
## 44  3642.34444 3596.77147 3534.21109 3517.73990 3953.20086 3487.20375
## 45  1252.01398 1208.66993 1151.11511 1135.51046 6101.04393 1108.86879
## 46   115.15207  102.08820  137.36448  160.96273 7223.40875  151.65421
## 47   172.23530  135.56917  127.55391  133.75350 7139.52288  113.41076
## 48   189.86048  174.16372   86.51012  108.70143 7179.00306  120.92973
## 49   101.01485  109.24285  148.11820  165.14539 7258.87746  178.67009
## 50  6436.05050 6390.37104 6291.37497 6279.89068 1063.26761 6249.89720
## 51  6913.63334 6868.20384 6766.70355 6755.75688  921.63550 6725.89838
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## 52   614.60963  572.17742  461.09218  447.08836 6705.29395  426.93091
## 53  6984.86736 6938.97197 6840.00490 6828.56288  731.58321 6798.55845
## 54  2763.48331 2717.89845 2648.68439 2633.58482 4705.10212 2602.53165
## 55  2341.07326 2296.02178 2226.28502 2210.40064 5037.00407 2182.80989
## 56   505.08910  468.95309  360.13331  352.72369 6837.64908  330.84437
## 57  7000.87994 6954.98023 6855.18089 6844.00300  774.92193 6814.19276
## 58  7572.49008 7526.64387 7428.08475 7416.50275  699.73138 7386.82103
## 59  6960.83960 6914.97057 6820.90485 6808.87002  532.92213 6779.51245
## 60   452.84876  410.21945  313.83594  312.30914 6861.68041  277.67067
## 61  6095.05390 6050.43453 5948.46896 5936.99747 1488.10282 5908.76171
## 62   242.27670  214.74636  132.87964  147.61098 7088.07294  133.13527
## 63    58.18935   72.34639  199.64719  210.91942 7260.55301  215.30676
## 64  4414.19053 4368.79434 4305.42832 4288.74469 3302.16989 4259.10354
## 65  4643.22496 4597.37121 4526.54913 4510.50385 2951.72458 4480.83128
## 66  1612.74487 1569.27499 1513.08195 1496.81829 5778.71162 1469.90034
## 67   182.34583  163.81697   99.80481  119.67874 7168.11886  124.36639
## 68    90.60353  103.01942  169.57594  193.11655 7274.50005  204.72909
## 69   127.27529  115.25190  219.45842  225.45066 7228.75003  223.19946
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## 71  7218.66490 7172.82476 7074.08326 7062.54926  657.60398 7033.00960
## 72   635.69490  596.62048  464.24239  461.65247 6737.21975  440.58711
## 73   654.56780  613.97801  485.69950  482.95548 6713.20557  461.93290
## 74  7109.19503 7063.56921 6962.54530 6951.75553  824.71935 6921.65334
## 75   124.96800   93.42912  174.02299  179.76095 7199.93340  185.55862
## 76  6766.30645 6720.97917 6619.74441 6608.17585  966.59764 6579.44739
## 77  7442.14122 7396.19889 7303.50074 7290.54943  386.60057 7261.18689
## 78  7190.48844 7144.09336 7056.80558 7042.53797  532.23115 7013.67657
## 79   223.51286  200.28979  227.37854  231.09089 7127.65487  212.95774
## 80   169.94705  157.37217  202.21523  206.07523 7176.81893  200.84073
## 81   274.77627  237.05274  124.20547  122.79658 7045.67243  107.10275
## 82  7251.82460 7206.21412 7109.64732 7097.38071  499.11321 7068.40017
## 83    98.07650   73.05477  168.07141  169.64080 7211.05062  173.72968
## 84    69.97857   74.46476  214.09811  217.19576 7258.94352  226.22113
## 85  7886.22267 7840.23086 7747.47849 7734.76819  672.74512 7705.33860
## 86  6356.12728 6310.90659 6209.02641 6197.44068 1245.84590 6168.36826
## 87  7525.30983 7479.44944 7380.59422 7369.04207  705.85409 7339.41544
## 88   137.98188  122.97561  104.89042  125.53087 7219.04322  141.35063
## 89  6674.08720 6628.48339 6528.96875 6516.78318  975.72230 6488.32436
## 90  6446.51735 6400.94774 6299.56959 6288.04206 1178.60850 6259.37816
## 91  7462.25609 7416.50241 7321.02090 7308.60896  455.76968 7279.23849
## 92  7759.03654 7712.92889 7623.59371 7610.26879  599.38302 7580.97612
## 93  7996.43327 7950.31817 7862.86373 7848.94515  853.09261 7819.77966
## 94  6432.22085 6387.17394 6285.43348 6273.61236 1200.42201 6245.10753
## 95   605.11239  563.05861  525.52069  517.08220 6727.18344  478.19034
## 96   116.08617   87.34987  158.15499  174.81705 7214.61877  170.34964
## 97   257.09726  227.46208   94.25497   97.50897 7105.50385  101.64645
## 98   400.03250  362.98760  227.85302  218.97717 6954.77476  204.90242
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## 100  113.48128   93.78699  138.31486  148.46885 7199.86430  146.02397
## 101 1530.95689 1487.41689 1376.11991 1365.46402 5839.61720 1343.30786
## 102  585.54846  542.78633  500.38785  486.89219 6754.11660  455.59192
## 103 7723.12476 7677.41740 7583.83551 7570.62369  562.61888 7541.78911
## 104   71.69379   73.03424  181.47452  194.95384 7272.32466  207.28965
## 105 7822.68662 7776.37075 7681.63596 7669.49405  711.29459 7639.69967
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## 106 7791.80197 7745.61411 7650.67468 7638.47518  682.99780 7608.74701
##             20         21         22         23         24         25
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## 21   339.80877
## 22  6329.13264 6528.07828
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## 25  1815.09394 1745.49706 7969.03332 7691.97153  190.06052
## 26  6300.31229 6497.95652   87.24105  316.96530 8071.76920 7943.47739
## 27   521.12091  645.77473 6123.58220 5857.56340 2157.31268 2003.37241
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## 30  1364.55048 1239.99032 7620.54132 7351.65764  811.99323  659.96364
## 31  6120.40129 6321.26759  231.11036  148.13507 7883.70224 7756.27888
## 32  1175.20807 1136.12147 7323.45561 7049.21740  855.69679  708.04237
## 33  1642.13063 1591.34754 7770.43738 7493.25063  390.91431  301.58083
## 34  1649.50599 1618.60125 7726.07869 7447.72489  471.89935  415.30471
## 35  1309.84503 1190.01933 7571.84046 7302.16598  831.41145  681.63920
## 36  1202.03910 1165.40808 7336.95700 7061.69087  851.05993  721.34874
## 37  5409.45367 5610.00455  953.20460  706.22872 7179.45430 7050.23539
## 38  2991.78107 3148.26619 3490.01633 3248.14932 4846.04261 4701.27132
## 39  5931.53673 6127.05198  439.84088  321.82604 7714.24261 7584.26120
## 40  6135.70844 6334.40084  214.63457  165.79505 7902.58660 7774.77009
## 41  6334.04113 6533.25991   58.48932  310.25312 8100.92007 7973.31782
## 42  6235.20497 6438.28494  186.29815  214.41315 7988.50036 7862.46456
## 43  2588.03304 2863.08383 4627.49425 4327.78107 3820.03063 3733.61554
## 44  3235.16074 3491.69171 3654.17036 3353.45508 4696.00319 4593.90368
## 45  5177.24338 5393.42350 1268.34341  986.89057 6899.05022 6776.80035
## 46  6259.06031 6457.51036   91.42210  266.21044 8028.37611 7900.33898
## 47  6177.86055 6377.55400  169.73803  195.95152 7944.43075 7816.70551
## 48  6208.67917 6404.99703  187.98670  305.47831 7986.10806 7856.82818
## 49  6293.66507 6491.77849   94.35041  309.83221 8064.40841 7936.09337
## 50   375.77653  451.47093 6444.27932 6175.83695 1809.64334 1655.95229
## 51   679.04344  479.86040 6921.55098 6658.21598 1537.87613 1369.81021
## 52  5738.33817 5937.11807  622.65159  420.13212 7512.76074 7383.64551
## 53   756.43506  637.96865 6993.30466 6725.45552 1344.29052 1182.42886
## 54  3869.14047 4104.87174 2773.56630 2473.78253 5479.69242 5365.24594
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## 55  4156.45955 4388.61163 2355.01592 2066.47720 5826.98730 5709.62310
## 56  5872.66064 6071.65011  511.71965  354.49260 7647.29978 7518.33193
## 57   754.71584  623.61046 7009.03617 6742.37629 1360.41942 1194.55264
## 58  1311.69127 1167.45707 7581.17339 7313.02988  887.06990  726.71315
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## 60  5892.77201 6090.67320  454.97033  283.64414 7668.63215 7539.65211
## 61   569.86841  620.67544 6105.43250 5844.70051 2289.21449 2132.88865
## 62  6126.21678 6325.31248  254.93921  229.11133 7895.12527 7767.00238
## 63  6301.81260 6501.83974   75.90784  277.45270 8064.81165 7937.61249
## 64  2725.17155 2997.66209 4427.02135 4126.88769 4007.86190 3918.23404
## 65  2357.07700 2629.75208 4655.44155 4358.28062 3671.64854 3576.69009
## 66  4879.76946 5103.77311 1628.49624 1341.74886 6572.15497 6453.19967
## 67  6200.76205 6398.20006  178.17968  260.65303 7974.61027 7845.93296
## 68  6311.83357 6510.70449   75.70337  302.29952 8079.51645 7951.67800
## 69  6274.18847 6476.18290  139.16537  240.16869 8031.21049 7904.71777
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## 71   952.69670  818.85835 7227.80347 6960.30481 1171.14730 1003.08773
## 72  5758.29176 5953.20502  635.63826  479.15551 7545.05593 7414.15255
## 73  5735.85138 5930.52333  654.19645  492.59821 7521.71536 7391.00352
## 74   859.56850  663.73187 7117.21069 6853.06151 1363.54941 1195.18869
## 75  6243.12734 6444.09567  137.61904  233.54443 8002.95558 7875.86713
## 76   546.35062  399.61857 6775.25608 6511.68051 1634.35125 1465.55143
## 77  1324.05740 1298.33509 7452.58921 7176.80751  710.75805  586.45716
## 78  1320.71193 1395.11147 7202.03319 6919.59883  988.77955  908.11673
## 79  6178.15604 6381.43887  239.65392  199.93999 7931.43965 7805.34496
## 80  6223.52087 6425.52737  189.21945  225.87164 7981.70539 7854.99612
## 81  6079.55064 6277.59221  281.40718  227.03304 7852.64166 7724.05295
## 82  1035.11738  943.65566 7262.33764 6992.40588 1062.61705  907.06119
## 83  6252.26767 6452.41885  122.15973  252.27961 8016.26996 7888.87704
## 84  6302.56416 6503.32277  101.82829  279.02867 8063.67261 7936.79173
## 85  1719.04683 1639.29131 7896.76491 7621.54459  371.80775  278.72567
## 86   392.85112  357.99162 6364.87816 6101.98836 2021.94040 1862.64704
## 87  1260.72360 1112.38932 7534.05986 7266.76049  947.29879  785.52276
## 88  6252.86694 6450.90544  122.84543  277.10106 8024.06680 7895.70953
## 89   534.53718  454.01542 6683.73025 6417.42347 1678.42218 1514.24701
## 90   408.63431  345.93641 6455.27877 6190.94710 1905.54061 1741.86165
## 91  1237.62030 1142.61630 7472.48004 7201.01021  840.95125  695.61484
## 92  1712.59861 1685.11187 7770.31312 7491.49051  420.48187  384.88180
## 93  1977.88397 1950.69449 8007.97509 7726.65665  334.81637  417.41945
## 94   404.48857  357.54860 6441.56681 6178.82068 1956.02710 1793.75500
## 95  5785.70419 5990.93090  613.71573  351.82240 7526.99369 7401.79039
## 96  6253.71873 6453.70560   99.24717  243.54466 8018.06679 7890.60847
## 97  6133.75179 6330.17938  251.92658  286.46291 7912.64242 7783.25388
## 98  5980.99624 6177.37614  399.93499  312.52200 7762.01140 7632.15160
## 99  6358.95322 6558.17871   61.20457  334.77455 8125.10929 7997.52249
## 100 6237.76659 6437.04932  117.80492  241.81811 8005.46095 7877.67466
## 101 4853.14795 5045.52406 1540.20745 1310.15610 6651.17373 6517.99164
## 102 5812.80440 6019.73612  595.16468  307.81650 7553.79415 7428.95733
## 103 1535.87304 1456.93308 7733.85234 7459.30211  541.21253  411.10096
## 104 6311.71435 6511.15397   80.42388  299.27245 8076.60455 7949.05932
## 105 1612.84221 1514.05680 7832.02886 7559.20088  495.66218  360.39423
## 106 1577.53605 1478.72580 7801.32860 7528.67810  530.61568  395.26194
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## 27  6093.24856
## 28  7045.12874 1026.01901
## 29   560.02589 5619.48334 6564.95164
## 30  7592.16932 1561.32412  652.58409 7117.47624
## 31   219.97954 5915.94194 6866.63411  451.65695 7410.93489
## 32  7297.28888 1363.85373  667.93113 6830.09121  493.13893 7111.48508
## 33  7744.85261 1824.46102 1057.97212 7283.61964  579.81807 7557.91962
## 34  7700.76626 1818.63493 1099.50625 7242.75887  670.56245 7513.24670
## 35  7543.74754 1515.46593  621.55772 7069.50904  200.88305 7361.87789
## 36  7310.90576 1388.02341  707.49276 6845.62108  553.27931 7125.09902
## 37   927.24646 5206.22935 6156.45620  511.67079 6700.82913  746.76837
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## 39   398.59127 5726.62876 6676.89284  237.38997 7227.01072  285.22973
## 40   196.75619 5928.53970 6880.72671  465.24832 7426.57849  112.30316
## 41    92.21714 6128.38086 7079.83496  609.84998 7625.51736  234.21998
## 42   206.73171 6028.44507 6980.22528  591.75755 7521.61346  186.29278
## 43  4614.87291 2438.61067 3206.83255 4237.98926 3593.86908 4412.87129
## 44  3642.62076 3046.92993 3927.98409 3283.01584 4381.54482 3440.72071
## 45  1255.29678 4976.82027 5921.54296  942.30409 6452.13786 1060.67761
## 46    87.85784 6053.18519 7004.97438  535.60620 7550.71175  172.20918
## 47   155.48633 5972.00218 6923.68580  489.29643 7469.28169  110.33132
## 48   131.98485 6002.52364 6953.35480  450.53635 7501.85864  200.08248
## 49    60.58878 6087.75418 7038.86965  554.45469 7585.44606  210.95023
## 50  6416.19724  588.03826  732.51212 5939.91852 1240.76831 6234.37615
## 51  6891.91359  901.01221  324.74298 6409.35426  838.65011 6713.95085
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## 53  6964.69260  969.05469  339.87351 6487.71547  715.46069 6783.98953
## 54  2761.15211 3668.16044 4596.40120 2391.12777 5094.53531 2558.75595
## 55  2338.18284 3963.35880 4894.93534 1955.78935 5408.58725 2141.44204
## 56   478.64810 5666.20278 6617.12241  248.68253 7163.57711  342.35216
## 57  6980.33287  993.39166  313.35922 6501.16636  701.06205 6799.95426
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## 58  7552.77982 1525.67526  602.87810 7076.06331  217.94495 7371.73738
## 59  6943.84281 1011.19187  621.63172 6476.11010  769.39847 6760.63592
## 60   427.38624 5688.35503 6638.26589  308.59358 7186.15210  283.73579
## 61  6073.45231  457.08533 1071.80968 5588.27558 1649.65875 5899.57456
## 62   217.47873 5918.17700 6870.70622  416.85249 7416.13700  161.51161
## 63   103.25212 6094.83166 7046.94047  602.33711 7591.44242  207.94711
## 64  4414.80566 2563.23565 3362.32033 4043.48018 3764.03533 4212.76133
## 65  4639.98610 2202.42344 2995.07095 4243.66681 3405.16138 4439.64593
## 66  1616.49405 4680.75411 5621.35446 1293.21383 6142.49965 1419.29983
## 67   142.02465 5995.49456 6946.04952  451.84953 7493.63290  164.86965
## 68    86.63717 6106.10432 7057.19356  575.01913 7602.99987  218.23153
## 69   168.40428 6068.65273 7020.22685  600.82859 7562.80232  185.59903
## 70    91.33455 6141.53027 7093.34780  618.68328 7639.05616  248.19146
## 71  7198.92770 1172.44872  299.43447 6720.65852  483.53800 7018.62102
## 72   597.59602 5556.61813 6503.88192  140.50623 7054.24822  461.66655
## 73   617.03322 5535.08031 6480.99383  150.64196 7031.04253  479.95521
## 74  7087.73187 1074.44358  299.07190 6605.82682  656.74196 6909.25423
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## 86  6334.38576  507.15579  795.77258 5850.35196 1377.49846 6158.02046
## 87  7505.36908 1471.44181  541.63641 7027.43979  234.06837 7324.86696
## 88    88.41945 6047.32784 6998.32730  510.49290 7545.13797  177.88479
## 89  6654.02367  694.88488  506.97140 6174.15857 1041.70821 6474.81567
## 90  6425.21151  543.81706  700.31279 5942.51159 1272.42446 6247.15055
## 91  7444.46418 1438.13456  590.19488 6973.34826  307.39226 7262.06789
## 92  7745.19328 1874.44952 1164.67592 7288.16191  709.17276 7557.40326
## 93  7983.77192 2140.66018 1419.37204 7531.61676  927.60175 7794.56381
## 94  6410.55395  507.14495  709.23268 5925.97190 1295.72412 6234.51658
## 95   609.31929 5577.63929 6529.91907  523.88453 7066.77869  421.16149
## 96   122.73141 6048.68953 6999.67985  549.69446 7544.10472  161.07762
## 97   202.33141 5929.21496 6879.40935  378.35830 7428.15690  183.12564
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## 41  7327.83140 7774.39168 7730.26824 7576.72033 7341.27094  959.48632
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## 43  3197.56736 3537.58957 3469.76671 3536.76137 3181.25463 3767.49970
## 44  4011.20244 4396.49451 4336.55589 4325.93146 4008.79321 2806.67348
## 45  6140.21897 6575.56218 6526.27007 6402.73403 6147.84019  571.01313
## 46  7254.09071 7701.68923 7657.92935 7501.84924 7268.11255  881.88605
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## 54  4750.67427 5166.72362 5112.92617 5040.07917 4755.10610 1913.42442
## 55  5082.64282 5506.31229 5455.60061 5357.53656 5085.51492 1512.64900
## 56  6870.98457 7318.88093 7274.57978 7115.57833 6883.46548  527.15273
## 57   627.86304 1054.36474 1114.38683  647.23103  698.65871 6087.42335
## 58   514.76402  663.99473  775.54239  222.65220  585.09315 6660.31193
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## 60  6892.32892 7341.07955 7297.99740 7137.27273 6906.11758  558.78887
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## 61   1503.34560 1956.95759 1947.64730 1607.64673 1515.86147 5190.86419
## 62   7121.01805 7567.81613 7523.03184 7367.78338 7133.45989  755.31781
## 63   7293.14377 7738.93785 7693.75461 7542.81605 7306.15124  931.61956
## 64   3372.27104 3721.87198 3655.19685 3707.31385 3358.76674 3571.94919
## 65   3018.02054 3378.91506 3315.47945 3347.63827 3006.28608 3776.82790
## 66   5822.04260 6250.87266 6199.40594 6092.70991 5827.14373  877.13682
## 67   7198.90103 7647.45611 7603.85823 7445.03271 7212.87848  823.30978
## 68   7306.04578 7753.19263 7708.65390 7554.44101 7319.55060  930.52996
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## 72   6763.73876 7216.25298 7175.09707 7005.79060 6779.53273  423.71807
## 73   6740.65805 7192.96823 7151.59835 6982.88372 6756.11109  412.47061
## 74    692.73805 1076.80314 1142.03853  605.81185  752.85324 6196.60738
## 75   7232.02579 7677.76061 7631.95552 7483.19310 7245.82300  873.20902
## 76    879.90625 1325.13924 1352.36053  918.00926  939.81647 5855.65095
## 77    339.51730  399.61106  446.00224  480.27076  319.32429 6533.66612
## 78    607.36480  721.83308  664.23490  927.72733  587.05536 6287.60567
## 79   7163.05982 7606.46271 7559.08804 7415.35623 7174.75588  819.29665
## 80   7211.63546 7656.05597 7609.46529 7462.35633 7223.75941  860.63058
## 81   7077.43654 7525.21182 7481.40869 7323.45328 7090.82619  711.24047
## 82    462.20558  758.84583  780.78550  431.00348  473.62855 6344.21461
## 83   7244.75983 7690.23758 7644.91197 7493.69462 7257.95157  885.80359
## 84   7293.19237 7737.84738 7692.18870 7543.19687 7305.66280  935.02834
## 85    642.46401  281.81732  365.29988  554.07039  617.97573 6977.94031
## 86   1243.14762 1696.67233 1701.07143 1328.76108 1254.94064 5446.64420
## 87    520.24610  712.07654  812.42600  236.02119  613.03507 6613.15726
## 88   7249.32100 7697.29888 7654.01195 7496.42348 7263.57123  872.66947
## 89    916.29253 1364.54571 1384.14739 1005.22137  978.61228 5763.19729
## 90   1119.24975 1583.43645 1606.43518 1222.05483 1162.56613 5535.38535
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## 93    891.50042  458.44956  399.98125  947.96150  859.90755 7093.84959
## 94   1183.26286 1632.16666 1643.20267 1250.59626 1208.04429 5523.38628
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## 98   6983.89476 7433.85264 7391.97091 7227.57041 6998.80676  620.17175
## 99   7352.29842 7798.99795 7754.20505 7601.18754 7365.74219  982.81077
## 100  7232.02406 7678.96770 7634.36310 7480.21591 7245.70521  866.35616
## 101  5866.68774 6321.80338 6281.62121 6103.55757 5881.79029  690.11883
## 102  6788.70673 7229.77828 7184.06786 7045.38934 6800.52946  510.28032
## 103   508.99705  332.36275  408.70038  417.21937  517.72966 6814.76992
## 104  7304.02382 7750.37889 7705.71645 7553.21812 7318.00355  939.00532
## 105   563.55479  355.61637  517.31035  416.89567  608.98604 6911.75774
## 106   545.86628  359.53720  515.69371  368.93902  583.88012 6881.17352
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## 39  3077.71864
## 40  3298.32200  291.24217
## 41  3494.62945  445.24488  208.43464
## 42  3411.87236  420.64118  192.04947  181.14635
## 43  2308.48240 4299.90244 4439.03897 4627.60175 4494.32976
## 44  1852.53313 3335.71806 3466.34462 3654.63514 3521.85463 1005.99105
## 45  2463.37472  965.88560 1089.23505 1268.16008 1137.27481 3394.46815
## 46  3419.31806  373.69239  148.13507   99.32271  183.24028 4566.25722
## 47  3342.32748  312.33155  111.51233  178.56091  164.57825 4477.69115
## 48  3356.46764  302.66979  184.38275  192.97668  256.19524 4550.65775
## 49  3449.91420  395.01139  194.30389   94.81034  201.90840 4607.83778
## 50  3123.15450 6049.99240 6249.74863 6449.12575 6348.17714 2593.00501
## 51  3552.19059 6522.98444 6727.64193 6927.03060 6830.04444 3125.92082
## 52  2901.60731  261.83010  453.66177  627.99204  556.50517 4094.28260
## 53  3652.75622 6597.86359 6799.07141 6998.41018 6897.70737 3076.59146
## 54  1693.43733 2449.91837 2585.17156 2774.46013 2646.36184 1895.67798
## 55  1720.04419 2017.19211 2167.18712 2354.89002 2228.18110 2322.23793
## 56  3031.53377  212.77453  360.29571  515.94961  463.22349 4224.22537
## 57  3664.64405 6612.75646 6815.02839 7014.12425 6914.46231 3118.25929
## 58  4239.72782 7186.52489 7387.21138 7586.30997 7484.54574 3584.77712
## 59  3677.21280 6582.50925 6776.92541 6976.15059 6867.86131 2924.52612
## 60  3056.10111  179.35997  286.47862  461.71311  414.10506 4239.75577
## 61  2707.58232 5702.67253 5911.21290 6110.14182 6013.42739 2569.13429
## 62  3282.19317  267.45841  158.34772  252.37868  231.26392 4443.25950
## 63  3466.75785  431.10092  187.54200   75.66373  133.53277 4587.59839
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## 64  2204.07214 4102.47754 4238.87155 4427.08290 4293.23666  211.45922
## 65  2148.05680 4313.58969 4464.19556 4656.27372 4527.38191  272.64079
## 66  2255.42236 1326.24658 1448.44952 1627.80681 1493.89591 3036.80572
## 67  3352.54858  299.18556  148.82876  176.58143  228.32652 4525.68017
## 68  3469.68572  417.25652  199.80741   74.12152  185.79020 4611.75628
## 69  3447.64891  429.54278  183.08741  126.15863  103.21822 4541.42951
## 70  3507.25291  453.25379  222.81158   45.09989  200.20490 4643.13773
## 71  3878.01973 6831.63150 7033.67592 7232.69562 7131.52382 3292.72106
## 72  2906.98779  264.64316  480.48725  641.12011  603.79384 4156.41733
## 73  2883.75224  278.28942  497.59120  660.88123  619.12680 4134.39500
## 74  3750.19439 6718.90482 6923.28751 7122.61504 7024.83615 3273.50745
## 75  3411.75483  388.26151  167.37682  143.96527  106.17909 4525.84953
## 76  3409.12965 6375.87994 6582.20290 6780.56399 6682.12137 3005.16256
## 77  4186.68270 7066.10770 7259.07625 7456.96835 7346.90105 3276.00992
## 78  4015.73455 6823.67130 7008.46374 7205.39929 7089.33163 2898.64468
## 79  3355.65493  376.12764  185.49933  236.50159  150.32299 4444.83014
## 80  3393.55595  385.96114  175.41380  181.67829  133.97388 4501.77509
## 81  3234.78438  209.84756  158.01899  284.88068  263.38375 4407.96518
## 82  3927.31486 6868.12755 7067.86361 7266.78051 7162.06458 3260.15628
## 83  3415.07145  377.33937  164.56306  125.32358  133.06389 4544.58183
## 84  3468.35451  436.33359  204.80479  102.96601  126.22203 4581.34434
## 85  4610.08937 7509.62596 7702.62228 7901.21048 7791.27826 3701.03486
## 86  2984.65375 5963.86335 6170.79784 6369.93336 6273.58151 2708.64006
## 87  4187.49424 7138.54047 7340.18324 7539.24074 7438.09445 3560.63379
## 88  3408.51742  354.67591  164.71491  124.90797  207.56686 4567.76379
## 89  3321.88320 6285.98536 6489.31722 6688.51411 6588.50537 2874.44377
## 90  3085.82112 6056.05334 6260.93795 6460.13382 6362.66634 2740.79678
## 91  4150.61610 7080.28672 7277.99698 7476.99893 7371.48771 3405.01145
## 92  4534.39632 7388.65468 7576.19139 7774.31470 7660.40958 3501.14824
## 93  4791.95419 7629.42036 7813.91611 8011.76522 7895.74569 3676.54185
## 94  3058.09238 6040.09396 6247.39602 6446.35339 6349.55542 2776.38974
## 95  2995.27845  429.37280  440.33964  618.60812  494.26511 4040.41978
## 96  3420.49529  382.07591  157.56269  111.21601  144.70314 4546.00143
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## 98  3132.70331  149.93332  270.88743  405.52312  390.00897 4343.80582
## 99  3518.85749  466.90042  234.23492   53.97222  195.22039 4652.16627
## 100 3398.74918  352.35493  131.15640  113.76731  160.86951 4541.02973
## 101 2006.68558 1135.19646 1358.55327 1545.03269 1463.13499 3379.73224
## 102 3025.83162  419.69632  423.02128  594.12288  475.10209 4050.18814
## 103 4435.95153 7344.96167 7539.45595 7738.15941 7629.68472 3577.61247
## 104 3472.98978  425.93192  201.26102   74.95999  160.37768 4605.89286
## 105 4530.61364 7442.75514 7637.97080 7836.72183 7730.01016 3693.74349
## 106 4494.99522 7411.53425 7607.21894 7805.99366 7699.43953 3672.26864
##             44         45         46         47         48         49
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## 45  2450.87862
## 46  3592.80545 1211.46812
## 47  3505.01155 1124.67773  122.05327
## 48  3581.15261 1194.16666  139.81774  159.01572
## 49  3635.57932 1248.91913   85.58037  152.27935  118.97479
## 50  3280.00625 5285.56761 6373.93473 6292.65055 6326.09192 6409.73494
## 51  3812.89509 5779.88901 6850.84447 6770.73844 6799.80610 6885.75994
## 52  3132.96760  774.94258  555.83721  479.05532  495.40690  581.26844
## 53  3806.01642 5836.66283 6922.95139 6841.94059 6874.21639 6958.44437
## 54   934.49826 1590.53796 2710.61395 2624.06269 2696.19825 2753.71785
## 55  1421.35182 1109.75718 2293.26318 2205.63166 2269.45786 2331.63290
## 56  3263.60292  887.23052  447.23707  385.75381  389.02699  471.62379
## 57  3840.03008 5855.73608 6938.43931 6858.02020 6888.95442 6973.66001
## 58  4358.14536 6420.13699 7510.90860 7430.00410 7461.98707 7545.97747
## 59  3704.29845 5791.71693 6902.36119 6819.52674 6855.86005 6937.53155
## 60  3272.18902  908.92299  389.89486  320.62595  350.44400  422.63459
## 61  3135.23380 4970.16569 6035.37455 5954.50409 5980.50792 6068.27801
## 62  3474.61465 1086.14778  195.82645  163.47171  160.94409  212.93191
## 63  3614.20296 1229.08055  101.43964  150.13660  198.94974  108.40664
## 64   802.04302 3198.05550 4366.07879 4277.39068 4351.61338 4407.78720
## 65  1070.21166 3423.56788 4592.21755 4503.93839 4571.03435 4633.11817
## 66  2101.56299  377.34467 1571.55019 1484.44165 1557.20808 1610.19688
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## 67  3555.99845 1168.07662  121.21056  139.93927  104.73299  137.98551
## 68  3639.62828 1251.00679   87.90336  161.23895  166.04819   85.39906
## 69  3567.74186 1187.59589  137.22609  154.22062  231.32661  155.16121
## 70  3669.91635 1284.78559  109.53082  186.48324  195.14610   91.30717
## 71  4039.49799 6068.57660 7157.68824 7076.42127 7107.87662 7192.36408
## 72  3199.37791  861.97622  563.93085  510.34400  492.68550  590.35413
## 73  3177.19971  846.21983  583.81932  527.01992  516.02326  611.36323
## 74  3982.12368 5972.87494 7046.47870 6966.39139 6995.69375 7081.45734
## 75  3552.47773 1170.52894  126.81088  114.45523  190.05789  134.83694
## 76  3681.98235 5629.96892 6705.03475 6624.19203 6652.28472 6738.50198
## 77  4113.99016 6260.43904 7383.87987 7300.46909 7339.35903 7419.47384
## 78  3777.90246 5991.71453 7134.71541 7049.30202 7094.63847 7171.11735
## 79  3473.94113 1090.26832  212.50412  171.97965  250.33378  229.21606
## 80  3530.67062 1144.17874  169.05029  157.70542  210.63001  175.72706
## 81  3438.74905 1055.94744  216.34232  162.10182  164.63596  238.95606
## 82  4034.53802 6090.65325 7192.87147 7110.45224 7143.54751 7227.08821
## 83  3572.67631 1184.35721  116.03017  112.96902  169.83521  119.09240
## 84  3608.56980 1222.20293  136.15800  156.00000  215.74058  136.14331
## 85  4550.36207 6706.49066 7827.91492 7744.71542 7783.03045 7863.39418
## 86  3322.34240 5228.99656 6294.41959 6213.88936 6241.72148 6328.27939
## 87  4326.42635 6374.89082 7463.75147 7382.94325 7414.21439 7498.67508
## 88  3596.10039 1210.93270   78.93668  128.21856  110.31772   86.23804
## 89  3556.15790 5534.21042 6613.66343 6532.08106 6562.24207 6647.71841
## 90  3379.30836 5314.02757 6384.83406 6304.07083 6332.80041 6419.01223
## 91  4204.56002 6295.98944 7403.02600 7320.58188 7355.59209 7438.21444
## 92  4374.16266 6568.16512 7702.15476 7618.18738 7659.98192 7738.00801
## 93  4571.18453 6800.31499 7940.28772 7855.54161 7900.00620 7976.60774
## 94  3404.10047 5301.33040 6371.21095 6290.47025 6317.41719 6404.54753
## 95  3067.27827  726.48193  557.34460  477.35521  567.97007  599.55067
## 96  3572.88693 1190.05840   80.06248  115.17378  173.16755  117.17508
## 97  3512.21042 1129.95310  189.28022  174.59668  107.75899  199.56703
## 98  3378.67770 1006.61164  330.97734  283.99120  252.99209  350.42546
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## 102 3075.80038  737.22046  537.50628  454.22682  551.48345  581.00861
## 103 4411.77731 6547.50479 7664.81481 7581.73575 7618.90340 7700.05753
## 104 3632.57911 1247.40651   96.55050  157.33086  174.20964   78.15369
## 105 4521.25679 6651.53930 7762.51055 7680.42505 7716.80536 7798.26994
## 106 4497.16411 6621.07083 7731.81466 7649.67588 7685.74297 7767.48814
##             50         51         52         53         54         55
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## 51   579.45837
## 52  5854.93271 6330.90144
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## 54  3944.69619 4460.89946 2245.37146 4488.42043
## 55  4252.72665 4763.35029 1815.03636 4804.24666  718.62925
## 56  5990.23297 6464.72188  230.03043 6538.82948 2370.79755 1941.54088
## 57   628.50457  307.27187 6419.35316  246.81977 4516.53230 4825.28714
## 58  1182.16327  759.10869 6991.43054  638.35100 5064.00928 5380.52600
## 59   779.34716  695.63065 6383.14288  562.05338 4420.79472 4741.90088
## 60  6009.00974 6484.56614  267.50514 6557.77782 2392.33944 1959.76274
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## 62  6242.19529 6718.89604  423.02128 6790.75018 2587.17104 2165.20992
## 63  6416.25046 6894.73422  601.96262 6965.08851 2735.99123 2317.24621
## 64  2738.64748 3273.99175 3897.44673 3234.09431 1697.63777 2131.71293
## 65  2372.36549 2906.65443 4108.01789 2867.83891 1927.54974 2341.52130
## 66  4981.92473 5484.73573 1130.40214 5533.38052 1269.22969  762.11023
## 67  6316.90193 6792.51780  492.47640 6865.77425 2669.87116 2246.74120
## 68  6426.93014 6904.17359  599.94000 6976.07504 2756.58611 2337.55834
## 69  6387.54186 6868.58945  585.44342 6937.23756 2690.45665 2275.04923
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## 70  6462.71994 6940.27240  640.39675 7011.91522 2789.78583 2370.18417
## 71   851.89612  449.84997 6636.93257  377.19093 4725.34792 5035.23376
## 72  5876.16593 6348.28150  201.66556 6424.60256 2306.96467 1877.40725
## 73  5853.26464 6325.51081  211.29127 6402.08833 2285.82042 1854.56059
## 74   755.77642  281.05515 6526.63673  284.67525 4645.29655 4950.84377
## 75  6357.12993 6836.38128  543.65062 6906.01513 2673.69314 2256.82188
## 76   544.77610  328.65636 6183.21518  447.98326 4322.38973 4614.87475
## 77  1167.09811  968.64338 6865.70543  770.34148 4866.31544 5196.67182
## 78  1173.04731 1182.93702 6620.47377  983.26599 4564.54247 4914.43201
## 79  6291.29700 6772.80739  501.49078 6840.83299 2593.51576 2176.22425
## 80  6337.85752 6817.72675  535.66314 6887.28423 2649.43994 2230.75951
## 81  6195.29725 6671.58407  371.86691 6744.40361 2554.93014 2127.72743
## 82   926.81498  616.88248 6671.91329  503.70031 4738.25770 5047.65431
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## 84  6416.99860 6896.04735  603.50808 6965.98213 2731.29402 2311.73074
## 85  1561.92830 1259.09650 7309.76887 1078.46187 5309.86666 5642.74100
## 86   474.49341  638.10657 5772.64108  771.11348 3933.34641 4221.98697
## 87  1140.53058  699.58273 6943.49573  590.32364 5024.38315 5338.74255
## 88  6368.37177 6844.61781  538.56290 6917.18476 2711.70113 2291.93695
## 89   518.87956  436.85925 6092.65599  503.46201 4209.34080 4513.40182
## 90   433.51355  565.01681 5864.32562  654.16817 4007.90818 4300.62763
## 91  1109.55757  782.59824 6883.25076  623.29126 4929.18594 5244.81763
## 92  1550.59021 1348.19212 7186.32444 1145.20828 5154.90543 5495.51090
## 93  1812.21329 1603.26885 7426.29369 1404.00819 5369.78649 5722.89568
## 94   514.23049  599.65240 5848.77936  733.69680 4011.86328 4295.77246
## 95  5895.07362 6379.44911  355.89605 6443.67248 2192.14165 1785.97004
## 96  6367.90562 6846.47559  554.12183 6917.27157 2692.51295 2274.42608
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## 99  6473.82406 6951.44561  650.46983 7023.03752 2798.53676 2380.15252
## 100 6353.60181 6830.76584  534.16196 6902.16937 2689.29433 2264.22393
## 101 4974.87980 5443.68046  952.17120 5522.63062 1622.19019 1191.57585
## 102 5921.14136 6407.51052  358.22898 6470.71580 2197.72132 1799.47298
## 103 1393.62836 1082.46940 7145.76525  912.45603 5159.14392 5488.81153
## 104 6426.42793 6904.21103  604.49566 6975.26114 2753.44039 2333.23938
## 105 1460.70736 1119.82945 7244.63049  959.63952 5266.78650 5593.82445
## 106 1429.94475 1084.67507 7213.45015  927.09870 5238.51258 5564.68382
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## 57  6554.30462
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## 59  6515.51479  607.90131  802.49237
## 60   241.67126 6573.42118 7146.03289 6540.56588
## 61  5642.59417 1091.60341 1614.00682 1141.12050 5667.98589
## 62   299.04682 6806.51438 7378.13852 6766.23485  297.51302 5897.91149
## 63   497.98695 6981.38396 7552.83483 6940.77244  440.01818 6077.89190
## 64  4027.34863 3274.32558 3752.49117 3090.01942 4041.45444 2685.88421
## 65  4240.19681 2907.88686 3391.38969 2731.99378 4256.36629 2327.09261
## 66  1245.59664 5553.66996 6113.44338 5476.38677 1266.89068 4683.21876
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## 71  6770.36690  348.78790  436.22586  584.03681 6792.21223 1244.25841
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## 73   241.73539 6416.00101 6989.96474 6388.25289  304.48153 5507.37224
## 74  6661.09668  267.97388  559.41934  669.07100 6680.45597 1133.21048
## 75   449.79551 6922.40240 7493.14827 6879.74527  394.56178 6020.03995
## 76  6316.70230  409.11490  892.86337  687.29833 6338.82931  814.14925
## 77  6998.84112  790.61748  566.40621  652.71740 7022.19610 1644.74436
## 78  6753.52464 1020.40776 1021.68684  726.72966 6776.74775 1637.01497
## 79   392.91857 6857.77355 7427.42324 6809.86696  372.89945 5954.63500
## 80   414.17267 6903.47246 7473.91477 6858.54052  389.30194 5998.70169
## 81   285.44176 6760.06391 7332.73523 6723.98706  238.55398 5852.81881
## 82  6803.83002  509.35842  479.34747  497.97992 6827.86467 1297.19929
## 83   448.62234 6932.38480 7503.78711 6891.12901  394.88099 6026.76007
## 84   500.98703 6982.59586 7553.69890 6940.00137  446.27458 6078.01555
## 85  7443.10426 1088.99954  614.53722 1002.40112 7466.01969 2018.02874
## 86  5905.59675  784.01275 1319.68860  961.23046 5927.00886  425.83565
## 87  7077.83420  572.33731  171.47595  762.75684 7098.65790 1552.69443
## 88   430.52062 6932.44582 7505.12138 6897.23053  383.63133 6027.73515
## 89  6227.41158  499.46772  993.47471  683.20568 6247.90669  759.24568
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## 91  7016.64143  632.68001  360.31930  643.29775 7038.75259 1530.01569
## 92  7319.45620 1170.40762  818.36544  949.58096 7343.07429 2013.09662
## 93  7559.76058 1426.81498 1029.59652 1217.80171 7582.52854 2280.65583
## 94  5980.16940  728.47169 1252.86432  885.20337 6004.52305  486.45966
## 95   396.83498 6462.12233 7029.66976 6410.26638  358.96379 5569.15945
## 96   449.51418 6932.99171 7504.80433 6894.15216  388.03608 6031.35159
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## 99   535.81060 7038.79947 7610.73334 7000.00957  484.91443 6134.49623
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## 101 1084.32652 5536.19445 6110.56683 5511.16911 1121.90953 4620.01223
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## 103 7279.35224  920.85069  492.03455  850.54571 7302.06491 1834.96267
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## 106 7347.26037  921.82319  426.19010  923.15221 7368.73218 1887.77382
##             62         63         64         65         66         67
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## 63   236.55866
## 64  4243.91046 4386.81114
## 65  4466.47848 4617.12248  380.95669
## 66  1447.09433 1588.05919 2840.69710 3071.68162
## 67   148.88250  184.09237 4326.55174 4548.40324 1530.12549
## 68   222.55561   91.28527 4411.59189 4639.30178 1611.52257  141.06382
## 69   239.72276   98.14785 4340.34019 4573.60809 1543.94398  205.97815
## 70   262.52428   90.27181 4442.53700 4671.56237 1644.04410  186.22298
## 71  7023.83250 7199.46741 3453.89215 3088.87666 5764.36961 7099.80514
## 72   439.43145  628.07165 3961.56030 4164.30955 1211.82672  485.68508
## 73   461.27757  646.78822 3939.48283 4142.11866 1192.82731  506.67643
## 74  6914.16503 7090.12369 3426.94762 3059.47283 5675.06916 6988.33027
## 75   201.80932  100.18483 4325.00208 4555.32798 1529.27924  177.90166
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## 76   6571.13346 6748.02112 3150.29887 2782.50660 5335.00431 6645.75451
## 77   7248.98351 7421.53825 3455.79513 3107.09639 5938.93492 7328.64135
## 78   7000.66425 7168.82319 3085.97991 2747.68193 5660.00989 7081.35496
## 79    176.96327  201.66309 4243.99623 4476.30763 1445.61890  224.72205
## 80    169.67027  156.88212 4301.13613 4531.78309 1501.55053  187.44599
## 81    133.38666  267.06179 4208.88061 4429.09133 1417.07445  166.21673
## 82   7056.55100 7232.62290 3426.07180 3064.67617 5781.05267 7134.96952
## 83    199.89247   91.09885 4344.07148 4572.75092 1544.71324  161.75599
## 84    245.99797   62.74552 4380.45146 4611.97712 1580.92536  203.36421
## 85   7693.22208 7865.66285 3883.75308 3537.35183 6385.01026 7772.68589
## 86   6160.92217 6337.86746 2839.03117 2474.74120 4938.48479 6234.72325
## 87   7330.64376 7505.88716 3726.54478 3364.20392 6069.61860 7406.35781
## 88    183.25665  134.82211 4368.03285 4592.50814 1572.02513   95.82797
## 89   6480.50230 6655.57616 3020.58802 2653.27647 5236.98597 6554.92029
## 90   6252.15963 6427.79978 2876.94039 2509.72827 5020.54579 6325.66929
## 91   7268.34300 7442.53465 3577.42952 3219.19307 5982.47942 7346.22706
## 92   7567.30956 7737.80886 3687.66132 3350.67426 6239.67243 7648.33812
## 93   7806.10332 7974.68777 3867.24954 3538.48908 6468.80198 7887.44807
## 94   6235.31178 6414.28320 2909.43311 2542.32826 5010.08313 6310.72603
## 95    469.92446  575.47459 3839.55219 4069.59961 1070.11261  539.68046
## 96    204.41624  101.50862 4345.50987 4574.22627 1549.42957  146.25321
## 97    152.74489  258.09494 4282.15215 4499.72610 1491.98961  135.11847
## 98    226.36254  395.91413 4146.43654 4359.14281 1367.17665  257.30527
## 99    270.30723   83.85702 4451.49907 4681.06206 1652.16706  194.13397
## 100   174.60813  102.51829 4341.04987 4567.29417 1540.25063  109.25200
## 101  1333.15378 1519.20341 3194.23747 3356.50726  845.68256 1401.26978
## 102   450.73163  555.02342 3849.01767 4082.29090 1079.48182  516.24122
## 103  7529.98167 7702.94775 3756.96260 3406.06195 6228.77291 7608.86562
## 104   233.16089   67.36468 4405.26276 4634.26780 1607.26787  173.64331
## 105  7629.84023 7802.14278 3872.17097 3521.69419 6334.41678 7707.02303
## 106  7598.31573 7771.44208 3850.04649 3498.39763 6304.70999 7676.05406
##              68         69         70         71         72         73
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## 69   137.62267
## 70    74.96666  135.83078
## 71  7210.13148 7171.69213 7246.24027
## 72   607.79520  618.37125  651.93941 6658.04919
## 73   628.49821  636.55165  672.43141 6635.54700  127.62837
## 74  7099.86373 7063.60963 7135.90296  334.92238 6544.60190 6521.80611
## 75   130.85106   99.57911  152.22352 7140.06190  580.59538  596.66574
## 76  6757.51552 6721.77060 6793.88534  546.19777 6201.29624 6178.98730
## 77  7435.06066 7389.63226 7470.85089  647.56390 6895.45488 6872.07938
## 78  7184.62003 7133.69336 7219.82389  944.29286 6656.20327 6633.26925
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## 79   219.34220  163.07974  242.22923 7074.35743  550.03273  567.39140
## 80   169.22470  132.29135  189.11637 7120.37085  569.91403  587.22653
## 81   257.58300  268.51629  292.47393 6978.43156  389.51508  408.55477
## 82  7244.35663 7203.88097 7280.39168  326.76138 6697.00844 6673.68496
## 83   121.67991  120.31625  134.12681 7150.02434  583.02573  600.30326
## 84   120.35780  109.56277  115.95689 7200.02778  637.47392  654.76561
## 85  7879.30765 7834.16549 7915.10777  888.05236 7339.18279 7315.95537
## 86  6347.07090 6312.35733 6383.31520  958.14143 5790.83897 5767.49122
## 87  7516.57056 7477.89442 7552.66609  376.30440 6964.70071 6941.91868
## 88    91.97826  167.74981  130.37638 7151.56878  543.78580  563.94149
## 89  6665.96137 6628.27662 6702.10885  658.10106 6112.39004 6090.79658
## 90  6437.40188 6401.53911 6473.61221  860.90592 5881.42500 5860.14889
## 91  7454.96526 7413.10428 7490.77706  443.96734 6909.53602 6886.04088
## 92  7752.81555 7704.05666 7788.35214  992.88015 7220.03573 7196.74517
## 93  7990.58421 7939.94395 8025.90842 1244.51356 7462.37596 7439.16326
## 94  6423.28405 6388.85984 6459.64875  887.95608 5866.77075 5843.83710
## 95   601.43163  532.40210  631.96994 6679.11177  468.72700  464.20254
## 96   102.10289  109.67680  125.15191 7151.89800  573.88152  589.62785
## 97   231.88359  275.37248  265.40535 7033.44581  415.93148  439.06947
## 98   375.87631  395.50095  414.92288 6881.06249  267.39110  294.79993
## 99    74.82647  140.73024   44.64303 7257.23412  663.59777  683.27227
## 100   99.06059  132.69891  123.34910 7136.10895  550.97005  570.56901
## 101 1518.04414 1499.72131 1557.22124 5753.13532  961.23930  936.52229
## 102  579.52222  512.56024  606.97941 6706.58870  466.68726  468.16450
## 103 7716.18099 7672.01440 7752.04392  707.29979 7174.31153 7151.27758
## 104   79.29060  113.93419   70.49113 7209.44679  623.02969  642.54572
## 105 7814.67638 7771.20049 7850.45088  763.78073 7270.50356 7247.44541
## 106 7783.89844 7740.71295 7819.71924  725.93181 7239.30245 7216.23177
##             74         75         76         77         78         79
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## 75  7031.58695
## 76   457.57076 6688.59731
## 77   845.73518 7360.44299 1031.55853
## 78  1135.02775 7106.87871 1183.15299  551.28940
## 79  6967.45147  157.96519 6625.07094 7289.43180 7033.40558
## 80  7012.76194  132.98496 6669.77413 7338.68585 7084.04263   85.65045
## 81  6867.47261  225.72328 6524.86912 7206.47695 6959.19895  233.27237
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## 82   517.55000 7172.45000  663.29104  517.89960  812.08743 7104.01436
## 83  7040.85258   86.49855 6697.74410 7372.51918 7120.23644  174.69688
## 84  7091.51063  105.74498 6748.66639 7420.49021 7166.29702  199.06532
## 85  1082.78622 7804.62459 1355.83443  497.09959  884.13008 7733.94401
## 86   835.27959 6280.04180  562.67842 1389.30630 1430.65160 6216.01239
## 87   508.31290 7446.31620  830.49564  596.20802 1022.59767 7380.50757
## 88  7040.36526  140.16419 6698.25141 7378.94362 7130.61786  225.34196
## 89   565.02035 6596.31981  354.31483 1081.64227 1155.63056 6532.49202
## 90   721.63564 6369.23394  466.43006 1285.83669 1334.98689 6306.41800
## 91   625.85621 7382.73899  881.41534  393.94416  794.51495 7314.00916
## 92  1203.21237 7675.86640 1411.05670  472.87419  688.08575 7602.92799
## 93  1452.05337 7912.45840 1678.41235  717.59668  864.06886 7838.79640
## 94   779.25413 6355.98804  472.92706 1322.01778 1382.63842 6291.07376
## 95  6572.79005  513.74507 6233.39843 6889.44548 6630.90831  445.46717
## 96  7041.82334   95.39916 6700.09589 7374.59233 7122.86403  182.42259
## 97  6921.12094  231.96983 6578.10611 7265.23799 7021.01196  268.80662
## 98  6768.51564  354.06355 6425.07113 7113.88305 6871.02394  348.66316
## 99  7147.09130  152.77762 6804.89317 7481.25858 7229.95007  244.33174
## 100 7026.13777  116.21101 6683.75142 7361.02520 7110.45203  182.78950
## 101 5640.23289 1459.06203 5293.33487 5998.69894 5767.84544 1408.33270
## 102 6601.47461  497.42738 6261.87991 6914.48053 6653.78501  432.27191
## 103  919.28668 7642.27708 1168.92215  390.42285  786.45280 7572.46961
## 104 7099.65548  102.73753 6757.21429 7432.75534 7181.48856  213.63520
## 105  937.00800 7741.43740 1237.72331  489.51609  924.55178 7672.86739
## 106  904.52529 7710.87161 1202.02537  462.49757  907.82597 7641.73750
##             80         81         82         83         84         85
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## 81   233.97008
## 82  7150.84680 7013.26244
## 83   140.26760  220.94569 7182.41819
## 84   160.05312  272.81312 7232.03892   73.75636
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## 85  7783.05506 7650.47829  782.50879 7816.48252 7864.66280
## 86  6260.22971 6113.58324 1055.05261 6288.00374 6338.79618 1751.97089
## 87  7426.85182 7285.19156  442.00679 7456.42455 7506.60929  671.85415
## 88   182.73752  199.99250 7186.94156  130.95037  159.05974 7823.23149
## 89  6577.38869 6433.56977  759.45112 6605.34200 6655.64580 1415.04982
## 90  6350.93048 6204.60877  970.79761 6378.40019 6428.91165 1643.14211
## 91  7361.49536 7224.22896  349.71989 7392.97890 7442.01659  557.11938
## 92  7653.47215 7525.91151  846.88842 7689.00442 7736.16333  346.35820
## 93  7890.03403 7764.89337 1102.60918 7925.95786 7972.47653  482.35775
## 94  6334.96559 6190.13901  959.35134 6363.71951 6414.85978 1685.62896
## 95   499.76895  450.80484 6708.38826  537.99442  576.23953 7332.96413
## 96   145.09997  226.01327 7185.87267  102.62066  119.39430 7818.92819
## 97   249.76989  116.63190 7069.81987  219.81356  270.22583 7709.36891
## 98   351.92329  166.75731 6917.76481  348.47525  403.27782 7558.11643
## 99   188.34012  305.34243 7291.10650  136.68577  111.11706 7925.37734
## 100  139.02518  196.95685 7169.93236   90.33825  124.63948 7805.05554
## 101 1444.99239 1283.42861 5790.98152 1465.06246 1518.90092 6440.90095
## 102  485.76023  426.56887 6736.61065  518.58461  552.36944 7359.79327
## 103 7620.92514 7486.77821  593.79626 7653.48006 7701.96800  282.31897
## 104  164.59040  260.24604 7242.87567   99.63433   92.94622 7876.96477
## 105 7721.33071 7585.37184  707.11102 7753.32180 7802.12695  296.56702
## 106 7690.07113 7554.35490  660.84945 7722.43207 7771.28220  305.00820
##             86         87         88         89         90         91
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## 87  1264.71815
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## 88  6287.55302 7457.82602
## 89   508.25781  935.95726 6606.84668
## 90   363.87635 1154.87835 6377.76050  330.70228
## 91  1264.04114  365.55437 7397.52060  941.05366 1162.00344
## 92  1771.81828  856.41228 7698.20310 1444.47222 1666.20347  661.31989
## 93  2034.14036 1076.23185 7936.69037 1700.62253 1928.89580  907.92345
## 94   333.57608 1188.20747 6363.75659  459.35498  367.24243 1186.72280
## 95  5826.41133 6983.68162  569.13267 6138.96156 5914.57987 6916.00282
## 96  6290.51333 7457.83038  107.78219 6608.43416 6380.34294 7395.60707
## 97  6167.23674 7339.98113  163.02147 6488.83865 6258.79837 7281.52972
## 98  6014.55668 7187.48857  306.94462 6335.48940 6105.53634 7129.83562
## 99  6394.44274 7563.59716  140.84389 6713.16513 6485.01164 7501.49558
## 100 6273.40179 7442.55762   94.69424 6592.03838 6363.81301 7380.33373
## 101 4884.34059 6061.71898 1457.00103 5206.08461 4975.77964 6006.34656
## 102 5854.51680 7011.96599  542.56428 6166.17702 5941.16007 6943.30548
## 103 1572.19719  519.29568 7659.60227 1228.45187 1464.19876  394.33488
## 104 6347.27012 7515.64701  117.61377 6665.19512 6437.10168 7453.42995
## 105 1647.58551  504.09920 7757.49799 1304.26148 1526.82677  489.35059
## 106 1608.86295  469.82337 7726.69431 1269.28326 1492.79335  444.80333
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## 93   330.99396
## 94  1707.39304 1971.94320
## 95  7199.59853 7434.25504 5903.01160
## 96  7691.66835 7928.95132 6367.19192  533.72558
## 97  7586.84605 7827.00850 6243.49966  521.70873  213.49707
## 98  7436.74149 7677.39207 6090.48791  457.02626  344.09882  171.95057
## 99  7798.29545 8035.73873 6470.80528  636.65768  128.89531  278.35768
## 100 7678.47915 7916.43019 6349.26114  536.22290   90.04443  188.66637
## 101 6328.20567 6572.67221 4956.37327 1091.38261 1469.92347 1331.09354
## 102 7227.06573 7460.24276 5930.88594  207.19073  516.56655  500.75343
## 103  418.53076  606.79403 1493.83567 7172.29329 7656.33437 7545.19059
## 104 7749.74451 7987.06974 6423.57307  594.26509  101.15335  240.80490
## 105  513.83266  661.59580 1579.94715 7271.76877 7754.30951 7642.80806
## 106  521.28303  680.00809 1535.95540 7241.64450 7723.70623 7611.73009
##              98          99         100         101         102         103
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## 99   427.74058
## 100  320.16246  129.83066
## 101 1179.86143 1568.20184 1449.77688
## 102  435.99656  616.48925  515.53952 1114.09605
## 103 7393.59723 7762.45013 7641.80659 6274.49145 7199.10515
## 104  385.34400   79.17070  105.55567 1521.74867  572.64212 7714.13508
## 105 7491.09845 7861.12664 7740.65114 6371.65559 7299.07796  299.21564
## 106 7459.92118 7830.38824 7709.84098 6339.72405 7268.97744  276.43444
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## 105 7812.63502
## 106 7781.95194  100.00000
```

Do the MDS

```
rollcall.mds = lapply(rollcall.dist, function(d) as.data.frame((cmdscale(d,
    k = 2)) * -1))

## Add identification information about Senators back into MDS data frames
congresses = 101:113

for (i in 1:length(rollcall.mds)) {
    names(rollcall.mds[[i]]) = c("x", "y")

    congress = subset(rollcall.data[[i]], state < 99)

    congress.names = sapply(as.character(congress$name), function(n) strsplit(n,
        "[, ]")[[1]][1])

    rollcall.mds[[i]] = transform(rollcall.mds[[i]], name = congress.names,
        party = as.factor(congress$party), congress = congresses[i])
}

head(rollcall.mds[[1]])
```

```
##            x        y      name party congress
## 2  -11.44068 293.0001    SHELBY   100      101
## 3  283.82580 132.4369    HEFLIN   100      101
## 4  885.85564 430.3451   STEVENS   200      101
## 5 1714.21327 185.5262 MURKOWSKI   200      101
## 6 -843.58421 220.1038 DECONCINI   100      101
## 7 1594.50998 225.8166    MCCAIN   200      101
```

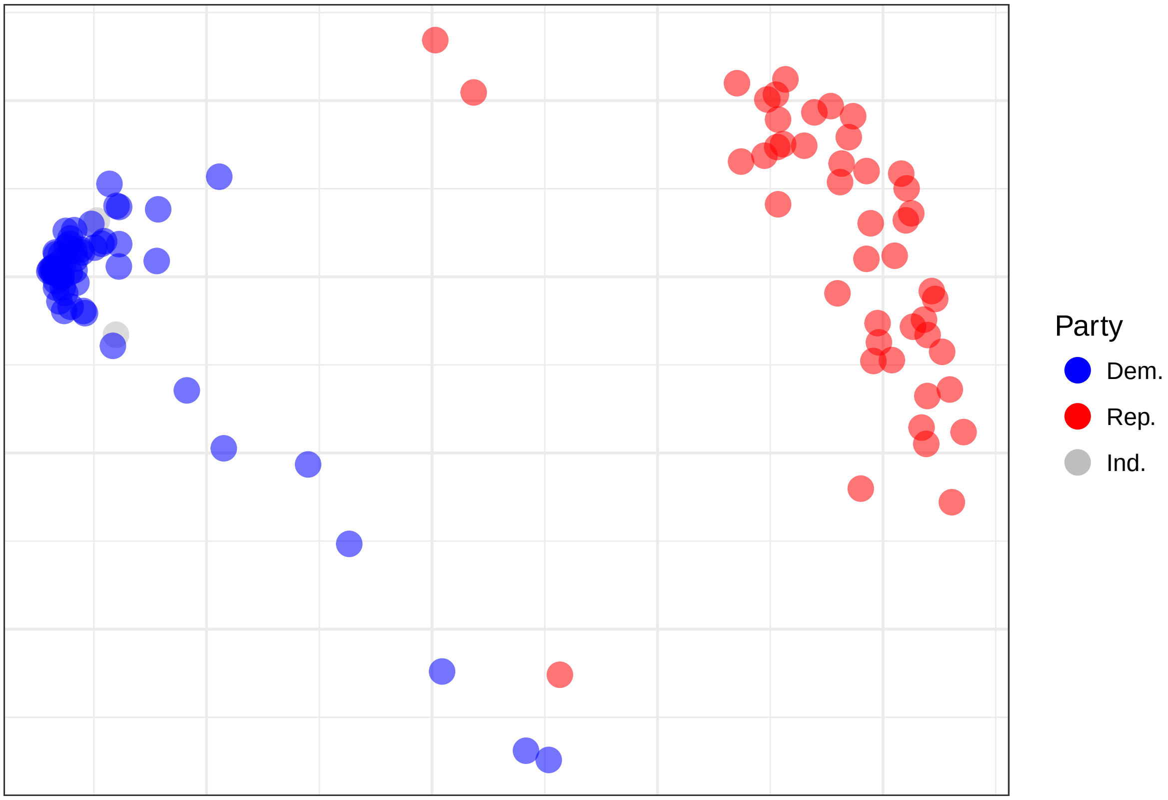
2) Generate an MDS plot to project the senators on the two dimensional space.

Use shapes or colors to differentiate the senators' party affliation

```
cong.113 <- rollcall.mds[[13]]

base.113 <- ggplot(cong.113, aes(x = x, y = y)) + scale_alpha(guide = "none") +
    theme_bw() + theme(axis.ticks = element_blank(), axis.text.x = element_blank(),
    axis.text.y = element_blank()) + xlab("") + ylab("") + scale_shape(name = "Party",
    breaks = c("100", "200", "328"), labels = c("Dem.", "Rep.", "Ind."), solid = FALSE) +
    scale_color_manual(name = "Party", values = c(`100` = "blue", `200` = "red",
        `328` = "grey"), breaks = c("100", "200", "328"), labels = c("Dem.",
        "Rep.", "Ind."))
```

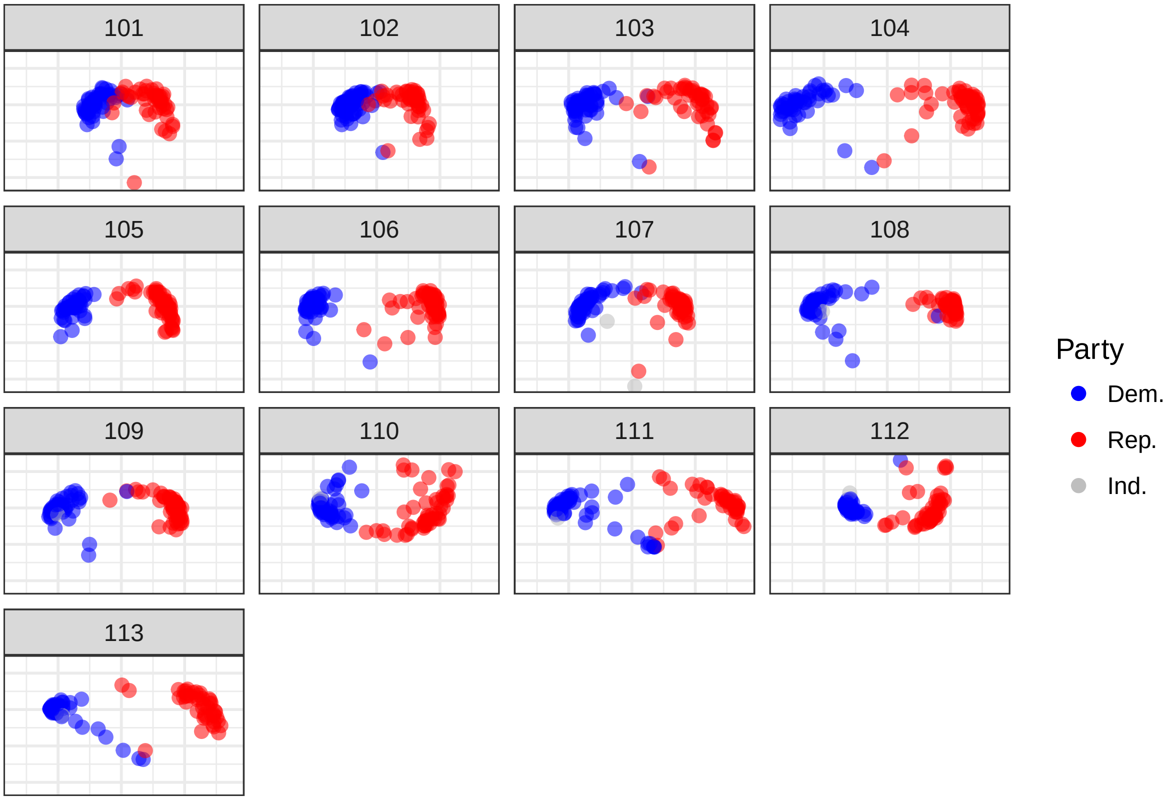
```
print(base.113 + geom_point(aes(color = party, alpha = 0.75), size = 4))
```



Create a single visualization of MDS for all Congresses on a grid.

```
all.mds <- do.call(rbind, rollcall.mds)
all.plot <- ggplot(all.mds, aes(x = x, y = y)) + geom_point(aes(color = party,
    alpha = 0.75), size = 2) + scale_alpha(guide = "none") + theme_bw() + theme(axis.ticks = element_bla
    axis.text.x = element_blank(), axis.text.y = element_blank()) + xlab("") +
    ylab("") + scale_shape(name = "Party", breaks = c("100", "200", "328"),
    labels = c("Dem.", "Rep.", "Ind."), solid = FALSE) + scale_color_manual(name = "Party",
    values = c(`100` = "blue", `200` = "red", `328` = "grey"), breaks = c("100",
        "200", "328"), labels = c("Dem.", "Rep.", "Ind.")) + facet_wrap(~congress)

print(all.plot)
```



2. Use k-means and hierarchical clustering to group the senators, and color the senators on the MDS plots based on the clustering results.

(you will use k-means, h-clustering with single-link, h-clustering with complete-link, h-clustering with average-link and k=2).

1) K-means

```
set.seed(1)  ## fix the random seed to produce the same results
grpSen113_k = kmeans(sen113kh[, c(10:666)], centers = 2, nstart = 10)
# grpSen113_k
```

Create a function for generating MDS plots for all clustering.

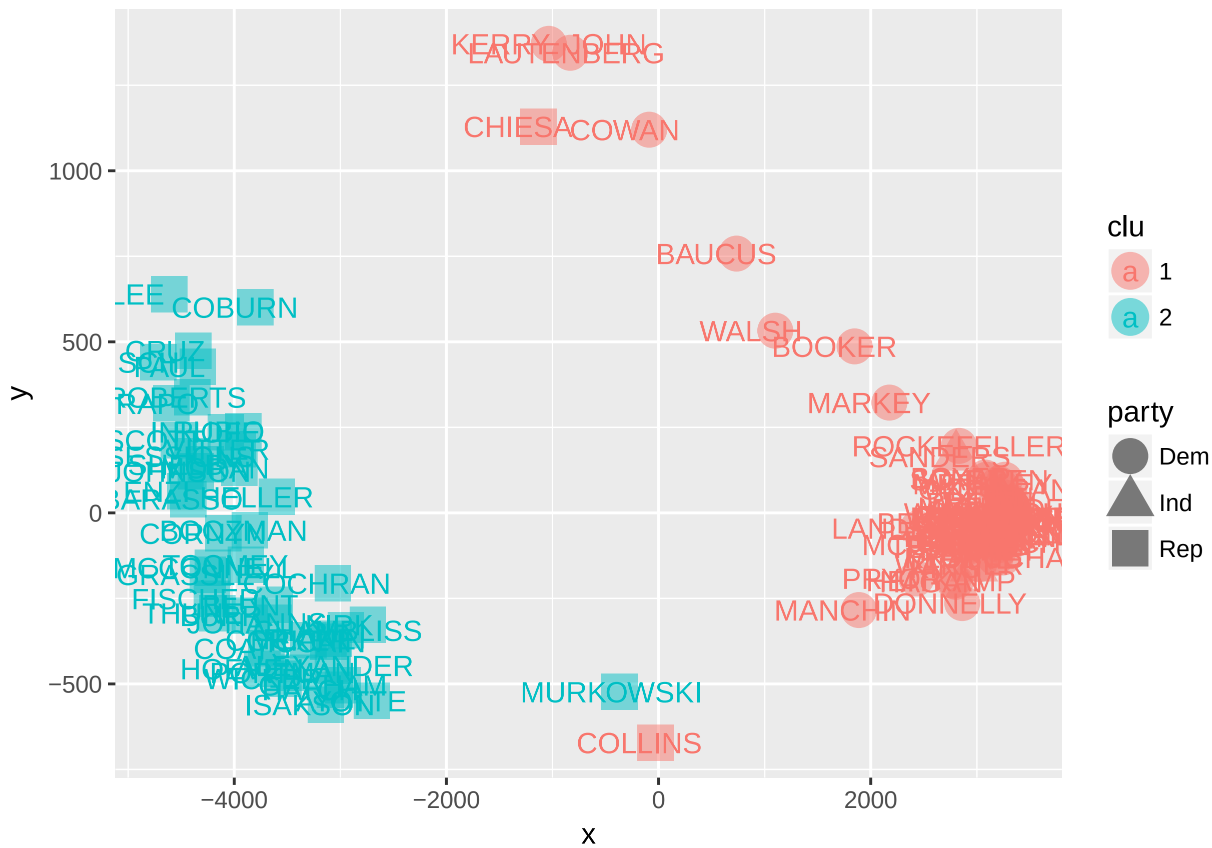
```
library(plyr)
clusterMDS <- function(grpName) {
    lbls = sen113kh$name
    party = mapvalues(sen113kh$party, from = c(100, 200, 328), to = c("Dem",
        "Rep", "Ind"))
    data.mds = cmdscale(sen113_dist)

    data2 = data.frame(x = data.mds[, 1], y = data.mds[, 2], name = lbls, party = party,
        clu = factor(grpName$cluster))
```

```r
    p = ggplot(aes(x = x, y = y, shape = party, color = clu), data = data2) +
        geom_point(size = 6, alpha = 0.5) + geom_text(aes(x = x, y = y, shape = party,
        color = clu, label = name), size = 4)
    print(p)
    return(data2)
}
```

MDS plots for k-means clustering.

```r
kmeans_MDS = clusterMDS(grpSen113_k)
```



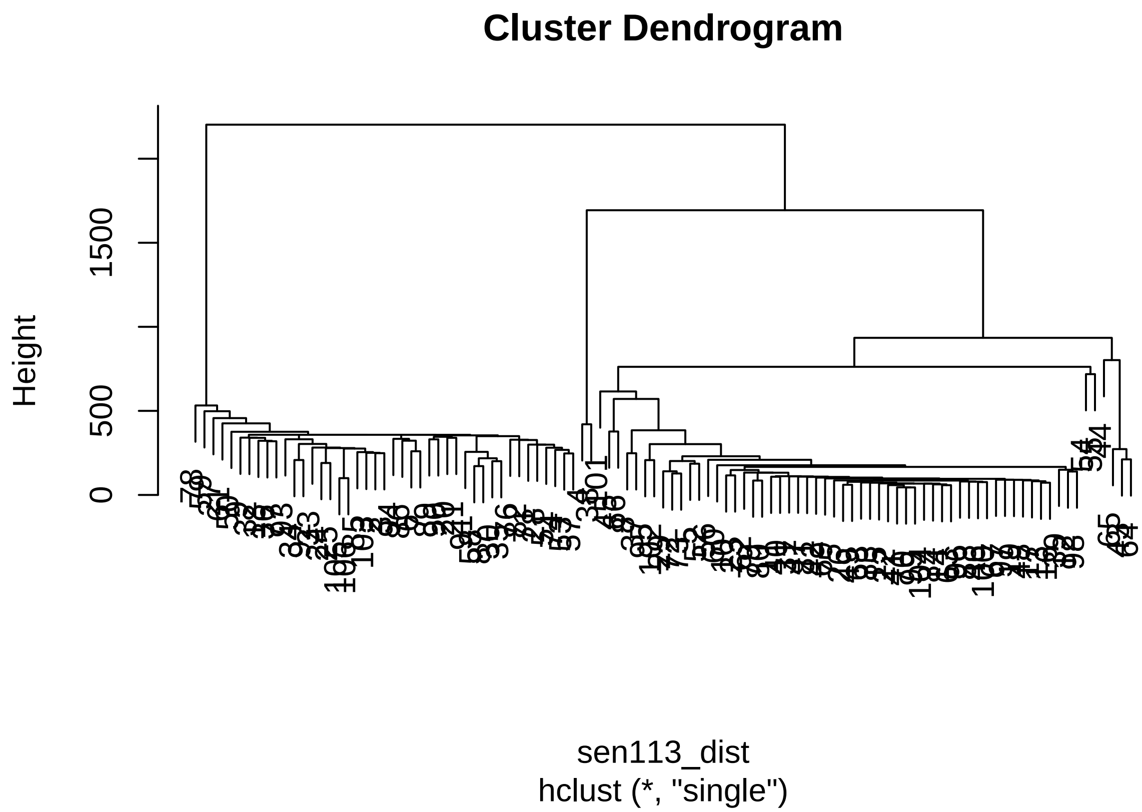
2)h-clustering with single-link

Create a function for generating MDS plot for all hclust.

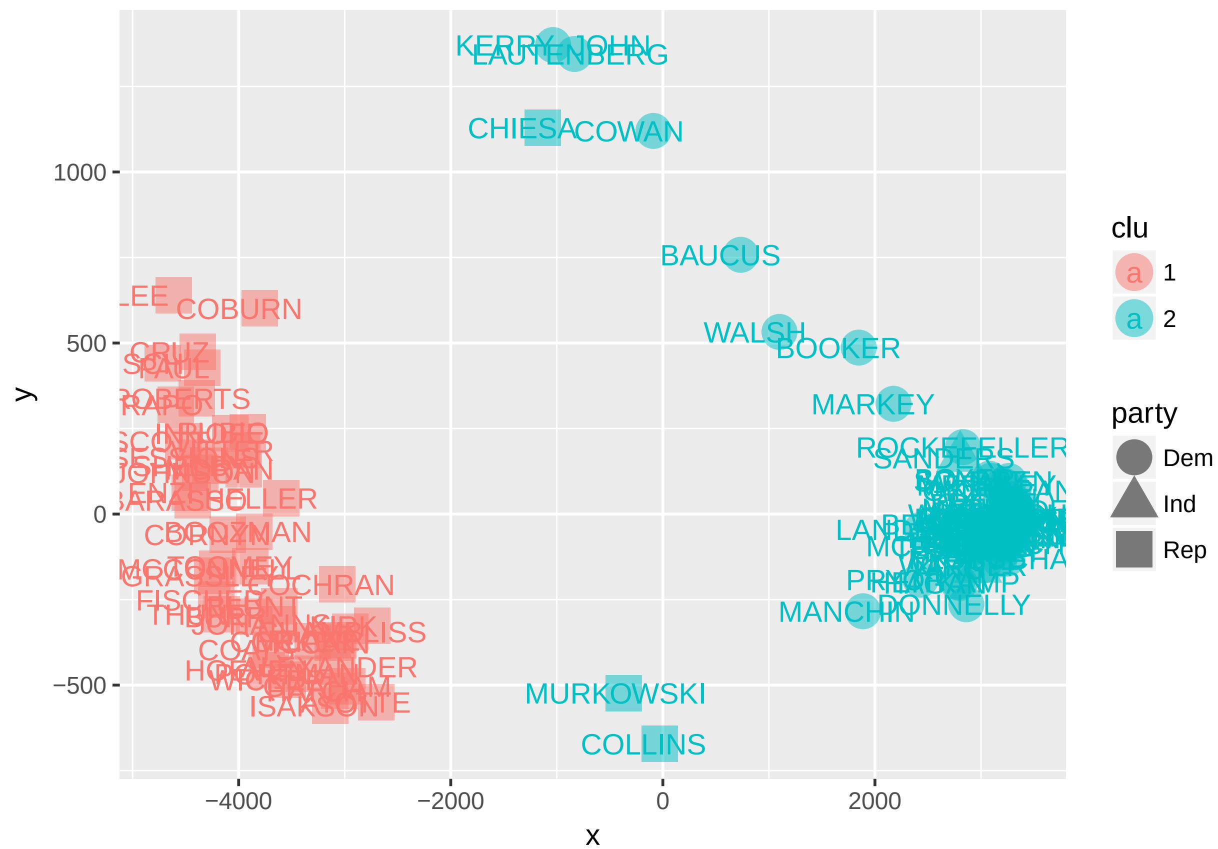
```r
library(cluster)
hclusterMDS <- function(hc_agg) {
    hc1 = cutree(hc_agg, k = 2)
    hc1 <- as.data.frame(hc1)
    names(hc1)[names(hc1) == "hc1"] <- "cluster"
    data3 = clusterMDS(hc1)
    return(data3)
}
```

Dendrogram and MDS plot for single method

```r
hc_s = hclust(sen113_dist, method = "single")
plot(hc_s)
```

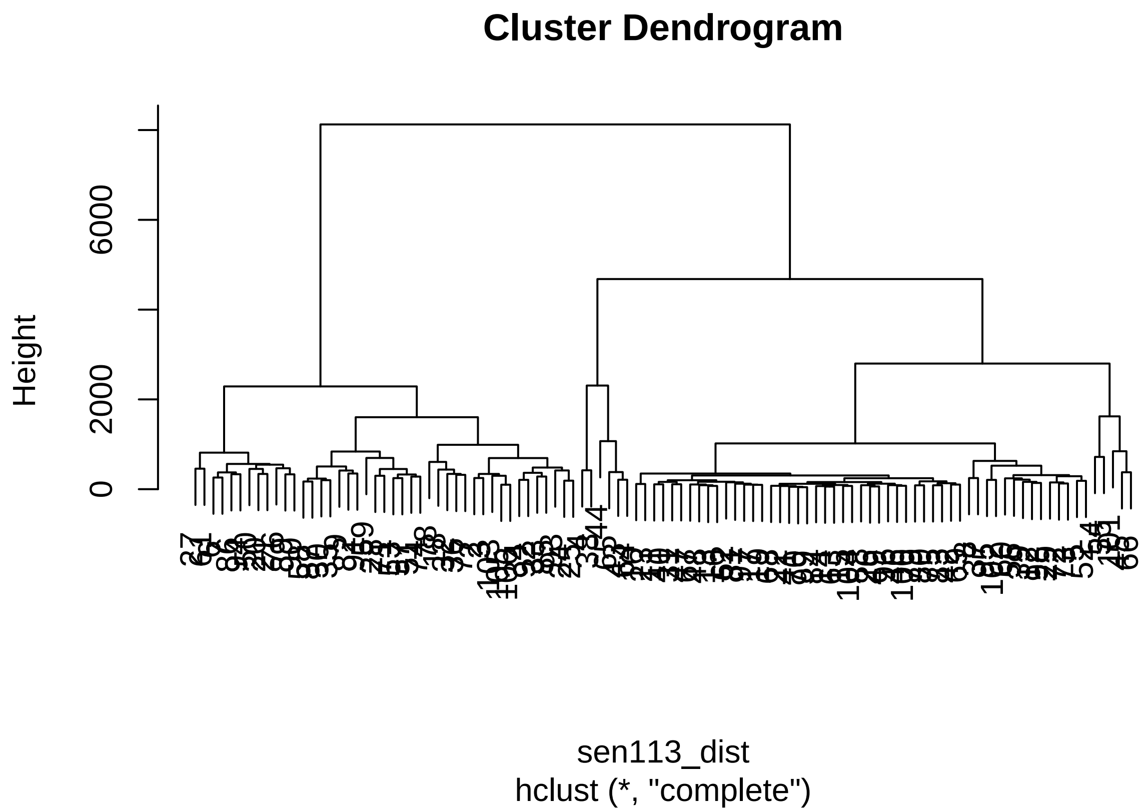



```r
grpSen2 <- hclusterMDS(hc_s)
```

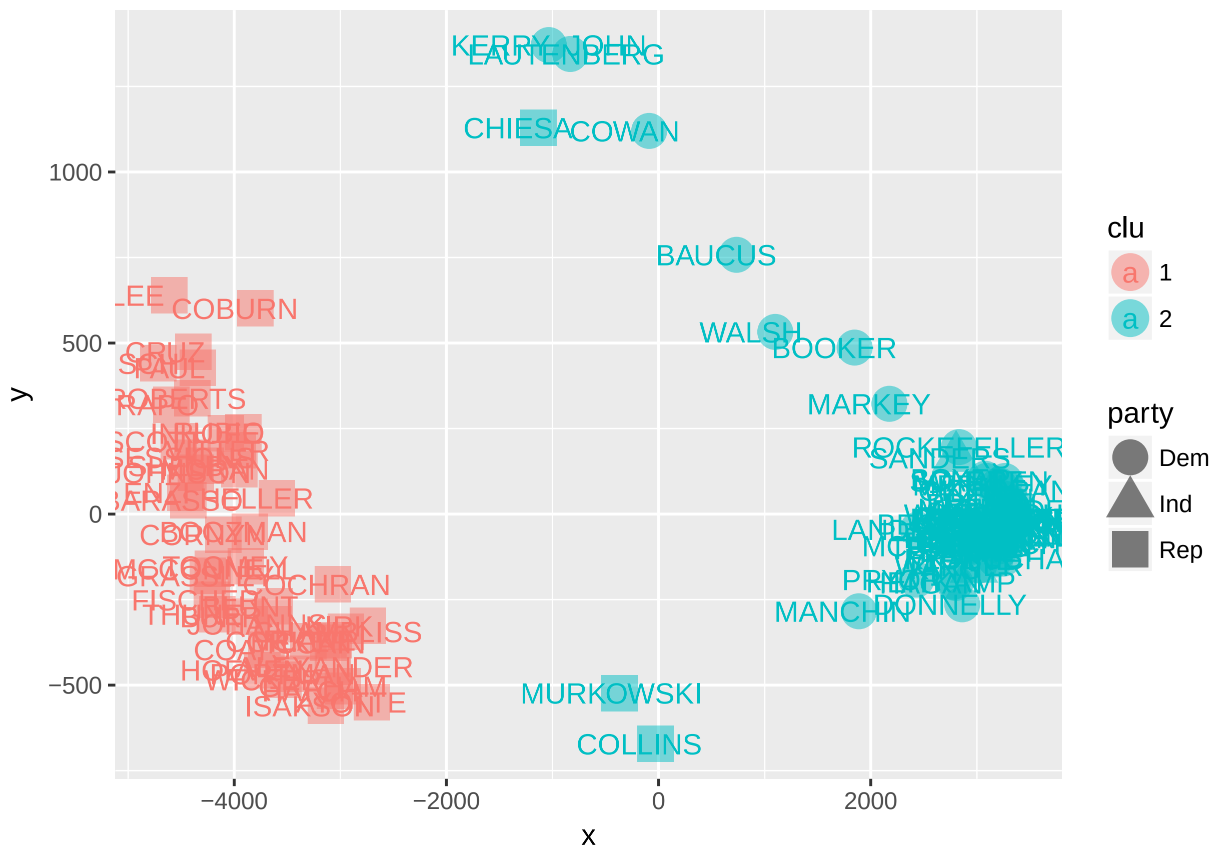


3) h-clustering with complete-link

```
hc_c = hclust(sen113_dist, method = "complete")
plot(hc_c)
```

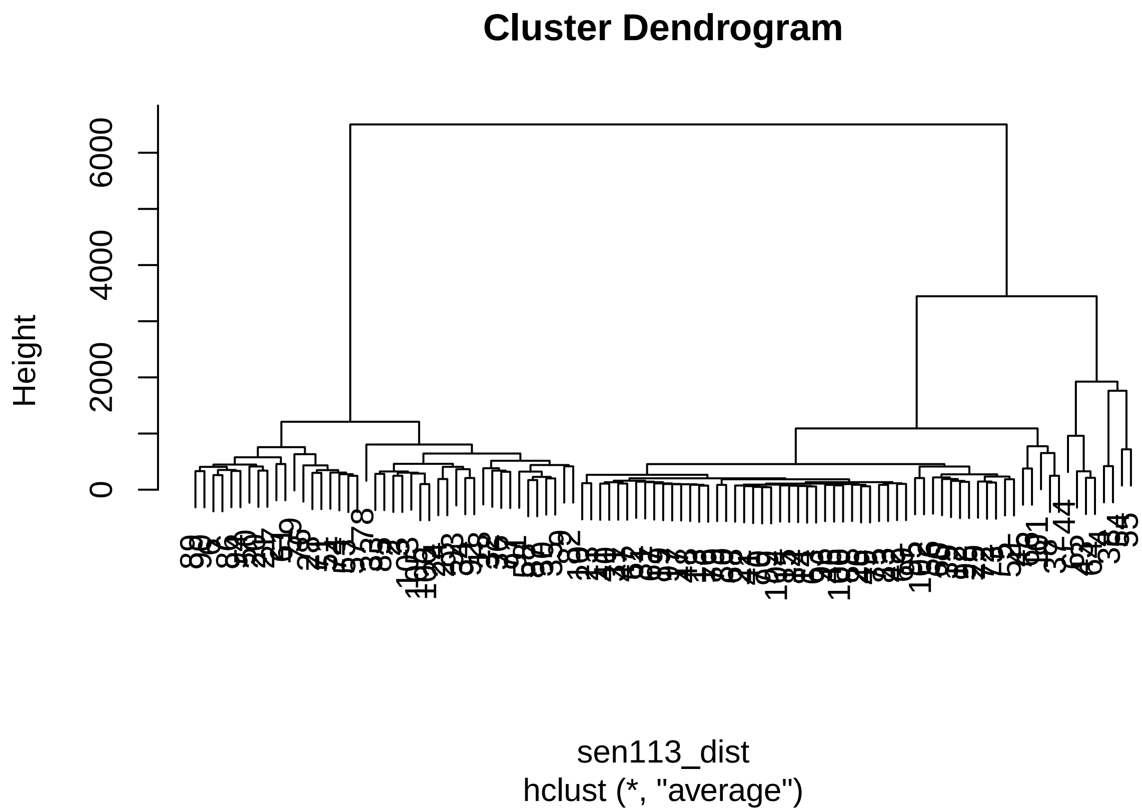



```
grpSen3 <- hclusterMDS(hc_c)
```

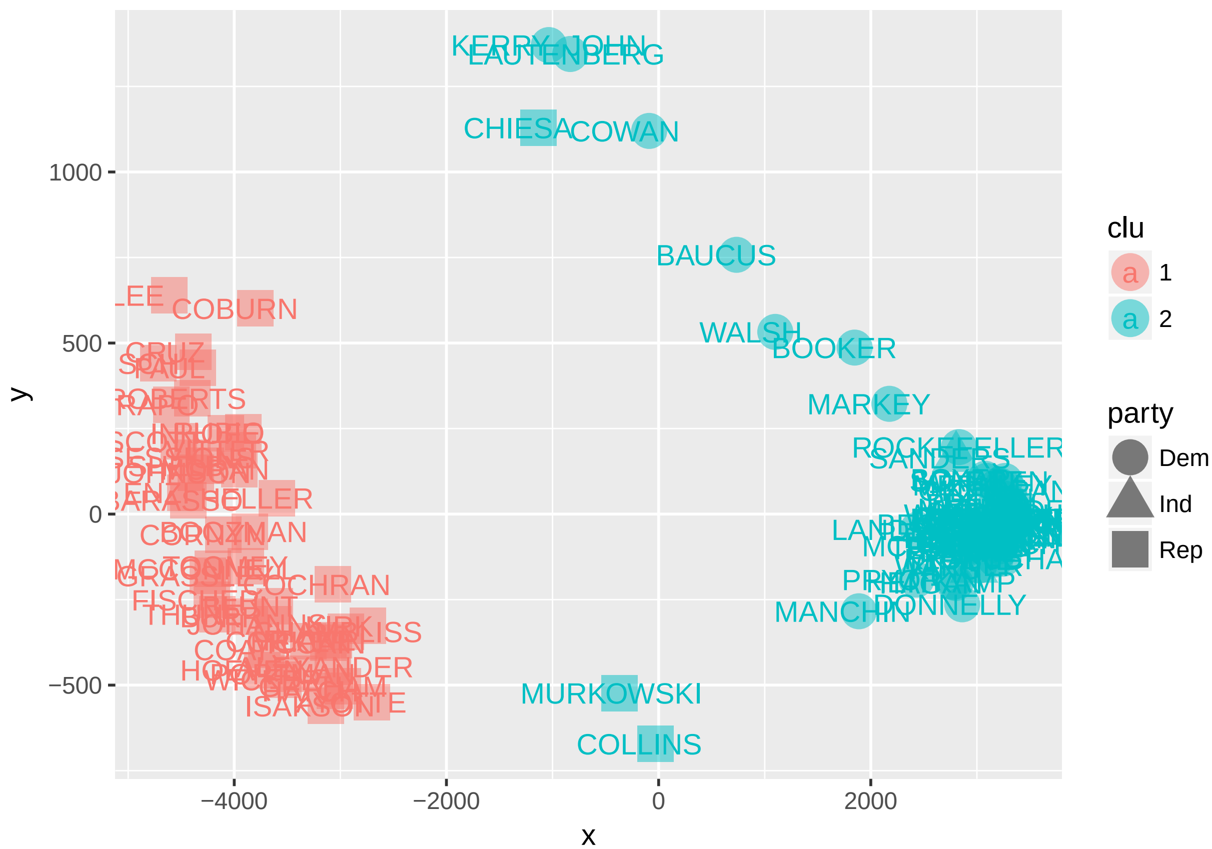


4) h-clustering with average-link

```
hc_a = hclust(sen113_dist, method = "average")
plot(hc_a)
```




```
grpSen4 <- hclusterMDS(hc_a)
```



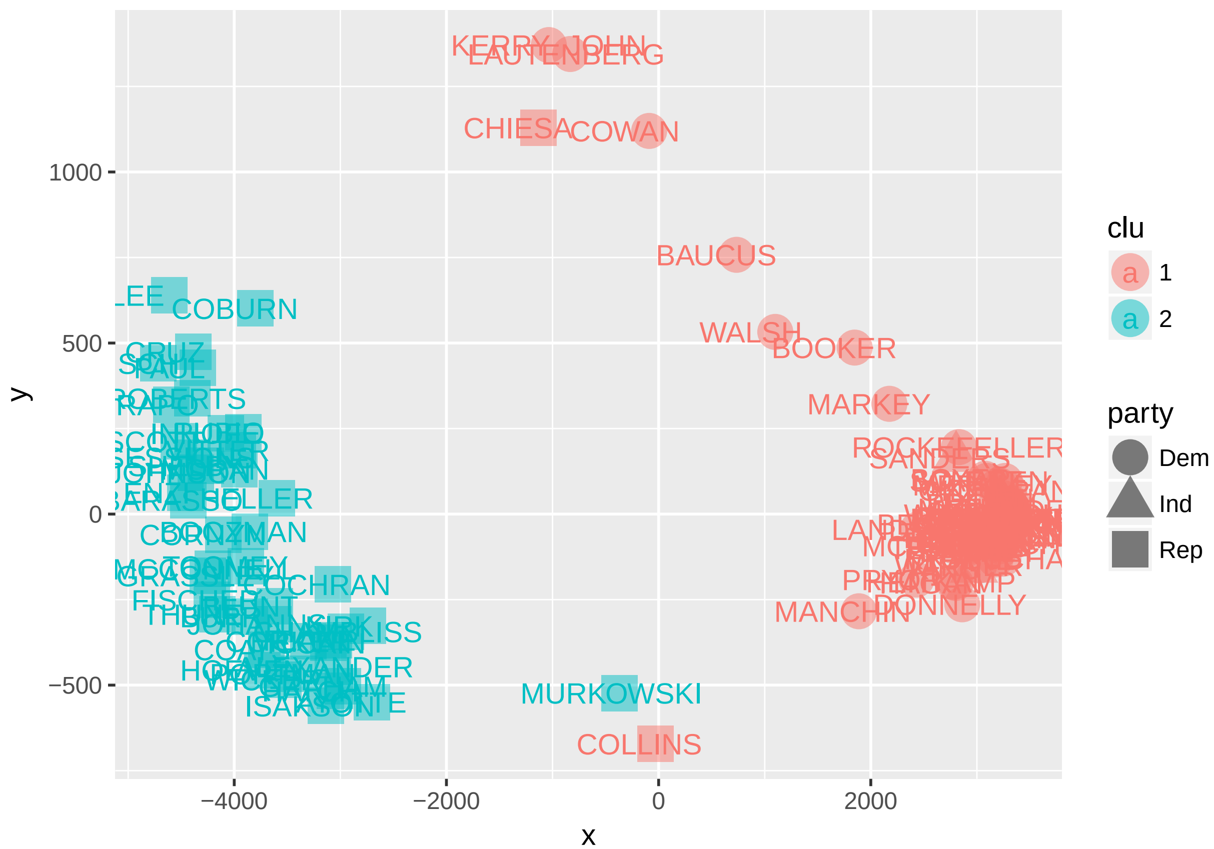

```
# grpUnemp8
```

3. Compare the clustering results with the party labels and identify the party members who are assigned to a seemly wrong cluster.

Requirements:Specifically, based on the k-means results, which Republicans are clustered together with Democrats, and vice versa? And based on the three variants (single-link, complete-link and average-link), which Republicans are clustered together with Democrats, and vice versa?

1) K-means result

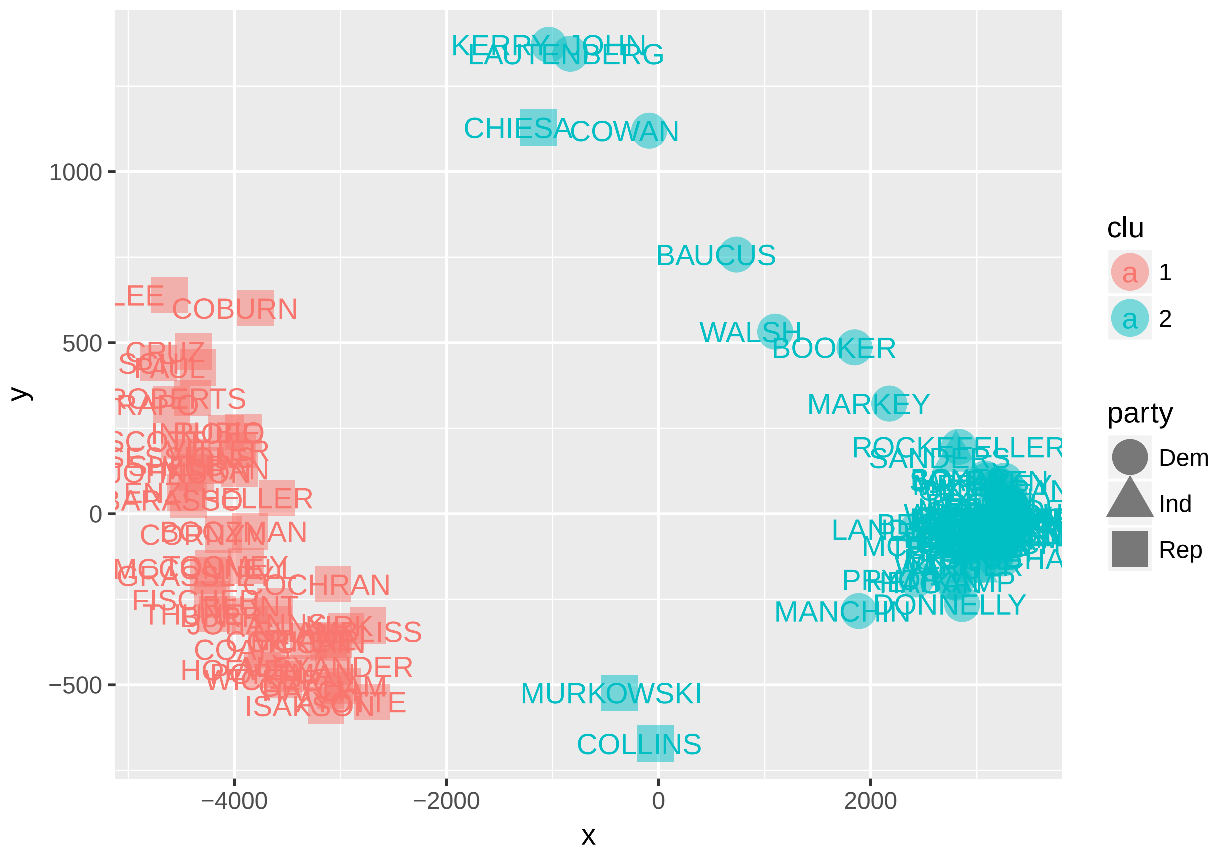
```
kmeans_MDS = clusterMDS(grpSen113_k)
```



Based on the k-means MDS map we can see, cluster 1 should belong to Democrats while cluster 2 should belong to Republican. That means, if the point is red, then its shape should be a circle. If the point is blue, then its shape should be square. However, in this graph, Collins and Chiesa are red square, that means they should be republicans but we clustered them together with Democrats. This graph doesn't have any blue circle, that means all Democrats are clustered correctly

2) h-clustering with single link

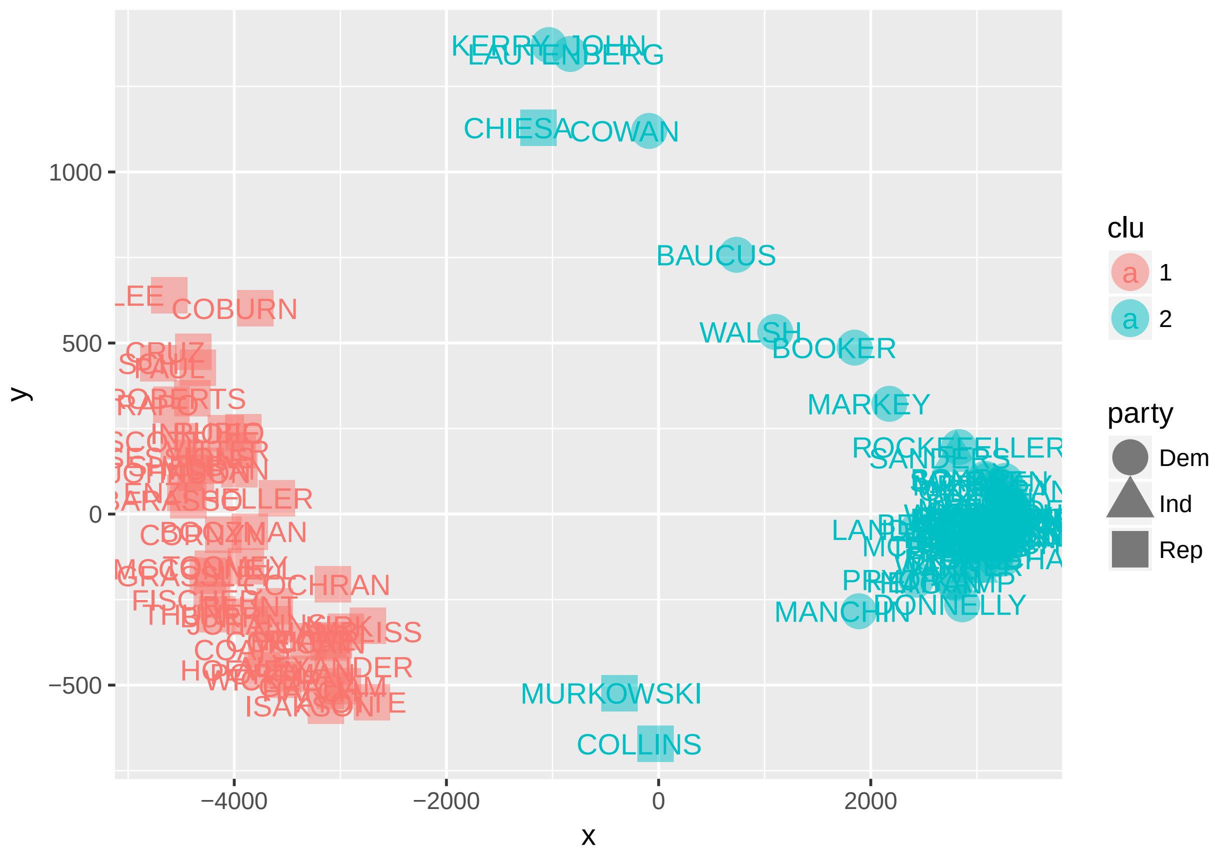
```
grpSen2 <- hclusterMDS(hc_s)
```



Based on the h-clustering with single link MDS map we can see, cluster 1 should belong to Republican while cluster 2 should belong to Dempcrats. That means, if the point is red, then its shape should be square. If the point is blue, then its shape should be a circle. However, in this graph, Murkowski, Collins and Chiesa are all blue square, that means they should be republicans but we clustered them together with Democrats. This graph doesn't have any red circle, that means all Democrats are clustered correctly.

3) h-clustering with complete link

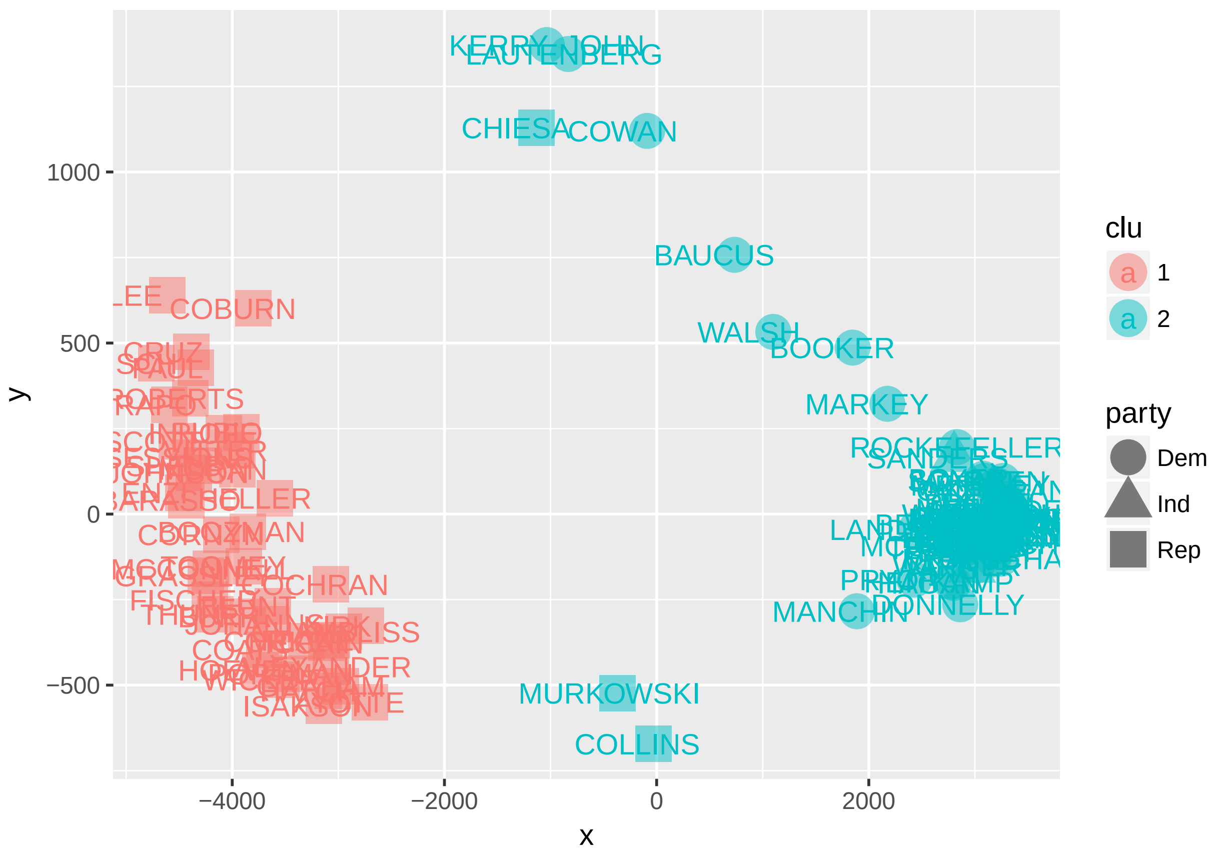
```
grpSen3 <- hclusterMDS(hc_c)
```



Based on the h-clustering with complete link MDS map we can see, cluster 1 should belong to Republican while cluster 2 should belong to Dempcrats. That means, if the point is red, then its shape should be square. If the point is blue, then its shape should be a circle. However, in this graph, Murkowski, Collins and Chiesa are all blue square, that means they should be republicans but we clustered them together with Democrats. This graph doesn't have any red circle, that means all Democrats are clustered correctly. The MDS cluster result is very similar to hclustering with single link.

4) h-clustering with average link

```
grpSen4 <- hclusterMDS(hc_a)
```



Based on the h-clustering with complete link MDS map we can see, cluster 1 should belong to Republican while cluster 2 should belong to Dempcrats. That means, if the point is red, then its shape should be square. If the point is blue, then its shape should be a circle. However, in this graph, Murkowski, Collins and Chiesa are all blue square, that means they should be republicans but we clustered them together with Democrats. This graph doesn't have any red circle, that means all Democrats are clustered correctly. The three variants of h-clustering show the same result, no Democrats are clusterd wrongly, Murkowski, Collins and Chiesa, this three republican are clustered together with Democrats

4. Compute the purity and entropy for these clustering results with respect to the senators' party labels.

Create two function to calculate the purity and entropy of the cluster results.

```
cluster.purity <- function(clusters, classes) {
    sum(apply(table(classes, clusters), 2, max))/length(clusters)
}

cluster.entropy <- function(clusters, classes) {
    en <- function(x) {
        s = sum(x)
        sum(sapply(x/s, function(p) {
            if (p) -p * log2(p) else 0
        }))
    }
    M = table(classes, clusters)
    m = apply(M, 2, en)
```

```
    c = colSums(M)/sum(M)
    sum(m * c)
}
```

Get the purity of four methods.

```
p1 <- cluster.purity(kmeans_MDS$clu, kmeans_MDS$party)
p2 <- cluster.purity(grpSen2$clu, grpSen2$party)
p3 <- cluster.purity(grpSen3$clu, grpSen3$party)
p4 <- cluster.purity(grpSen4$clu, grpSen4$party)

purity <- c(p1, p2, p3, p4)
```

Get the entropy of four method.

```
e1 <- cluster.entropy(kmeans_MDS$clu, kmeans_MDS$party)
e2 <- cluster.entropy(grpSen2$clu, grpSen2$party)
e3 <- cluster.entropy(grpSen3$clu, grpSen3$party)
e4 <- cluster.entropy(grpSen4$clu, grpSen4$party)

entropy <- c(e1, e2, e3, e4)
```

Generate the summary table

```
result = rbind(purity, entropy)
result = as.data.frame(result)
colnames(result) = c("k-means", "hclust-single", "hclust-complete", "hclust-average")

library(knitr)
kable(result, caption = "Summary of Clustering (k=2)")
```

Table 1: Summary of Clustering (k=2)

	k-means	hclust-single	hclust-complete	hclust-average
purity	0.9619048	0.9523810	0.9523810	0.9523810
entropy	0.2409547	0.2850529	0.2850529	0.2850529

5. Based on your observation on both measures and mis-classified members, choose two clustering methods that generate the most meaningful results and explain why

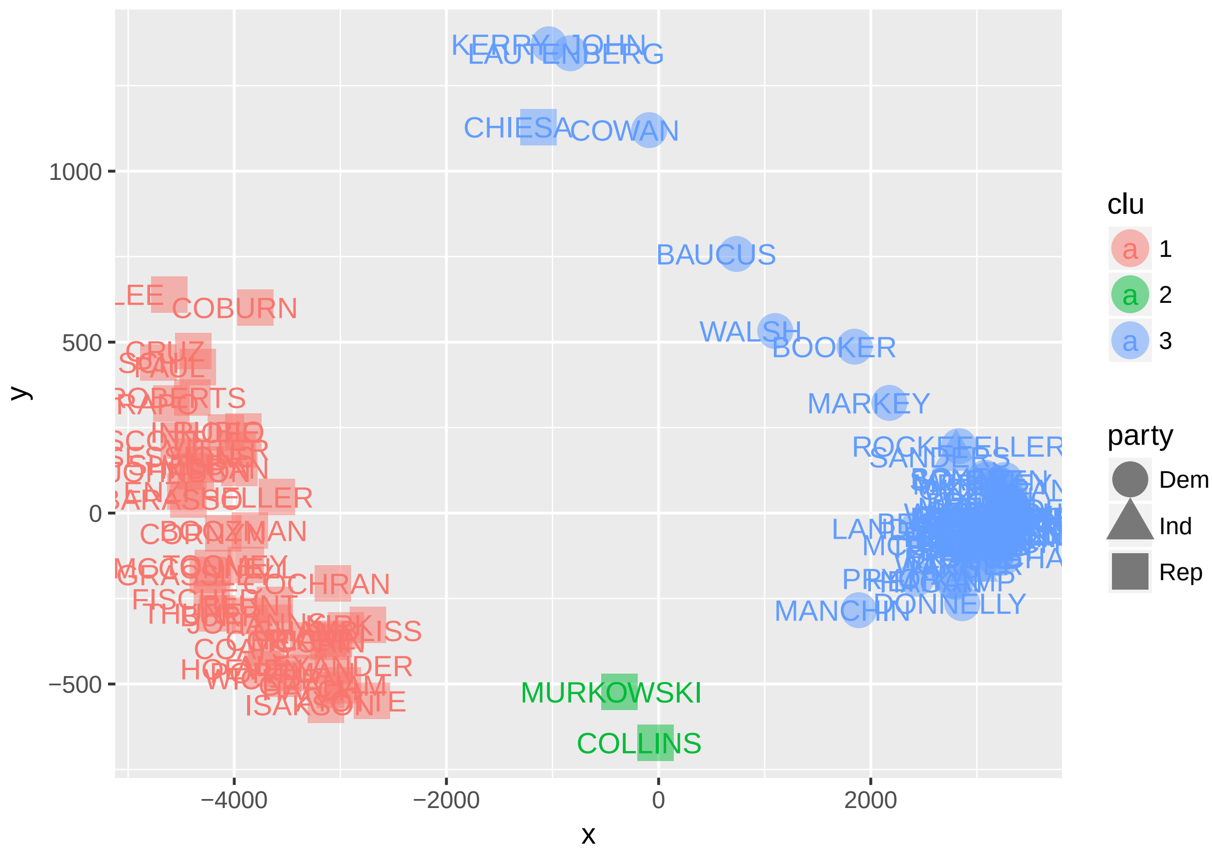
Based on the observation and measure table, k-means only have two republican together with democrats and k-means shows the highest purity and lowest entropy. This means the similarity in each of the k-means cluster is very high and difference inside each cluster is very low. Therefore, k-means is the most efficient method here.

However, all results from three variants of hierarchical clustering are showing the same. Therefore,I'm going to increase the k value in order to select the best approach in hclust.

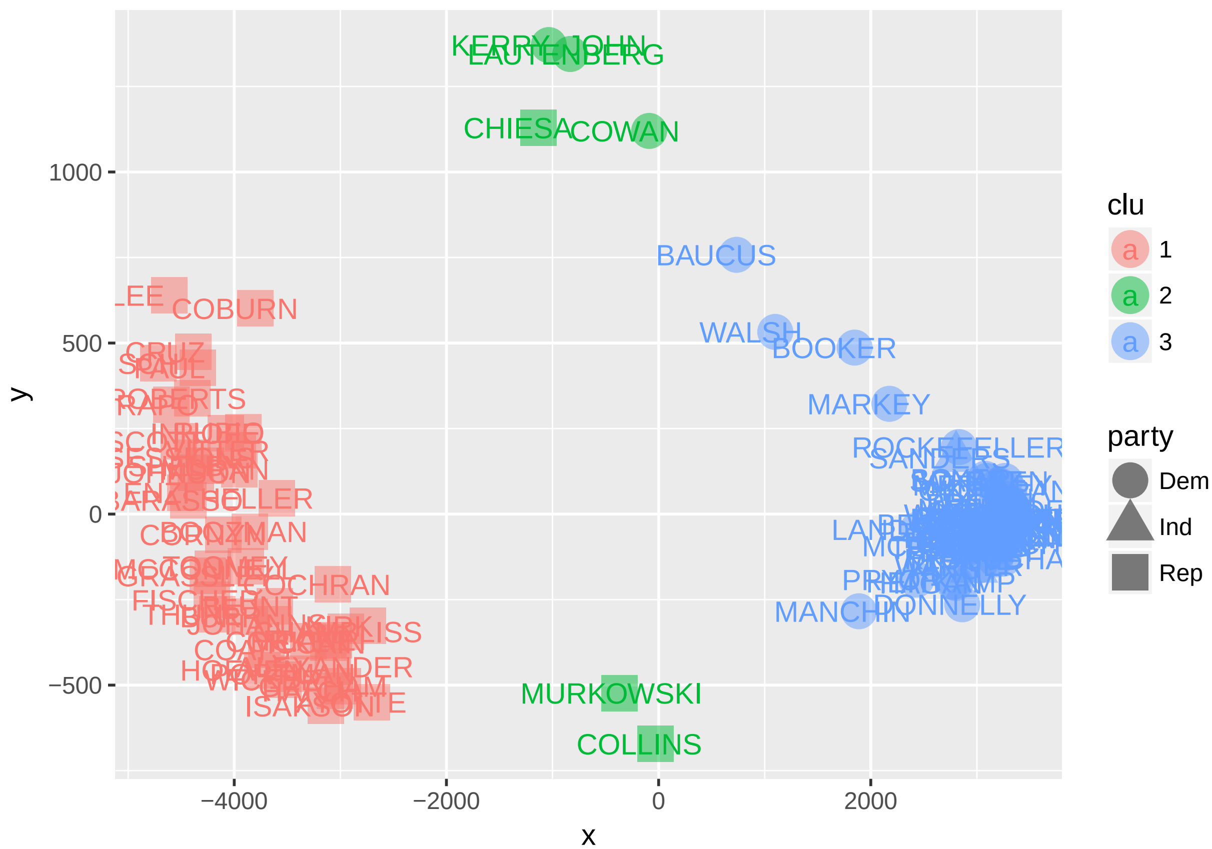
Increase the k value to 3 and check the result.

```
hclusterMDS <- function(hc_agg) {
    hc1 = cutree(hc_agg, k = 3)
    hc1 <- as.data.frame(hc1)
    names(hc1)[names(hc1) == "hc1"] <- "cluster"
```

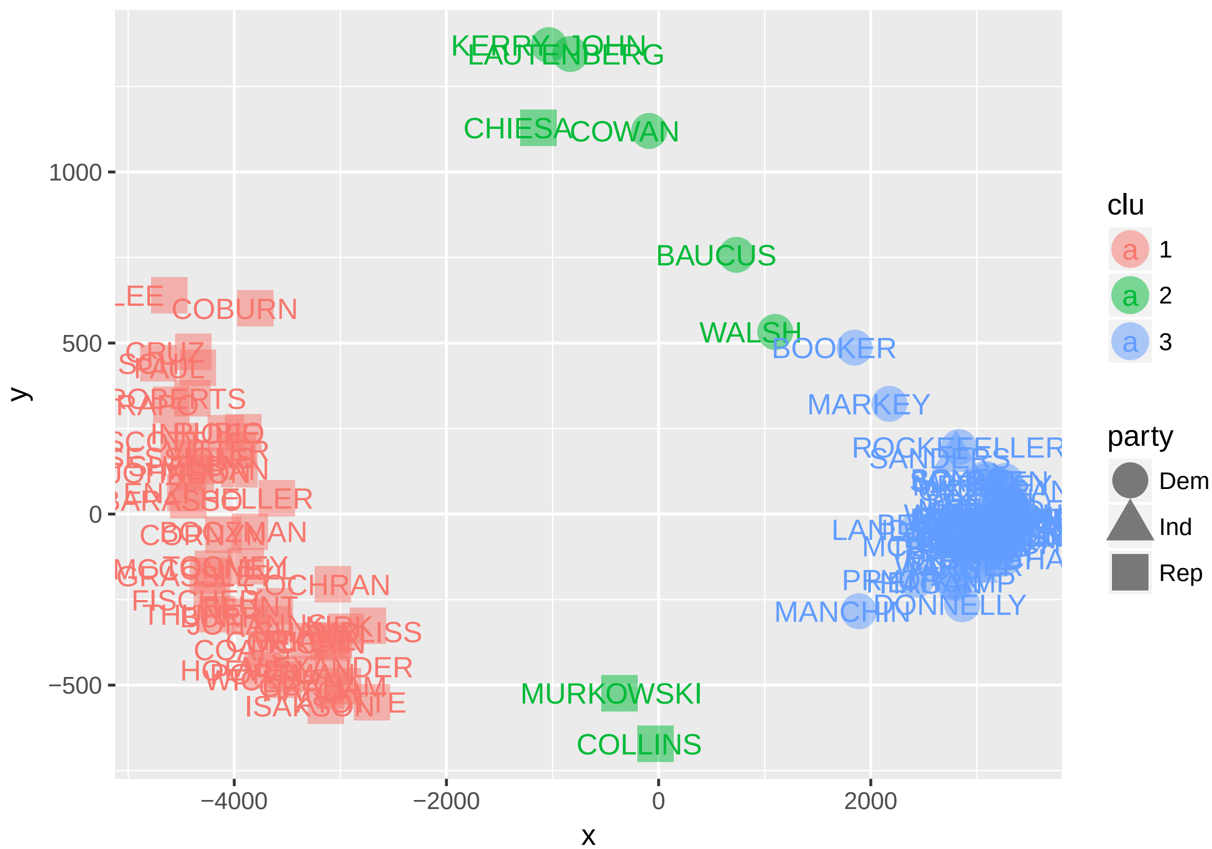
```r
    data3 = clusterMDS(hc1)
    return(data3)
}

# single link
hc_s = hclust(sen113_dist, method = "single")
grpSen2 <- hclusterMDS(hc_s)
```




```r
# complete link
hc_c = hclust(sen113_dist, method = "complete")
grpSen3 <- hclusterMDS(hc_c)
```




```r
# average link
hc_a = hclust(sen113_dist, method = "average")
grpSen4 <- hclusterMDS(hc_a)
```




```r
p2 <- cluster.purity(grpSen2$clu, grpSen2$party)
p3 <- cluster.purity(grpSen3$clu, grpSen3$party)
p4 <- cluster.purity(grpSen4$clu, grpSen4$party)

purity <- c(p2, p3, p4)

e2 <- cluster.entropy(grpSen2$clu, grpSen2$party)
e3 <- cluster.entropy(grpSen3$clu, grpSen3$party)
e4 <- cluster.entropy(grpSen4$clu, grpSen4$party)

entropy <- c(e2, e3, e4)

result = rbind(purity, entropy)
result = as.data.frame(result)
colnames(result) = c("hclust-single", "hclust-complete", "hclust-average")

kable(result, caption = "Summary of Clustering (k=3)")
```

Table 2: Summary of Clustering (k=3)

	hclust-single	hclust-complete	hclust-average
purity	0.9714286	0.9523810	0.9523810
entropy	0.1898924	0.1756948	0.1902527

When I increase the k to 3, the hclustering with single method shows the highest purity and hclustering with complete method shows the lowest entropy. I would like to choose the hclustering with single moethod to be the second most meaningful clustering method for this dataset, since it shows the highest purity and media entropy among the three variants of hclustering method.

In conclusion, k-means and h-clustering with single link are the two method here that generate the most meaningful results.