APS Assignment-3

"Floyd–Warshall algorithm"

Soumen Ghosh

Ph.D. Scholar
IIIT Chittoor

October 12, 2017

Problem Definition

Find the all pairs shortest path from Directed graphs with non-negative weights.

Methodology

In graph theory, the shortest path problem is the problem of finding a path between two vertices (or nodes) in a graph such that the sum of the weights of its constituent edges is minimized. The Floyd–Warshall algorithm is use for finding the shortest path between all the possible pair of a directed or undirected graph. In this work, **Floyd–Warshall** algorithm is implemented using C language for finding all pair shortest path of a given graph. The Johnson's algorithm also use for finding shortest path between all possible pair of a graph.

Algorithm

The pseudo code of the Floyd-Warshall algorithm is given bellow:

```
Floyd-Warshall(W)
    n = W.rows
    D^(0) = W
    for k = 1 to n
        let D^(k) = (d_ij^(k)) be a new matrix
        for i = 1 to n
            for j = 1 to n
                d_ij^(k) = min(d_ij^(k-1), d_ik^(k-1) + d_kj^(k-1))
    return D^(n)
```

Experiment

This result is obtained for two different graph. These two graph have four and five vertices.

```
soumen@iiits:~/Desktop/PhDCourseWork/APS/assign$ gcc APSPath
soumen@iiits:~/Desktop/PhDCourseWork/APS/assign$ ./a.out
Adjacency matrix for the input directed Graph
      0       5     Inf      10
    Inf       0       3     Inf
    Inf     Inf       0       1
    Inf     Inf     Inf       0
Shortest distances between every pair of vertices
      0       5       8       9
    Inf       0       3       4
    Inf     Inf       0       1
    Inf     Inf     Inf       0
Shortest path between every pair of vertices
      0       0       1       2
     -1       0       1       2
     -1      -1       0       2
     -1      -1      -1       0
Time required for execution: 383.000000 seconds
```

```
soumen@iiits:~/Desktop/PhDCourseWork/APS/assign$ gcc APSPath
soumen@iiits:~/Desktop/PhDCourseWork/APS/assign$ ./a.out
Adjacency matrix for the input directed Graph
      0       5     Inf       2     Inf
    Inf       0       2     Inf     Inf
      3     Inf       0     Inf       7
    Inf     Inf       4       0       1
      1       3     Inf     Inf       0
Shortest distances between every pair of vertices
      0       5       6       2       3
      5       0       2       7       8
      3       8       0       5       6
      2       4       4       0       1
      1       3       5       3       0
Shortest path between every pair of vertices
      0       0       3       0       3
      2       0       1       0       3
      2       0       0       0       3
      4       4       3       0       3
      4       4       1       0       0
Time required for execution: 374.000000 seconds
```

Complexity Analysis

- The time complexity of Floyd–Warshall algorithm is $\theta(n^3)$.

- The space complexity of Floyd–Warshall algorithm is $\theta(n^2)$.