Note that the equation for the untested susciptable compartment is $dS_u/dt = -\Lambda S_u - F_S S_u + \omega S_n$, where Λ is FoI, F_Z in general form is the weighted testing rate defined as $F_Z = \sigma W_Z$, and σ is the testing rate. Let $W = W_S S_u + W_I I_u + W_R R_u$ be the weighted number of people available for tests.

On testing rate

The testing rate, σ, should be formulated such that people from the _u compartments will not be tested if they're not there.

- The initial approache and the singularity issue;

$\sigma = \frac{\rho N_0}{W}$ was used initially. The issue was that the population in S compartments appeared to blow up when the DFE is achieved. This is once the only untested people are susceptibles, the FoI will become $\Lambda = 0$, testing rate $F_s = \rho N_0/S_u$. Thus, eq(1) of the model will be $dS_u/dt = -\rho N_0 + \omega S_n$ which is no longer dependent on S_u and a linear rate of leaving the S_u compartment.

- The secondary approache;

One natural way is to define

$$\sigma = \frac{\tau \rho N_0}{\tau W + \rho N_0}.$$

Where τ (1/day) is a rate for the maximum rate of testing the whole untested population. In general, we want to test at a rate of ρ across the whole population. This won't always be possible. So we impose a maximum rate of τ per testable person. It is consistent to think of both τ and ρ as pure rates, but it might be clearer to think of ρ as tests per capita per unit time, and τ as tests per testable person per unit time. It's not that we're switching through time, it's that we're imposing both of these as limitations. Note that, in general, since $\tau \gg \rho$, thus $\sigma = \rho \frac{\tau N0}{(\tau+\rho)N_0} \approx \rho \frac{\tau N0}{(\tau)N_0} = \rho$. This generally collapses to the original form. On the other hand, when W is super-small, however, it collapses instead to τ. In other words, at the beginning, we expect the answer to be close to ρN_0 since τ should be very fast. Once W becomes small, the limitation imposed by τW will become important.

On the testing strategies

- The testing weights, W_S, W_I and W_R, are representing different testing strategies. - The only thing that matters is the relative proportions of W's. - Right now there are no restictions on the weights. If we impose $\sum_Z W_Z = 1$, we are just reducing a degree of freedom. i.e., if we increase the W_I, the other two weights will decrease.

- Two things to explore are: (i) Random testing; this scenariocan be explored when $W_S = W_I = W_R$.

(ii) Testing and tracing; this scenariocan be explored when $W_S \ll W_I$ and for example $W_I = W_R = 1$.

Questions

1- When $\sigma = \rho$? $\frac{\tau N0}{\tau W + \rho N_0} = 1$. Mathematically, this can be achieved when $W \approx N_0$ and $\tau \gg \rho$.

2- What is the most interesting thing in the testing strategies? We intuitively know testing/tracing/isolation (TTI) strategy is more effective than random testing in the sense that it lowers the R_0. Obvious things to ask are

(i) How much TTI is lowering R_0? (ii) How fast TTI is lowering R_0?