Home assignment number 1, 2024, in SF2863 Systems Engineering.

Instructor: Per Enqvist.

The home assignments are a mandatory part of the course. In total, there are two home assignments and you need to collect 4.0 out of 6.0 points in order to pass this part of the course. Home assignment 1 can give up to 2.0 points.

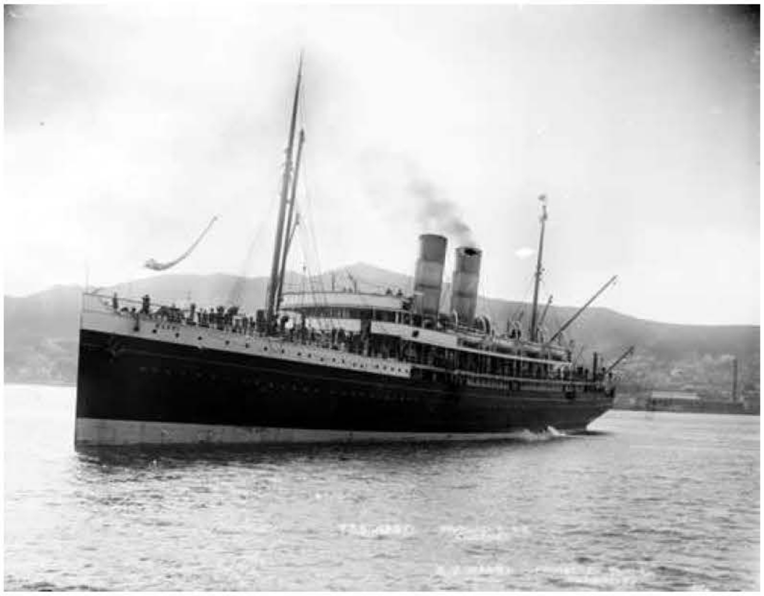
The home assignments can, in addition, give bonus points on the exam. If a home assignment is handed in before the deadline, then the points awarded on the assignment will also count as bonus points on the exam (hence, home assignment 1 can give up to 2.0 bonus points on the exam). In order to get bonus points on the exam the report should be submitted to canvas before 19:00, Friday, November **15**, *2024.*

This home assignment should be done in groups of 2 persons, and one report per group is to be submitted to canvas. See the information on canvas on how to form groups and submit your results. State your name, and email address on the front of the report.

In the report you should describe in your own words how the problem was solved. In particular, carefully describe how you defined your states, modelled the problem and implemented the simulations. Do not explain the code, but rather the mathematical ideas. You should not need more than 4 pages for the report. Add a screenshot of your results from Matlab grader as a last page. The answers to the questions in the assignment should be given in the main report and numbered as in this text. In particular, it should not be necessary to look at your code to understand how you define and calculate things. Relevant print-outs and plots should be included in the report. We use automated plagiarism-control tools and will also make random checks that you have not copied parts of another groups report and written the computer code yourselves!

Questions should be posed in the discussion group in canvas.

System and data
Consider a ferry equipped with two engines, each powering a separate propeller. If one engine fails, the ferry can still operate at a reduced speed, but if both engines fail, the ferry will come to a complete stop.



We model the ferry's operation as a continuous-time Markov process. The failure of engine 1 occurs after a random period, represented by an exponential random variable with parameter λ_1. Similarly, the time until engine 2 fails is modeled by an exponential random variable with parameter λ_2.
Three repair technicians are available to repair the engines as needed. The repair time for engine k (where $k = 1$ or 2) follows an exponential distribution with rate parameter $n_k\mu_k$, where n_k represents the number of technicians assigned to engine k.
Let $V(t)$ represent the ferry's speed at any time t. When both engines are operational, the ferry travels at speed v. If only engine 1 is functional, the speed is reduced to v_1; if only engine 2 is operational, the speed is v_2. All relevant parameters are provided in the assignment grader.

Theoretical solution
To begin, calculate the average speed of the ferry using Markov process theory. The steps for this approach are as follows:

1. Define the States of the Markov Process: Identify the four possible states.

2. Determine the Intensity Matrix: Construct the intensity matrix based on the transition rates between these states.

3. Justify the Existence of a Stationary Distribution: Explain why a stationary distribution exists for this Markov process, allowing for long-term average behavior.

4. Calculate the Stationary Distribution: Use MATLAB to find numerical solutions to the stationary distributions for each repair strategy, as specified in the intensity matrix.

5. Compute the Ferry's Average Speed: Derive the ferry's average speed from the stationary distribution obtained.

Repeat Steps 4 and 5 for each of the four repair strategies:

- Strategy (i): All 3 repairers are assigned to engine 1.
- Strategy (ii): Assign 2 repairers to engine 1 and 1 repairer to engine 2.
- Strategy (iii): Assign 1 repairer to engine 1 and 2 repairers to engine 2.
- Strategy (iv): All 3 repairers are assigned to engine 2.

General assumptions:

If only one engine is non-operational, all three repairers should work on repairing that engine.
If both engines are down, follow repairer distributions (i) to (iv) as outlined above.

Next, conduct a numerical simulation of the system to estimate the average speed using ergodic averages. This simulation will focus on repair strategy (i) (where all three repairers are assigned to engine 1 when both engines are down) and will be implemented in two distinct ways:

- **Direct Simulation of the Continuous-Time Process:** Simulate the continuous-time Markov process directly, capturing the behavior of the system over time.
- **Time Discretization Approach:** Approximate the continuous-time process by discretizing time, allowing for a stepwise simulation.

These two approaches will provide complementary estimates of the average speed and enable comparison of their accuracy and computational efficiency.
By simulating the process we mean that you start the process in some state and then you determine one realization of the process by using the probability description of the process to determine how long you remain in a state and where you transition. This will give you a description of how the state of the ferry is changing over the simulation horizon for that particular realization.

Matlab tips:
An exponentially distributed variable can be generated using the command `exprnd`, where you specify the *mean* of the variable (which is inversely proportional to the intensity).
A stochastic variable with uniform probability in [0, 1] can be generated using the command `rand`.

Continuous time approach

To simulate the continuous-time process, we use random generators to determine the dwell time, i.e. time between transitions, and the next state to obtain a realization of the Markov process.

There are (at least) two ways to do this and you can choose which one you want. Begin the simulation at $t = 0$ with both ferry engines operational.

6a. Describe in detail how you determine the time to the next jump and where you jump.

7a. Run the simulation for a time interval $[0,\ T]$.

Keep track of how long the system is in the different states and take ergodic estimates (time averages) to approximate the stationary state probabilities.

If you run the simulation twice you would get different results. Comparing the two different results can give an indication on if the averages have converged or not. Choose (the magnitude of) T large enough so that the errors are less than 1%.

8a. Now I want you to start a simulation from a fully functioning system and run the simulation until it first reaches the state when all engines are not working. Repeat this 10000 times and determine the average as an estimate of the expected time to failure.

Discretization approach

Now a discretization of the continuous time process will be made.
Start the process at time 0 with both engines of the ferry functioning. Then make a discretization of the time axis so that we only consider the process at times $t_k = kh$ for $k = 0, 1, 2, \cdots, N$. Then the probability of a jump to another state during the time interval $[t_k,\ t_{k+1}]$ can be determined (approximatively) using the intensity matrix for the continuous time Markov process. You can use the matrix exponential command `expm`, or the linear approximative expressions used in the lecture. If the time step h is small enough this approximation is good, and it is reasonable to assume that there is at most one jump of the process in this time frame.

6b. Determine the transition matrix of a discrete time Markov chain that will approximate the continuous time process.
Hint: Check if the transition matrix you obtain has row sums equal to one, and that the two different approaches mentioned above give similar results.

7b. Now use this discretization to simulate the process.

If you run the simulation twice you would get different results. Comparing the two different results can give an indication on if the averages have converged or not. Choose (the magnitude of) N large enough so that the errors are less than 1%.

8b. Take averages to determine estimates of the expected speed of the ferry.

9b. What is the probability that none of the engines are working after 10 steps, if we know that both engines were working from the start.

What is the average number of time steps until both engines are not working, if we know that both engines were working from the start.

The idea is that you should use the transition matrix for the Markov chain approximation to answer question 9b. No simulation is necessary to solve this question.

Good luck!