Planning of Hydro Power Generation

Project Assignment 1A

SF2812 Applied Linear Optimization

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Abstract

This report investigates an optimization problem to maximize the profits of the fictional company PowerCompany AB by optimizing electricity generation across its hydropower plants. The focus is on strategically plan water discharge and spillage from their reservoirs over a 9-hour period, employing both linear and stochastic programming approaches.

The linear programming model optimizes electricity production based on known electricity prices, with the optimal profit achieved being 1.203 billion €. The discharge schedule, as outlined in the results, prioritizes minimal discharge during low-price hours, while maintaining efficient reservoir levels and avoiding spillage.

Sensitivity analysis of the marginals from the linear progamming model identifies that constraints such as minimum reservoir levels and maximum discharge capacities significantly influence profitability. For example, relaxing the minimum reservoir level constraint at Ahlen results in an increase of 31.15 million €, while tightening the same constraint at Forsen leads to a profit reduction of 30.3 million €. Further analysis through marginal values for both minimum reservoir levels and maximum discharge levels reveals that increasing discharge capacity at Kärret results in a slight profit increase (64,800 €), highlighting the importance of optimizing capacity utilization.

The study also investigates the impact of saving water after the ninth hour to sell at a future electricity price. This development shows that as the future increases, the strategy shifts toward conserving more water, prioritizing storage for to sell at a high future price. This behavior is most apparent when the future price reaches exceed the prices offered during the nine hours. In this case, discharge and spillage rates are kept at a minimum throughout simply in order to not exceed the reservoir limits as water builds up.

Using stochastic programming, we model price uncertainty by considering multiple pricing scenarios and two differenet models - namely a EV model and a 2-stage model. The EV model results in a slightly lower profit of 0.988 billion € and the 2-stage model achieves a slightly better profit of 1.021 billion €. Despite these reduction, the stochastic models provide a more robust strategy across varying market conditions.

Overall, the optimization approach developed in this study effectively maximizes profit while providing valuable insights into the importance of managing constraints and considering price uncertainty in the operation of hydropower plants.

1 Background

In this project, we are to help PowerCompany AB who operates four hydroelectric power plants. They need to plan electricity production for the next nine hours.

Each plant has a reservoir and a turbine for generating electricity. Each reservoir starts at certain water level and each reservoir has both a minimum and maximum allowed water level. These values are found in Table 1.

Table 1: *Constants for each reservoir level.*

Reservoir	Ahlen	Fjället	Forsen	Kärret
Start level (Mm^3)	5800	1000	20	13
Max level (Mm^3)	7160	1675	27	13
Min level (Mm^3)	5800	1000	10	6

Each power plant has a certain discharge capacity that the discharge level is not allowed to exceed. The discharge level determines the amount of electricity generated during the hour subject to an efficiency or power conversion factor. Each power plant can also spill water. In so doing, the water passes through the reservoir without generating electricity and onto the next reservoir if there is one down stream. The spillage is also limited. The respective spillage and discharge levels during each hour are our controls. Each power plant receives water from the other reservoirs upstream subject to some time delay. They also each have a local inflow from rain, small streams, etc. All relevant parameter from this paragraph is found in Table 2.

Table 2: *Parameters for the power plants*

Parameter	Ahlen	Fjället	Forsen	Kärret
Discharge capacity (m^3/s)	540	135	975	680
Power conversion $\left(\frac{\mathrm{MW}}{\mathrm{m}^3/\mathrm{s}}\right)$	0.52	1.17	0.29	0.05
Spillage capacity (m^3/s)	820	930	360	400
Local inflow of water (m^3/s)	177	28	8	29
Time delay for water to reach the next power plant (h)	2	1	1	N/A

As mentioned, the electricity price varies hourly. The price for each hour is found in Table 3.

Table 3: *Electricity prices per hour.*

Hour	1	2	3	4	5	6	7	8	9
El. Price (€/MWh)	45	55	95	80	140	150	80	70	130

The reservoirs are located as in Figure 1. This configuration determines the water flow between the reservoirs. Having passed Kärret, the water exits our system. At each reservoir, there is also a local inflow.

At a later point, we shall also consider the following developments of the problem.

First, we assume, in addition, that PowerCompany may opt to store the water that remains after the ninth hour and generate electricity to be sold at 95 €/MWh. To be precise, we shall assume that all local inflows cease after the ninth hour and only the remaining water in the reservoirs that exceed the reservoir minimum level is available to us. This assumption is made since otherwise the profit will be infinite and or unrealistic. We shall consider how this affects our plan also when varying the future price at which we can produce electricity after the ninth hour.

In another development, we shall incorporate uncertainty in the electricity price. We shall assume that the deal with the power grid company allows us to know the price for the first two hours but not the future prices after that. Moreover, we shall assume that our analysts have given us three possible pricing scenarios that are equally likely. These are given in Table 4.

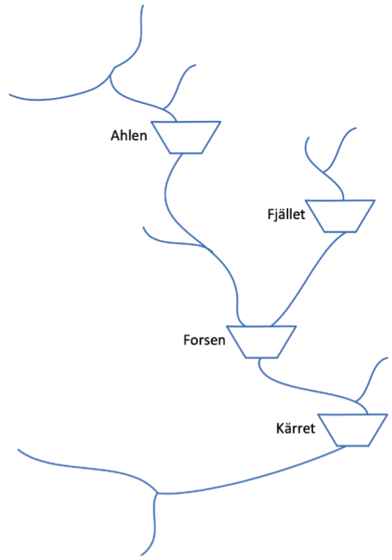

Figure 1: *The four hydro power plants. Water flows downwards in the figure.*

Table 4: *Electricity prices for each of the three equally likely scenarios.*

Hour	1	2	3	4	5	6	7	8	9
Scenario 1 (€/MWh)	45	55	80	80	110	110	80	30	70
Scenario 2 (€/MWh)	45	55	95	80	80	130	130	60	95
Scenario 3 (€/MWh)	45	55	120	90	140	105	80	90	120
Average Price (€/MWh)	45	55	96.67	83.33	110	115	96.67	60	95

2 Mathematical Formulation

The objective is to maximize profit from the generated electricity by controlling the water flow through spillage and discharge at each reservoir during every one of the nine hours.

To model this problem mathematically as an LP problem, let us note that the power plants and the flow of water may be represented by a graph $\mathcal{G} = (N, E)$ with nodes $N = \{1, 2, 3, 4\}$ that denote the power plants and with edges $E = \{(1, 3), (2, 3), (3, 4)\}$ that determine the flow of water where Ahlen is assigned to node 1, Fjället to node 2, Forsen to node 3, and Kärret to node 4. Of course, this is without considering local inflow to each reservoir and the outflow from the entire system at Kärret. Moreover, let us make the following definitions.

- $V_{i,t}$ - Water discharge level through turbine i during hour t in m^3/s.
- η_i - Power conversion factor of turbine i in $\frac{\mathrm{MW}}{\mathrm{m}^3/\mathrm{s}}$.
- P_t - Price during hour t in €/MWh.

Then, the objective function (the profit) is

$$\max \sum_{t=1}^{9} \sum_{i=1}^{4} 3600 P_t \eta_i V_{i,t}, \tag{1}$$

where we have multiplied by 3600 to account for difference in units of time.

Next, let us also define:

- $R_{i,t}$ - Water level of reservoir i at the start of hour t in $\mathrm{Mm}^3 = 10^6\ \mathrm{m}^3$.

- Q_i - Local inflow to $R_{i,t}$ during each hour in m^3/s.
- $S_{i,t}$ - Spillage from $R_{i,t}$ during hour t in m^3/s.
- $\delta_{i,j}$ - Time delay from reservoir i to reservoir j in number of hours.

Then, the water level in reservoir i at hour $t+1$ is governed by

$$R_{i,t+1} = R_{i,t} + 3600 \cdot 10^{-6} \left(Q_i - V_{i,t} - S_{i,t} + \sum_{j:j \to i} V_{j,t-\delta_{j,i}} + S_{j,t-\delta_{j,i}} \right). \tag{2}$$

The sum in (2) is taken over all reservoirs j such that j is directly upstream from reservoir i (i.e. j such that $(j,i) \in E$) and if the time delay $\delta_{j,i}$ is such that $t - \delta_{j,i} < 1$ then we set $V_{j,t-\delta_{j,i}} = S_{j,t-\delta_{j,i}} = 0$. Since (2) holds for all $t = 1, \dots, 9$, it may be restated as

$$R_{i,t+1} = R_{i,1} + \sum_{\tau=1}^{t} 3600 \cdot 10^{-6} \left(Q_i - V_{i,\tau} - S_{i,\tau} + \sum_{j:j \to i} V_{j,\tau-\delta_{j,i}} + S_{j,\tau-\delta_{j,i}} \right). \tag{3}$$

for $t = 1, \dots, 9$. The above constraint shall be referred to as the *water balance constraint* or *equation*. Before continuing, let us add that the inclusion of $t = 9$ in the water balance constraint means that we care about the reservoir levels at the start of the 10th hour. That is, our actions during hour 9 are not allowed to make reservoir levels in hour 10 violate the water balance and, as we shall see below, the bounds of the reservoir levels. Moreover, $R_{i,t}$ is simply a function of $V_{j,\tau}, S_{j,\tau}$ for j such that $j \to i$ and $\tau < t$ and so they are not variables in a strict sense. The discharge and spillage $V_{j,\tau}, S_{j,\tau}$ are our variables.

Moreover, we have the following constraints determined by the values found in Table 1 and Table 2 that we shall vary at a later point.

$$R_i^{(min)} \leq R_{i,t} \leq R_i^{(max)}, \quad \forall i,t \qquad\qquad 0 \leq V_{i,t} \leq V_i^{(max)}, \quad \forall i,t,$$
$$R_{i,1} = R_i^{(start)}, \quad \forall i \qquad\qquad 0 \leq S_{i,t} \leq S_i^{(max)}, \quad \forall i,t.$$

Finally, this may all be summarized as the LP problem (LP) that we can implement in GAMS:

$$\begin{aligned} \max_{V_{i,t},S_{i,t}} \quad & \sum_{t=1}^{9} \sum_{i=1}^{4} 3600 P_t \eta_i V_{i,t}, \\ \text{s.t.} \quad & R_{i,1} = R_i^{(start)}, \quad \text{for } i = 1, \dots, 4 \\ & R_i^{(min)} \leq R_{i,t} \leq R_i^{(max)}, \quad \text{for } i = 1, \dots, 4, \text{ and } t = 2, \dots 10, \\ & 0 \leq V_{i,t} \leq V_i^{(max)}, \quad \text{for } i = 1, \dots, 4, \text{ and } t = 1, \dots 9, \\ & 0 \leq S_{i,t} \leq S_i^{(max)}, \quad \text{for } i = 1, \dots, 4, \text{ and } t = 1, \dots 9. \end{aligned} \tag{LP}$$

Note that $R_{i,t}$ is merely as dummy variable to illustrate how we derive the constraints. They are not actually needed in the problem and instead we could have just used the water balance equation in their place in (LP). Similarly, they may or may not be be declared as variables in the GAMS implementation. Either way, they are, strictly speaking, not variables - they are completely determined by our discharge and spillage levels through the water balance equation. We have done both implementations in GAMS, i.e., with (`HA1.gms` or `Project1A_1_Rs.gms`) and without (`Project1A_1.gms`) the explicit reservoir levels, and they yield the same solution. With that being said, the explicit use of the reservoir levels as variables allows for improved interpretability, especially with regard to duality, as we shall soon see.

Before continuing with the more advanced models mentioned at the end of Section 1, let us add that we shall be varying the constants found above while analyzing the results in the next section, Section 3. Additionally, let us touch on the interpretation of the dual problem. Since we are maximizing profit under constraints, the dual problem has a nice interpretation with the dual variables representing shadow prices as seen in Table 5. We shall return to these interpretations when conducting sensitivity analysis having obtained some initial results.

Table 5: *Interpretation of dual variables. Note, that spillage is not to be seen as a cost. It is rather an opportunity to optimize electricity generation down stream or avoid reaching maximum reservoir capacity.*

Primal Constraint	Dual variable	Interpretation of dual variable
$R_{i,t} \leq R_i^{(max)}$	$\rho_{i,t}^{(max)}$	Marginal value of increasing the maximum reservoir capacity by 1 Mm^3.
$R_{i,t} \geq R_i^{(min)}$	$\rho_{i,t}^{(min)}$	Marginal value of decreasing the minimum reservoir capacity by 1 Mm^3.
$V_{i,t} \leq V_i^{(max)}$	$\gamma_{i,t}^{(max)}$	Marginal value of increasing the turbine discharge capacity by 1 m^3/s.
$S_{i,t} \leq S_i^{(max)}$	$\sigma_{i,t}^{(max)}$	Marginal value of increasing spillage capacity by 1 m^3/s.
Water Balance (2)	$\lambda_{i,t}$	Marginal value of water at reservoir i during hour t (€/Mm^3).

Now, let us return to the development that allows us to generate electricity using the remaining water. We assume that all locals inflows cease after the ninth hour. Moreover, we assume that we have another $T-9$ hours to utilize the remaining water. Then the problem becomes

$$\begin{aligned} \max_{V_{i,t},S_{i,t}} \quad & \sum_{t=1}^{T}\sum_{i=1}^{4} 3600 P_t \eta_i V_{i,t}, \\ \text{s.t.} \quad & R_{i,1} = R_i^{(start)}, \quad \text{for } i=1,\dots,4 \\ & R_i^{(min)} \leq R_{i,t} \leq R_i^{(max)}, \quad \text{for } i=1,\dots,4, \text{ and } t=2,\dots T, \\ & 0 \leq V_{i,t} \leq V_i^{(max)}, \quad \text{for } i=1,\dots,4, \text{ and } t=1,\dots T, \\ & 0 \leq S_{i,t} \leq S_i^{(max)}, \quad \text{for } i=1,\dots,4, \text{ and } t=1,\dots T. \end{aligned} \tag{LP'}$$

where P_t is given by the prices in Table 3 for $t \leq 9$ but is set to the future price $P = 95$ €/MWh for $t \geq 10$ and

$$Q_{i,t} = \begin{cases} Q_i & \text{for } t \leq 9, \\ 0 & \text{for } t \geq 10. \end{cases}$$

We shall not implement (LP') in GAMS. Instead, let us try a different approach.

Another way to model the ability to store the remaining water is by letting go of the constraints after the 10nth hour. In that case, we can discharge all the remaining water at maximum capacity and we do not have to extend the time horizon. This allows us to easily calculate the monetary value of the remaining water that exceeds the minimum allowed reservoir levels. The remaining water M_i that can pass through each power plant i is the sum of $R_{i,10}$ and the $R_{j,10}$ for all j that are upstream from i less the corresponding $R_i^{(min)}$ and $R_j^{(min)}$. That is,

$$\begin{gathered} M_1 = R_{1,10} - R_1^{(min)}, \quad M_2 = R_{2,10} - R_2^{(min)}, \\ M_3 = R_{3,10} - R_3^{(min)} + R_{2,10} - R_2^{(min)} + R_{1,10} - R_1^{(min)} = R_{3,10} - R_3^{(min)} + M_1 + M_3, \\ M_4 = R_{4,10} - R_4^{(min)} + R_{3,10} - R_3^{(min)} + R_{2,10} - R_2^{(min)} + R_{1,10} - R_1^{(min)} = R_{4,10} - R_4^{(min)} + M_3. \end{gathered}$$

Then, the time it takes for all water to pass through reservoir i with maximal discharge is $\frac{M_i}{V_i^{(max)}}$ so that the total value of the remaining water is

$$\sum_{i=1}^{4} \frac{10^6 M_i}{V_i^{(max)}} \cdot P \cdot \eta_i \cdot V_i^{(max)} = \sum_{i=1}^{4} 10^6 M_i P \eta_i$$

where $P = 95$ €/MWh is the future price. Then, we just have to add this to the objective without changing the constraints. We get the new problem

$$\begin{aligned} \max_{V_{i,t},S_{i,t}} \quad & \sum_{t=1}^{9}\sum_{i=1}^{4} 3600 P_t \eta_i V_{i,t} + \sum_{i=1}^{4} 10^6 M_i P \eta_i, \\ \text{s.t.} \quad & R_{i,1} = R_i^{(start)}, \quad \text{for } i=1,\dots,4, \\ & R_i^{(min)} \leq R_{i,t} \leq R_i^{(max)}, \quad \text{for } i=1,\dots,4, \text{ and } t=2,\dots,10, \\ & 0 \leq V_{i,t} \leq V_i^{(max)}, \quad \text{for } i=1,\dots,4, \text{ and } t=1,\dots 9, \\ & 0 \leq S_{i,t} \leq S_i^{(max)}, \quad \text{for } i=1,\dots,4, \text{ and } t=1,\dots 9. \end{aligned} \tag{LP''}$$

(LP”) is easily solved in GAMS. See `Project1A_3.gms`. Note, however, that there is no guarantee that this would be possible in practice. That is, using maximal discharge might violate the constraints of the system.

Continuing, let us consider the development of incorporating price uncertainty. We shall model this in two different ways. The first of which is the expected value problem. This model is rather rudimentary, we simply replace P_t in (LP) with

$$\overline{P}_t = \sum_{s=1}^{3} \frac{1}{3} P_{t,s}$$

for $t = 1, \ldots, 10$ where $P_{t,s}$ is found in Table 4 and $s = 1, 2, 3$ denotes the scenario. This gives the following LP problem.

$$\begin{aligned}
\max_{V_{i,t}, S_{i,t}} \quad & \sum_{t=1}^{9} \sum_{i=1}^{4} 3600 \overline{P}_t \eta_i V_{i,t}, \\
\text{s.t.} \quad & R_{i,1} = R_i^{(start)}, \quad \text{for } i = 1, \ldots, 4 \\
& R_i^{(min)} \leq R_{i,t} \leq R_i^{(max)}, \quad \text{for } i = 1, \ldots, 4, \text{ and } t = 2, \ldots 10, \\
& 0 \leq V_{i,t} \leq V_i^{(max)}, \quad \text{for } i = 1, \ldots, 4, \text{ and } t = 1, \ldots 9, \\
& 0 \leq S_{i,t} \leq S_i^{(max)}, \quad \text{for } i = 1, \ldots, 4, \text{ and } t = 1, \ldots 9.
\end{aligned} \tag{EV}$$

This is rather simple to implement in GAMS, see `Project1A_5_EV.gams`. Effectively, it amounts to only changing the given prices in (LP).

Next, let us consider the 2-stage problem. Here, we simply extend all relevant quantities to also vary over the three scenarios after the second hour. In particular, the water balance is now given by (3) for $t = 1, 2$ but

$$R_{i,t+1,s} = R_{i,3} + \sum_{\tau=1}^{t} 3600 \cdot 10^{-6} \left(Q_i - V_{i,\tau,s} - S_{i,\tau,s} + \sum_{j:j \to i} V_{j,\tau-\delta_{j,i},s} + S_{j,\tau-\delta_{j,i},s} \right).$$

for $i = 1, \ldots, 4$, $t = 3, \ldots, 9$, and $s = 1, 2, 3$. Denote $x = (V_{i,1}, S_{i,1}, V_{i,2}, S_{i,2})$. The second stage problem (4) is

$$\begin{aligned}
Q(x, s) = \max_{V_{i,t,s}, S_{i,t,s}} \quad & \sum_{i=1}^{4} \sum_{t=3}^{9} 3600 P_t \eta_i V_{i,t,s}, \quad \text{for } s = 1, 2, 3, \\
\text{s.t.} \quad & R_i^{(min)} \leq R_{i,t,s} \leq R_i^{(max)}, \quad \text{for } i = 1, \ldots, 4, \quad \text{and } t = 4, \ldots, 10, \\
& 0 \leq V_{i,t,s} \leq V_i^{(max)}, \quad \text{for } i = 1, \ldots, 4, \quad \text{and } t = 3, \ldots, 9 \\
& 0 \leq S_{i,t,s} \leq S_i^{(max)}, \quad \text{for } i = 1, \ldots, 4, \quad \text{and } t = 3, \ldots, 9,
\end{aligned} \tag{4}$$

and the first stage problem (5) is

$$\begin{aligned}
\max_{V_{i,t}, S_{i,t}} \quad & \sum_{i=1}^{4} \sum_{t=1}^{2} 3600 P_t \eta_i V_{i,t} + \sum_{s=1}^{3} \frac{1}{3} Q(x, s) \\
\text{s.t} \quad & R_{i,1} = R_1^{(start)}, \text{for } i = 1, \ldots, 4, \\
& R_i^{(min)} \leq R_{i,t} \leq R_i^{(max)}, \quad \text{for } i = 1, \ldots, 4, \text{ and } t = 2, 3, \\
& 0 \leq V_{i,t} \leq V_i^{(max)}, \quad \text{for } i = 1, \ldots, 4, \text{ and } t = 1, 2, \\
& 0 \leq S_{i,t} \leq S_i^{(max)}, \quad \text{for } i = 1, \ldots, 4, \text{ and } t = 1, 2.
\end{aligned} \tag{5}$$

Aggregating (4) and (5) yields the aggregate recourse problem (RP) which is rather easy to implement in GAMS by extending all variables to also vary over the scenario and then forcing equality across scenarios for each variables during hour 1 and 2, see `Project1A_5_RP.gms`.

$$
\begin{aligned}
\max_{V_{i,t},S_{i,t},V_{i,t,s},S_{i,t,s}} \quad & \sum_{i=1}^{4}\sum_{t=1}^{2} 3600 P_t \eta_i V_{i,t} + \sum_{i=1}^{4}\sum_{t=3}^{9}\sum_{s=1}^{3} \frac{1}{3} 3600 P_{t,s}\eta_i V_{i,t,s} \\
\text{s.t.} \quad & R_{i,1} = R_i^{(start)}, \quad \text{for } i = 1,\dots,4, \\
& R_i^{(min)} \le R_{i,2} \le R_i^{(max)}, \quad \text{for } i = 1,\dots,4, \\
& R_i^{(min)} \le R_{i,t,s} \le R_i^{(max)}, \quad \text{for } i = 1,\dots,4, \quad t = 3,\dots,10, \quad \text{and } s = 1,2,3, \\
& 0 \le V_{i,t} \le V_i^{(max)}, \quad \text{for } i = 1,\dots,4, \text{ and } t = 1,2, \\
& 0 \le V_{i,t,s} \le V_i^{(max)}, \quad \text{for } i = 1,\dots,4, \quad t = 3,\dots,9, \quad \text{and } s = 1,2,3, \\
& 0 \le S_{i,t} \le S_i^{(max)}, \quad \text{for } i = 1,\dots,4, \text{ and } t = 1,2, \\
& 0 \le S_{i,t,s} \le S_i^{(max)}, \quad \text{for } i = 1,\dots,4, \quad t = 3,\dots,9, \quad \text{and } s = 1,2,3.
\end{aligned} \tag{RP}
$$

3 Results and Analysis

Solving the initial (LP) problem with the original input data yields the optimal plan found in Table 6 with optimal profit $1.203189 \cdot 10^9$ € and reservoir levels in Table 7. With the original input data, no spillage is used.

The initial results are intuitive. For instance, we use minimal discharge for the earlier power plants when prices are low, the reservoir levels are kept at a rather minimal level, the reservoir levels add up (e.g. for Ahlen we get a local inflow of $177 \cdot 3600 \cdot 10^{-6} = 0.637$ m^3/h), and we have minimal spillage since we start at the minimum reservoir levels for the early plants and the local inflows are not large enough to merit spillage.

Table 6: *Turbine discharge $V_{i,t}$ of power plant i during hour t (m^3/s)*

Power plant / Hour	1	2	3	4	5	6	7	8	9
Ahlen	0	0	0	0	522.000	540.000	0	0	531.000
Fjället	0	0	0	0	33.000	135.000	0	0	84.000
Forsen	0	0	908.778	0	975.000	975.000	246.000	0	975.000
Karret	29.000	521.222	680.000	680.000	680.000	680.000	680.000	680.000	680.000

Table 7: *Reservoir level $R_{i,t}$ of power plant i at the start of hour t (Mm^3)*

Power plant / Hour	1	2	3	4	5	6	7	8	9	10
Ahlen	5800.000	5800.637	5801.274	5801.912	5802.549	5801.307	5800.000	5800.637	5801.274	5800.000
Fjället	1000.000	1000.101	1000.202	1000.302	1000.403	1000.385	1000.000	1000.101	1000.202	1000.000
Forsen	20.000	20.029	20.058	16.815	16.844	13.362	10.000	11.508	13.481	10.000
Kärret	13.000	13.000	11.228	8.884	9.812	7.469	8.635	9.802	8.344	6.000

The marginal values (also known as shadow prices or dual values) in (LP) model indicate how much the objective function would change if the right-hand side of a constraint were increased by one unit. By looking at Table 7 we can interpret that until the values are not equal to our bounding constraints, the marginals are negligible. In contrast, when we reach a limiting boundary, the large marginal values show that the reservoir level at this specific time instance is a binding constraint. As an example, the first row of Table 8 tells that if the minimum reservoir level constraint in Ahlen and at ($t = 7$) relaxed by ($1Mm^3$) , the total objective function (profit) would increase by 31.15 million euros.

Furthermore, if we increase the minimum reservoir level at Forsen, the total profit during the first nine hours will decrease to 1,172,888,900 € which is 30,300,000 € less than the previous amount. This value is equal to the total lost values at hours 7 and 10 in this site based on Table 8. So changing the constraint at each site will only change the objective function by adding or subtracting values at certain hours.

Table 8: *Marginal Values for Minimum Reservoir Levels*

Site(Time)	Lower Limit (Mm^3)	Level (Mm^3)	Marginal (€/Mm^3)
Ahlen(7)	5800.0000	5800.0000	-31,150,000
Ahlen(10)	5800.0000	5800.0000	-67,600,000
Fjället(7)	1000.0000	1000.0000	-42,000,000
Fjället(10)	1000.0000	1000.0000	-152,100,000
Forsen(7)	10.0000	10.0000	-4,350,000
Forsen(10)	10.0000	10.0000	-25,950,000
Kärret(10)	6.0000	6.0000	-2,750,000

With regard to the limits of the maximum discharge level, Table 9 shows that when the discharge level is fully utilized at a given time and site, how much we can increase the profit by adding one unit ($1m^3/s$) to the discharge level in and one hour period.

One way of showing that is to increase the discharge capacity of Kärret by one unit. This will affect the generated profit by this site from third hour forward. Simulation in GAMS with the new constraint shows the total profit of 1,203,253,700 € which is 64,800 € more than the previous value. This number is compatible with the summation of values in Table 9 for the Kärret plant.

Going of the marginal values presented in Table 9, we would advice management at PowerCompany AB to focus on increasing the maximal discharge capacity of Forsen as that has a very high marginal contribution to the profit, given our original data at least.

Table 9: *Marginal Values for Maximum Discharge Constraints*

Site(Time)	Level (Mm^3)	Upper Limit (m^3/s)	Marginal (€/(m^3/s))
Ahlen(5)	540.000	540.000	18,720.000
Fjället(6)	135.000	135.000	26,460.000
Forsen(5)	975.000	975.000	46,980.000
Forsen(6)	975.000	975.000	57,420.000
Forsen(9)	975.000	975.000	42,300.000
Kärret(3)	680.000	680.000	7,200.000
Kärret(4)	680.000	680.000	4,500.000
Kärret(5)	680.000	680.000	15,300.000
Kärret(6)	680.000	680.000	17,100.000
Kärret(7)	680.000	680.000	4,500.000
Kärret(8)	680.000	680.000	2,700.000
Kärret(9)	680.000	680.000	13,500.000

Moving on, let us look at some results from (LP”). Using $P = 95$ €/MWh, we obtain the optimal profit of $1.2440440 \cdot 10^9$ € which is comparable to the original optimum for (LP) of $1.293189 \cdot 10^9$ €. Let us also look at the resulting plan. See Table 10 for the discharge plan and Table 11 for the reservoir levels. Notably, there was no spillage used.

Table 10: *Turbine discharge $V_{i,t}$ of power plant i during hour t in m^3/s for* (LP”)

Power plant / Hour	1	2	3	4	5	6	7	8	9
Ahlen	0	0	0	0	522.000	540.000	0	0	0
Fjället	0	0	0	0	33.000	135.000	0	0	84.00
Forsen	0	0	0	0	975.000	975.000	0	0	975.000
Kärret	29.000	29.000	680.000	0	680.000	680.000	84.000	0	680.000

Let us investigate the effect that the future price P has. We vary the price with increments of 50 €, from 50 € to 250 €. The resulting profits are found in Table 12. Moreover, as one would expect, we find that the plan prioritizes saving the water more as the future price increases. For instance, at $P = 200$ €, we find that the optimal plan is to only discharge at Kärret so that we remain within the maximum reservoir capacity and otherwise we save as much

Table 11: *Reservoir level $R_{i,t}$ of power plant i at the start of hour t in Mm^3 for* (LP”)

Power plant / Hour	1	2	3	4	5	6	7	8	9
Ahlen	5800.000	5800.637	5801.274	5801.912	5802.549	5801.307	5800.000	5800.637	5801.274
Fjället	1000.000	1000.101	1000.202	1000.302	1000.403	1000.385	1000.000	1000.101	1000.202
Forsen	20.000	20.029	20.058	20.086	20.115	16.634	13.272	15.666	17.638
Kärret	13.000	13.000	13.000	10.656	10.761	8.417	9.584	12.896	13.000

water as possible. This is intuitive seeing as $P = 200$ is then the best price at which we can sell. Conversely, we do not prioritize saving the water when $P = 50$. In fact, this makes the optimal plan identical to that of the original problem (LP) found in Table 6 as this makes the saved water very low valued and thus it is not worth it to change our otherwise optimal plan.

Table 12: *Resulting total profit depending on the chosen future price P for* (LP”)

Future price P (€)	50	100	150	200	250
Total profit (M €)	1203.189	1260.495	1526.366	2033.362	2540.359

Finally, let us look at the results for considering price uncertainty. Intuitively, we should, of course, get a slightly lower profit for both stochastic models. We begin by looking at the results of the EV problem (EV). Indeed, the optimal profit $9.859140 \cdot 10^8$ € is slightly lower than $1.203189 \cdot 10^9$ € for the original problem (LP). The full plan is found in Table 15 for the discharge and Table 16 for the resulting reservoir levels. Notably, no spillage is used. Before moving on to the more interesting recourse problem (RP), let us also add that the EEV is $\text{EEV} = 9.876919 \cdot 10^8$. The EEV is found by utilizing the common plan in each of the three scenarios and then averaging the resulting profit.

Table 13: *Reservoir level $R_{i,t}$ of power plant i at the start of hour t (Mm^3) for* (EV)

Power plant / Hour	1	2	3	4	5	6	7	8	9
Ahlen	5800.000	5800.637	5801.274	5801.912	5802.549	5801.307	5800.000	5800.000	5800.637
Fjället	1000.000	1000.101	1000.202	1000.302	1000.403	1000.385	1000.000	1000.000	1000.101
Forsen	20.000	20.029	20.058	16.815	16.844	13.362	10.000	10.000	12.074
Kärret	13.000	12.103	9.759	7.416	8.344	6.000	7.166	8.333	8.383

Table 14: *Turbine discharge $V_{i,t}$ of power plant i during hour t (m^3/s) for* (EV)

Power plant / Hour	1	2	3	4	5	6	7	8	9
Ahlen	0	0	0	0	522.000	540.000	177.000	0	354.000
Fjället	0	0	0	0	33.000	135.000	28.000	0	56.000
Forsen	0	0	908.778	0	975.000	975.000	665.000	0	761.000
Kärret	278.222	680.000	680.000	680.000	680.000	680.000	680.000	680.000	680.000

For the recourse problem (RP), we again find that, indeed, the optimal value $1.020554 \cdot 10^9$ € is slightly lower than the optimal value of $1.293189 \cdot 10^9$ € for the original problem (LP). It is also notably higher than optimal profit for the EV problem (EV)

Let us look at the resulting plan, see Table 15 for the discharge and Table 16 for the reservoir level. The spillage is 260 m^3/s during hour 8 for Kärret in scenario 1, otherwise 0. Indeed, there are slight differences across scenarios. For the first two hours, the plan is quite similar to that of the original problem (LP) without price uncertainty. The only difference is that we use slightly higher discharge at Kärret, but the difference is only slight.

By the above, we should find that the WS is rather similar to our $\text{RP} = 1.020554 \cdot 10^9$ € and we should get a low EVPI. Indeed, $\text{WS} = 1.021559233 \cdot 10^9$ € so that $\text{EVPI} = \text{WS} - \text{RP} = 1\,005\,233$ € which is comparatively small. For transparency, the WS was found by inputting each of the three price scenarios in the original problem (LP), solved in `Project1A_1_Rs.gms` and then averaging the resulting profits.

Before concluding, let us note that the $\text{VSS} = \text{RP} - \text{EEV} = 32\,862\,100$ €. Comparatively, this number is also not very big but it is big engough to show that management should prefer the recourse solution, they should expect to make 32 862 100 € by in so doing, compared to the EV solution.

Table 15: *Discharge level $V_{i,t,s}$ in m^3/s for* (EV). Note that for $t = 1, 2$, the discharge coincides across every scenario for each site.

Plant(Time) / Scenario	1	2	3
Ahlen(1)	0	0	0
Ahlen(2)	0	0	0
Ahlen(3)	0	159.000	345.000
Ahlen(4)	0	0	0
Ahlen(5)	540.000	0	540.000
Ahlen(6)	522.000	540.000	177.000
Ahlen(7)	177.000	540.000	177.000
Ahlen(8)	0	0	0
Ahlen(9)	354.000	354.000	354.000
Fjället(1)	0	0	0
Fjället(2)	0	0	0
Fjället(3)	0	0	5.000
Fjället(4)	0	0	0
Fjället(5)	135.000	0	135.000
Fjället(6)	33ä000	135.000	28.000
Fjället(7)	28.000	61.000	0
Fjället(8)	0	0	0
Fjället(9)	56.000	56.000	84.000
Forsen(1)	0	0	0
Forsen(2)	0	0	0
Forsen(3)	975.000	975.000	975.000
Forsen(4)	0	0	356.778
Forsen(5)	975.000	202.778	975.000
Forsen(6)	975.000	975.000	975.000
Forsen(7)	616.778	975.000	0
Forsen(8)	0	182.000	0
Forsen(9)	743.000	975.000	975.000
Kärret(1)	47.222	47.222	47.222
Kärret(2)	680.000	680.000	680.000
Kärret(3)	680.000	680.000	680.000
Kärret(4)	680.000	680.000	680.000
Kärret(5)	680.000	529.000	680.000
Kärret(6)	680.000	680.000	680.000
Kärret(7)	680.000	680.000	680.000
Kärret(8)	680.000	680.000	680.000
Kärret(9)	680.000	680.000	680.000

Table 16: *Reservoir level $R_{i,t,s}$ in Mm^3 for* (EV). Note that for $t = 1, 2$, the reservoir level coincides across every scenario for each site.

Plant(Time) / Scenario	1	2	3
Ahlen(1)	5800.000	5800.000	5800.000
Ahlen(2)	5800.637	5800.637	5800.637
Ahlen(3)	5801.274	5801.274	5801.274
Ahlen(4)	5801.912	5801.339	5800.670
Ahlen(5)	5802.549	5801.976	5801.307
Ahlen(6)	5801.242	5802.614	5800.000
Ahlen(7)	5800.000	5801.307	5800.000
Ahlen(8)	5800.000	5800.000	5800.000
Ahlen(9)	5800.637	5800.637	5800.637
Fjället(1)	1000.000	1000.000	1000.000
Fjället(2)	1000.101	1000.101	1000.101
Fjället(3)	1000.202	1000.202	1000.202
Fjället(4)	1000.302	1000.302	1000.284
Fjället(5)	1000.403	1000.403	1000.385
Fjället(6)	1000.018	1000.504	1000.000
Fjället(7)	1000.000	1000.119	1000.000
Fjället(8)	1000.000	1000.000	1000.101
Fjället(9)	1000.101	1000.101	1000.202
Forsen(1)	20.000	20.000	20.000
Forsen(2)	20.029	20.029	20.029
Forsen(3)	20.058	20.058	20.058
Forsen(4)	16.576	16.576	16.576
Forsen(5)	16.605	16.605	15.339
Forsen(6)	13.124	16.476	13.100
Forsen(7)	10.129	12.995	10.104
Forsen(8)	10.000	10.000	12.178
Forsen(9)	12.009	11.537	12.844
Kärret(1)	13.000	13.000	13.000
Kärret(2)	12.934	12.934	12.934
Kärret(3)	10.591	10.591	10.591
Kärret(4)	8.247	8.247	8.247
Kärret(5)	9.414	9.414	9.414
Kärret(6)	7.070	7.614	8.354
Kärret(7)	8.236	6.000	9.521
Kärret(8)	9.403	7.166	10.687
Kärret(9)	8.344	8.333	8.344

4 Conclusions

The results from the LP model illustrate how the revenue efficiently can be maximized by minimizing the spillage and optimizing discharge during high-price periods. However, the sensitivity analysis of the duals implies that constraints for minimum reservoir levels and maximum discharge limits still have a significant impact on the expected profitability. We believe that the analysis of the marginals is of high interest as it shows where the biggest profit increases can be made per unit of improved constraint.

When incorporating the future value of stored water, the model accounted for potential profits from electricity generated after the 9-hour horizon. This extension encouraged strategic water storage, especially in upstream reservoirs where water can generate cumulative energy as it flows downstream through multiple turbines. The model prioritized water conservation as the future price increased, demonstrating adaptive planning based on market conditions.

Finally, the stochastic programming enabled solutions that hedged against market volatility by introducing price uncertainty for three equally likely scenarios. This solution balanced profitability across the different price scenarios, reducing reliance on any single forecasted outcome. As anticipated, the stochastic models gave slightly lower profit compared to the deterministic LP model, reflecting the cost of hedging against uncertainty. Moreover, the EV model performed worse, albeit very slightly comparatively, than the 2-stage model.