Project 2- Group 2A

Ali Ghasemi	David Selin	Raouf Boussersoub
980422-8352	20010426-4932	19990129-1618
aghasemi@kth.se	dseli@kth.se	raoufb@kth.se

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Abstract

In power production, finding a balance between operational efficiency and cost minimization is a complex challenge. Power plants must carefully plan their production to meet fluctuating demand while adhering to technical constraints, startup costs, and unit dependencies. This study optimizes the 24-hour operation of a power plant with four generation units, aiming to develop a cost-effective and reliable scheduling strategy.

The deterministic model focuses on minimizing production costs under fixed demand values. The optimal schedule results in a total cost of 4026.5 kkr, with the strategy favoring extended operation of fewer units to minimize startup expenses.

Recognizing the uncertainties in real-world electricity demand, a stochastic model was implemented, assuming demand follows a normal distribution. The results show an increase in total cost to 4106.2 kkr, reflecting the price of accounting for unpredictable variations. The model also demonstrates how incorporating uncertainty affects unit commitment and external power reliance.

To further refine the strategy, the impact of external power price fluctuations was analyzed. A 50% reduction in purchase costs led to a slight decrease in total cost (4056.2 kkr), while a 50% increase resulted in a small rise to 4108.0 kkr, emphasizing the sensitivity of the system to market changes.

Finally, long-term investment strategies were evaluated by increasing the maximum output capacity of two units by 15 MW each. The most cost-effective expansion was found to be in Units 1 and 3, reducing total costs to 3994.2 kkr. This strategic investment improves efficiency and reduces dependence on external power sources.

These findings provide valuable insights into optimizing power plant operations, demonstrating the importance of balancing cost, reliability, and adaptability in power generation planning.

Problem Description

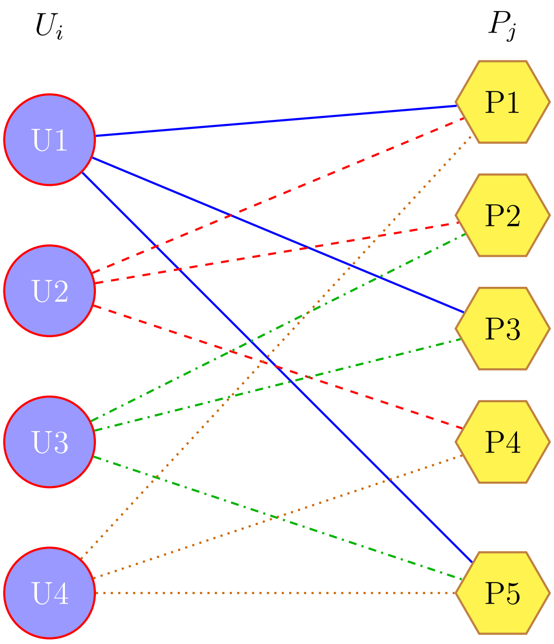
The Varme Power Plant is responsible for meeting electricity demand over a 24-hour period using four power generation units. Given the constraints on production limits, startup costs, and unit dependencies, the plant must schedule its operations to ensure a cost-optimal, cyclic production plan. The objective of this study is to design a strategy that balances economic efficiency with operational feasibility, while also addressing potential uncertainties in demand and future investment decisions.

The 24-hour period is divided into five time intervals, each with a specific power demand. These demands vary throughout the day, creating the need for a dynamic and responsive scheduling approach. The table 1 presents the expected demand for each period:

Time Period (hours)	**Expected Demand (MW)**
00:00 - 05:00	50
05:00 - 10:00	65
10:00 - 15:00	85
15:00 - 20:00	75
20:00 - 24:00	60

Table 1: Expected Demand for Each Time Period

To clarify the structure of the problem, we can represent it as a network that illustrates the possible relationships between units and demand in each period. Units are responsible for supplying the demand each period.



The primary challenge in this problem lies in determining which units should operate during each period, considering that starting a unit incurs a fixed cost and keeping it running requires an operational cost per megawatt-hour (MWh). The four available units exhibit different startup and running costs, making some units more cost-effective than others. The cost structure for each unit is outlined in table 2 :

Unit	Startup Cost (kkr)	Running Cost (kkr per MW-hour)
1	15	2.6
2	15	2.6
3	20	2.3
4	1	6.5

Table 2: Startup and Running Costs for Each Unit

It is evident that Unit 4 has a low startup cost but a high running cost, whereas Unit 3 has the highest startup cost but a lower operational expense. This creates a trade-off where the model must carefully balance when and how long to run each unit.

Furthermore, each unit has a minimum and maximum power output constraint, meaning that when a unit is turned on, it must generate at least a certain amount of power, up to a specified limit. These operational constraints are presented in table 3:

Unit	Minimum Supply (MW)	Maximum Supply (MW)
1	10	50
2	12	45
3	15	55
4	2	35

Table 3: Minimum and Maximum Supply Constraints for Each Unit

The presence of technical constraints further complicates scheduling. Each unit can run for at most three consecutive periods before it must be shut down for at least one period. Additionally, Unit 4 cannot operate unless at least one of Unit 2 or Unit 3 is running. These constraints introduce a layer of dependency between units, requiring a coordinated scheduling approach.

Beyond the Deterministic Model: Introducing Stochastic Demand

In real-world scenarios, demand is rarely known with complete certainty. External factors such as weather, industrial activity, and unforeseen power grid fluctuations can cause demand to deviate from expectations. To account for these uncertainties, the demand in each period is assumed to follow a normal distribution with the mean values given above and a standard deviation of 10 MW.

If demand exceeds the available production, additional power can be purchased from external suppliers at a cost of 10 kkr/MWh. This introduces the challenge of determining an optimal schedule that is not only cost-effective under normal conditions but also robust against demand fluctuations.

By incorporating stochastic demand modeling, the study aims to compare the cost differences between a deterministic and a probabilistic approach, evaluating how the variability in demand affects scheduling decisions.

Evaluating the Impact of External Power Prices

In addition to demand uncertainty, the cost of external power may change due to market fluctuations. To assess the impact of these variations, this study examines two alternative pricing scenarios:

1. An increase of 50% in the cost of external power.
2. A decrease of 50% in the cost of external power.

By analyzing these cases, the study seeks to determine how sensitive the optimal production schedule is to changes in external power pricing. This can provide insights into how the power plant should adapt its operations under different market conditions.

Long-Term Planning: Investment in Capacity Expansion

Finally, to support long-term strategic decision-making, this study investigates how increasing the maximum power generation capacity of two units by 15 MW each could influence the optimal scheduling decisions. This investment could provide more flexibility in meeting demand internally, potentially reducing reliance on external power purchases and improving cost efficiency.

A critical question arises: *Which two units should be upgraded to maximize cost savings while maintaining operational feasibility?* By analyzing different capacity expansion scenarios, this study aims to provide recommendations on the most effective investment strategy for the future.

Mathematical Formulation

This scenario can be formulated as a mixed-integer linear programming optimization problem. The objective is to minimize the total cost of generated power while meeting the demand during different time intervals throughout the day. Additionally, this plan must be cyclic and adhere to certain constraints, as outlined below.

Unit constraints

Verme's power plant has four units, which can be switched on or off during each of the five periods of the day. This can be formulated as a binary variable as follow:

$$u_{it} = \begin{cases} 1, & \text{if unit } i \text{ works at period } t \\ 0, & \text{if unit } i \text{ rests at period } t \end{cases} \tag{1}$$

There are two key constraints to consider regarding the operation of the units. First, each unit can operate for a maximum of three consecutive time periods before it must be shut down for at least one time period. Second, for technical reasons, Unit 4 can only operate when either Unit 2 or Unit 3 is running. These technical details can be articulated as follows:

$$u_{4,t} \leq u_{2,t} + u_{3,t}, \quad \forall j. \tag{2}$$

$$\sum_{t=1,2,3,4} u_{it}, \quad \sum_{t=2,3,4,5} u_{it}, \quad \sum_{t=3,4,5,1} u_{it}, \quad \sum_{t=4,5,1,2} u_{it}, \quad \sum_{t=5,1,2,3} u_{it} \leq 3. \tag{3}$$

Production constraints

For each unit at each period, if it operates, then the total generated power by this unit can range between certain minimum and maximum thresholds.This can be formulated with a new variable $P(i,t)$:

$$P_{it}(\text{Power output of unit i at period t})$$

$$u_{it}.P_{i,\min} \leq P_{it} \leq u_{it}.P_{i,\max}, \quad \text{for each } i \text{ and every } t \text{ in MW} \tag{4}$$

Demand constraints

Regardless of the configuration of working units, the power plant should meet a certain demand during each period. Using $P(i,t)$ this can be formulated as:

$$\sum_i P(i,t) = Demand_t, \quad \text{for every } t \tag{5}$$

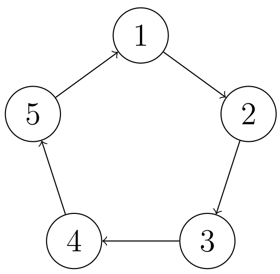
In practice, we formulate the problem in GAMS in way that production should be always greater than or equal to demand.

Warm start and Cold start

There is an initial cost when the unit is switched off during one period (warm start). If the unit is switched off during two or more periods, the initial cost increases by 50% (cold start). This can be formulated using the variable α_{it}:

$$\alpha_{it} = \begin{cases} 1, & \text{if } u_{i,t-1} = 0 \text{ and } u_{i,t-2} = 1 \\ 1.5, & \text{if } u_{i,t-1} + u_{i,t-2} = 0 \\ 0, & \text{if } u_{i,t-1} = 1 \end{cases} \tag{6}$$

To enable Verme to use our plan on a daily basis, consecutive periods should be modeled in a loop. (This is the same case for (3)):



In practice, to implement the algorithm in GAMS, we used three additional binary variables S_{it},$\mathrm{S}_{warm_{it}}$,$\mathrm{S}_{cold_{it}}$

$$S_{it} = \begin{cases} 1, & \text{if unit } i \text{ starts at period } t \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

$$\mathrm{S}_{warm_{it}} = \begin{cases} 1, & \text{if unit } i \text{ has a warm start at period } t \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

$$\mathrm{S}_{cold_{it}} = \begin{cases} 1, & \text{if unit } i \text{ has a cold start at period } t \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

Then to achieve the same results as α_{it}, we can define our constraints as follow:

Startup constraint

A startup ($S_{i,t} = 1$) occurs if unit i is off in $t-1$ and on in t:

$$S_{i,t} \geq U_{i,t} - U_{i,t-1}, \quad \forall i,t \qquad \text{(Lower bound)} \tag{10}$$

$$S_{i,t} \leq U_{i,t}, \quad \forall i,t \qquad \text{(Upper bound 1)} \tag{11}$$

$$S_{i,t} \leq 1 - U_{i,t-1}, \quad \forall i,t \qquad \text{(Upper bound 2)} \tag{12}$$

Warm Start:

Unit i was off in $t-1$ but on in $t-2$:

$$\mathrm{S}_{warm_{it}} \leq S_{i,t}, \quad \forall i,t \tag{13}$$

$$\mathrm{S}_{warm_{it}} \leq U_{i,t-2}, \quad \forall i,t \tag{14}$$

$$\mathrm{S}_{warm_{it}} \geq S_{i,t} + U_{i,t-2} - 1, \quad \forall i,t \tag{15}$$

Cold Start:

Unit i was off in both $t-1$ and $t-2$:

$$\mathrm{S}_{cold_{it}} \leq S_{i,t}, \quad \forall i,t \tag{16}$$

$$\mathrm{S}_{cold_{it}} \leq 1 - U_{i,t-2}, \quad \forall i,t \tag{17}$$

$$\mathrm{S}_{cold_{it}} \geq S_{i,t} - U_{i,t-2}, \quad \forall i,t \tag{18}$$

Cost Formulation

The total cost of production for each unit during each period consists of two components. The first component is the initial cost, which applies only if the unit has been idle for at least the previous period.This cost itself can be divided into to parts: Warm start and Cold start. The second component is the production cost incurred during that specific period which is the running cost. The sum of these two sentences across all periods and units will provide us with the objective function:

$$\text{TotalCost} = \sum_{i,t} (S_{warm_{it}} \cdot \mathrm{I}_i + S_{cold_{it}} \cdot 1.5 \cdot \mathrm{I}_i) + \sum_{i,t} (P_{i,t} \cdot \mathrm{R}_i \cdot \mathrm{D}_t) \tag{19}$$

Where:

- i (Unit's number)
- t (period's number)
- D_t (Duration of each period in Hours)
- R_i (Running cost for each unit in $\frac{kkr}{\text{MW}\cdot\text{h}}$)
- I_i (Initial cost for each unit in kkr)
- P_{it} (Production of unit i at period t in MW)

Stochastic Problem

The objective of the Stochastic problem is to identify an optimal cyclic unit commitment schedule that minimizes the expected cost of power production while accommodating stochastic demand across five time periods. Since demand is uncertain, a **two-stage stochastic programming** approach is used:

- **First-stage**: Deciding which units are on or off in each period before knowing the demand.
- **Second-stage**: Given the realized demand in each scenario, determining how much each active unit generates and whether additional power must be purchased to meet demand.

Unlike the deterministic case where demand was fixed, in the stochastic problem demand follows a normal distribution:

$$Demand_t^s \sim \mathcal{N}(\mu_t, \sigma^2) \tag{20}$$

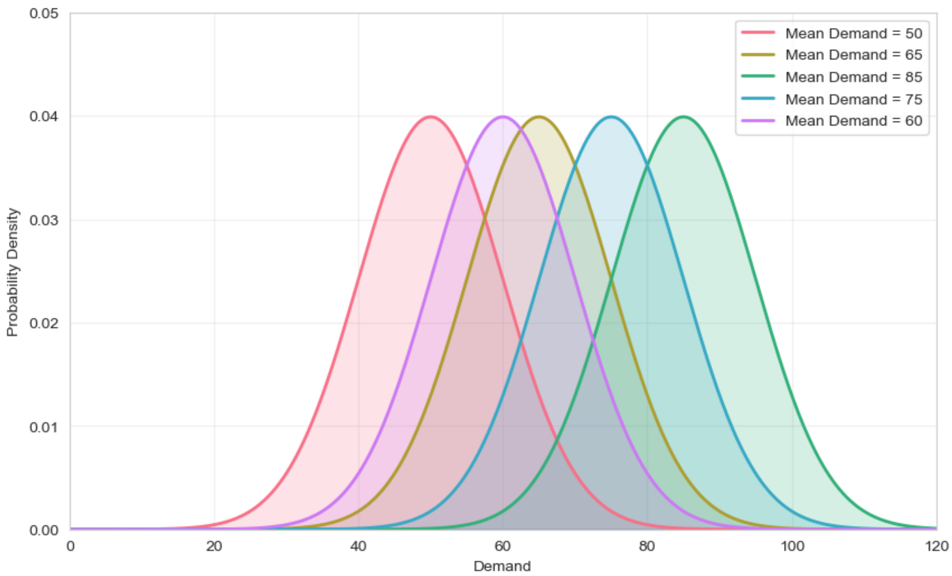


Figure 1: Distribution of demand over different periods

To solve the problem numerically, we approximate the continuous Normal distribution by generating a discrete set of 100 demand scenarios:

$$D_t^s = \text{randnorm}(\mu_t, \sigma) \tag{21}$$

where each scenario has an equal probability:

$$p_s = \frac{1}{100}, \quad \forall s. \tag{22}$$

(This ensures that demand remains random but follows the assumed normal distribution.)

First-stage decisions are consist of desicion variables $\mathrm{u}_{it}\mathrm{S}_{it}$,$\mathrm{S}_{warm_{it}}$,$\mathrm{S}_{cold_{it}}$. These decisions define the unit Commitment and are the same as the deterministic Case.

Second-stage decisions will determine power generation and purchases at each period and scenario. For this part we need to define new decision variables:

- $P_{i,t}^s \geq 0$: Power generated by unit i in period t, under demand scenario s.
- $B_t^s \geq 0$: Power purchased in period t in scenario s.

We also need to change some constraints to be compatible with different scenarios:

Demand Satisfaction for Each Scenario
In each period t and for each demand scenario s, the total power from committed units plus purchased power must meet (or exceed) demand:

$$\sum_{i \in U} P_{i,t}^s + B_t^s \geq D_t^s, \quad \forall t, \forall s. \tag{23}$$

Production constraint for each scenario)
Each unit can only generate power (if it is on) in a bounded amount:

$$u_{i,t} \cdot P_i^{\min} \leq P_{i,t}^s \leq u_{i,t} \cdot P_i^{\max}, \quad \forall i, \forall t, \forall s. \tag{24}$$

Purchasing Power at 10 kkr/MWh
If total available generation is insufficient in scenario s, we can buy power:

$$B_t^s \geq 0, \quad \forall t, \forall s. \tag{25}$$

Stochastic cost formulation

$$\begin{aligned}\text{TotalCost} = \sum_{i,t} &\underbrace{(S_{\text{warm},i,t} \cdot I_i + S_{\text{cold},i,t} \cdot 1.5 \cdot I_i)}_{\text{First-stage cost}} \\ &+ \sum_{s \in S} p_s \Bigg[\sum_{i,t} \underbrace{R_i \cdot P_{i,t}^s \cdot D(t)}_{\text{Second-stage cost: Power production}} \\ &+ \sum_t \underbrace{B_t^s \cdot 10 \cdot D(t)}_{\text{Second-stage cost: power purchase}} \Bigg]\end{aligned} \tag{26}$$

Results and analysis

Deterministic Model

Using the mixed-integer method to solve the deterministic model results in an optimal cost of 4026.5 kkr and the corresponding optimal plan is shown in Table 4.

Table 4: Power produced by each unit in different periods (MW)

Unit	Period 1	Period 2	Period 3	Period 4	Period 5
1	50.000	10.000	–	–	48.000
2	–	–	30.000	20.000	12.000
3	–	55.000	55.000	55.000	–
4	–	–	–	–	–

Figures 2 and 3 can help us gain a better understanding of the solution. From Figure 2 it is evident that units 1 and 3 are the primary contributors to production. This conclusion is consistent with the defined constraints. Unit 3 boasts the lowest running cost (2.3) among all units, despite its higher initial cost compared to units 1 and 2. By employing unit 3 for three consecutive periods, we effectively pay the initial cost only once.

Furthermore, while units 1 and 2 have identical initial and running costs, unit 1 has a higher maximum output capacity than Unit 2. This distinction, combined with fluctuating demand across various periods, clearly leads to a greater contribution from Unit 1.

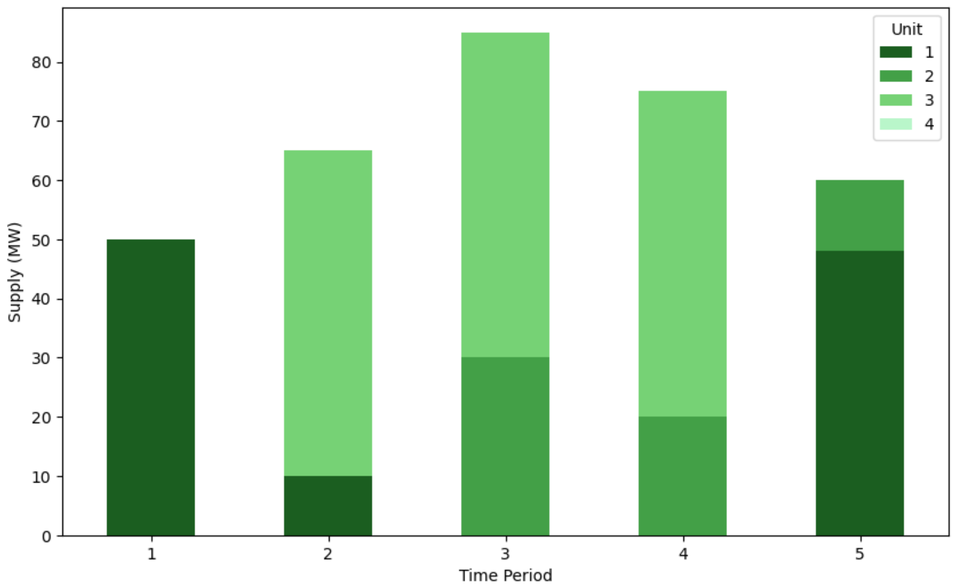


Figure 2: Contribution of units to satisfy demand at each period

Figure 3 shows that Unit 4 does not contribute to production. This is due to the fact that its running cost is more than twice that of the other units. It is also evident that for units 1, 2, and 3, we need to pay the initial cost of cold start (1.5 times of the initial cost)

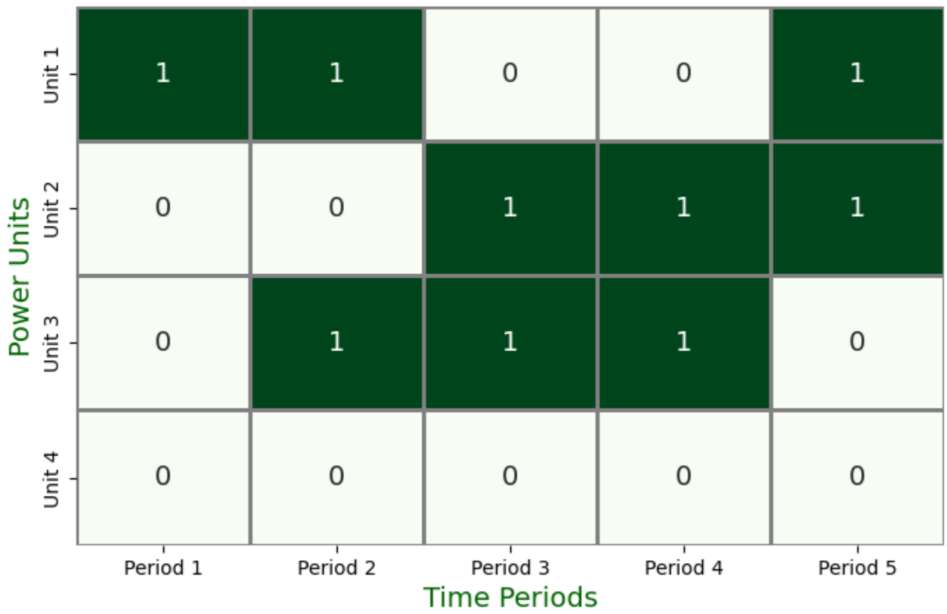


Figure 3: Unit Commitment Heatmap (ON/OFF Status)

Stochastic Model

In the deterministic model, the primary assumption is that the company has perfect information about demand in each period, and that these values remain constant. In this part, company take a more realistic approach and model the demand as a stochastic process with known distribution. Then to minimize long-run cost, it seeks a solution that minimize the expected cost. The final solution for this model is 4106.19 kkr and unit configuration is as Figure 4. The difference between this solution and the deterministic solution illustrates that it is impossible under uncertainty, to find a solution that is ideal under all circumstances. Furthermore, under this formulation we only buy extra electricity for one scenario (out of 100) in period 4 with the amount of $7.342MW$.

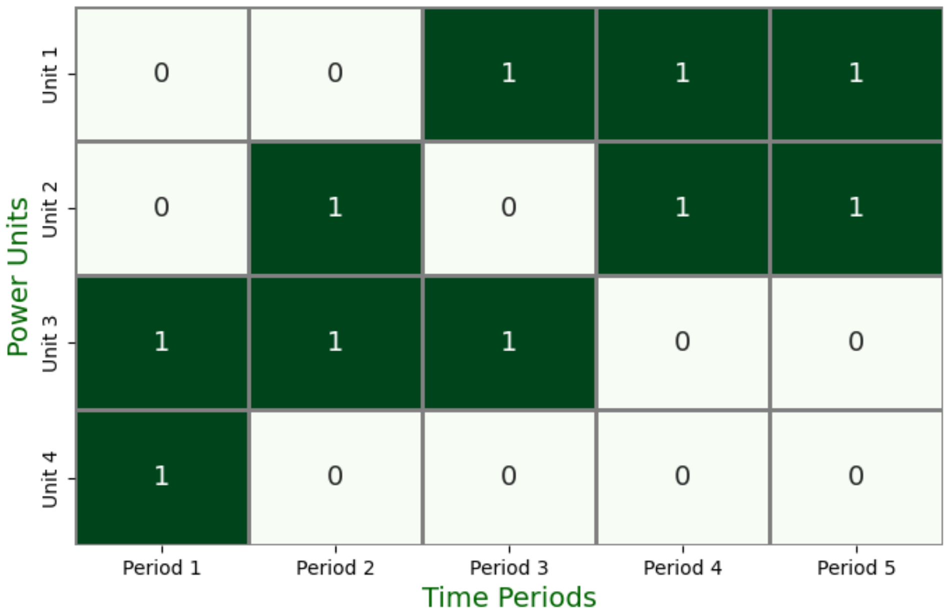


Figure 4: Unit Commitment Heatmap (ON/OFF Status)

It can be seen that the cost for stochastic model is higher than of the deterministic one. This is mainly due to the fact that model should also account for more critical scenarios (where demand is higher than mean value). This can be observed from Figure 5. In the deterministic model Unit 4 was not being used due to the high initial cost. But here, in some extreme situtaions in period 1, we need to use this unit (shown by warmer color). As a result, this unit should always be operational during this period, and the company will incur the initial cost consistently.

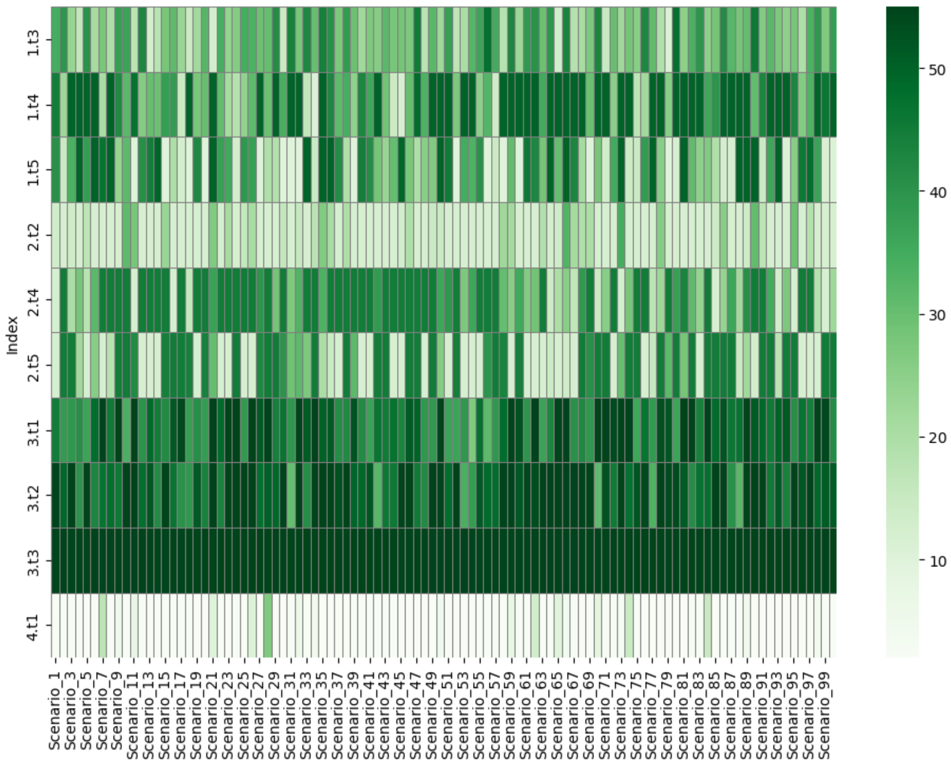


Figure 5: Power produced by unit i in period t in different scenarios (MW)

The difference between the total cost of the stochastic model and the deterministic model shows the **VSS** (Value of the Stochastic Solution).

$$\text{VSS} = \text{Cost}_{\text{stochastic}} - \text{Cost}_{\text{deterministic}} = 4106.194608 - 4026.500000 \approx 79.69 \text{ kkr} \quad (27)$$

VSS value means that when we incorporate demand uncertainty into the model, the optimal solution has an expected cost that is 79.69 kkr higher than the solution obtained from a deterministic model (which uses the average demand). This difference represents the cost of hedging against uncertainty. In other words, if we were to ignore the variability in demand and plan based solely on average values, we would expect to achieve a cost of 4026.5 kkr. However, because uncertainty exists, the best we can do is 4106.19 kkr. Thus, the additional 79.69 kkr is the price of being robust to demand fluctuations. It means that we are accepting a slightly higher expected cost in exchange for a solution that performs better across different scenarios, and reducing the risk of significant cost deviations when actual demand differs from the average.

Impact to External Power Prices

By changing the cost of purchasing additional power the optimal strategy for the different stochastic scenarios changes. First the cost was reduced by 50%, which yielded the following power purchases for the different scenarios and periods.

Case 1: Cost of additional power 5 kkr/MWh

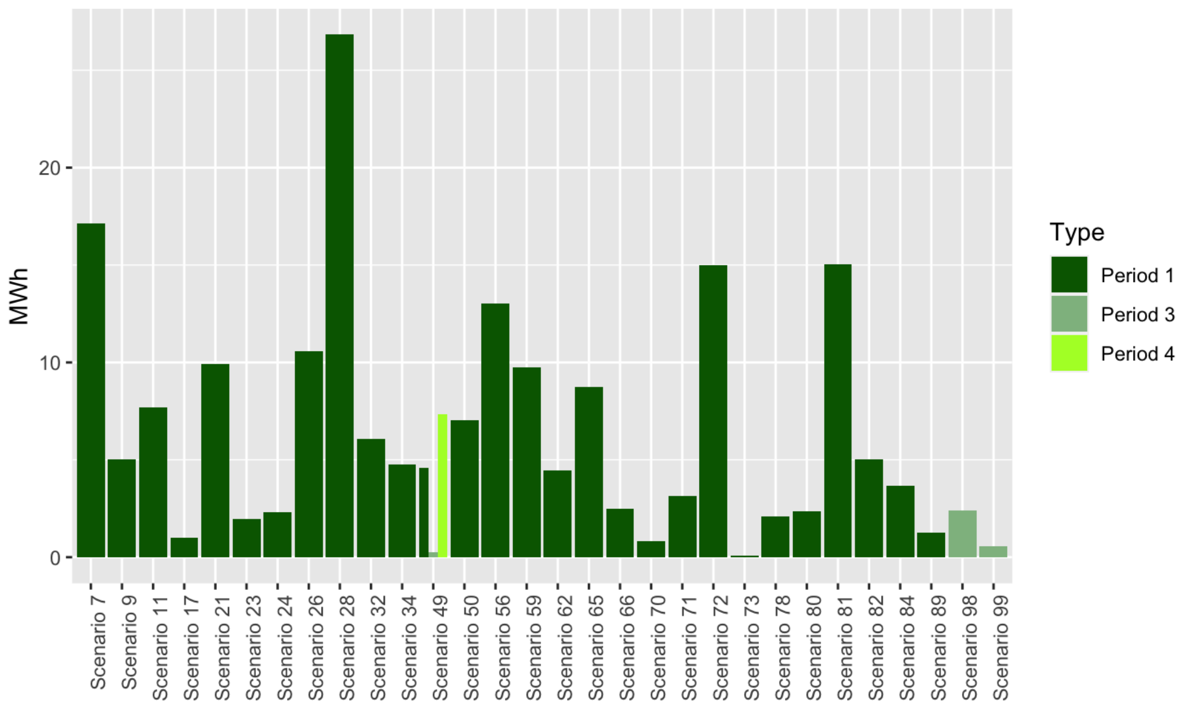


Figure 6: Additional power purchased when cost is 5 kkr/MWh

In many scenarios, as seen in Figure 6, the demand will be higher than the maximum level. In these scenarios it would be more cost efficient to buy extra power rather than turning on and running another unit. This is the case for the first period when only Unit 3 is switched on, which is shown in Figure 7.

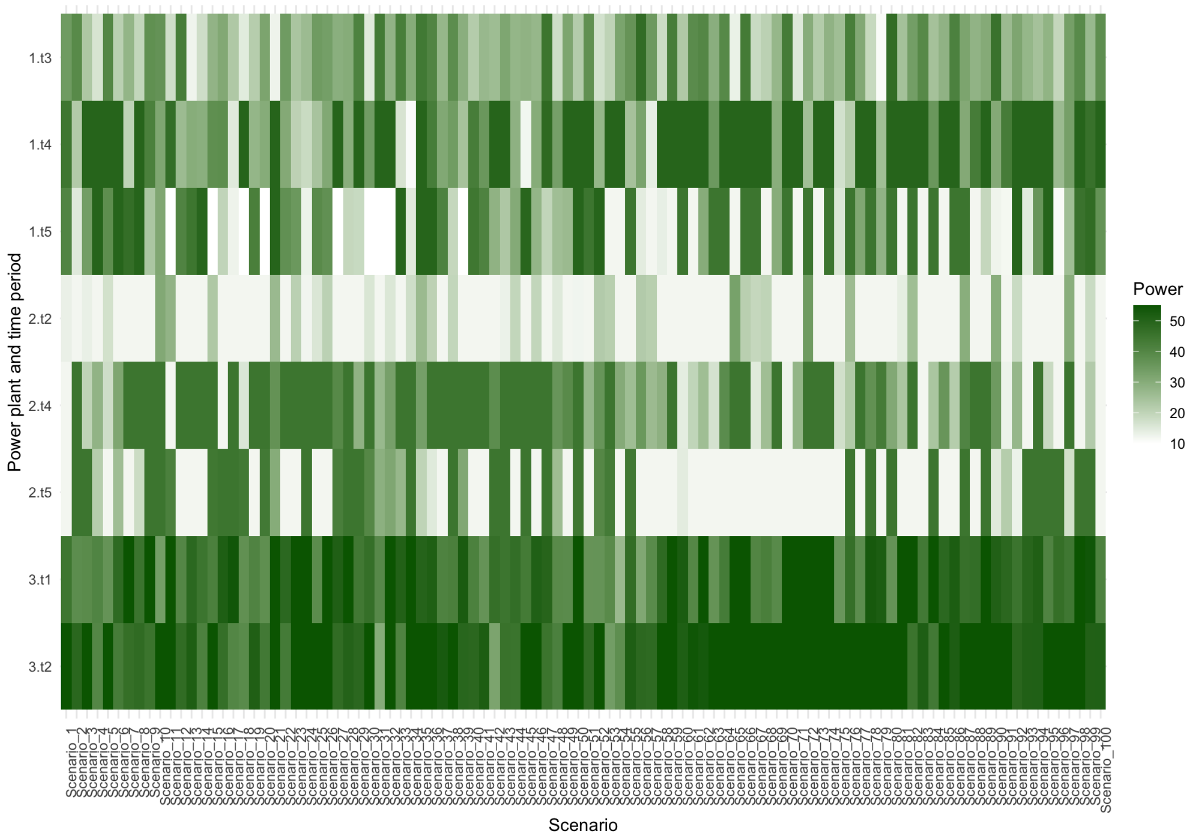


Figure 7: Power produced by unit i in period t when additional power costs 5 kkr/MWh

Figure 7 also shows that Unit 4 stopped producing electricity when additional power cost 5 kkr/MWh. If more power was needed it was cheaper to purchase additional power instead of running Unit 4. The final solution for this model is 4056.2 kkr, which is a slightly lower cost compared to the stochastic model with a 10 kkr/MWh cost for extra power. The result is intuitive, if additional power is cheaper it creates opportunities for reducing the cost of operation.

Case 2: Cost of additional power 15 kkr/MWh

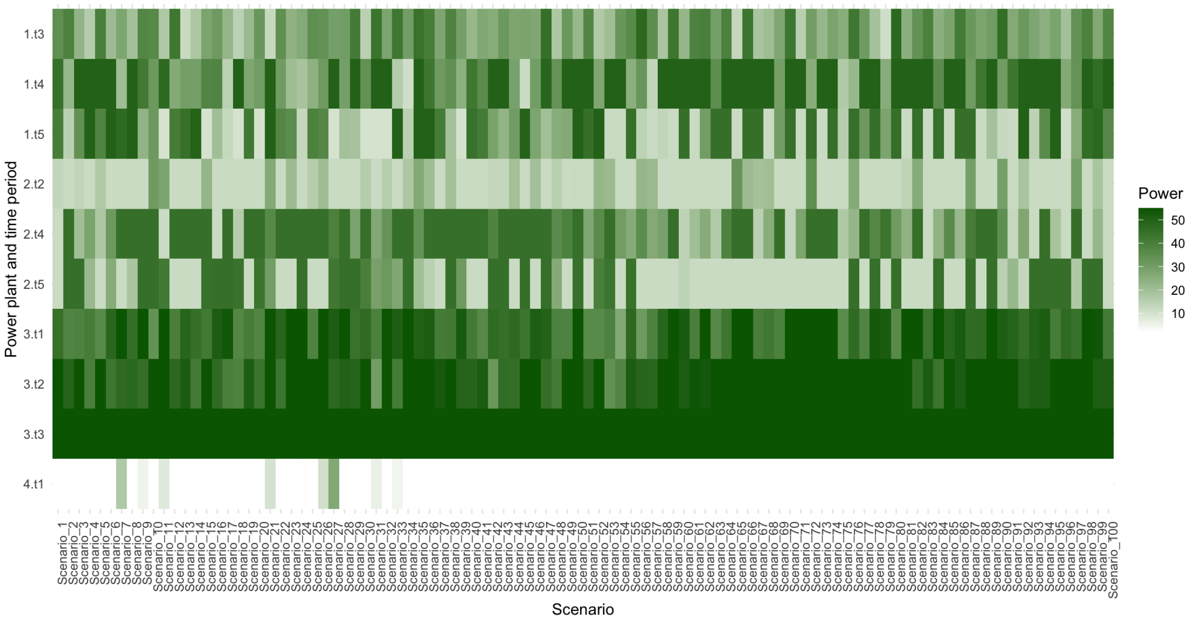


Figure 8: Power produced by unit i in period t when additional power costs 15 kkr/MWh

Figure 8 shows that when increasing the cost to purchase additional power by 50%, Unit 4 was used in some scenarios. Unit 4 has a relatively low startup cost but a high running cost, which lowers the barrier to turning it on but makes it very costly to increase production. Unit 3 had no production in period 3 for the 5 kkr/MWh case, in this case the production was at max level however. This is the case since a relatively high initial cost and a low running cost favors maximizing production if a power plant is turned on.

When looking at the additional power bought when cost of additional power, it was only in Scenario 49 that additional power was bought, 7.342 MWh in period 4. For that scenario in that period, the strategy for purchasing extra power did not change regardless if the additional power cost 5 kkr or 15 kkr. The total cost became 4108.0 kkr, a small increase in cost compared to the base case of 10 kkr/MWh which is not surprising.

Investment in Capacity Expansion

By increasing the maximum capacity of two units by 15 MW each, the optimal strategy for the different scenarios changes. The goal is to determine which two units should be expanded to achieve the lowest total cost.

To evaluate the impact of increasing capacity different combinations of two units where tested, the table below presents the total cost for each scenario.

Power plants	Cost
1,2	4029.9
1,3	3994.2
1,4	4032.9
2,3	4009.8
2,4	4049,3
3,4	4012,5

Table 5: Total cost when increasing the max capacity of two power plants by 15 MW

Table 5 show that the lowest total cost (3994.2) was achieved when the capacity of Unit 1 and 3 was increased. From the previous optimal solution, Unit 3 was consistently at its maximum production level (meaning the capacity constraint for Unit 3 was active) due to its low running cost making the model prioritize it. However due to its binding upper limit additional demand had to be met by either using other less cost efficient units or buying external power. By relaxing this constraint the model was able to shift more production to Unit 3, effectively reducing reliance on alternatives and improving cost efficiency. Similarly Unit 1 was utilized across multiple periods, indicating that its capacity constraint was active in the optimal solution. Compared to Unit 2 which has a lower maximum output, relaxing the constraint on Unit 1s production capacity allowed the model to distribute power more efficiently across time periods. By relaxing these constraints the model was able to better allocate production to the most cost effective resources reducing the reliance on less efficient units and external power.

Conclusion

This study focused on optimizing the operations of a power plant over the course of a full day while ensuring that the production plan remains consistent, allowing the same schedule to be applied every day. Developing such a plan required balancing cost efficiency with technical constraints, minimizing startup costs while effectively meeting fluctuating demand.

To address demand uncertainty, a more flexible scheduling approach was deemed necessary. Although having flexibility resulted in slightly higher costs, it ensured that the system remained robust against unexpected fluctuations, thereby reducing the risk of power shortages or unnecessary expenses while maintaining the consistency of the schedule.

The impact of external power prices also highlighted the sensitivity of the scheduling strategy to market variations. When external costs were lower, the plant was more likely to purchase power from outside sources. Conversely, higher prices led to increased reliance on internal production, underscoring the importance of adaptability in planning.

To enhance long-term efficiency, expanding the capacity of selected units emerged as a strategic decision. By increasing internal production capabilities, the plant could sustain a reliable cyclic schedule while reducing dependence on external power sources.

A key takeaway from this study is the importance of developing a cyclic, cost-effective, and adaptable production plan that can be consistently utilized each day. By integrating deterministic optimization, accounting for demand uncertainty, and making strategic investments, power plants can achieve a reliable and sustainable operational model.