Advance Neuroscience HW5

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Rescorla-Wagner Rule

Figure 1 shows the extinction paradigm. It is similar to the plot in the lecture slides. first ω rises exponentially and then after the reinforcement is gone, it reduces exponentially.

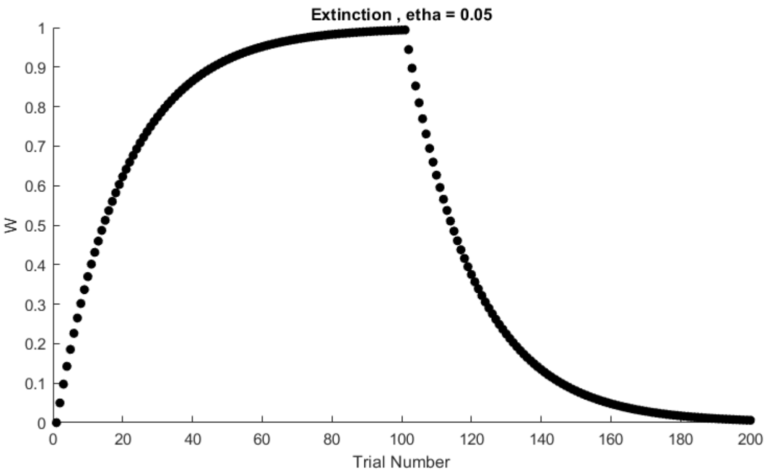

Figure 1: Extinction

figure 2 is the partial paradigm.

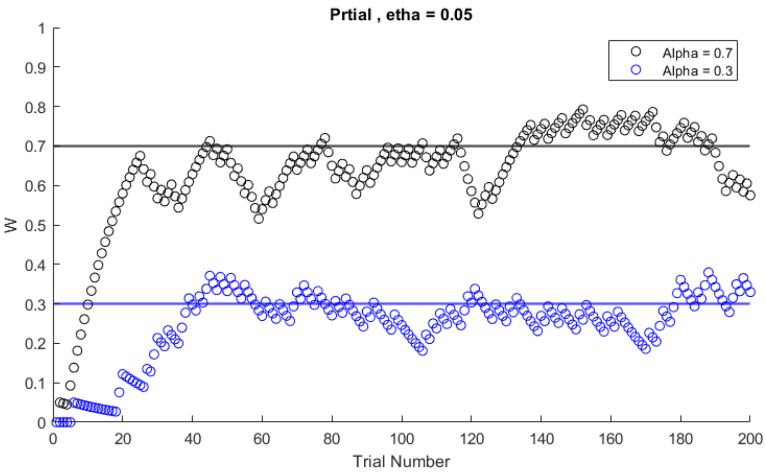

Figure 2: Partial

This figure is made for α of 0.3 and 0.7 and we can see that ω has converged to the values.

In blocking (figure 3) after the value of ω_1 is learned, the onset of simulus 2 will not affect the value of ω_1 or ω_2. Yet ω_2 rises a tiny bit.

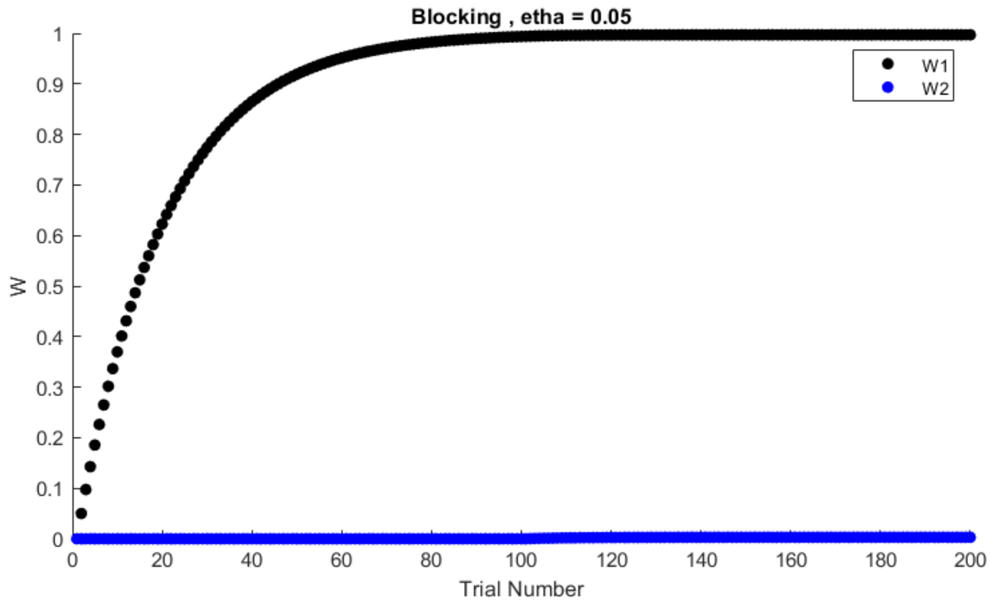

Figure 3: Blocking

In inhibitory (figure 4) of course the system will understand that the onset of stimulus 2 is reducing the reward so its value will be reduced accordingly.

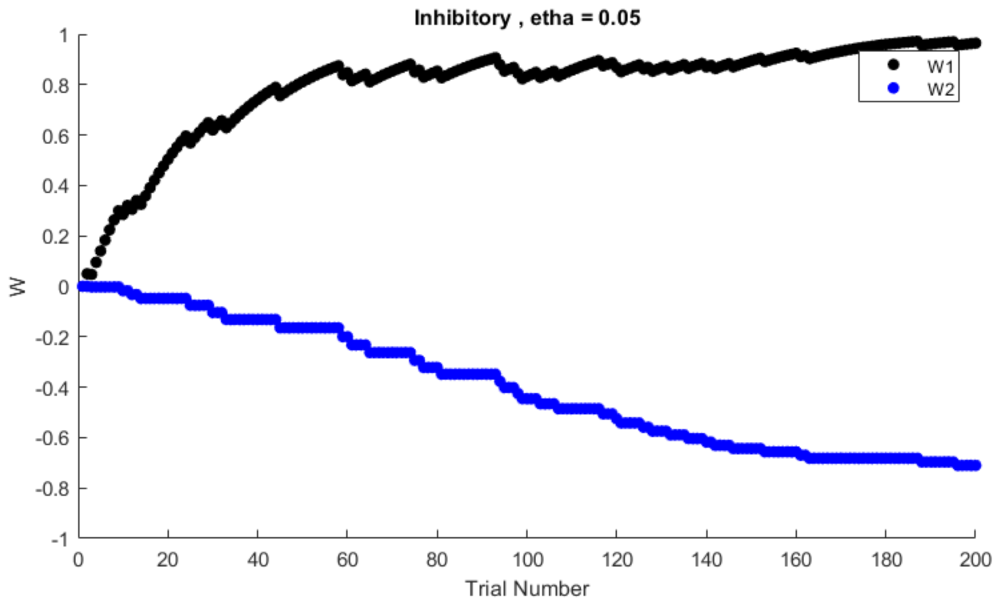

Figure 4: Inhibitory

Figure 5 is the overshadow paradigm. In this paradigm, because we have the same parameters for both S1 and S2, the values will rise similarly and in the 50% of the reward will depend on each stimulus.

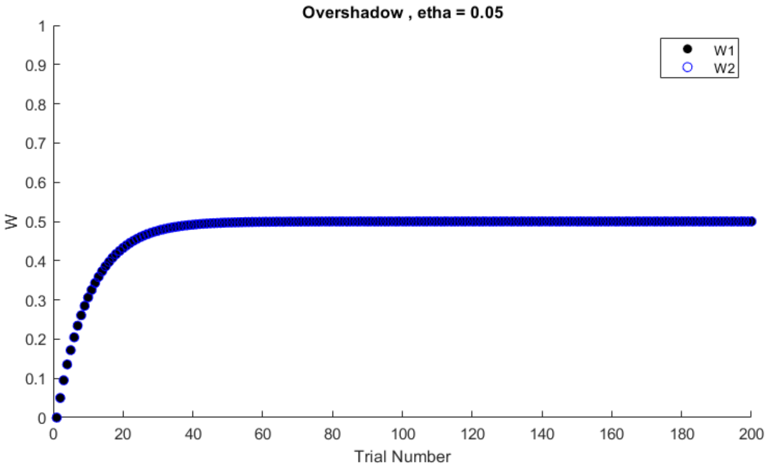

Figure 5: Overshadow

In overshadowing paradigm, most of the time two stimulus will end up having different values becuase one of the stimuli will be more silent than the other one. Of course in the delta rule both stimuli have the same associability because of the same parameters. But, we can change the associability by changing the learning rate of each stimuli so that it implies that the subject will learn more from one stimulus comparing with the other one.

Kalman Filter

Figure 1 and 2 of the paper are simulated here. The parameters are set in way that either it is noted in the paper or it is implied from the figures in the paper. Figure 6 is similar to figure 1B of the paper. Becuase of the randomness of the process, we have run the code several times to reach the one similar to paper.

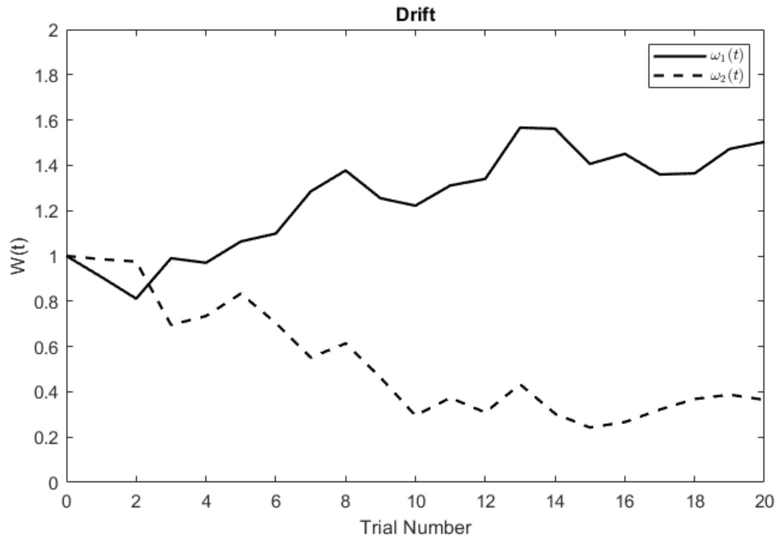

Figure 6: Drift - w0 is 1 and variance of gaussian process is 0.1

Figure 7 shows the blocking paradigm.

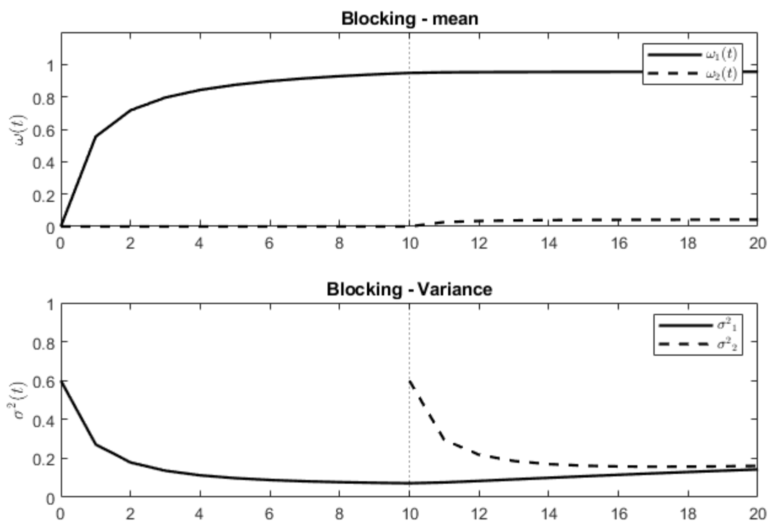

Figure 7: Blocking - W = 0.01 , τ = 0.7 , $\Sigma_0 = 0.6 \times I$

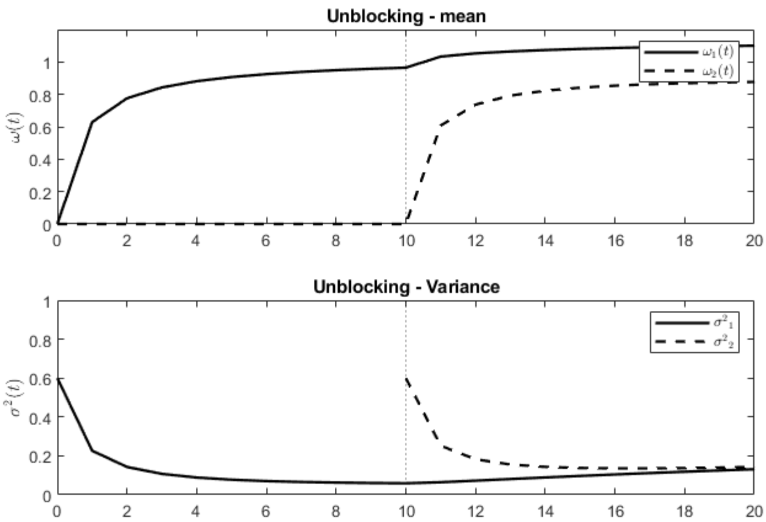

Figure 8: Unblocking - W = 0.01 , $\tau = 0.7$, $\Sigma_0 = 0.6 \times I$

In figure 8 we have unblocking and it has behaved like we expected. In case of backwar blocking we have figure 9 and figure 10.

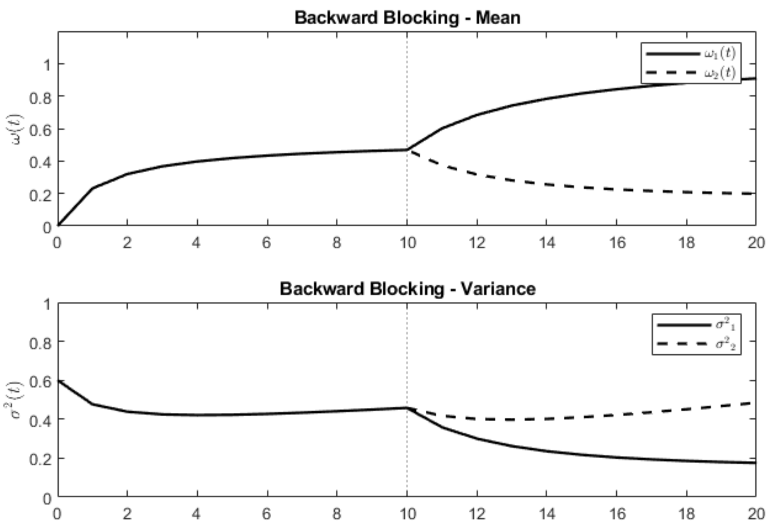

Figure 9: Backward blocking - W = 0.02 , $\tau = 1.2$, $\Sigma_0 = 0.6 \times I$

In figure 10 we have made a sample of data points (denoted by red color) using the joint distribution of w1 and w2 and also made the ellipses by matrix transformation using covariance matrix of each trial.

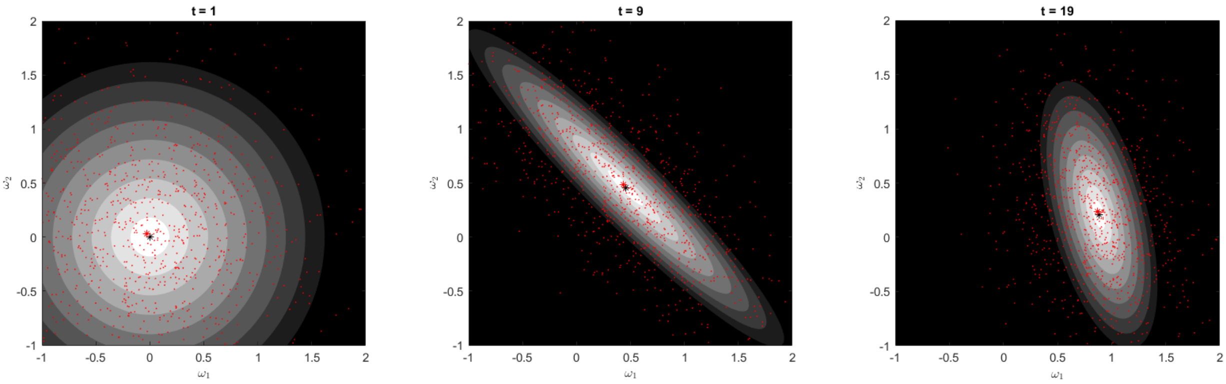

Figure 10: Backward blocking. The red points are data made by joint distribution of w1 and w2. the ellipses are made by multiplication of covariance matrix and a circle around mean w1 and w2. The black star is the mean w1 and w2 from the Kalman filter output and the red star is the mean of w1 and w2 from the sample data.

In blocking paradigm, when the process noise rises (figure 17), first we learn the w1 a tiny bit faster, because we rely more on the measurment data so we learn faster that w1 is r. Also, we can see that w2 reduces. That is because again we put more trust in measurment and the measurment implies that the S2 does not have any effect on reward (reinforcment). But if the actor wanted to make conclusion by only its process it would have thought that S2 might have some contribution in the value of reward. In addition, when process noise increases, the uncertanity about the results rises so we have a rise in variances.

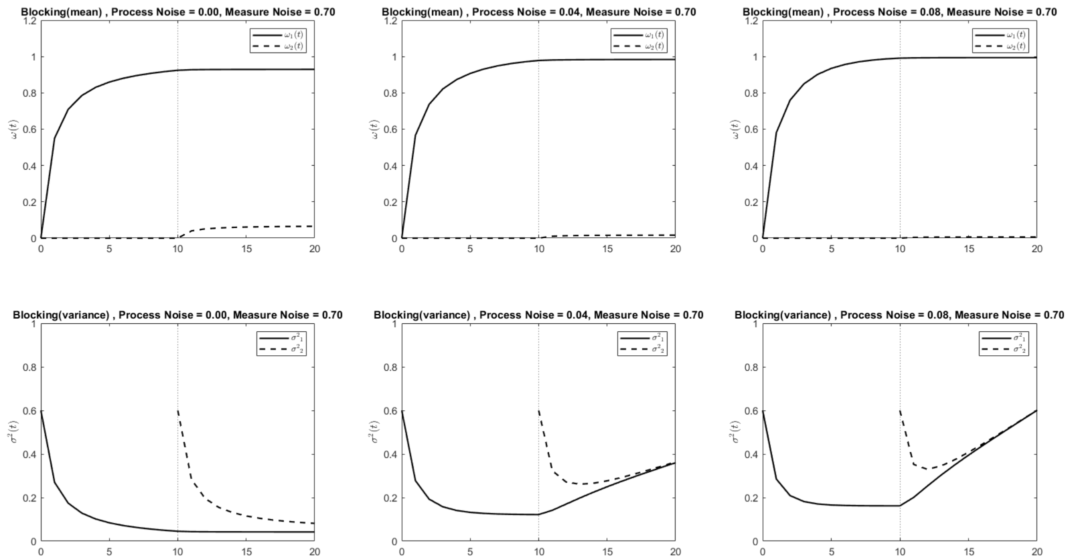

Figure 11: Bloking paradigm. Effect of process noise.

When we increase the measurment noise in the blocking paradigm, we can put less trust in what we see from the environment so we learn slower than before. Also because when the environment noise rises, we trust the self estimate more than before, w2 also increases more than before. Because, we believe that it might have some contribution to the reward. Morever, the uncertainty also rises because we have a more noisy place.

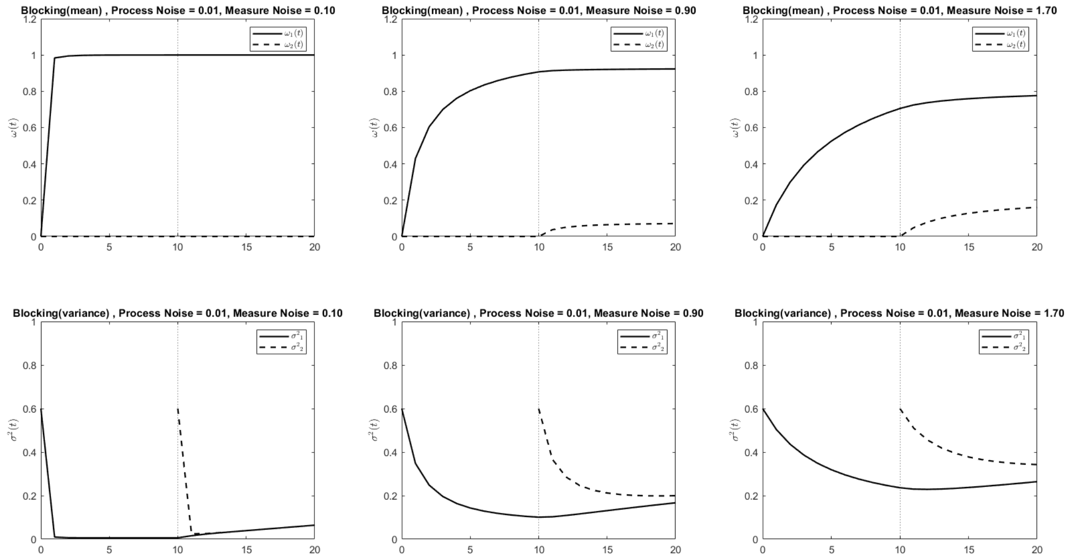

Figure 12: Bloking paradigm. Effect of measurment noise.

We also have all the same dynamics that we discussed above, for the unblocking paradigm.

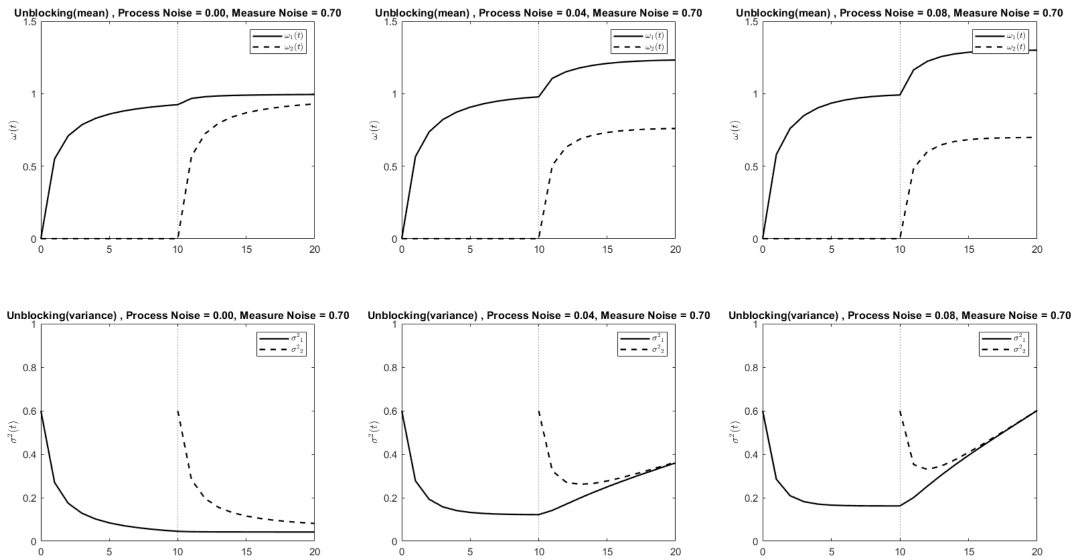

Figure 13: Unbloking paradigm. Effect of process noise.

We can see the same that we discussed before.

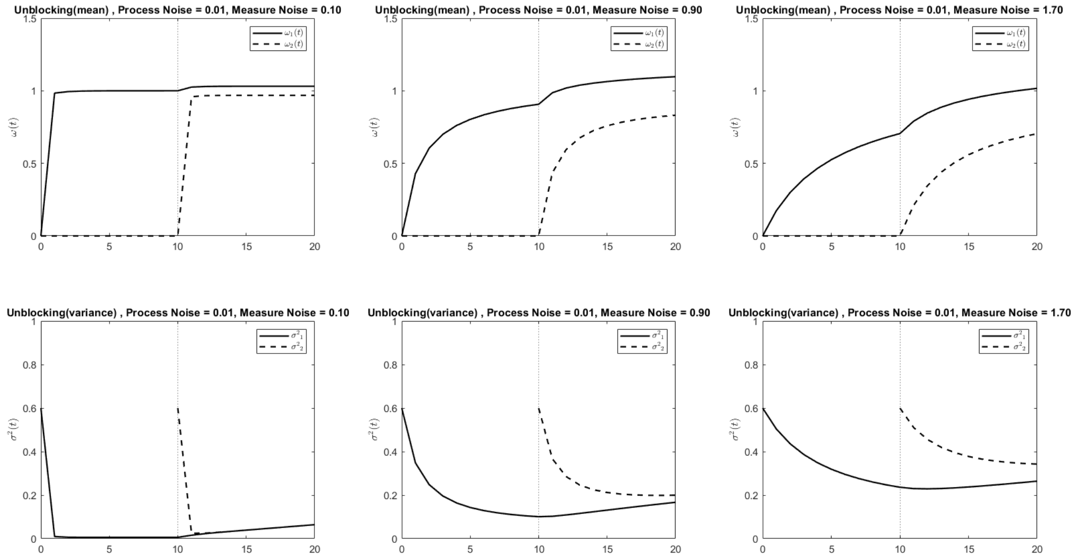

Figure 14: Unbloking paradigm. Effect of process noise.

Figure 15 shows the effect of process noise on backward blocking uncertainty. As can be seen as the nosie variance increases, the width of the ellipes become larger which means that we are more uncertain about the w1 and w2.

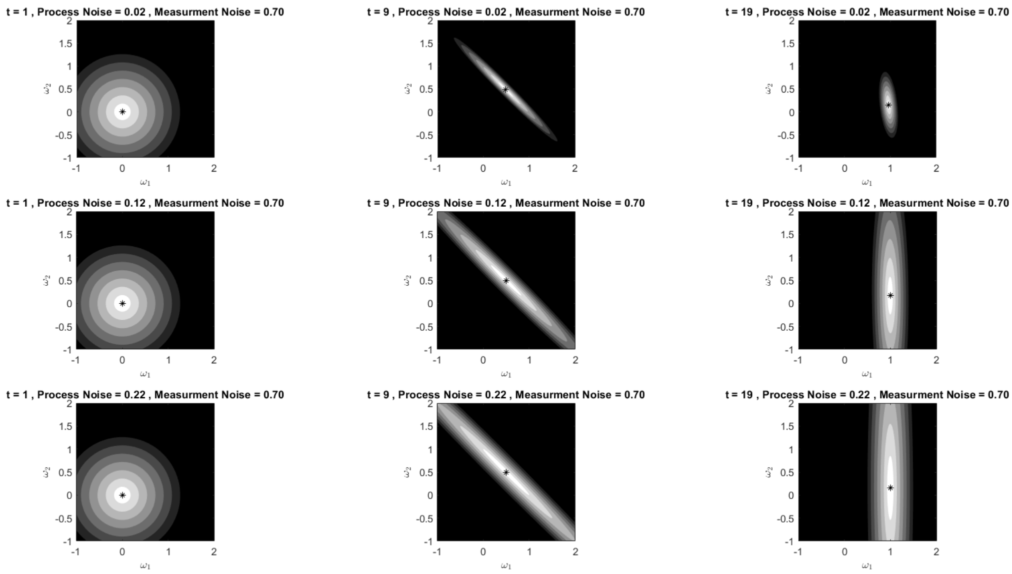

Figure 15: Unbloking paradigm. Effect of process noise.

The same can be seen for measurment noise.

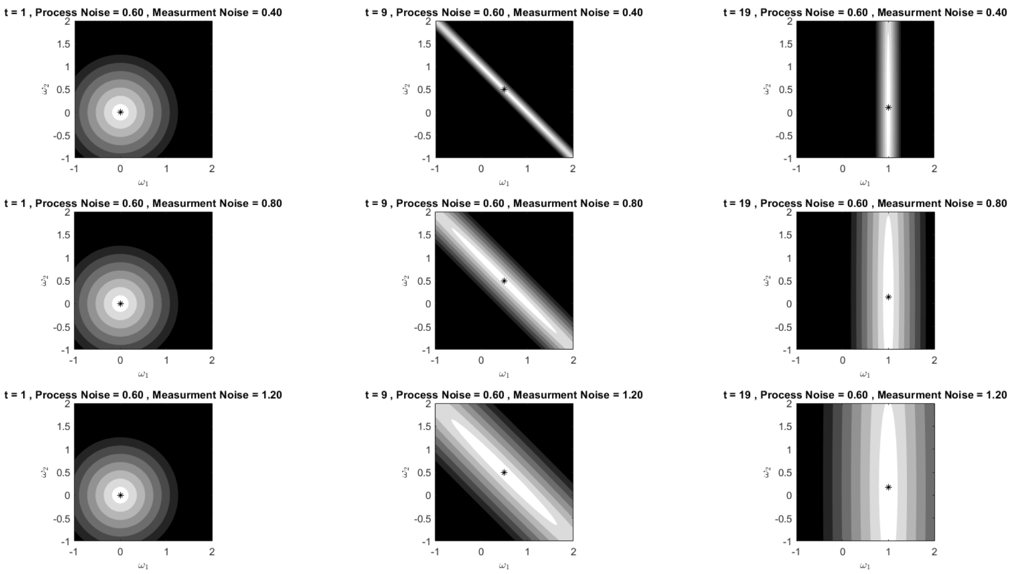

Figure 16: Unbloking paradigm. Effect of measurment noise.

Steady-State Kalman Gain:

$$\Sigma^-(t+1) = A\Sigma(t)A^T + W \tag{1}$$

$$\Sigma(t) = \Sigma^-(t) - GC\Sigma^-(t) \tag{2}$$

taking (2) in (1) we get:

$$\Sigma^-(t+1) = W + A\Sigma^-(t)A^T - AGC\Sigma^-(t)A^T = W + A\Sigma^-(t)A^T - A\Sigma^-(t)C^T(C\Sigma^-(t)C^T + V)^{-1}C\Sigma^-(t)A^T$$

Lemma:

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}$$

So,

$$\Rightarrow \Sigma^-(t+1) = W + A((Sigma^-(t))^{-1} + C^TV^{-1}C)^{-1}A^T$$

$$\Rightarrow \Sigma_\infty = W + (\Sigma_\infty^{-1} + C^TV^{-1}C)^{-}1$$

So we have:

$$G_\infty = \Sigma_\infty C^T(C\Sigma_\infty C^T + V)^{-1}$$

Question 4

In the simple Kalman filter model, changes to Σ do not depend on the observed error. So if we have a stimulus associated with a reward of 1, after some trials the model will learn that the stimulus is associated with the reward and w will converge to 1. But then if we change the value of reward (even a drastic change) it will not rise the uncertainty about the stimulus. But, in reallity it must depend on the error becuase when the reward related to a given stimulus changes we must rise our uncertainy about the stimulus and start learning about it again.

Question 5

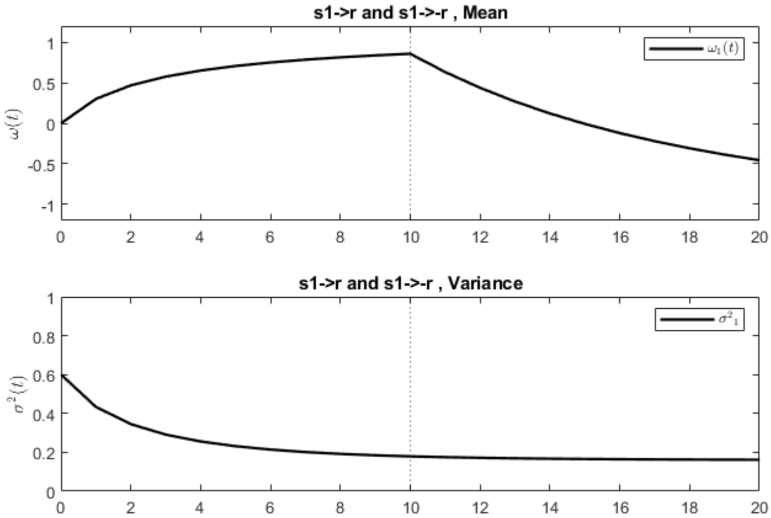

Figure 17: Error does not affect the uncertainty

in figure 17 we can see that although the reward changes in the second stage and there is an error, the incertainty about the stimulus does not change and it keep converging to its steady state value. It seems that learning rate rises in the second stage. because we go to -0.5 from about 0.7 in 10 trials.

The Improved Kalman Filter Model

By adding the NE signal to the model we get the bottom plots.

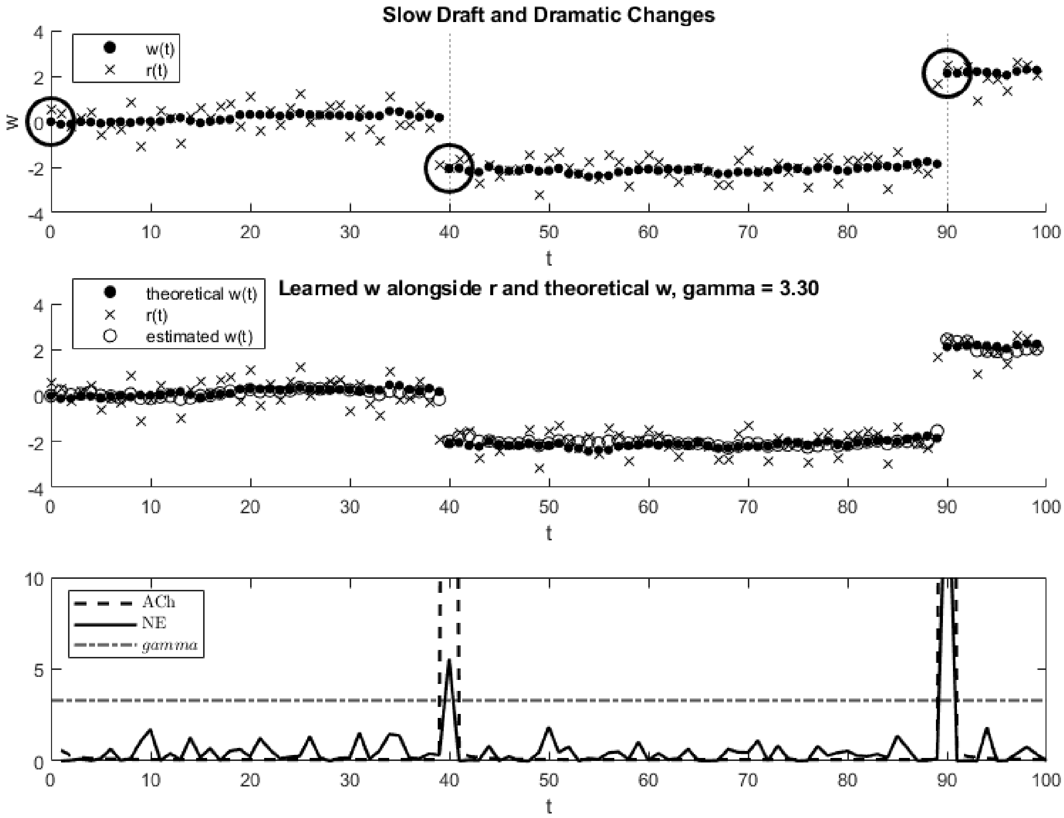

Figure 18: Similar to fiugre 3 of the paper

Also we investigate the effect of gamma on learning by calculating the MSE.

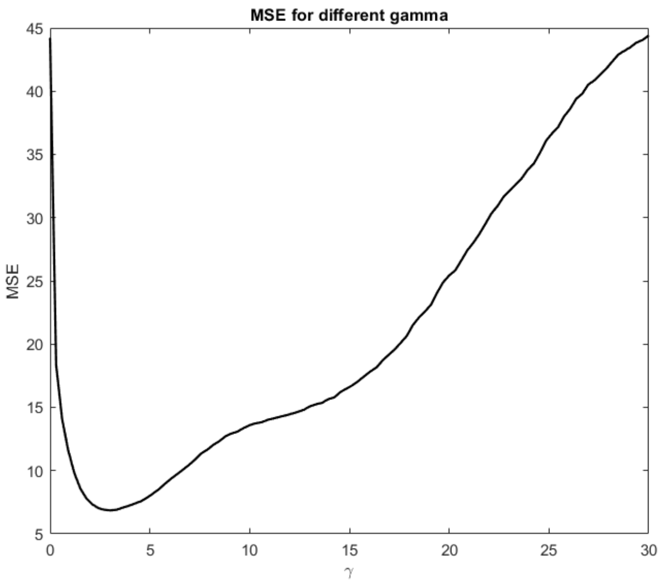

Figure 19: MSE value calculated over gamma