In the name of God

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Project : **Questions**

Motor Control

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Part 1 : Minimum Jerk Trajectories and Force Fields

In this part we are going to model straight line hand movement trajectory and understand meaning of force fields used in experiments and their effects.

Trajectories

Neville Hogan (1984) noted that smoothness can be quantified as a function of jerk, which is the time derivative of acceleration. Hence, jerk is the third time derivative of location (*i.e.*, position). If the location of a system is specified by variable ***x(t)***, then the jerk of that system is

$$Jerk \equiv \dddot{x}(t) = \frac{d^3x}{dt^3}$$

For your CNS to move your hand or some other end effector smoothly from one point to another, it should minimize the sum of the squared jerk along its trajectory. For a particularly trajectory $x_1(t)$ that starts at time t_i and ends at time t_f , you can measure smoothness by calculating a jerk cost:

$$\int_{t=t_i}^{t_f} \dddot{x}_1(t)^2 \, dt$$

Optimizing the functional above results in $\frac{d^6 x_1(t)}{dt^6} = 0$ (easy to prove. You can try). A good choice for a function with zero sixth derivative is a ***5*** degree polynomial.

1 . Assuming the ***5th*** degree polynomial as $x_1(t) = \sum_{i=0}^{5} a_i t^i$ compute a_is for a movement on a ***10 cm*** straight line occurring in ***0.5 s*** duration. Plot $x_1(t)$, $v_1(t)$ and $a_1(t)$. (the two last are velocity and acceleration respectively)

Hint: you need 6 equations. You can use initial and final states of the hand movement. Like:

$$x_1(0) = 0, \quad x_1(0.5) = 10, \quad v_1(0) = 0, \quad v_1(0.5) = 0, \quad a_1(0) = 0, \quad a_1(0.5) = 0$$

Approve conditions above in your plots.

Force Fields

Sometimes, in Motor Control experiments, there are perturbations in the environment. These perturbations are usually forces proportional to hand velocity magnitude with orientations perpendicular or parallel with velocity's orientation. (or sometimes a combination of these orientations). Model for such force fields in 2 dimensions follow notation below:

$$\boldsymbol{f} = \boldsymbol{BV} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

Which ***f*** is force vector **generated by the hand**, ***B*** is field matrix and ***V*** is velocity vector.

2 . Consider following field matrices. Using ***quiver()*** function, plot forces generated in these **fields** (read reference 1, figure.1 to find out which field is perpendicular and which is parallel to velocity) for mesh-grid of:

$$\dot{x} = -\frac{1}{2} : \frac{1}{10} : \frac{1}{2} \qquad , \qquad \dot{y} = -\frac{1}{2} : \frac{1}{10} : \frac{1}{2}$$

Field matrices:

$$\boldsymbol{B}_{standard\ curl\ field} = \begin{bmatrix} 0 & 13 \\ -13 & 0 \end{bmatrix}$$

$$\boldsymbol{B}_{opposite\ standard\ curl\ field} = \begin{bmatrix} 0 & -13 \\ 13 & 0 \end{bmatrix}$$

$$\boldsymbol{B}_{curl\ assist\ field} = \begin{bmatrix} 11 & 11 \\ -11 & 11 \end{bmatrix}$$

$$\boldsymbol{B}_{saddle\ field} = \begin{bmatrix} 11 & 11 \\ 11 & -11 \end{bmatrix}$$

3 . Now consider the movement in question 1 (***10 cm*** in ***0.5 s***); this straight line can be navigated in different directions. In an experiment we ask subjects to move their hand in 8 directions **starting from the origin**. If they follow the trajectory you found in question 1 without deviation from the straight line, plot forces applied by the field during the motion. (plot for all 4 fields in question 2). Directions are 8 main directions i.e. their angles in polar representation are $\theta_i = \frac{i\pi}{4}.\ for\ \ 0 \leq i \leq 7.\ i \in \mathbb{Z}$

You should plot all 8 trajectories and forces applied to them in 1 figure for each force field. Use ***quiver()*** function to show forces in each point.

Discuss and compare your results.

Part 2 : Learning Rules in Motor Control

In this section, we are going to study motor learning and its different properties based on the paper "Interacting Adaptive Processes with Different Timescales Underlie Short-Term Motor Learning" (Smith et al. 2006).

Savings

Savings is a fundamental property of memory. It refers to the ability of prior learning to speed up relearning even after behavioral manifestations of the prior learning have been washed out.

A typical experiment that demonstrates savings has three parts. First, a novel response to a stimulus is gradually learned over the course of many trials. Next, this stimulus-response relationship is unlearned or extinguished so that the stimulus no longer evokes the learned response. Finally, the initially learned stimulus response relationship is relearned under the original learning conditions. If savings is present, relearning will proceed more quickly than initial learning.

In order to simulate the experimental paradigm that demonstrates savings in saccade adaptation, we want to use three different proposed models and compare their results. The models are:

a) A single-state, single time-constant model:

$$x(n+1) = A.x(n) + B.e(n)$$

b) A two-state, gain-specific model

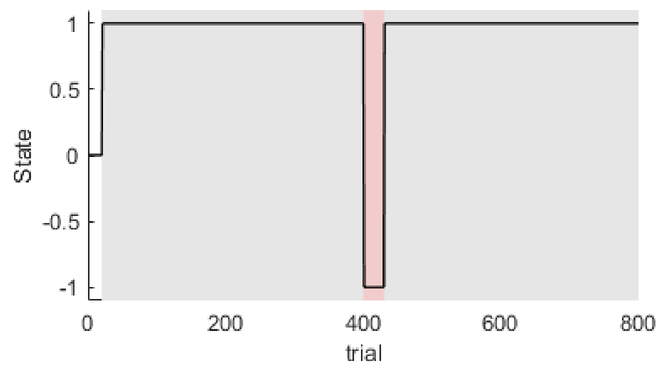
$$x_1(n+1) = \min(0, [A.x_1(n) + B.e(n)])$$
$$x_2(n+1) = \max(0, [A.x_2(n) + B.e(n)])$$
$$x = x_1 + x_2$$

c) A two-state, gain-independent, multi-rate model

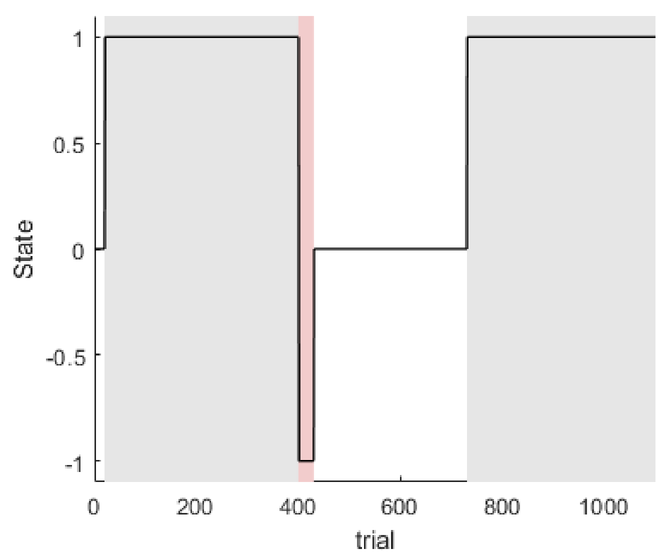
$$x_1(n+1) = A_f.x_1(n) + B_f.e(n)$$
$$x_2(n+1) = A_s.x_2(n) + B_s.e(n)$$
$$x = x_1 + x_2$$

Where $x(n)$ is the net motor output, x_1 and x_2 are the internal states, $e(n)$ is the error on trial n, A is the decay factor and B is the learning rate.

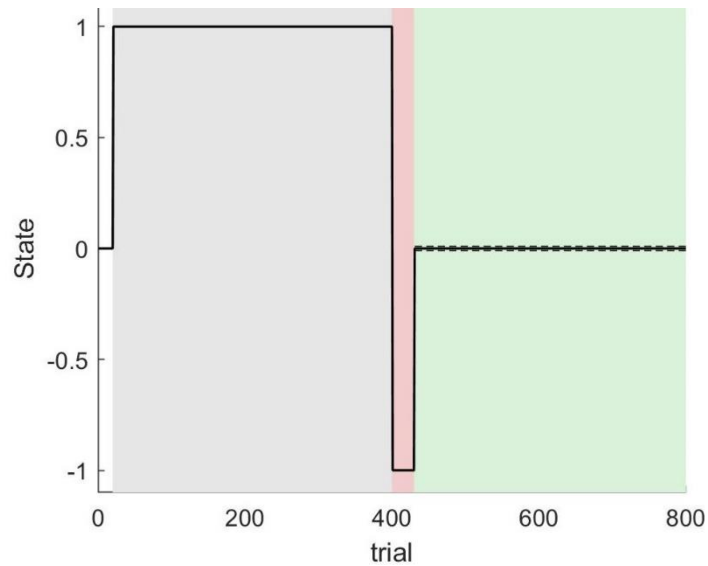
1 . Simulate the progression of motor output in a learn-unlearn-relearn paradigm (shown in the next figure) with the three models described above and compare the results.



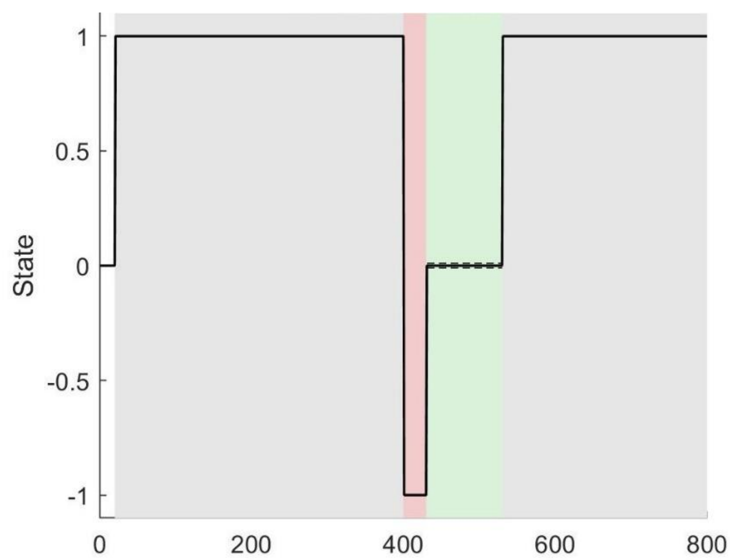
2 . Now add 1 to 300 washout trials after the unlearning trials and show the effect of the number of washout trials on the amount of savings in each model. (Washout trial is a trial where no stimulus is present). How is the amount of savings measured in the paper?



3 . Instead of relearning progresses in question 1, insert error-clamp trials (in error clamp trials force fields help subject's hand to reach the goal so the error is zero in these trials). Simulate this paradigm for the proposed models and compare the results. Explain the long-term memory's effect in the multi-rate model in this paradigm.

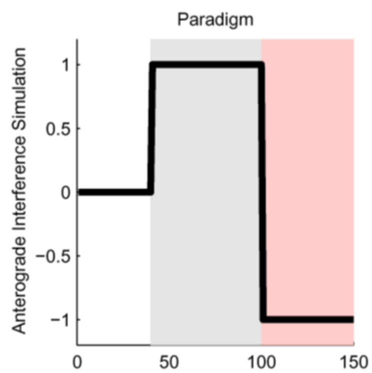


4 . Now insert relearning step after 100 error-clamp in paradigm above. Simulate this paradigm for the models and explain your observations.

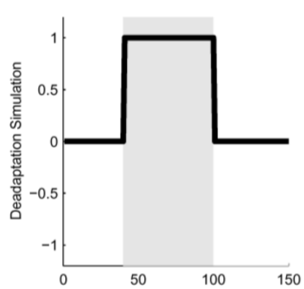


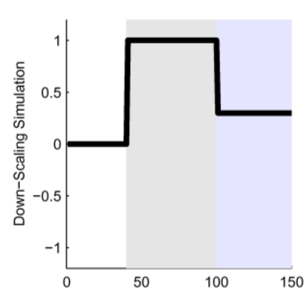
5 . ***Anterograde interference*** is a paradigm, which the force field applies exactly opposite of its forces in the learning phase in order to see the effect of learning block on the unlearning block.

Simulate this paradigm for the three models and compare the results. Simulate each model with applying opposite force field after ***150***, ***300*** and ***600*** trials and plot the amount of learning in each phase. What do you see?



6. Repeat question 5 for ***De-adaptation*** and ***Down-scaling*** paradigms. De-adaptation is a paradigm, which the force field does not apply any forces after initial learning and down-scaling is when the force filed perturbs in a same orientation but with a smaller magnitude.





7 . In the multi-rate model, we have 4 parameters. In this question we intend to see how these parameters affect spontaneous recovery paradigm. Plot amount of spontaneous recovery magnitude as 2D heat-maps of all possible choices for 4 parameters (6 plots). Plot your heat-maps parameters in range below:

$$(1 - A_s) = logspace(0.0008, 0.08, 10) \ / \ B_s = logspace(0.0018, 0.18, 10)$$

$$(1 - A_f) = logspace(0.047, 0.94, 10) \ / \ B_f = logspace(0.021, 0.84, 10)$$

Propose a way to measure spontaneous recovery magnitude.

Part 3 : Memory of Errors in Motor Learning

"The previous view of motor learning [Part 2] suggests that when we revisit a task, the brain recalls the motor commands it previously learned. In this view, motor memory is a memory of motor commands, acquired through trial-and-error and reinforcement. Here we show that the brain controls how much it is willing to learn from the current error through a principled mechanism that depends on the history of past errors." Herzfeld et al. 2014

In this part we are going to simulate and discuss mathematic model suggested by Herzfeld et al. To do so, consider model below:

$current\ trial\ number:$	n
$action\ in\ trial\ n:$	$u^{(n)}$
$perturbation\ in\ trial\ n:$	$x^{(n)}$
$sensory\ consequence\ in\ trial\ n:$	$y^{(n)} = u^{(n)} + x^{(n)}$
$beleif\ about\ environment\ \ in\ trial\ n:$	$\hat{x}^{(n)}$
$prediction\ of\ sensory\ consequence\ in\ trial\ n:$	$\hat{y}^{(n)} = u^{(n)} + \hat{x}^{(n)}$
$prediction\ error\ in\ trial\ n:$	$e^{(n)} = y^{(n)} - \hat{y}^{(n)}$

We want to learn the perturbation in environment. Learning rule used for this purpose is:

$$\hat{x}^{(n+1)} = a\hat{x}^{(n)} + \eta^{(n)} e^{(n)}$$

In which $\boldsymbol{a}$ is a constant and $\eta^{(n)}$ is ***Error-Sensitivity*** in trial $\boldsymbol{n}$. Error-Sensitivity is calculated from equation below:

$$\eta^{(n)} = \sum_i w_i{}^{(n)} g_i(e^{(n)})$$

Where $\boldsymbol{g_i()}$ s are some basis functions and $\boldsymbol{w_i}$ s are their weights. Basis functions are Gaussian functions with identical $\boldsymbol{\sigma}$ s and centered around $\check{e}_i$ s which are called preferred errors.

$$g_i\left(e^{(n)}\right) = \exp\left(-\frac{(e^{(n)} - \check{e}_i)^2}{2\sigma^2}\right)$$

Weights are also updating during learning and follow updating rule below:

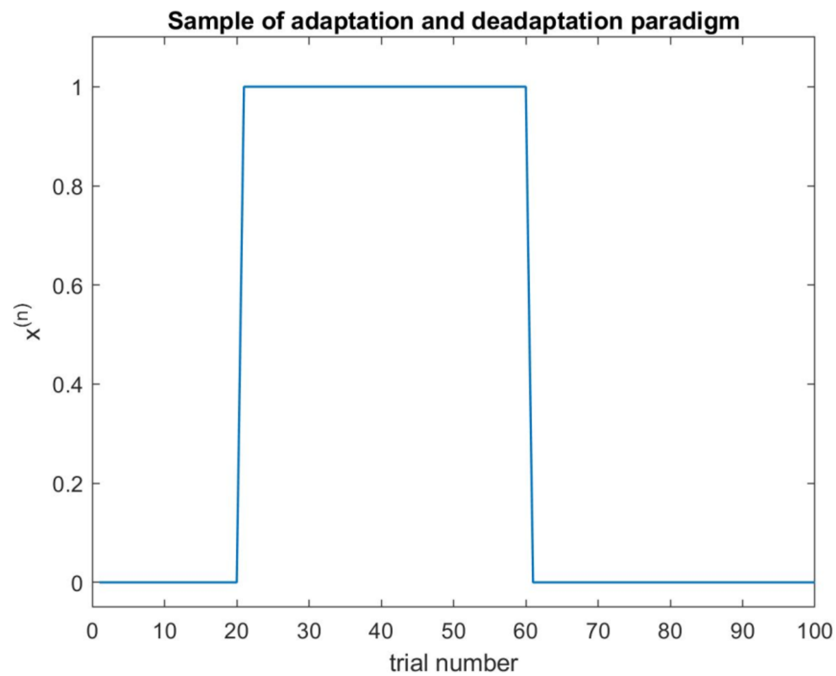
$$\boldsymbol{w}^{(n+1)} = \boldsymbol{w}^{(n)} + \beta\ sign\left(e^{(n-1)} e^{(n)}\right) \frac{\boldsymbol{g}\left(e^{(n-1)}\right)}{\boldsymbol{g}^{\boldsymbol{T}}(e^{(n-1)})\boldsymbol{g}(e^{(n-1)})}$$

Where β is a constant. (the last equation is written with the notation of vectors for brevity)

1 . Read reference 3 and explain why η is called error sensitivity? Explain how it's is related to ***Memory of Errors***. Then simulate this model for an adaptation-deadaptation learning paradigm like below: 20 null trials, 40 adaptation to 1 and 40 deadaptation to 0 trials. Assume:

$$a = 1 \qquad \beta = 0.001 \qquad \sigma = 1$$

And 10 basis functions with preferred errors distributed uniformly between *-5* and *5*. i.e. $\check{e} = linspace(-5.5.10)$. Initialize weights with $\boldsymbol{w}^{(0)} = 0.05\ ones(1.10)$.



2 . Now we want to simulate model in switching paradigms. Assume 100 trials with binary ***x(n)*** and the first trial ***x(0) = 0***. In each trial the perturbation ***x(n)*** changes with probability of ***z***. Simulate model in environments with slow (***z = 0.9***), medium (***z = 0.5***) and fast (***z = 0.1***) switching perturbations. Compare your results. (*Hint*: you must observe something like figure 3.C of reference 3)

3 . Change model parameters to settings below:

$$a = 1 \qquad \beta = 0.001 \qquad \sigma = 0.1$$

$$\check{e} = linspace(-1.1.10) \qquad \boldsymbol{w}^{(0)} = 0.05\ ones(1.10)$$

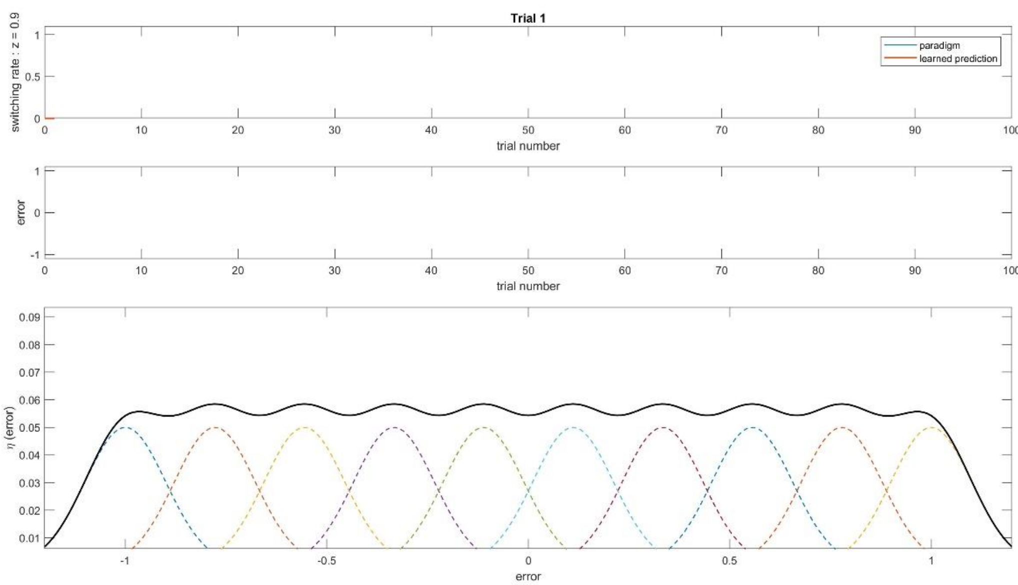
Note that sigma and preferred errors are changed.

Simulate learning in two environments with z_1 = *0.9* and z_2 = *0.1* switching probabilities for 100 trials. Save results and make a demo for each environment consisting of 3 plots:

Plot 1 must show environment perturbation and learned value of each trial together.

Plot 2 must show error in each trial.

Plot 3 must show all 10 basis functions multiplied by their weight in each trial and their summation (which is $\eta^{(n)}$) as a function of error together. A Sample for the first trial is shown below.



Analyze demos and compare them. Explain more about error sensitivity and how it is observed in the demos. Explain about $\beta\ sign\left(e^{(n-1)}e^{(n)}\right)$ effect. Why this term was added to updating rule and how it is observed in demos?

References

1 . Quantifying Generalization from Trial-by-Trial Behavior of Adaptive Systems that Learn with Basis Functions: Theory and Experiments in Human Motor Control, Donchin et al. 2003

2 . Interacting Adaptive Processes with Different Timescales Underlie Short-Term Motor Learning, Smith et al. 2006

3 . A memory of errors in sensorimotor learning, Herzfeld et al. 2014