Advance Neuroscience HW1

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Integrate and Fire Neuron

Figure 1 shows the plots of part a,b and c, before and after the removing procedure.

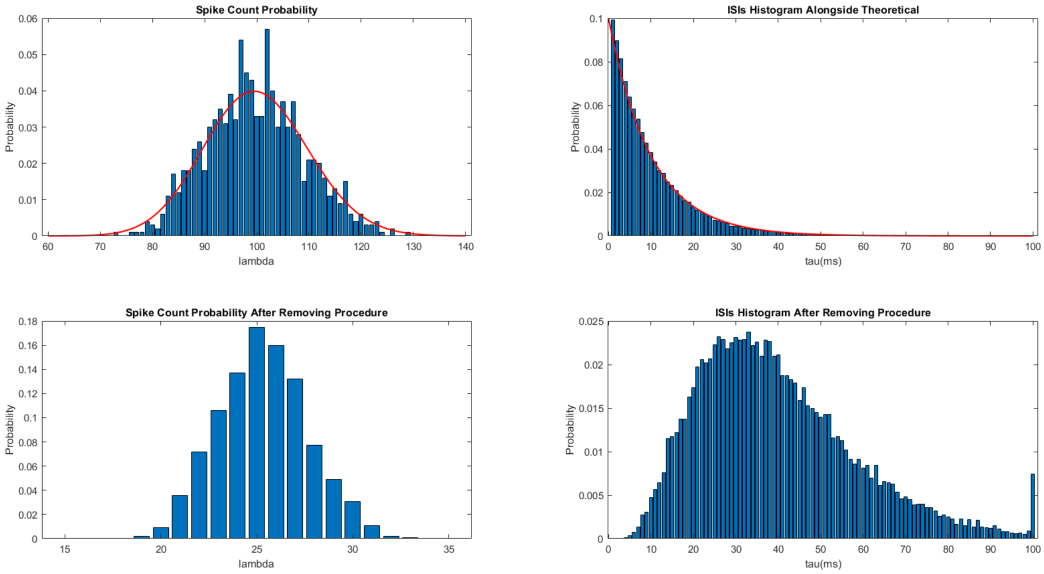

Figure 1: The top figures are before the removing procedure and the bottom figure are after the procedure.

The red lines that are fitted to the top figures are the theoretical probabilities as expressed bellow:

- **Poisson PDF**: $P(X=k)=\dfrac{\lambda^k e^{-\lambda}}{k!}$
- **ISI PDF**: $f_\tau(\tau_0)=\lambda e^{-\lambda\tau_0}$

The spike deleting procedure is similar to integration over postsynaptic input. Actually the postsynaptic neuron needs some spikes to fire a spike itself. So after getting a spike train as an input, after every k spikes in the sequence, the post synaptic neuron gets enough input to make an action potential.
If you look at the bottom right figure of the figure 1, it can be seen that the ISI distribution after the procedure looks like the gamma distribution.

Part d.
C_v poisson process theoretically equals to 1. By calculating the coefficient of variation from the simulated data, we get the number 0.9524 before the procedure and 0.4779 after the procedure.

Part e.
Let the intra spike interval (ISI) of poisson process be the sequence $\boldsymbol{X} = (X_1, X_2, ...)$ and also the ISI after the procedure be $\boldsymbol{T} = (T_1, T_2, ...)$. By the definition we know:

$$T_k = \sum_{i=1}^{k} X_i$$

The X_is are i.i.d variables with exponential distributions, So:

$$E[T_k] = E[X_1 + X_2 + X_3 + ...] = \frac{k}{\lambda}$$

Also,

$$Var[T_k] = Var[X_1 + X_2 + X_3 + ...] = \frac{k}{\lambda^2} \Rightarrow std[T_k] = \frac{\sqrt{k}}{\lambda}$$

So we have:

$$C_v = \frac{std[T_k]}{E[T_k]} = \frac{\sqrt{k}/\lambda}{k/\lambda} = \frac{1}{\sqrt{k}}$$

Part f.
As reported in the figure 3 of the Softky and Koch paper, the system is non-stationary. CV value changes depending on the ISI average. At lower firing rates in V1, the CV is between 0.5 and 1 and at high firing rates it is less than 0.5. So it is obvious that the variability of the real data is different than the variability of simulated data. It could be due to the firing adaptation of the neurons.

Part g.
Based on the equation [14] of the paper, we can reconstruct the figure 6 which can be seen in figure 2

$$C_v = \frac{1}{\sqrt{N_{th}}}(\frac{\overline{\Delta t} - t_0}{\overline{\Delta t}})$$

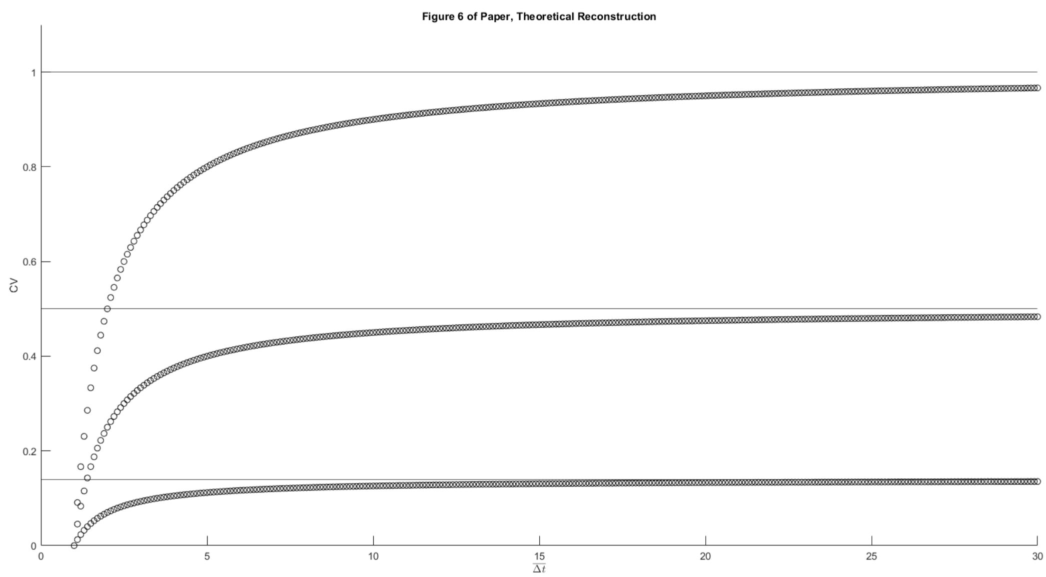

Figure 2: Reconstruction of the figure 6 of the paper based on the equation [14]

In the next step, the figure 6 of the paper is reconstructed based on simulation data. A vector of mean frequency in the range $[30, 999]Hz$ is defined and for each frequency the CV is calculated. The same procedure has been done for $N_{th} = 1, 4$ and 51 and the result is shown in figure 3.

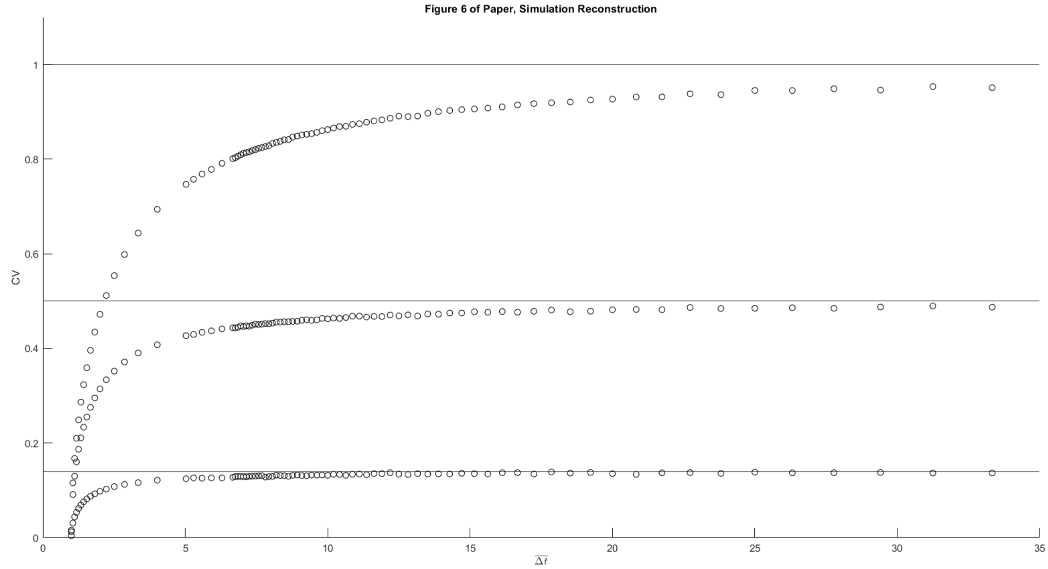

Figure 3: Reconstruction of the figure 6 of the paper based on simulation. Respectively from the top we have: $N_{th} = 1, 4, 51$

Leaky Integrate and Fire Neuron

Part a.

Result of the simulation can be seen in fiugre 4. Here $\tau_m = 13ms$ as it is mentioned in the paper.

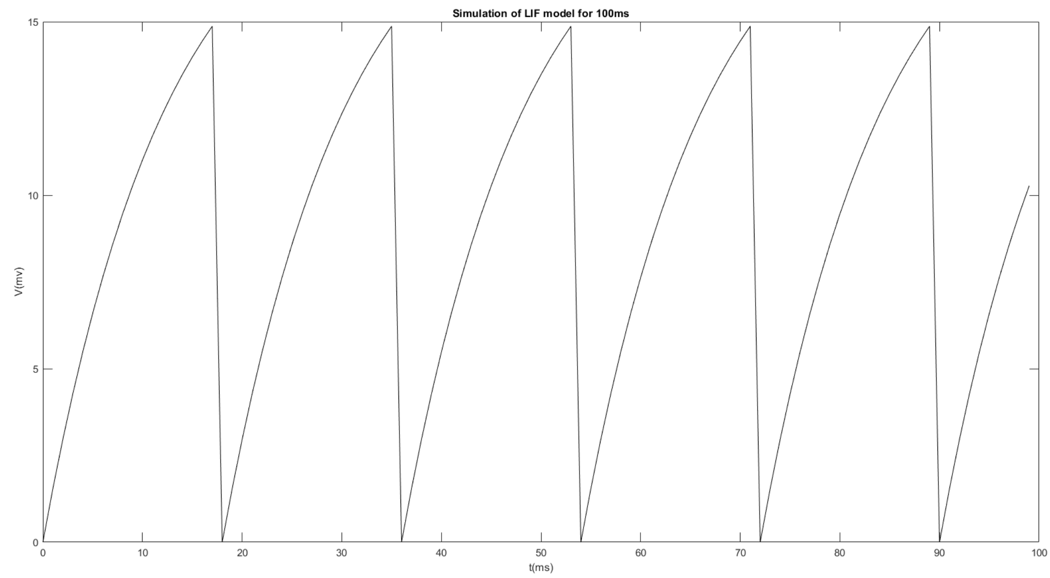

Figure 4: LIF Neuron Simulation, $\tau_m = 13ms$ and $RI = 20mV$

Part b.
By solving the differential equation:

$$v(t) = RI(1 - e^{\frac{-t}{\tau_m}}) \rightarrow \text{ Reaching the threshold: } v(t) = RI(1 - e^{\frac{-t}{\tau_m}}) = v_{th}$$

$$\Rightarrow 1 - e^{\frac{-t}{\tau_m}} = \frac{V_{th}}{RI} \Rightarrow e^{\frac{-t}{\tau_m}} = 1 - \frac{v_{th}}{RI} \Rightarrow \frac{-t}{\tau_m} = ln(1 - \frac{v_{th}}{RI}) \Rightarrow t = -\tau_m ln(1 - \frac{v_{th}}{RI})$$

$$\Rightarrow T = t + \Delta t_r = -\tau_m ln(1 - \frac{v_{th}}{RI}) + \Delta t_r \Rightarrow f = \frac{1}{T} = \frac{1}{-\tau_m ln(1 - \frac{v_{th}}{RI}) + \Delta t_r}$$

Part c.
One trial of the simulation can be seen in figure 5.

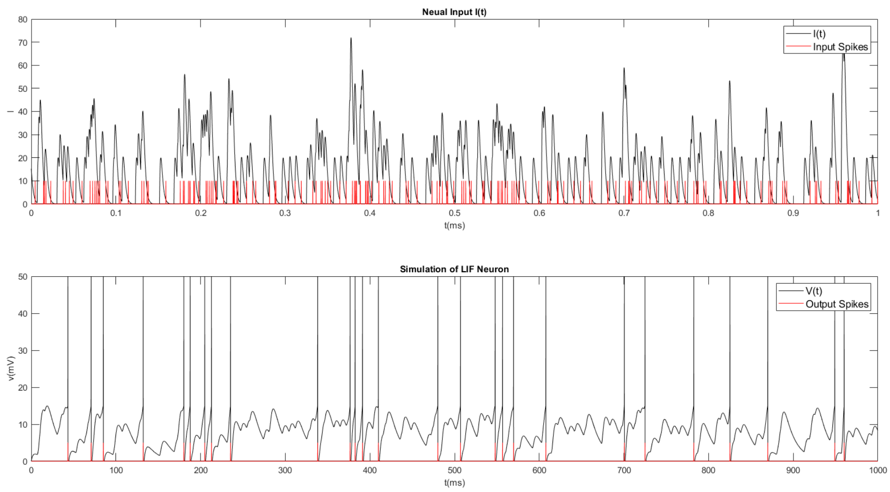

Figure 5: LIF Neuron Simulation With General Input $I(t)$, $\tau_m = 13ms, R = 1m\Omega, v_{th} = 15mV, v_r = 0, t_r = 1ms, \tau_{peak} = 1.5ms, f_r = 200$

In order to reconstruct figure [8] of the paper, first we have to make sure the output mean firing rate stays around $200Hz$. A set of number of frs are found so that the mean firing rate stays at $200Hz$ for each τ_m. Then for every τ_m we simulate the model for 10 seconds and 15 trials. In each trial, every N_{th} spikes are held and the others are deleted. The the CV is calculated. The result can be seen in figure 6.

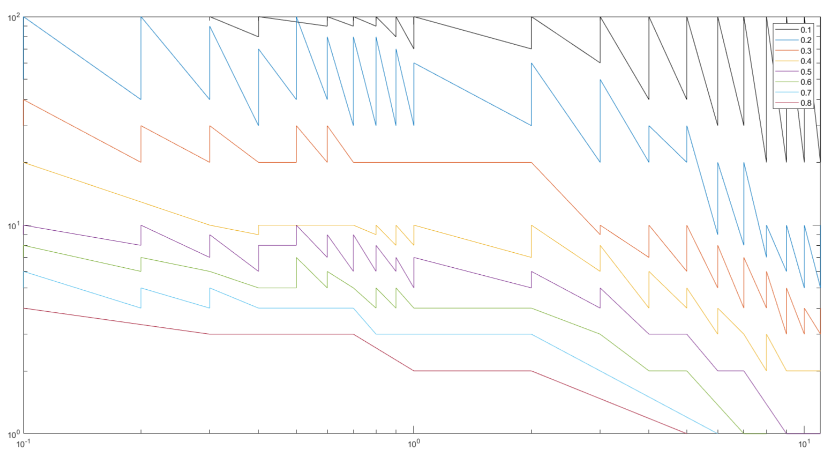

Figure 6

As N_{th} becomes smaller, CV becomes larger. (fig. 6) This means that N_{th} has an opposite relation with CV. Also just like the figure [8] of the paper, if we move from top right to the bottom left in figure 6, CV becomes larger.

Also figure 7 explains the effect of width and magnitude of the EPSCs on CV. The top panel shows that CV slightly increases as the magnitude of EPSCs gets bigger and it marginally gets near $CV = 1$. The bottom shows that τ_{peak} or width of EPSCs does not have any effects on CV.

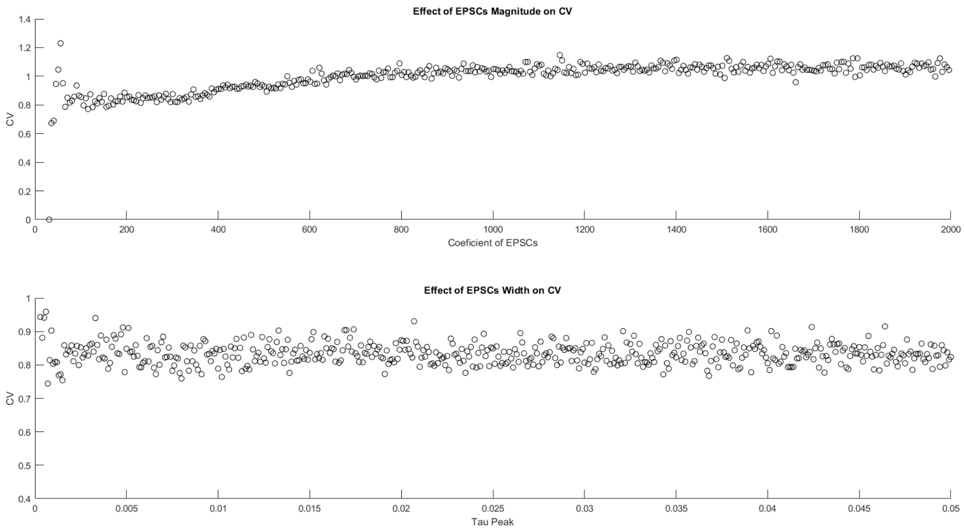

Figure 7: τ_{peak} in the top panel equals to $1.5ms$. Magnitude coefficient in bottom panel equals to 100

Part d.

Figure 8 show the effect of the percentage of inhibitory inputs on CV. Inhibitory and excitatory inputs are constructed using poisson generator function and $f_r = 100$ in both of them. An important thing is that the output neuron does not fire if the percentage of inhibitory neurons is greater than 50%. So the plot only contains the number of percentages up to 40%. As the figure declares, CV rises exponentially as percentage of inhibitory neuron increases.

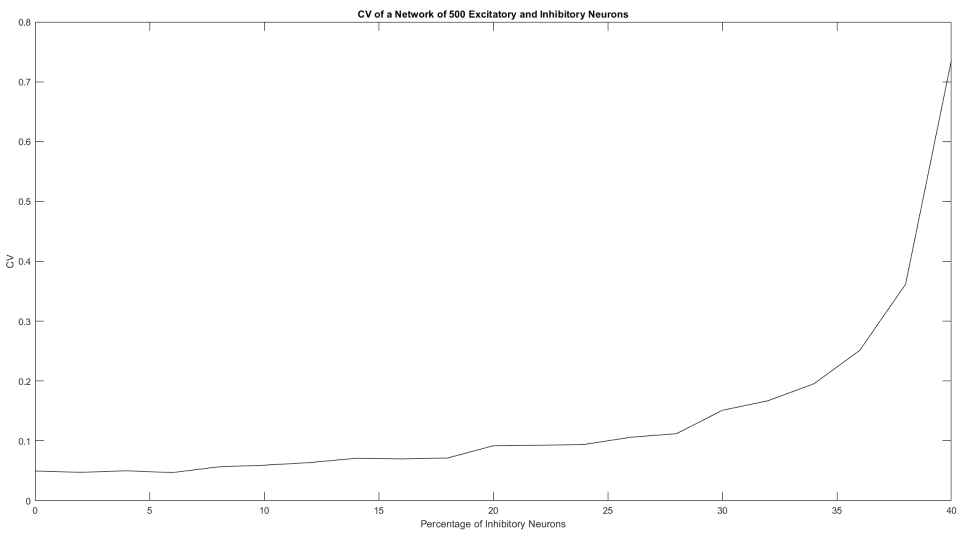

Figure 8: Effect of inhibitory neurons on CV.

Part e.
Figure 9 shows the effect of N/M and the window D on CV. The x axis of the plot is in logarithmic form. Increasing D causes a decrease in CV. Looking at input spikes through a larger window will increase the probability of spiking significantly therefore firing becomes more regular (because in every window there is a spike) and CV decreases. Also if we move upward in the plot at every D we will reach higher CVs. This could be due to the fact that needing more inputs to make an action potential has a less probability so the spiking becomes more stochastic and iregular Therfore CV rises.

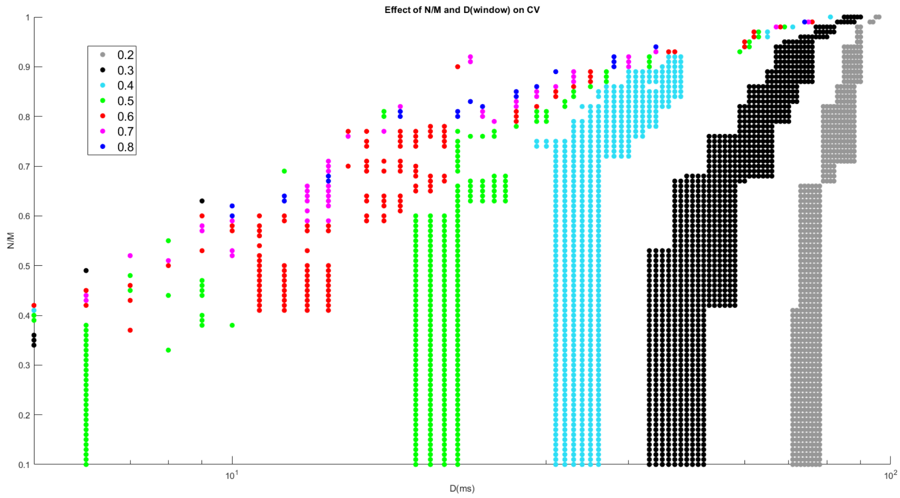

Figure 9: Effect of inhibitory neurons on CV.

Part f.
Figure 10 shows the effect of the threshold and the window D on CV. The figure was made with the same procedure as figure 9. As discussed in the top, when D increases, CV drops and we can see the same effect here. Also the threshold works as same as the N/M in the last question. Moving upward in the plot (increasing threshold) leads to an increase in CV. This happens because increasing the threshold reduces the probability of output neuron to fire. For example think of the extreme threshold that all excitatory neurons must fire and all inhibitory ones must remain silent. This example clarifies how the probability of firing decreases.

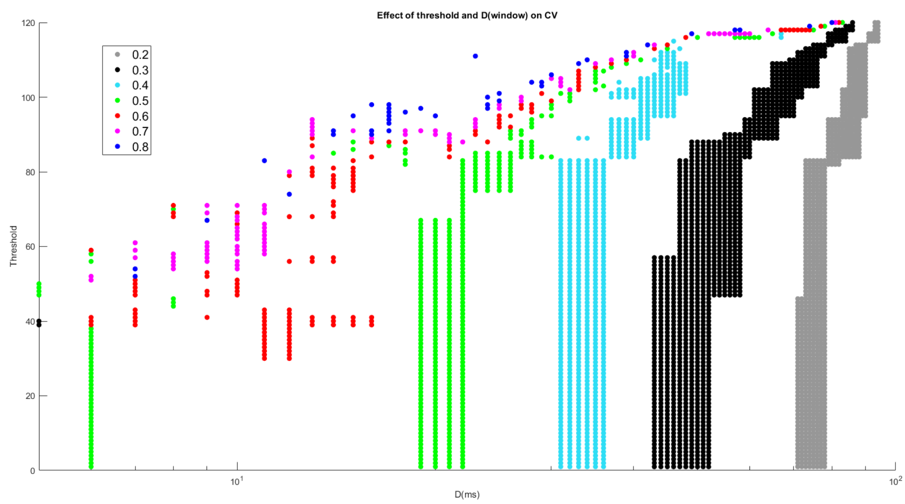

Figure 10: Effect of inhibitory neurons on CV.