Statistical Pattern Recognition
Assignment 1

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Abstract

This is an introductory assignment to the world of *Statistics* and *Probability* in the world of *Pattern Recognition*. We'll introduce some key concepts like *Probability Distribution Function, Cumulative Distribution Function, Probability Density Function, Probability Mass Function, Joint Probability Density Function, Joint Cumulative Density Function, Marginal Density* & more details as the probabilistic point of view. Furthermore, we'll review the concepts of *Expected Value, Variance, Standard Deviation, Covariance & Correlation of Random Variables(e.g. Random Vectors), Univariate & Multivariate Gaussian Distribution, Total Probability & Bayes Theorem, Geometric & Mahalanobis Distances, Central Limit Theorem, Independence & Correlation* as the statistics point of view. Also, a principal concept called *Linear Transformation* is discussed. The relationship between these fields is far more important than each separately.

Key Words. *PDF, PMF, JPDF, JPMF, CDF, JCDF, Covariance Matrix, Correlation Coefficient, Correlation, Variance, Expected Vector, Gaussian Distribution, Marginal Probability, Linear Tranformation, Eigenvector, Eigenvalue, Rank.*

Practical & Theoretical Problems

1. Expectation & Variance

A random variable X has $E(X) = -4$ and $E(X^2) = 30$. Let $Y = -3X + 7$. Compute the following.

(a) $V(X)$

(b) $V(Y)$

(c) $E((X+5)^2)$

(d) $E(Y^2)$

Solution

The main equation to calculate the *Variance* of a random variable X is given in 1.1.

$$V(X) = E[(X - E[X])^2] \tag{1.1}$$

Expanding the equation 1.1, we'll have the equation 1.2 using simple calculus.

$$V(X) = E[X^2 + E[X]^2 - 2XE[X]]$$
$$V(X) = E[X^2] + E[X]^2 - 2E[X]^2$$
$$V(X) = E[X^2] - E[X]^2 \tag{1.2}$$

(a) The equation 1.2 can be conducted directly to compute the *Variance*. Replacing the values from the problem description we get the following as result.

$$V(X) = E[X^2] - E[X]^2$$

$$V(X) = 30 - 16 = 14$$

(b) It is important to mention the *Linearity* of *Expectation* operator as formally described in 1.3.

$$E[aX + b] = aE[X] + b \tag{1.3}$$

Using property 1.3, we can write the *V(Y)* as

$$V(Y) = V(-3X + 7) = E[(-3X + 7)^2] - E[-3X + 7]^2$$
$$V(Y) = E[9X^2 + 49 - 42X] - E[-3X + 7]^2$$
$$V(Y) = 9E[X^2] + E[49] - 42E[X] - 9E[X]^2 - E[49]$$
$$V(Y) = 9 * 30 + 49 - 42 * (-4) - 9 * 30 - 49$$
$$V(Y) = 168$$

(c) Expanding the internals of the expectation, we'll get the following.

$$E[(X + 5)^2] = E[X^2 + 10X + 25]$$
$$E[(X + 5)^2] = E[X^2] + 10E[X] + E[25]$$
$$E[(X + 5)^2] = 30 + 10 * (-4) + 25$$
$$E[(X + 5)^2] = 15$$

(c) Same as above, we'll use 1.2 to get the following.

$$E[Y^2] = E[(3X + 7)^2]$$
$$E[Y^2] = E[9X^2 + 49 - 42X]$$
$$E[Y^2] = 487$$

2. Eigenvector & Eigenvalue

(a) Compute eigenvalues and eigenvectors of $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 9 & -5 \end{bmatrix}$ and compare your results with Matlab outputs.

(b) A $2 * 2$ matrix A has $\lambda_1 = 2$ and $\lambda_2 = 5$, with corresponding eigenvectors $V_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$ and $V_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$. Find A.