Statistical Pattern Recognition
Assignment 4

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Abstract

Keywords. *KNN Classifier, Kernel Density Estimation.*

1 Parzen Windows

Given dataset $X = 2, 3, 4, 4, 4, 5, 5, 10, 11, 11, 11, 12, 14, 16$ use Parzen windows to estimate density $p(x)$ at $x = 5$ and $x = 12$; assuming $h = 4$ in the following conditions.

(a) If you use standard kernel function

$$K(u) = \begin{cases} 1 & |u| \leq \frac{1}{2} \\ 0 & o.w. \end{cases}$$

(b) If you use Gaussian Kernel, $N(0, 10)$.

(c) For the same dataset and the same sample points, i.e. $x = 5$ and $x = 12$, estimate the density using KNN approach. Take $K = 5$ in your estimations.

Solution

(a) We'll use (1.1) to estimate the density using different kernels.

$$\hat{p}_{(x)} = \frac{1}{n * h^d} \sum_{i=1}^{k} \Phi(\frac{x - x_i}{h}) \tag{1.1}$$

In this case, we'll compute the distance of each of the given points from our dataset. For every distance less than 2 we'll consider the effect of that point in our estimation.

$$x = 5 \quad \rightarrow \quad \hat{p}_{(x)} = \frac{1}{14 * 4} * (6) = \frac{3}{28} = 0.107$$

Doing the same for the point $x = 12$ and we'll have the following results:

$$x = 12 \quad \rightarrow \quad \hat{p}_{(x)} = \frac{1}{14 * 4} * (6) = \frac{3}{28} = 0.107$$

(b) In this case, the value of items in the $\sum$ are no longer 1. They will have different outputs, specially on the center of the curve. This can lead to an smoother and more realistic estimation of the density. The Gaussian Kernel equation is given in (1.2).

$$\phi(u) = \frac{1}{\sqrt{2\pi}} \exp(\frac{-u^2}{2}) \tag{1.2}$$

$$x = 5 \quad \rightarrow \quad \hat{p}_{(x)} = \frac{1}{14 * 4} * \frac{1}{\sqrt{2\pi}}(\exp(-2) + 3\exp(\frac{-1}{2}) + 2)) = 0.028$$

$$x = 12 \quad \rightarrow \quad \hat{p}_{(x)} = \frac{1}{14 * 4} * \frac{1}{\sqrt{2\pi}}(2\exp(-2) + 3\exp(\frac{-1}{2}) + 1)) = 0.021$$

(c) In this case, we have to consider the window size as a dynamic variable. In case $h = 2$ then $x = 4$, $x = 4$, $x = 4$ and $x = 5$ will be in our window of estimation as well as the centered point $x = 5$ (K = 5). To estimate the density we'll have the following equation:

$$x = 5 \quad \rightarrow \quad \hat{p}_{(x)} = \frac{1}{14 * 2} * (5) = 0.178$$

Note that we have considered the kernel to be the same as part a. In the second case, choosing the $h = 2$ yields 5 points.

$$x = 12 \quad \rightarrow \quad \hat{p}_{(x)} = \frac{1}{14 * 2} * (5) = 0.178$$