

# STATISTICAL PATTERN RECOGNITION

## ASSIGNMENT 1

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### Abstract

This is an introductory assignment to the world of *Statistics* and *Probability* in the world of *Pattern Recognition*. We'll introduce some key concepts like *Probability Distribution Function*, *Cumulative Distribution Function*, *Probability Density Function*, *Probability Mass Function*, *Joint Probability Density Function*, *Joint Cumulative Density Function*, *Marginal Density* & more details as the probabilistic point of view. Furthermore, we'll review the concepts of *Expected Value*, *Variance*, *Standard Deviation*, *Covariance* & *Correlation of Random Variables*(e.g. *Random Vectors*), *Univariate* & *Multivariate Gaussian Distribution*, *Total Probability* & *Bayes Theorem*, *Geometric* & *Mahalanobis Distances*, *Central Limit Theorem*, *Independence* & *Correlation* as the statistics point of view. Also, a principal concept called *Linear Transformation* is discussed. The relationship between these fields is far more important than each separately.

**Key Words.** *PDF, PMF, JPDF, JPMF, CDF, JCDF, Covariance Matrix, Correlation Coefficient, Correlation, Variance, Expected Vector, Gaussian Distribution, Marginal Probability, Linear Transformation, Eigenvector, Eigenvalue, Rank.*

## Practical & Theoretical Problems

### 1. Expectation & Variance

A random variable  $X$  has  $E(X) = -4$  and  $E(X^2) = 30$ . Let  $Y = -3X + 7$ . Compute the following.

- (a)  $V(X)$
- (b)  $V(Y)$
- (c)  $E((X + 5)^2)$
- (d)  $E(Y^2)$

## Solution

The main equation to calculate the *Variance* of a random variable  $X$  is given in 1.1.

$$V(X) = E[(X - E[X])^2] \quad (1.1)$$

Expanding the equation 1.1, we'll have the equation 1.2 using simple calculus.

$$\begin{aligned} V(X) &= E[X^2 + E[X]^2 - 2XE[X]] \\ V(X) &= E[X^2] + E[X]^2 - 2E[X]^2 \\ V(X) &= E[X^2] - E[X]^2 \end{aligned} \quad (1.2)$$

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(a) The equation 1.2 can be conducted directly to compute the *Variance*. Replacing the values from the problem description we get the following as result.

$$\begin{aligned} V(X) &= E[X^2] - E[X]^2 \\ V(X) &= 30 - 16 = 14 \end{aligned}$$

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(b) It is important to mention the *Linearity* of *Expectation* operator as formally described in 1.3.

$$E[aX + b] = aE[X] + b \quad (1.3)$$

Using property 1.3, we can write the  $V(Y)$  as

$$\begin{aligned} V(Y) &= V(-3X + 7) = E[(-3X + 7)^2] - E[-3X + 7]^2 \\ V(Y) &= E[9X^2 + 49 - 42X] - E[-3X + 7]^2 \\ V(Y) &= 9E[X^2] + E[49] - 42E[X] - 9E[X]^2 - E[49] \\ V(Y) &= 9 * 30 + 49 - 42 * (-4) - 9 * 30 - 49 \\ V(Y) &= 168 \end{aligned}$$

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(c) Expanding the internals of the expectation, we'll get the following.

$$\begin{aligned} E[(X + 5)^2] &= E[X^2 + 10X + 25] \\ E[(X + 5)^2] &= E[X^2] + 10E[X] + E[25] \\ E[(X + 5)^2] &= 30 + 10 * (-4) + 25 \\ E[(X + 5)^2] &= 15 \end{aligned}$$

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(c) Same as above, we'll use 1.2 to get the following.

$$\begin{aligned} E[Y^2] &= E[(3X + 7)^2] \\ E[Y^2] &= E[9X^2 + 49 - 42X] \\ E[Y^2] &= 487 \end{aligned}$$

## 2. Eigenvector & Eigenvalue

A random variable  $X$  has  $E(X) = -4$  and  $E(X^2) = 30$ . Let  $Y = -3X + 7$ . Compute the following.