Muon Physics

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This lab introduces the student to key concepts of muon physics. The experiment results leverages useful data analysis techniques in order to formulate results. The results from the experiments verified relationships between the discriminator threshold value to that of the efficiency of the muon detection. Some of the concepts that are demonstrated are the operations of a plastic scintillator to detect muons, plotting the decay histogram of the detected muons, measuring the decay time based off the data from the detector and more.

I. INTRODUCTION

A. Historical Background

Muons where discovered, in 1936, by two physicists Carl D. Anderson and Seth Neddermeyer, at California Institute of Technology, while studying cosmic radiation. Anderson, in particular, observed an odd phenomena when some particles, moving in the same velocity and negatively charged, curved less sharply from electrons when it passed through a magnetic field. It was initially assumed that the magnitude of the particles charge was equal to that of the electron but of greater mass in order to account for the difference in the curvature. This newly discovered particle is what is now known as a muon. The two physicists were able to identify and separate both negatively and positively charged muons from cosmic rays. In the very next year, two physicists named J.C. Street and E.C. Stevenson were independently able to confirm the existence of muons in a cloud chamber and realized muon's were not only real, but were very common. A few years later, in 1941, two physicists in Colorado named Bruno Rossi and D. B. Hall used muons to measure and confirm special relativity equations for time dilation and length contraction for the very first time. [1]

B. Muon Production

High energy charged particles that are produced in different areas of the universe showers the Earth's atmosphere. These high energy particles are known as primary cosmic rays. The composition of these particles are approximately 98% protons or nuclei and 2% electrons. Illustrated in FIG. 1, when these particles reach the Earth's atmosphere (about 10 kilometers above Earth's surface), they interact with the molecules in the atmosphere (mainly oxygen and nitrogen) which in turn produces different particles which are called secondary cosmic rays that include: protons, neutrons, pions, kaons, photons, electrons and positrons. This process of *particle showers* is known as *cascade*. These interactions can occur several times and not all the particles that are produced during the cascade live down to sea-level. This is due to collisions and interactions with other atmospheric particles. The particle of particular interest is the pions. Pions decay in a relatively short distance into muons and neutrino or antineutrinos via the weak force

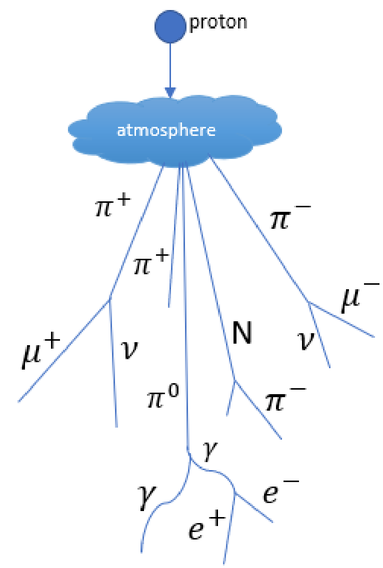

FIG. 1. The figure illustrates the cascade affect when a proton collides with a molecule in the atmosphere.

$$\begin{aligned} \pi^+ &\to \mu^+\nu_\mu \\ \pi^- &\to \mu^-\bar{\nu_\mu}. \end{aligned} \tag{1}$$

Muons behave similar to an unstable heavy electron which are still not fully understood. Muons have the same charge as an electron but are about 200 times their mass and are one of nature's fundamental building blocks of matter. The muons formed from these decay generally continue in the same direction as the original primary cosmic ray at a velocity near the speed of light. When a charge particle travels through matter, it interacts with the electric field which in turn causes loosely bound outer electrons to be knocked loose. A muon, however, interacts with matter only through the weak and electromagnetic forces thus muon can travel at larger distances even reaching the ground.[2] Muons do lose energy as it travels due to inelastic collisions with atomic electrons. These energy lost due to these collisions are relatively small, however, many collisions can occur therefore the cumulative effect of the loss in energy is substantial. Since charge must be conserved, the muon eventually decays into an electron, a neutrino and an anti-neutrino. [3][4] The mean literature lifetime for a muon to decay at rest has been experimentally measured to be approximately $\tau_\mu \approx 2.1969811 \pm 0.00000022\mu s$.

With a half life of only about $2.2\mu s$ causes the muons to decay a kilometer or two before reaching the earth's surface. But then how does the muon reach the surface of the earth? Since muon travel close to the speed of light, special relativity needs to be considered. Time dilation and length contraction are important concepts to consider when discussing muons. In special relativity, time and length are changed by the Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}},$$

where v is the speed of the moving observer and c is the speed of light. The change in length and time (proper time and proper length) is given by $\Delta t_0 = \frac{\Delta t_\mu}{\gamma}$ and $L_0 = \gamma L_\mu$, respectively, where Δt_μ and L_μ are the time and length observed in the observers reference frame, respectively. Therefore the muon experiences time dilation and length contraction effects which is why the muon can reach earth's surface.

Positive and negative muons have different lifetimes when interacting with matter. The reason for this is due to negative muon formation when it interacts with matter, it binds to the nuclei (mainly carbon and hydrogen in this experiment) which behaves very similar to electrons. Since muons are not electrons then the Pauli exclusion principle does not apply to it therefore the negative muons can interact with protons before decaying. This gives the negative muons two different interactions in which it decays which is not the case for positive muons thus the lifetime for a negative muon is less. Also, the lifetime of negative muons depends on the atomic number Z of the material. This concept is further explored in the muon decay section.

C. PMT Plastic Scintillator Muon Detector

In the process of discovering muon's, the scientific community quickly started engineering instrument's that provided the capability to detect muons. One such detector is the, Photomultiplier Tube (PMT) Scintillator, which was largely invented in 1944, by two physicists named, Samuel Curran and Baker.[5] FIG. 2 describes the five main components of the scintillator that will be discussed: scintillator, photomultiplier, amplifier, discriminator and the FPGA time. The first to be discussed is the plastic scintillator which is used to detect incoming particles due to its sensitivity to energy. The plastic scintillator is a special material medium, which consists of an organic substance made by combining one or more fluors with a solid plastic solvent. A fluors is a chemical compound that is a primary fluorescent emitter. When a charged particles enters the the scintillator, the kinetic energy of the particles gets absorbed via ionization and atomic excitation due to the solvent molecules. The absorbed energy is re-emitted in the form of light with a decay time of just a few nanoseconds. The scintillators name is derived due to this re-emiited light which is known as *scintillation* hence the name. The particle of particular interest for the scintillator to detect is the muon. During the cosmic ray showers there are many particles with different energy ranges. Muons have relatively high energies at ground level therefore it can be easily distinguished from other particles.[2] When the muons enters the plastic scintillator, the only muons of particular interest for the experiment are the muons that enter, slow, then stop and eventually decay inside the scintillator. An important note to consider is that the measurements of the muon lifetime that occurs in the plastic scintillator is an average over both positively and negative charged muons. As the muons slows down inside the scintillator, it emits light which gets detected by the photomultiplier tube (PMT).[4] A typical PMT consists of a photocathode which is simply a surface on the PMT where the light gets focused and converts the light into measurable electron current using the photoelectric effect. The photoelectrons produced by the photocathode get directed by the electrode voltages towards the PMT dynodes. There are a set of dynodes within a PMT where each time the photoelectrons strikes the surface of the dynode leads to emission of several secondary electrons. Now, when the muon stops after some time it decays into an electron, a neutrino and an anti-neutrino. Since the electron mass is roughly 200 times lighter than an electron, it therefore experiences very energetic affects which produces scintillator light along its path. The neutrino and anti-neutrino also produce some of the energy but escape detection due to their low energy. The timing clock by the PMT gets triggered when this secondary burst of scintillator light occurs. Therefore the PMT basically serves two purposes, which is that it amplifies the weak light signal from the muons and it converts the light into a voltage pulse that is recognizable to the system hardware. What this experiment is interested in is the time intervals between successive clock triggers for a set of muon decays. Only muons with a typical total energy of about 160 MeV will stop inside the plastic scintillator (due to the size of the scintillator). Thus, for this experiment, when the muon stops and then decays into an electron it also generates a second light pulse (which is what the scintillator material is good for) therefore the measurement that takes place is the time between this initial and stop light pulse.

The output from the PMT is transferred into a two-

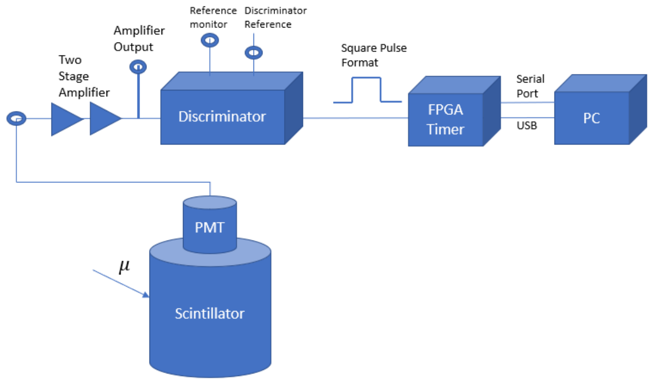

FIG. 2. The figure illustrates a schematic of the muon detector. The main components are shown.

stage amplifier which is a fast current feedback amplifier where the output then feeds to a voltage comparator which is also known as a *discriminator*. The two-stage amplifier amplifies the weak signal that it receives into something that can be sensed. The discriminator compares the two voltage signals and determines which one is greater with an adjustable threshold. When the discriminator receives the output from the amplifier, it filters certain signals that are not within a certain voltage range and converts the signals within the acceptable voltage into a TTL format (which is a square pulse of well defined shape). In other words, the scintillator has no way in distinguishing different particles that enter, however, fortunately the background noise from other particles have much less initial energy therefore the discriminator is easily able to filter the weak light pulses they create. The discriminator voltage threshold can be tuned. Now the TTL output pulse that is produced will trigger a timing circuit for the field programmable gate array (FPGA). A FPGA is a silicon chip with reprogrammable digital circuitry. The FPGA allows the user to run multiple tasks in parallel. In the case of this specific experiment, the FPGA is reconfigured to be a timing circuit. It acts when the TTL output arrives at a time interval which will cause a stop and reset of the timing circuit. The data is sent to the computer which communicates with the FPGA via a serial port or USB.

D. Muon Decay

Suppose a person observes a muon at its rest frame. The person, in that rest frame, will not be able to determine exactly when the muon will decay, even if the person knows the moment the muon was created. Therefore, the muon lifetime is not fixed at some time, however, the only criteria that can be deduced is that at some time t, there is a probability for the muon to decay.[6] The decay constant, λ, is defined as the probability of a muon decaying per unit time. Now suppose at some t there are $N(t)$ muons. Let dt be a small time interval and let λdt be the probability that a muon decays within dt then the change of the counts of muons is

$$dN = -N(t)\lambda dt \tag{2}$$

which is the familiar radioactivity decay equation. In other words, what this means is that the decay probability at time dt is equal to λdt i.e. suppose at time t there are $N(t)$ muons then at time $t+dt$ there will be $N(t)\lambda dt$ muons that will have been decayed therefore the number of muons has decreased by Eq. 2. Now integrating Eq. 2 gives

$$N(t) = N_0 \exp(-\lambda t), \tag{3}$$

where N_0 is the initial number of muons at $t = 0$. Now consider just one muon, then the probability distribution of the decay time will yield

$$D(t) = \lambda \exp(-\lambda t) \tag{4}$$

this is due to the fact that there is no cosmic muons enclosed in some box that can be used as a sample for some initial amount of muons N_0, instead the scintillator measures the random muons one at a time. Due to the PMT scintillator detecting both positive and negative muons, the expected decay derived from the measurements must consider both positive and negative muons using the following equation

$$\mu = \frac{\lambda^+ + \lambda^-}{2}, \tag{5}$$

where λ^+ are positive muons at rest and λ^- are negative muons at rest. Therefore the values that are computed from the experiment for the mean lifetime decay constant will be less than the literature free space value. With these consideration the muon charge ratio can be obtained. The muon charge ratio ρ is defined as the ratio of the number of positive to negative charge atmospheric muons arriving at the Earth's surface.[7]. The charge ratio, ρ, can be determined by the following equation

$$\rho = -\frac{\tau^+}{\tau^-}\frac{(\tau^- - \tau_{obs})}{(\tau^+ - \tau_{obs})}, \tag{6}$$

where τ^- is the lifetime for negative muon, τ^+ is equal to the free space lifetime since positive muons are not captured by the scintillator nuclei and τ_{obs} is the expected average lifetime.

Another important concept to consider is the Fermi coupling constant, G_F, which is the measurement of the strength due to the weak force. The equation that is used to determine G_F is

$$\frac{G_F}{(\hbar c)^3} = \sqrt{\frac{\hbar}{\tau_\mu} \cdot \frac{192\pi^3}{(m_\mu c^2)^5}}, \tag{7}$$

where m_μ is the mass of a muon which is ≈ 0.105GeV, c is the speed of light, $\hbar$ is reduced plank constant which is $\hbar \approx 6.582 \cdot 10^{-25}\,\mathrm{GeV}\cdot\mathrm{s}$ and τ_μ is the mean muon decay time that will need to be determined based off the datasets that are given. The value for the Fermi coupling constant at ground level is measured to be $\approx 1.1663787 \cdot 10^{-5}\mathrm{Ge}V^{-2}$.[8]

E. Muon Stopping Rate

The concept of time dilation, earlier discussed, will be demonstrated and used in order to determine the stopping rate of muons in the scintillator as a function of elevation above sea level. To measure the stopping rate then the total number of stopped muons in the scintillator at some fixed altitude above seal level is measured using the decay time histogram. Then a lower altitude is used where estimations are made for the new stopping rate where one incorporates time dilation effects and the other excludes time-dilation effects. The transit time t' which is measured in the muon's rest frame as it descends vertically from some height H down to sea-level is given by

$$t' = \frac{mc}{pC_0}\int_{\gamma_1}^{\gamma_2} \frac{d\gamma}{\sqrt{\gamma^2 - 1}}, \tag{8}$$

where γ_1 is the muon's gamma factor at height H and γ_2 is its gamma factor just before it enters the scintillator, m is the muon Lorentz mass and ρ is the pathlength-averaged mass density of the atmosphere. $\gamma_2 = 1.5$ since we want the muons to stop inside the scintillator. γ_1 can be computed by $\gamma_1 = E_1/mc^2$, where $E_1 = E_2 + \Delta E$ and $E_2 = 160\mathrm{MeV}$. Now, $R = \exp(-t'/\tau)$ is the ratio of muon stopping rates for the same detector at two different positions separated by a vertical distance H. Therefore the double ratio is $R_0 = R_{raw}/R$, where R_{raw} is the raw stopping rates. The quantity ΔE can be computed from the Bethe-Bloch equation or estimated via the standard atmosphere, $\Delta E = 2\mathrm{MeV/g}/cm^2 \cdot H \cdot p_{air}$, where p_{air} is the density of air.

II. EXPERIMENTAL PROCEDURE AND SETUP

The high voltage for the PMT can be set between –1100 and −1200 Volts using the knob at the top of the detector tube however the exact setting is not critical. Four different discriminator threshold values are measured: $148mV$, $190mV$, $260mV$ and $550mV$. All four data files includes the time interval of a decay which are the measured time between a pair of pulses in ns. However, most of the data in those data files are useless except for only the numbers less than 20000. Therefore, a program needs to be written that goes through all of the numbers in the datafiles and filters out any value that is ≥ 20000. Once the data files have been filtered a histogram should be plotted. Based off the discriminator settings, the relationship between the effect that the discriminator setting has on the data quality are described.

III. RESULTS

The experiment consisted of measuring the muon lifetime for four separate discriminator threshold settings: 148mV, 190mV, 260mV and 550mV. The mean decay time, histogram, uncertainty, exponential fit and the reduced chi-squared were all computed as well as determining the relationship between the discriminator settings and the accuracy of the results. It is important to understand that the values that were obtained from the measurements of the muon mean decay time were not the time of their creation in the atmosphere but instead the decay time in the scintillator however the student should understand the results should be the same regardless since the measurements are of a probability distribution for an exponential decay as discussed in the muon decay section.

Since the detector measures any particle that produces enough scintillation light, it is important to consider the background noise added to the measurements. The background noise was estimated by observing the decay time histogram and measuring the exponential fit which includes the background parameter. Once the background parameter value is estimated, then it can be subtracted from the original data which then produces a new distribution of data that provides a more accurate exponential fit. Eq. 3 was used as the exponential curve fit model function however a background constant was added to it.

FIG. 3 through FIG. 6 illustrates the decay time histogram based off the data that was provided. The histograms were first normalized in order to illustrate the probability distribution to better verify the distributional exponential model that was used to best fit. All the data files includes the time interval of a decay which are the measured time between a pair of pulses in ns. FIG. 3 describes the exponential decay histogram for the 148mV data file. The mean decay rate was measured at $2.09 \pm 0.08\mu s$ however the mean normalized amplitude was measured at $4.40 \pm .08\mu s$ this discrepancy between the normalized amplitude and the decay rate may mean the data is not reliable so further scrutiny is needed. Further examination for the 148mV data yielded a high mean value of the time interval with a value of 6334 ± 80 ns and a significantly high chi-squared, degree of freedom ratio with a value of $\chi = 11.6$. These statically inconsistency and the discrepancy between the normalized amplitude and decay rate forces the data to be rejected therefore not reliable.

FIG. 4 describes the exponential decay histogram for the 190mV data file. The mean value of the time interval was measured to be 4097 ± 64 ns. The mean decay rate was measured at $2.17 \pm 0.06\mu s$ and the mean normalized amplitude was measured at $2.81 \pm 0.06\mu s$ which is a better relationship than the previous 148mV measurements.

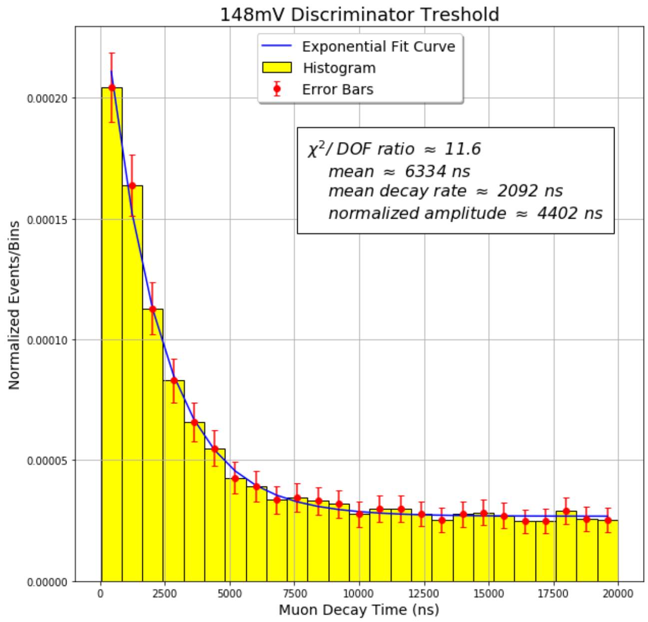

FIG. 3. The figure illustrates the probability density histogram for the decay time for the 148mV muon dataset.

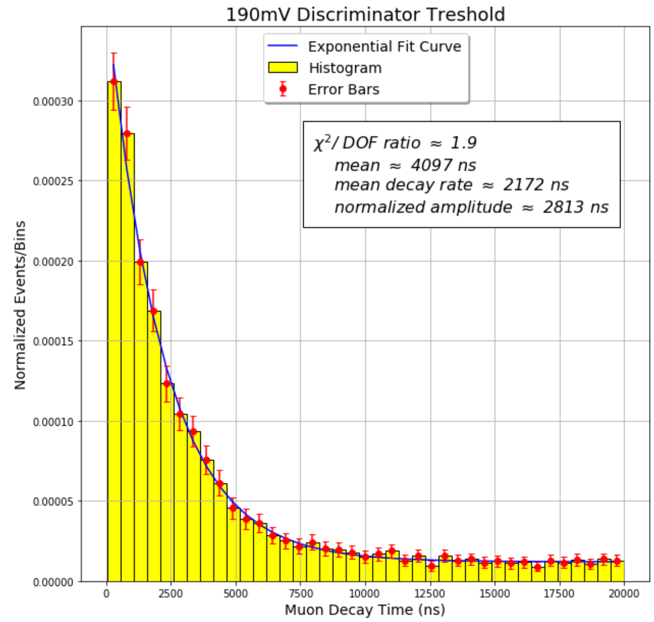

FIG. 4. The figure illustrates the probability density histogram for the decay time for the 190mV muon dataset. An exponential fit was plotted.

The chi-squared ratio was measured to be 1.9. The measurements obtained for the 190mV provided more accurate results compared to the 148mV data, therefore, there maybe a relationship between the increase of discriminator settings to that of the accuracy of the measurements so to verify that relationship the results for the 260mV and 550mV should be determined.

FIG. 5 describes the exponential decay histogram for the 260mV data file. The mean value of the time interval was measured to be 3482 ± 59 ns. The mean decay rate was measured at $2.12 \pm 0.06\mu s$ and the mean normalized amplitude was measured at $2.50 \pm .05\mu s$. The chi-squared ratio was measured at 1.6. Now FIG. 6 describes the exponential decay histogram for the 550mV data file. The mean value of the time interval was measured to be 2392 ± 48 ns. The mean decay rate was measured at $2.16 \pm 0.04\mu s$ and the mean normalized amplitude was measured at $2.17 \pm .04\mu s$. The chi-squared ratio was measured at 1.1. Here we clearly see improving results in our measurements and better statically consistency due to the chi-square ratio and the mean value for the time interval. Therefore the relationship between the discriminator settings to that of the accuracy in the measurements is verified. The reason why the increase in discriminator settings led to more accurate results was due to the PMT filtering more noise. With a low discriminator threshold, more particles are detected which leads to the data being skewed due to the unnecessary extra noise.

Another method in which the muon decay time can be computed is illustrated in FIG. 7. Taking the log of the counts that were measured and finding the slope of the

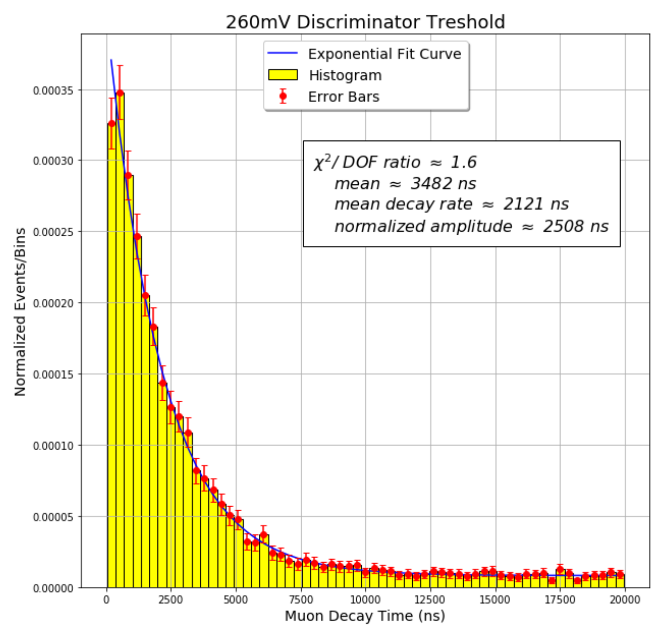

FIG. 5. The figure illustrates the probability density histogram for the decay time for the 260mV muon dataset.

linearized Eq. 3 gives

$$\log N = -\frac{1}{\tau}t + \log N_0.$$

This process gave a mean decay rate of $2.15 \pm 0.05\mu s$ for the 550mV data file which is very close to the exponential fit derived above of $2.16 \pm 0.04\mu s$. This gives confidence that our measurements for the dataset for the 550mV

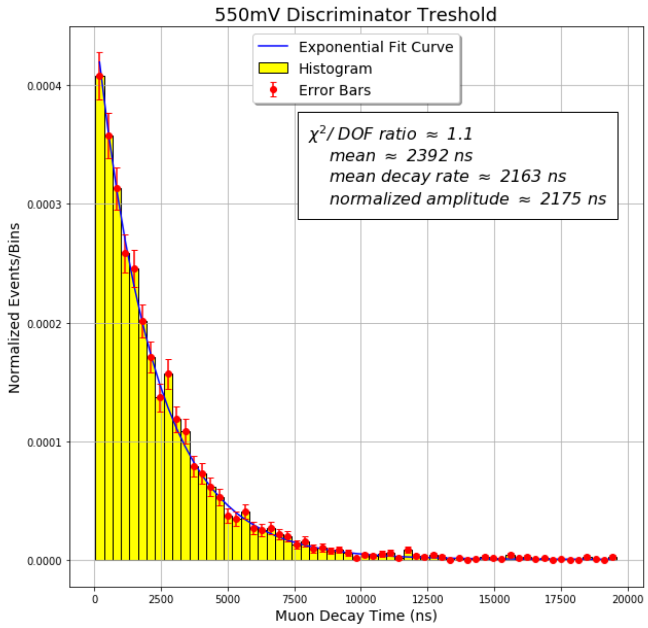

FIG. 6. The figure illustrates the probability density histogram for the decay time for the 550mV muon dataset.

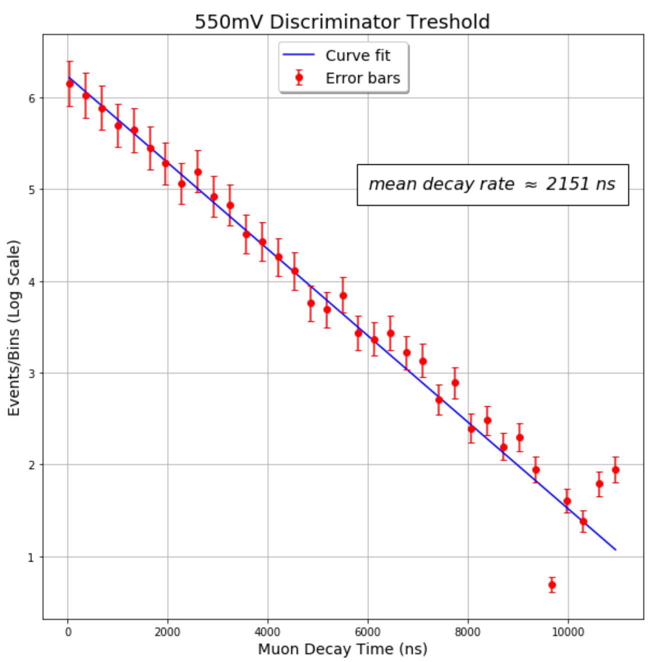

FIG. 7. The figure illustrates the count in log scale for the decay time for the 550mV muon dataset.

threshold is reliable. This method can also be applied to the other discriminator datasets.

The mean of the three datasets can now be computed. The mean decay rate is $2.14 \pm 0.02\mu s$ which is lower than the literature free space value of $\approx 2.2\mu s$. This is expected since, as already discussed in the muon decay section, the experiment consisted of measuring both negative and positive muons. Now that the mean decay time is determined then the Fermi coupling constant, the charge ratio and stopping rate can be computed. Using Eq. 7 where $\tau_\mu \approx 2.14 \pm .02$ the Fermi coupling constant is computed to $1.194 \cdot 10^{-5} \pm 0.028$ GeV^{-2}. This value is close to the expected value of $1.1663787 \cdot 10^{-5}$GeV^{-2}, where the deviation from the two values comes from the lower muon mean decay τ_μ value that was used. For the charge ratio, using Eq. 6 where $\tau^- \approx 2.043 \pm 0.003\mu s$ and $\tau^+ = \tau_\mu$ gives the computed charge ratio of 1.81 ± 0.02.

For the stopping rate where the altitude in Athens is $(H = 194m)$, muon energy is 160 MeV and the higher altitude is $(H = 15000m)$. First the raw muon stopping rate was measured at an upper and lower elevation. Then the transit time t' was calculated in the muon's rest frame, accounting for the energy loss between the two elevations. Thus the stopping rate in Athens is .02 muons/sec. Now, t' can now be computed using Eq. 8. The value obtained is $t' = 4.34\tau$. Thus $R = \exp(-t'/\tau) = 0.013$ therefore the double ratio is $R_0 \approx 1.6 \pm 0.03$. Now comparing to the different elevation gets, $R_\mu = R_0 R = 1.6 \exp(t'/\tau)$, thus $t' = 3.48\tau$ and $R_\mu = 0.049 \pm 0.04$ which shows good agreement with the expectation.

In the process of determining the mean decay time value for the four different datasets led to the rejection of one of the discriminator settings due to poor statistical results. This insured that our mean decay time value was obtained from the most reliable datasets. The results obtained were plausible due to the comparison of the mean decay time to that of the literature decay time value. It is crucial to understand that the results for the lifetime that was obtained with the exponential fit were not from the start time of the muon. The results does not mean that the muons decay in the scintillator at an absolute constant time of $2.15\mu s$ once they are created. Instead, the results illustrates that given a set of muons, after $2.15\mu s$ have passed then $\approx 67\%$ will decay, then waiting another $2.15\mu s$ about 67% of those remaining will have decayed, so on and so forth. This is what probability density for the exponential decay means.

IV. CONCLUSION

The experiments that were conducted in this lab are heavily reliant on the plastic scintillator instrument. The purpose of these experiments is to illustrate fundamental behavior of muon decay and familiarize the student with a powerful instrument. This may seem daunting to the student but in this lab the student is able to perform straight forward procedures and illustrate sophisticated concepts of the behavior of muons using a powerful instrument. The main purpose of the lab is the measure the average duration of the life of a muon i.e. the muon decay. The results obtained verified the relationship between the discriminator settings to that of the efficiency of the instrument filtering unwanted noise. The higher the discriminator settings led to better results. In this lab, many key concepts and properties of decay lifetime were demonstrated. The results correlated within an uncertainty with the theoretical predictions and varied with the expected results. The uncertainty was computed by using Poisson statistics by measuring the total counts but since the collected data was so large the uncertainty was small in comparison. The uncertainty of the mean decay time was measured at $\approx 0.02\mu s$ by measuring the standard deviation from the expected value of $\approx 2.2\mu s$. Some of the uncertainties can be contributed due to the data being affected by the unavoidable noise coming from the electrical device which is known as *dark current*. These uncertainties contributed to the slight deviation from the theoretical expected results. The experiments results can be improved by performing multiple trials of the same experiment using the same discriminator threshold settings which will provide a better mean decay rate. These experiments nevertheless demonstrated and accurately measured the strong relationship between muon lifetime to that of the exponential decay rate they are represented by.

[1] https://en.wikipedia.org/wiki/Muon#History.

[2] https://home.fnal.gov/~group/WORK/muonDetection.pdf.

[3] https://cosmic.lbl.gov/SKliewer/Cosmic_Rays/Muons.htm.

[4] http://web.physics.ucsb.edu/~phys128/experiments/muonphysics/muonphysics.pdf.

[5] https://en.wikipedia.org/wiki/Scintillator#History.

[6] https://www.hep.lu.se/muonlab/DOC/manual.pdf.

[7] https://arxiv.org/pdf/1005.5332.pdf.

[8] https://physics.nist.gov/cgi-bin/cuu/Value?gf.