Week11hw609

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Verify that the given function pair is a solution to the first-order system

$$x = -e^t, y = e^t$$
$$\frac{dx}{dt} = -y, \frac{dy}{dt} = -x$$

$$\frac{dx}{dt} = -e^t, y = e^t, \frac{dx}{dt} = -y$$
$$\frac{dy}{dt} = e^t, x = -e^t, \frac{dy}{dt} = -x$$

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Find and classify the rest points of the given autonomous system:

$$\frac{dx}{dt} = -(y-1), \frac{dy}{dt} = x - 2$$

Stationary point when $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are 0, happens at (2,1)

This point is stable. Any points close to (2,1) will orbit the point at a constant radius. Given any point, radius r from the point (2,1), the coordinates of the point will be $(rcos(\theta), rsin(\theta))$, with θ being the angle between the segment drawn from the point to (2,1), and the horizontal line y=1. The magnitude of the change at any point along the circle drawn around (2,1) with this radius will be r.

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Apply the first and second derivative tests to the function $f(y) = \frac{y^a}{e^{by}}$. Show that $y = \frac{a}{b}$ is a unique critical point that yields the relative maximum f(a/b). Show also that f(y) approaches 0 as y tends to infinity.

$$f(y) = \frac{y^a}{e^{by}} = y^a e^{-by}$$

$$f'(y) = a y^{a-1} e^{-by} - b y^a e^{-by} = 0$$

$$a y^{a-1} e^{-by} = b y^a e^{-by}$$

$$a = by$$

$$y = \frac{a}{b}$$ first derivative test – critical point

is the second derivative positive or negative?

$$f''(x) = a(a-1) y^{a-2} e^{-by} - bay^{a-1} e^{-by} - aby^{a-1} e^{-by} + b^2 y^a e^{-by}$$

$$= \frac{a(a-1)y^{a-2} - bay^{a-1} - aby^{a-1} + b^2 y^a}{e^{by}} \qquad y = \frac{a}{b}$$

term by term

$$\frac{a(a-1)a^{a-2}}{b^{a-2}} = \frac{a^{a-1}(a-1)b^2}{b^a}$$

$$-\frac{ba\,a^{a-1}}{b^{a-1}} = -\frac{ba^a}{b^{a-1}} = -\frac{b^2a^a}{b^a}$$

$$-\frac{ab\,a^{a-1}}{b^{a-1}} = -\frac{b^2a^a}{b^a}$$

$$\frac{b^2a^a}{b^a}$$ cancel

$$= \frac{a^{a-1}(a-1)b^2 - b^2a^a}{b^a e^{by}} = \frac{a^a b^2 - a^{a-1}b^2 - b^2a^a}{b^a e^{by}}$$

$$= -\frac{a^{a-1}b^2}{b^a e^{by}} = -\frac{a^{a-1}b^2}{b^a e^a}$$ The sign of this cannot be determined.

The first derivative of this clearly shows a local extrema at a/b, but the second derivative is unclear unless a and b are defined. As an example, the case where a=1 and b=1 indeed has a maxima when y = 1, but if a=2, negative values of y tend towards infinity.

f(y) does however approach 0 as y tends to infinity. The numerator has a constant exponent, a, while the denominator has an exponent that tends to infinity (by). This means the fraction will tend towards 0 overall.