Assignment 3

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Problem Set 1

1- What is the rank of Matrix A?

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{bmatrix}$$

Subtract -1 * row 1 from row 2 and -5 * row 1 from row 4:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 7 \\ 0 & 1 & -2 & 1 \\ 0 & -6 & -17 & -23 \end{bmatrix}$$

Subtract $-\frac{1}{2}$ * row 2 from row 3 and -3 * row 2 from row 4:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 7 \\ 0 & 0 & -4 & \frac{5}{2} \\ 0 & 0 & 0 & \frac{13}{6} \end{bmatrix}$$

The rank of this matrix is 4 because it has no zero rows when reduced to REF.

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2 - Given an m x n matrix where m > n, what can be the maximum rank of a non-zero matrix? The minimum rank?

Lets assume we can get the below matrix in REF:

$$\begin{bmatrix} a & e & i \\ b & f & j \\ c & g & k \\ d & h & i \end{bmatrix}$$

So it looks somthing like this:

$$\begin{bmatrix} w & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & y \\ 0 & 0 & z \end{bmatrix}$$

In this case, once we run out of columns, the rows below the nth column become multiples of eachother, and can be eliminated.

The minimum rank of an m x n matrix that is non-zero is 1. Even in a situation where every row is a multiple of the pivot row, there is still one pivot.

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3 - What is the rank of Matrix B?

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix}$$

Subtract 3 * row 1 from row 2, and 2 * row 1 from row 3:

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The other two rows zero out. There is only one pivot, so the rank is 1.

Problem Set 2

Compute the eigenvalues and eigenvectors of the matrix A. You'll need to show your work. You'll need to write out the characteristic polynomial and show your solution.

First lets define the Matrix:

```r
A <- matrix(c(1,0,0,2,4,0,3,5,6),nrow=3)

A
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    3
## [2,]    0    4    5
## [3,]    0    0    6
```

We're solving for the eigenvalues λ, where $Av = \lambda v$

Another way of writing this equation is $(\lambda I_3 - A)v = 0$

We can solve for λ by setting the determinant of the matrix $(\lambda I_3 - A)v$ to 0

So,

$$(\lambda I_3 - A) = \begin{bmatrix} \lambda - 1 & -2 & -3 \\ 0 & \lambda - 4 & -5 \\ 0 & 0 & \lambda - 6 \end{bmatrix}$$

and $det(\lambda I_3 - A) = 0$

so $(\lambda - 1)(\lambda - 4)(\lambda - 6) + -2 * -5 * 0 + -3 * 0 * 0 - 0 * (\lambda - 4) * -3 - 0 * -5 * (\lambda - 1) - (\lambda - 6) * 0 * -2 = 0$

Out of 6 terms, all terms multiply by at least one 0 except for the first, so the characteristic polynomial is:

$(\lambda - 1)(\lambda - 4)(\lambda - 6) = 0$

And this is solved when $\lambda = 1, 4, 6$

now, lets plug in each value of λ into the matrix:

$\lambda = 1$

$$(\lambda I_3 - A) = \begin{bmatrix} 0 & -2 & -3 \\ 0 & -3 & -5 \\ 0 & 0 & -5 \end{bmatrix}$$

```
A1 <- matrix(c(0,0,0,-2,-3,0,-3,-5,-5),nrow=3)

# eliminate A1[2,3] by subtracting row 3 from row 2
# eliminate A1[1,3] by subtracting 3/5 * row 3 from row

A1[2,] <- A1[2,] - A1[3,]

A1[1,] <- A1[1,] - (3/5)*A1[3,]

A1
```

```
##      [,1] [,2] [,3]
## [1,]    0   -2    0
## [2,]    0   -3    0
## [3,]    0    0   -5
```

```
# eliminate A1[1,2] by subtracting (2/3)*A1[2,]

A1[1,] <- A1[1,] - (2/3)*A1[2,]

A1
```

```
##      [,1] [,2] [,3]
## [1,]    0    0    0
## [2,]    0   -3    0
## [3,]    0    0   -5
```

```
# Get matrix into reduced row echelon form by making pivots = 1
# Then move the zero rows to the bottom

A1[2,] <- A1[2,]/-3
A1[3,] <- A1[3,]/-5

A1 <- A1[c(2,3,1),]
```

$$(\lambda I_3 - A)v = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$v_2 = 0$
$v_3 = 0$

Let $v_1 = t$

$$E_{\lambda=1} = t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

where t is real

$\lambda = 4$

$$(\lambda I_3 - A) = \begin{bmatrix} 3 & -2 & -3 \\ 0 & 0 & -5 \\ 0 & 0 & -2 \end{bmatrix}$$

```r
A4 <- matrix(c(3,0,0,-2,0,0,-3,-5,-2),nrow=3)

# Eliminate A4[2,3] by subtracting (5/2) * row 3 from row 2
# Eliminate A4[1,3] by subtracting (3/2) * row 3 from row 1

A4[2,] <- A4[2,] - (5/2)*A4[3,]
A4[1,] <- A4[1,] - (3/2)*A4[3,]

A4
```

```
##      [,1] [,2] [,3]
## [1,]    3   -2    0
## [2,]    0    0    0
## [3,]    0    0   -2
```

```r
# Divide row 1 by 3 to make the pivot 1
# Divide row 3 by -2 to make the pivot 1
# Move the zero row to the bottom

A4[1,] <- A4[1,]/3
A4[3,] <- A4[3,]/-2

A4 <- A4[c(1,3,2),]
```

$$(\lambda I_3 - A)v = \begin{bmatrix} 1 & -\frac{2}{3} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$v_1 - \frac{2}{3}v_2 = 0$
$v_3 = 0$

let $v_2 = t$

So: $\frac{2}{3}t = v_1$

$$E_{\lambda=4} = t \begin{bmatrix} \frac{2}{3} \\ 1 \\ 0 \end{bmatrix}$$

Where t is real

$\lambda = 6$

```r
A6 <- matrix(c(5,0,0,-2,2,0,-3,-5,0),nrow=3)

# Eliminate A6[1,2] by subtracting -1 * row 2

A6[1,] <- A6[1,] + A6[2,]

# Divide Row 1 by 5 and row 2 by 2 to make the pivots 1

A6[1,] <- A6[1,]/5
A6[2,] <- A6[2,]/2

A6
```

```
##      [,1] [,2] [,3]
## [1,]    1    0 -1.6
## [2,]    0    1 -2.5
## [3,]    0    0  0.0
```

$(\lambda I_3 - A)v = \begin{bmatrix} 1 & 0 & -1.6 \\ 0 & 1 & -2.5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$
$v_1 - 1.6v_3 = 0$
$v_2 - 2.5v_3 = 0$

let $v_3 = t$

So

$V_1 = 1.6t$
$v_2 = 2.5t$

$E_{\lambda=6} = t \begin{bmatrix} 1.6 \\ 2.5 \\ 1 \end{bmatrix}$