A is a $(m \times n)$ matrix $\quad A^T$ is a $(n \times m)$ matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & & \ddots & \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix} \quad \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & & a_{m2} \\ \vdots & & \ddots & \\ a_{1n} & a_{2n} & & a_{mn} \end{bmatrix}$$

If $m \neq n$, then $A^TA \neq AA^T$ simply based on the number of rows and columns

$A^TA = (n \times m) \times (m \times n) =$ an $(n \times n)$ matrix

$AA^T = (m \times n) \times (n \times m) =$ an $(m \times m)$ matrix

What if $m = n$?

$$AA^T = \begin{bmatrix} \text{row 1} \cdot \text{row 1} & \text{row 1} \cdot \text{row 2} & \cdots & \text{row 1} \cdot \text{row } m \\ \text{row 2} \cdot \text{row 1} & \text{row 2} \cdot \text{row 2} & & \text{row 2} \cdot \text{row } m \\ \vdots & & \ddots & \\ \text{row } m \cdot \text{row 1} & \text{row } m \cdot \text{row 2} & & \text{row } m \cdot \text{row } m \end{bmatrix}$$

$$A^TA = \begin{bmatrix} \text{col 1} \cdot \text{col 1} & \text{col 1} \cdot \text{col 2} & \cdots & \text{col 1} \cdot \text{col } n \\ \text{col 2} \cdot \text{col 1} & \text{col 2} \cdot \text{col 2} & & \text{col 2} \cdot \text{col } n \\ \vdots & & \ddots & \\ \text{col } n \cdot \text{col 1} & \text{col } n \cdot \text{col 2} & & \text{col } n \cdot \text{col } n \end{bmatrix}$$

This only works iff, for each m and n, row m = col n

In other words, if and only if $A = A^T$