CFerrari_Assignment14

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$$f(x) = \frac{1}{1-x}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Lets first try to arrive at a general derivative form for $\frac{1}{1-x}$

$$f^{(1)}(x) = (-1)(-1)\frac{1}{(1-x)^2}$$

$$f^{(2)}(x) = (-1)^2(-1)(-2)\frac{1}{(1-x)^3}$$

$$f^{(3)}(x) = (-1)^3(-1)(-2)(-3)\frac{1}{(1-x)^4}$$

The negative terms always cancel eachother out

$$f^{(n)}(x) = \frac{1}{n!(1-x)^{n+1}}$$

The n! terms cancel eachother out:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-a)^n}{(1-a)^{n+1}}$$

So for a = 0:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

And since this is a basic power series, the radius of convervence is -1 < x < 1, since the $\lim_{x\to\infty} p^x$ is 0 for that radius.

$$f(x) = e^x$$

The multiple derivatives of this is easy, $\frac{d}{dx}e^x$ is just e^x

$$e^x = \sum_{n=0}^{\infty} \frac{e^a}{n!}(x-a)^n$$

Lets take a = 0

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Lets try to find its radius of convergence by using the ratio test.

$$\frac{A_{n+1}}{A_n} = \frac{x^{n+1}}{(n+1)n!}\frac{n!}{x^n} = \frac{x}{n+1}$$

$$\lim_{x \to \infty} \frac{x}{n+1} = 0$$

No matter what value x takes, the ratio converges to 0, so the series converges for all values of x

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$$f(x) = ln(1+x)$$

Lets first try to arrive at a general derivative form for $ln(1+x)$

$$f(x) = ln(1+x)$$

$$f^{(1)}(x) = \frac{1}{1+x}$$

$$f^{(2)}(x) = (-1)\frac{1}{(1+x)^2}$$

$$f^{(3)}(x) = (-1)(-2)\frac{1}{(1+x)^3}$$

$$f^{(n)}(x) = (-1)^{n-1}(n-1)!\frac{1}{(1+x)^n}$$

$\frac{(n-1)!}{n!} = \frac{1}{n}$, so, for a=0:

$$ln(1+x) = \sum_{n=0}^{\infty} -1^{n-1}\frac{x^n}{n}$$

Lets try to find its radius of convergence by using the ratio test.

$$\frac{A_{n+1}}{A_n} = -\frac{x^{n+1}}{n}\frac{n}{x^n} = -\frac{xn}{n+1}$$

$$\lim_{n \to \infty} -\frac{xn}{n+1} = -x$$

So, this is valid when $|x| < 1$