Module - getLocalTopology

import mesh

In[1]:=

```
DumpGet["C:\\Users\\aliha\\Desktop\\wolfram-vertex-3D\\PREVIOUS
    CODE - slow heuns\\meshGen_noise.mx"];
```

In[2]:=

```
yLim[[1]] = 0.;
edges = SetPrecision[edges, 10];
faceListCoords = SetPrecision[faceListCoords, 10];
(*convert faceListCoords into an association*)
indToPtsAssoc = SetPrecision[indToPtsAssoc, 10];
ptsToIndAssoc = KeyMap[SetPrecision[#, 10] &, ptsToIndAssoc];
xLim = SetPrecision[xLim, 10];
yLim = SetPrecision[yLim, 10];
faceListCoords = Map[Lookup[indToPtsAssoc, #] &, cellVertexGrouping, {2}];
```

In[10]:=

```
Clear@periodicRules;
With[{xlim1 = xLim[[1]], xlim2 = xLim[[2]], ylim1 = yLim[[1]], ylim2 = yLim[[2]]},
  periodicRules = Dispatch[{
      (*bottom right half*)
      {x_ /; x ≥ xlim2, y_ /; y ≤ ylim1, z_} ⧴ SetPrecision[{x - xlim2, y + ylim2, z}, 10],
      (*right*)
      {x_ /; x ≥ xlim2, y_ /; ylim1 < y < ylim2, z_} ⧴ SetPrecision[{x - xlim2, y, z}, 10],
      (*bottom*)
      {x_ /; xlim1 < x < xlim2, y_ /; y ≤ ylim1, z_} ⧴ SetPrecision[{x, y + ylim2, z}, 10],
      (*bottom left half*)
      {x_ /; x ≤ xlim1, y_ /; y ≤ ylim1, z_} ⧴ SetPrecision[{x + xlim2, y + ylim2, z}, 10],
      (*left half*)
      {x_ /; x ≤ xlim1, y_ /; ylim1 < y < ylim2, z_} ⧴ SetPrecision[{x + xlim2, y, z}, 10],
      (*top-left half*)
      {x_ /; x ≤ xlim1, y_ /; y ≥ ylim2, z_} ⧴ SetPrecision[{x + xlim2, y - ylim2, z}, 10],
      (*top*)
      {x_ /; xlim1 < x < xlim2, y_ /; y ≥ ylim2, z_} ⧴ SetPrecision[{x, y - ylim2, z}, 10],
      (*top-right*)
      {x_ /; x ≥ xlim2, y_ /; y ≥ ylim2, z_} ⧴ SetPrecision[{x - xlim2, y - ylim2, z}, 10]
     }];

  transformRules = Dispatch[{
      {x_ /; x ≥ xlim2, y_ /; y ≤ ylim1, _} ⧴ SetPrecision[{-xlim2, ylim2, 0}, 10],
      {x_ /; x ≥ xlim2, y_ /; ylim1 < y < ylim2, _} ⧴ SetPrecision[{-xlim2, 0, 0}, 10],
      {x_ /; xlim1 < x < xlim2, y_ /; y ≤ ylim1, _} ⧴ SetPrecision[{0, ylim2, 0}, 10],
      {x_ /; x ≤ xlim1, y_ /; y ≤ ylim1, _} ⧴ SetPrecision[{xlim2, ylim2, 0}, 10],
      {x_ /; x ≤ xlim1, y_ /; ylim1 < y < ylim2, _} ⧴ SetPrecision[{xlim2, 0, 0}, 10],
      {x_ /; x ≤ xlim1, y_ /; y ≥ ylim2, _} ⧴ SetPrecision[{xlim2, -ylim2, 0}, 10],
      {x_ /; xlim1 < x < xlim2, y_ /; y ≥ ylim2, _} ⧴ SetPrecision[{0, -ylim2, 0}, 10],
      {x_ /; x ≥ xlim2, y_ /; y ≥ ylim2, _} ⧴ SetPrecision[{-xlim2, -ylim2, 0}, 10],
      {___Real} ⧴ SetPrecision[{0, 0, 0}, 10]}];
 ];
```

In[12]:=

```
(*
origcellOrient=<|MapIndexed[First[#2]→#1&, faceListCoords]|>;
boundaryCells=With[{ylim1=yLim[[1]],ylim2=yLim[[2]],xlim2=xLim[[2]]},
  Union[First/@Position[origcellOrient,
      {x_/;x≥xlim2,__}|{x_/;x<0,__}|{_,y_/;y>ylim2,_}|{_,y_/;y≤ylim1,_}]/.
    Key[x_]⧴ x]
 ];
*)
wrappedMat = AssociationThread[
   Keys[cellVertexGrouping] → Map[Lookup[indToPtsAssoc, #] /. periodicRules &,
     Lookup[cellVertexGrouping, Keys[cellVertexGrouping]], {2}]];
```

Miscellaneous F[x]

In[13]:=

```
triangulateFaces[faces_] := Block[{edgelen, ls, mean},
   (ls = Partition[#, 2, 1, 1];
      edgelen = Norm[SetPrecision[First[#] - Last[#], 10]] & /@ ls;
      mean = Total[edgelen * (Midpoint /@ ls)] / Total[edgelen];
      mean  = mean ~ SetPrecision ~ 10;
      Map[Append[#, mean] &, ls]) & /@ faces
  ];
```

In[14]:=

```
Clear@outerCellsFn;
outerCellsFn::Information = "the function finds the cells at the boundary";
outerCellsFn[faceListCoords_, vertexToCell_, ptsToIndAssoc_] :=
  With[{ylim1 = yLim[[1]], ylim2 = yLim[[2]], xlim2 = xLim[[2]]},
   Block[{boundaryCells, bcells, temp, res},
    temp = <|MapIndexed[First[#2] → #1 &, faceListCoords]|>;
    boundaryCells = Union[First /@ Position[temp,
         {x_ /; x ≥ xlim2, __} | {x_ /; x ≤ 0, __} |
          {_, y_ /; y ≥ ylim2, _} | {_, y_ /; y ≤ ylim1, _}] /. Key[x_] :→ x];
    bcells = KeyTake[faceListCoords, boundaryCells];
    res = Union@ (Flatten@Lookup[vertexToCell,
          Lookup[ptsToIndAssoc,
           DeleteDuplicates@Cases[bcells,
              {x_ /; x ≥ xlim2, __} | {x_ /; x ≤ 0,
                 __} | {_, y_ /; y ≥ ylim2, _} | {_, y_ /; y ≤ ylim1, _}, {3}]
             /. periodicRules
          ]
         ] ~ Join ~ boundaryCells);
    res
   ]
  ];
```

In[109]:=

```
Clear@boundaryPtsPairing;
boundaryPtsPairing::Information =
  "the function pairs the points at the boundaries with corresponding mirror points";
boundaryPtsPairing[vertexToCell_, indToPtsAssoc_, ptsToIndAssoc_] :=
  Block[{outerpts, mirrorpairs, pt, mirror},
   outerpts = Keys@Select[vertexToCell, Length[#] ≠ 3 &];
   mirrorpairs = <|
      (pt = Lookup[indToPtsAssoc, #];
         If[
          pt[[1]] ≤ xLim[[1]] ||
           pt[[1]] ≥ xLim[[2]] || pt[[2]] ≤ yLim[[1]] || pt[[2]] ≥ yLim[[2]],
          mirror = ptsToIndAssoc[pt /. periodicRules];
          # → mirror,
          Nothing]) & /@ outerpts
      |> // KeySort;
   Map[Sort@*Flatten][List @@@ Normal@GroupBy[Normal@mirrorpairs, Last → First]]
  ];
```

getLocalTopology

In[20]:=

```
𝒟 = Rectangle[{First@xLim, First@yLim}, {Last@xLim, Last@yLim}];
getLocalTopology[ptsToIndAssoc_, indToPtsAssoc_, vertexToCell_,
    cellVertexGrouping_, wrappedMat_, faceListCoords_][vertices_] :=
  Block[{localtopology = <||>, wrappedcellList = {}, vertcellconns,
    localcellunion, v, wrappedcellpos, vertcs = vertices, rl1, rl2,
    transVector, wrappedcellCoords, wrappedcells, vertOutofBounds,
    shiftedPt, transvecList = {}, $faceListCoords = Values@faceListCoords,
    vertexQ, boundsCheck, rules, extractcellkeys, vertind,
    cellsconnected, wrappedcellsrem},
   vertexQ = MatchQ[vertices, {__?NumberQ}];
   If[vertexQ,
    (vertcellconns =
      AssociationThread[{#} , {vertexToCell[ptsToIndAssoc[#]]}] &@vertices;
     vertcs = {vertices};
     localcellunion = Flatten[Values@vertcellconns]),
    (vertcellconns = AssociationThread[# ,
         Lookup[vertexToCell, Lookup[ptsToIndAssoc, #]]] &@vertices;
     localcellunion = Union@Flatten[Values@vertcellconns])
   ];

   If[localcellunion ≠ {},
    AppendTo[localtopology,
     Thread[localcellunion →
       Map[Lookup[indToPtsAssoc, #] &, cellVertexGrouping /@ localcellunion, {2}]]
    ]
   ];
```

```
(* condition to be an internal edge: both vertices should have 3 neighbours *)
(* if a vertex has 3 cells in its local neighbourhood then the entire
 network topology about the vertex is known → no wrapping required *)
(* else we need to wrap around the vertex because other cells
 are connected to it → periodic boundary conditions *)
With[{vert = #},
  vertind = ptsToIndAssoc[vert];
  cellsconnected = vertexToCell[vertind];
  If[Length[cellsconnected] ≠ 3,
   If[(𝒟~RegionMember~Most[vert]),
     (*Print["vertex inside bounds"];*)
     v = vert;
     With[{x = v[[1]], y = v[[2]]}, boundsCheck =
        (x == xLim[[1]] || x == xLim[[2]] || y == yLim[[1]] || y == yLim[[2]])];

     extractcellkeys = If[boundsCheck,
       {rl1, rl2} = {v, v /. periodicRules};
       rules = Block[{x$},
         With[{r = rl1, s = rl2},
          DeleteDuplicates[HoldPattern[SameQ[x$, r]] || HoldPattern[SameQ[x$, s]]]
         ]
        ];
       Position @@ With[{rule = rules},
         Hold[wrappedMat, x_ /; ReleaseHold@rule, {3}]
        ],
       Position[wrappedMat, x_ /; SameQ[x, v], {3}]
      ];
     (* find cell indices that are attached to the vertex in wrappedMat *)
     wrappedcellpos = DeleteDuplicatesBy[
       Cases[extractcellkeys,
        {Key[p : Except[Alternatives @@
              Join[localcellunion, Flatten@wrappedcellList]]], y__} ⧴ {p, y}],
       First];
     (*wrappedcellpos = wrappedcellpos/.
        {Alternatives@@Flatten[wrappedcellList],__} ⧴ Sequence[];*)
     (* if a wrapped cell has not been considered earlier (i.e. is new)
      then we translate it to the position of the vertex *)
     If[wrappedcellpos ≠ {},
      If[vertexQ,
       transVector = SetPrecision[(v - Extract[$faceListCoords,
              Replace[#, {p_, q__} ⧴ {Key[p], q}, {1}]]) & /@ wrappedcellpos, 10],
       (* call to function is enquiring an edge and not a vertex*)
       transVector =
        SetPrecision[(v - Extract[$faceListCoords, #]) & /@ wrappedcellpos, 10]
      ];
      wrappedcellCoords = MapThread[#1 → Map[Function[x,
            SetPrecision[x + #2, 10]], $faceListCoords[[#1]], {2}] &,
        {First /@ wrappedcellpos, transVector}];
      wrappedcells = Keys@wrappedcellCoords;
      AppendTo[wrappedcellList, Flatten@wrappedcells];
 AppendTo[transvecList, transVector];
 AppendTo[localtopology, wrappedcellCoords];
],
(* the else clause: vertex is out of bounds *)
(*Print["vertex out of bounds"];*)
vertOutofBounds = vert;
(* translate the vertex back into mesh *)
transVector = vertOutofBounds /. transformRules;
shiftedPt = SetPrecision[vertOutofBounds + transVector, 10];
(* ------------- CORE B ------------- *)
(* find which cells the
 shifted vertex is a part of in the wrapped matrix *)
wrappedcells = Complement[
  Union@Cases[Position[wrappedMat, x_ /; SameQ[x, shiftedPt], {3}],
     x_Key ⧴ Sequence @@ x, {2}] /.
   Alternatives @@ localcellunion → Sequence[],
  Flatten@wrappedcellList];

(*forming local topology now that we know the wrapped cells *)
If[wrappedcells ≠ {},
 AppendTo[wrappedcellList, Flatten@wrappedcells];
 wrappedcellCoords = AssociationThread[wrappedcells,
   Map[Lookup[indToPtsAssoc, #] &,
    cellVertexGrouping[#] & /@ wrappedcells, {2}]];
 With[{opt = (vertOutofBounds /. periodicRules)},
  Block[{pos, vertref, transvec},
    Do[
     With[{cellcoords = wrappedcellCoords[cell]},
      pos = FirstPosition[cellcoords /. periodicRules, opt];
      vertref = Extract[cellcoords, pos];
      transvec = SetPrecision[vertOutofBounds - vertref, 10];
      AppendTo[transvecList, transvec];
      AppendTo[localtopology,
       cell → Map[SetPrecision[# + transvec, 10] &, cellcoords, {2}]];
     ], {cell, wrappedcells}]
   ];
 ];
];
(* to detect wrapped cells not detected by CORE B*)
(* ------------- CORE C ------------- *)
Block[{pos, celllocs, ls, transvec, assoc, tvecLs = {}, ckey},
 ls = Union@Flatten@Join[cellsconnected, wrappedcells];
 If[Length[ls] ≠ 3,
  pos = Position[faceListCoords, x_ /; SameQ[x, shiftedPt], {3}];
  celllocs = DeleteDuplicatesBy[Cases[pos, Except[{Key[Alternatives @@ ls],
        __}]], First] /. {Key[x_], z__} ⧴ {Key[x], {z}};
  If[celllocs ≠ {},
   celllocs = Transpose@celllocs;
   assoc = <|
     MapThread[
                (transvec = SetPrecision[
                    vertOutofBounds - Extract[faceListCoords[Sequence @@ #1], #2], 10];
                  ckey = Identity @@ #1;
                  AppendTo[tvecLs, transvec];
                  ckey → Map[SetPrecision[Lookup[indToPtsAssoc, #] + transvec, 10] &,
                    cellVertexGrouping[Sequence @@ #1], {2}]
                 ) &, celllocs]
               |>;
             AppendTo[localtopology, assoc];
             AppendTo[wrappedcellList, Keys@assoc];
             AppendTo[transvecList, tvecLs];
            ];
           ];
          ];
         ];
        ];
      ] & /@ vertcs;

    transvecList = Which[
      MatchQ[transvecList, {{{__?NumberQ}}}], First[transvecList],
      MatchQ[transvecList, {{__?NumberQ} ..}], transvecList,
      True, transvecList //. {x___, {p : {__?NumberQ} ..}, y___} ⧴ {x, p, y}
     ];
    {localtopology, Flatten@wrappedcellList, transvecList}
   ];
```

```
In[ ]:= (* to fasten speed of pattern matching we
         only extract the outermost cells for the wrappedMat *)
```

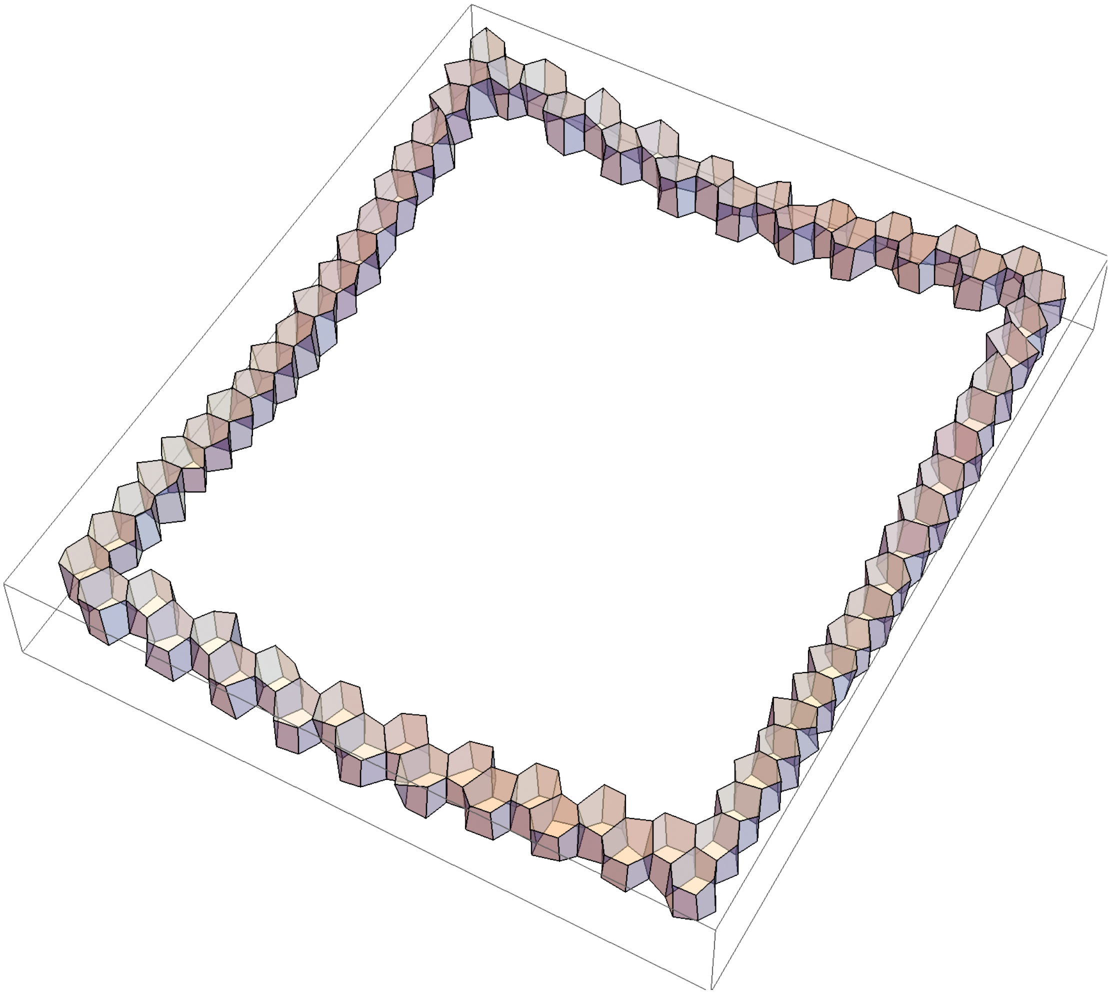
```
In[40]:= boundarycells = outerCellsFn[faceListCoords, vertexToCell, ptsToIndAssoc]
```

```
Out[40]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 40, 41, 60, 61,
          80, 81, 100, 101, 120, 121, 140, 141, 160, 161, 180, 181, 200, 201, 220, 221, 240,
          241, 260, 261, 280, 281, 300, 301, 320, 321, 340, 341, 360, 361, 380, 381, 382, 383,
          384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400}
```

```
In[41]:= wrappedMatSel = wrappedMat~KeyTake~boundarycells;
```

```
In[136]:= plt = Graphics3D[{Opacity[0.5], White, Polyhedron /@ Map[Lookup[indToPtsAssoc, #] &,
              Lookup[cellVertexGrouping, boundarycells], {2}]}, ImageSize → Large]
```

Out[136]=



```
In[•]:= (* neighbour-count for vertex *)
```

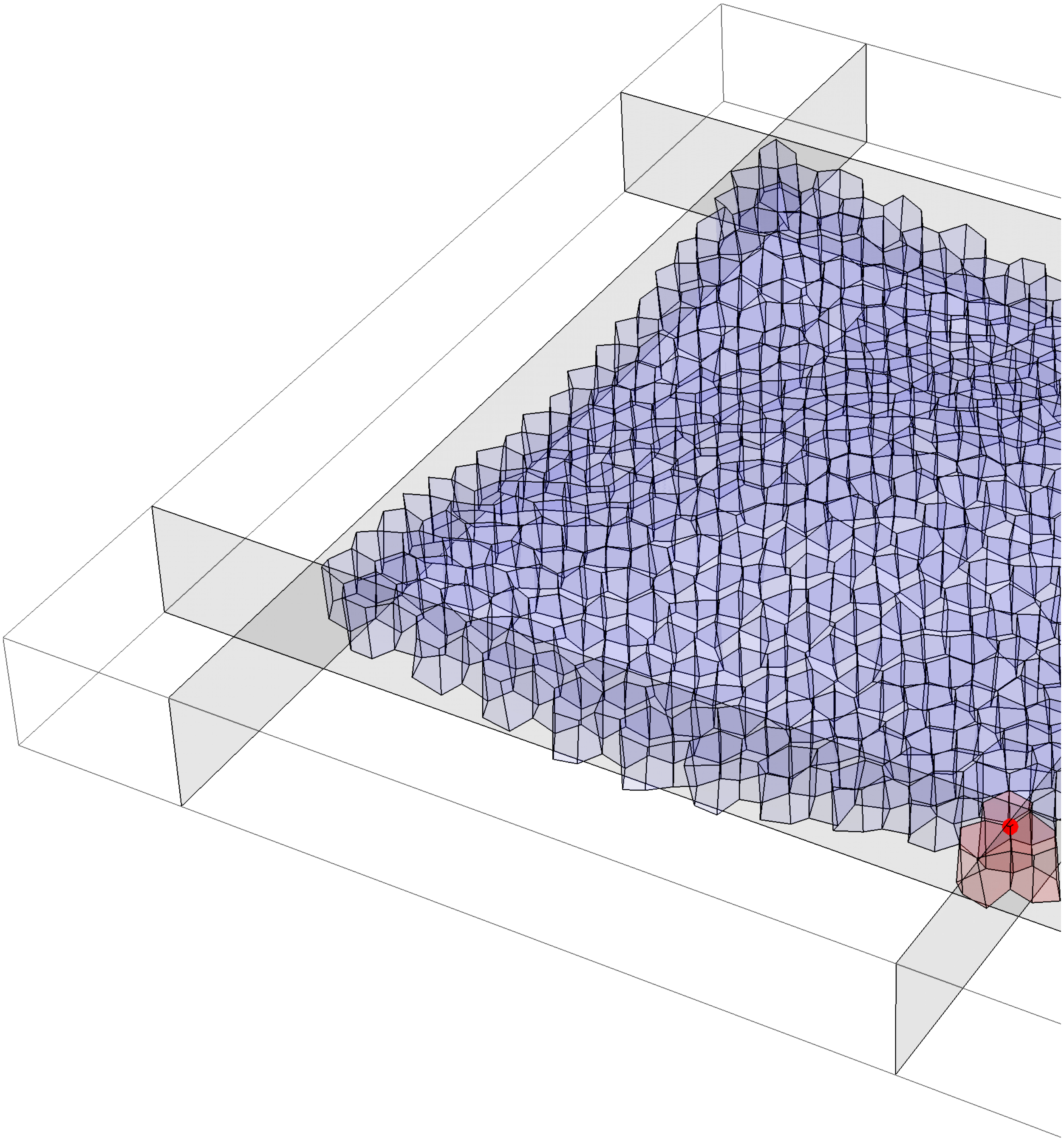
```
In[•]:= Keys[getLocalTopology[ptsToIndAssoc, indToPtsAssoc, vertexToCell, cellVertexGrouping,
                 wrappedMatSel, faceListCoords][indToPtsAssoc[#]] // First] & /@
            Range[Max@ptsToIndAssoc] // Counts@*Map[Length] // AbsoluteTiming

Out[•]= {1.04289, ⟨|3 → 1760|⟩}
```

```
In[•]:= (* neighbour-count for vertex is slow if slightly slower if we use wrappedMat *)
```

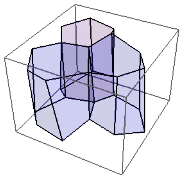
```
In[•]:= Keys[getLocalTopology[ptsToIndAssoc, indToPtsAssoc, vertexToCell, cellVertexGrouping,
                 wrappedMat, faceListCoords][indToPtsAssoc[#]] // First] & /@
            Range[Max@ptsToIndAssoc] // Counts@*Map[Length] // AbsoluteTiming

Out[•]= {3.3385, ⟨|3 → 1760|⟩}
```

selected vertex

```
In[•]:= Block[{xx, yy, zz, edgelen, edgesel, keys, dset, id = 1680, point},
         point = indToPtsAssoc[id];
         {xx, yy, zz} = getLocalTopology[ptsToIndAssoc, indToPtsAssoc, vertexToCell,
            cellVertexGrouping, wrappedMatSel, faceListCoords][indToPtsAssoc[id]];
         keys = Keys@xx;
         Print@Graphics3D[
           {{Opacity[0.05], Blue, Polyhedron /@ Values@faceListCoords}, Red, PointSize[0.01],
            Point@point, Opacity[0.1], Red, Polyhedron /@ Values[xx],
            Black, InfinitePlane[{{0, 0, 0}, {0, xLim[[2]], 0}, {0, xLim[[2]], 1}}],
            InfinitePlane[
             {{0, yLim[[2]], 0}, {xLim[[2]], yLim[[2]], 0}, {xLim[[2]], yLim[[2]], 1}}],
            InfinitePlane[{{xLim[[2]], yLim[[2]], 0}, {xLim[[2]], yLim[[2]], 1},
              {xLim[[2]], yLim[[1]], 1}}],
            InfinitePlane[{{xLim[[2]], yLim[[1]], 0}, {xLim[[2]], yLim[[1]], 1},
              {0, yLim[[1]], 1}}]},
           ImageSize → 1024];
         <|{"vertex" → id,
           "vertices in topology" → Length@DeleteDuplicates@Flatten[Values@xx, 2],
           "cell indices" → keys,
           "translated cell indices" → yy, (*"transvec"→zz,*)
           "localtopology" →
            Graphics3D[{Opacity[0.1], RandomColor[], Polyhedron /@ Lookup[xx, keys],
              Blue, If[yy ≠ {}, Polyhedron /@ Lookup[xx, yy]]}, ImageSize → Tiny]}|>
        ]
```



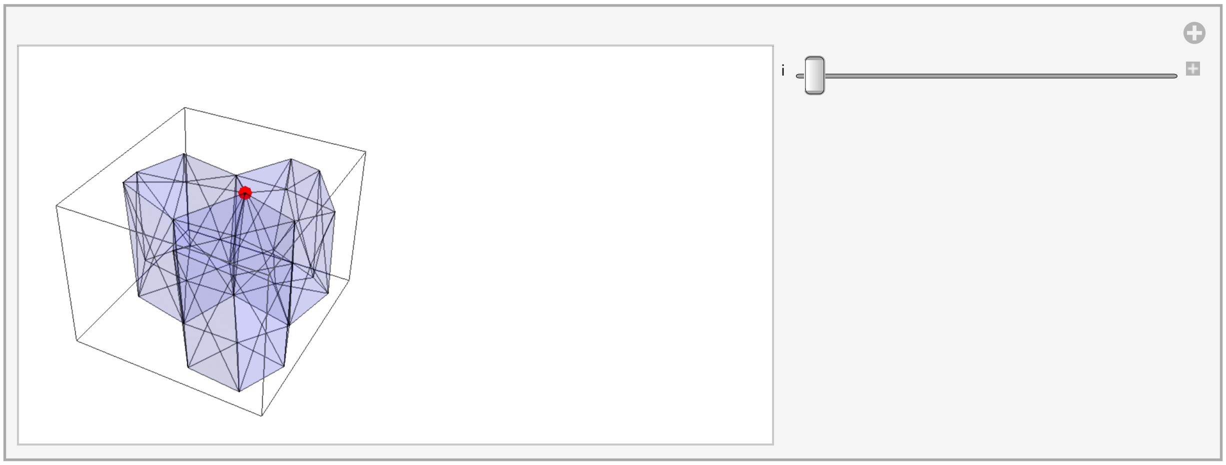
Out[]= ⟨| vertex → 1680, vertices in topology → 26, cell indices → {381, 20, 400},

translated cell indices → {20, 400}, localtopology →  |⟩

vertex to cell topology

```
In[•]:= Manipulate[
         Graphics3D[{{Opacity[0.1], Blue, EdgeForm[{Opacity[0.5], Black}],
            Values@Map[Polyhedron@* (Flatten[#, 1] &) @*triangulateFaces,
              First@getLocalTopology[ptsToIndAssoc, indToPtsAssoc, vertexToCell,
                 cellVertexGrouping, wrappedMatSel, faceListCoords][indToPtsAssoc[i]]]},
           {Red, PointSize[0.04], Point@indToPtsAssoc[i]}}, ImageSize → Small],
         {i, 1, Max[ptsToIndAssoc], 1}, SaveDefinitions → True
        ]
```

Out[•]=



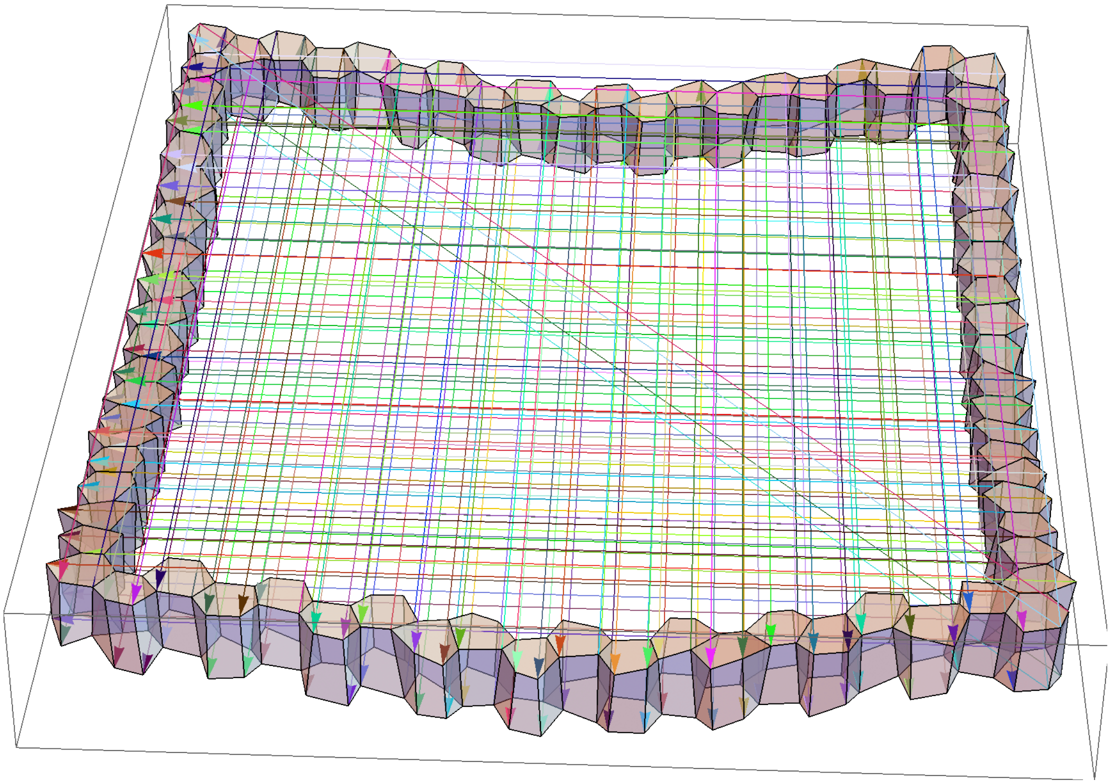
```
In[28]:= (* checking boundary pt pairing *)
```

```
In[77]:= {timing, arrows} =
          boundaryPtsPairing[vertexToCell, indToPtsAssoc, ptsToIndAssoc] // AbsoluteTiming
```

```
Out[77]= {0.0058381, {{4, 1678}, {6, 158}, {10, 1682}, {11, 163, 1681}, {12, 162}, {16, 1686},
   {32, 1694}, {40, 1698}, {52, 1704}, {56, 1706}, {72, 1714}, {84, 1720}, {88, 1722},
   {108, 1732}, {116, 1736}, {120, 1738}, {128, 1742}, {132, 1744}, {136, 1746},
   {140, 1748}, {144, 1750}, {152, 1754}, {5, 159, 1677}, {157, 167}, {168, 246},
   {169, 332}, {161, 171}, {172, 248}, {251, 328}, {165, 329}, {254, 331}, {327, 335},
   {336, 414}, {337, 500}, {330, 339}, {340, 416}, {419, 496}, {422, 499}, {333, 497},
   {495, 503}, {504, 582}, {498, 507}, {508, 584}, {587, 664}, {501, 665}, {590, 667},
   {505, 668}, {669, 833}, {663, 671}, {672, 750}, {666, 675}, {676, 752}, {755, 832},
   {758, 835}, {673, 836}, {831, 839}, {840, 918}, {834, 843}, {844, 920}, {923, 1000},
   {837, 1001}, {926, 1003}, {841, 1004}, {999, 1007}, {1008, 1086}, {1002, 1011},
   {1012, 1088}, {1091, 1168}, {1005, 1169}, {1094, 1171}, {1009, 1172}, {1173, 1337},
   {1167, 1175}, {1176, 1254}, {1177, 1340}, {1170, 1179}, {1180, 1256}, {1259, 1336},
   {1262, 1339}, {1341, 1505}, {1335, 1343}, {1344, 1422}, {1345, 1508}, {1338, 1347},
   {1348, 1424}, {1427, 1504}, {1430, 1507}, {1509, 1673}, {1503, 1511}, {1512, 1590},
   {1506, 1515}, {1516, 1592}, {1595, 1672}, {1598, 1675}, {1513, 1676}, {1671, 1679},
   {160, 1680, 1758}, {1674, 1683}, {164, 1684, 1760}, {3, 1685}, {9, 1687}, {20, 1688},
   {15, 1689}, {24, 1690}, {19, 1691}, {28, 1692}, {23, 1693}, {27, 1695}, {36, 1696},
   {31, 1697}, {35, 1699}, {44, 1700}, {39, 1701}, {48, 1702}, {43, 1703}, {47, 1705},
   {51, 1707}, {60, 1708}, {55, 1709}, {64, 1710}, {59, 1711}, {68, 1712}, {63, 1713},
   {67, 1715}, {76, 1716}, {71, 1717}, {80, 1718}, {75, 1719}, {79, 1721}, {83, 1723},
   {92, 1724}, {87, 1725}, {96, 1726}, {91, 1727}, {100, 1728}, {95, 1729}, {104, 1730},
   {99, 1731}, {103, 1733}, {112, 1734}, {107, 1735}, {111, 1737}, {115, 1739}, {124, 1740},
   {119, 1741}, {123, 1743}, {127, 1745}, {131, 1747}, {135, 1749}, {139, 1751},
   {148, 1752}, {143, 1753}, {147, 1755}, {156, 1756}, {151, 1757}, {155, 1759}}}
```

```
In[137]:= Show[plt, Graphics3D[{Arrowheads[Medium],
              Map[{RandomColor[], Arrow@Lookup[indToPtsAssoc, #]} &, arrows]}, ImageSize → Large]]
```

Out[137]=

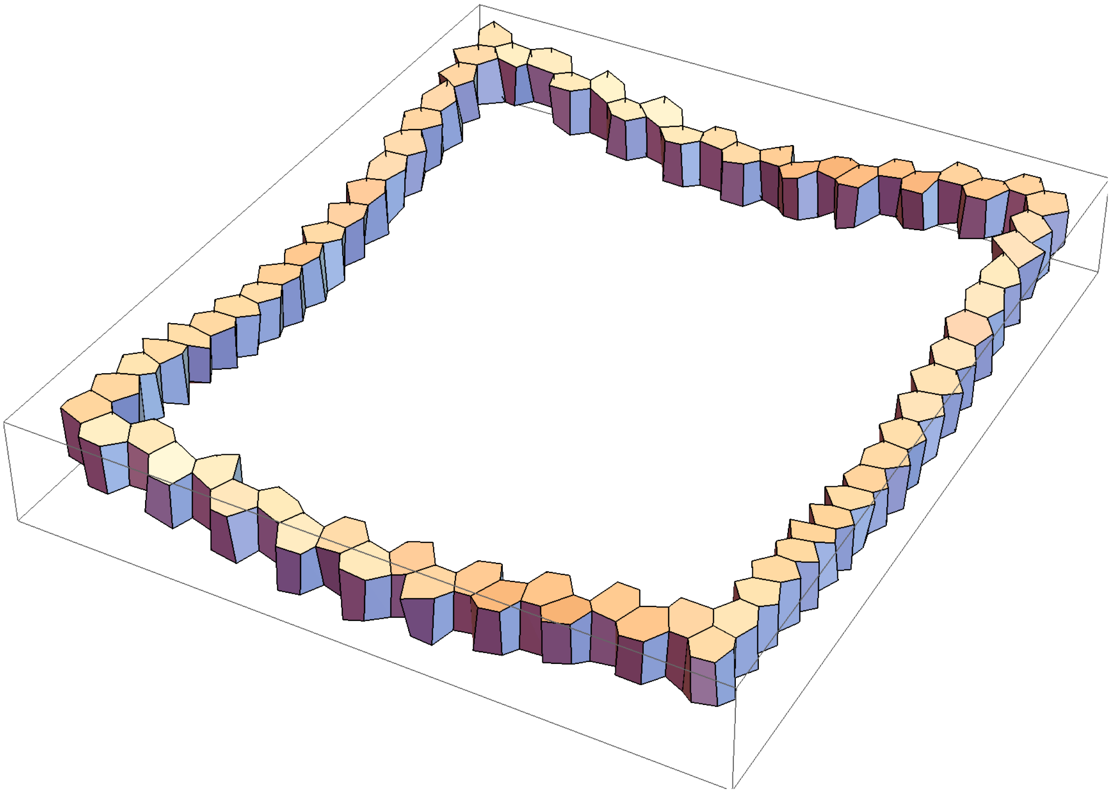


checks

```
In[122]:= indToPtsAssocC = indToPtsAssoc;
          indToPtsAssocC[328] = {2.02500000000000355271367880050092935 56`10.,
              15.6500000000000003553`10., 0.28026929672068318089017679994867648929`10.};
          indToPtsAssocC[329] = {1.57500000000000177635683940025046467 78`10.,
              15.6500000000000003553`10., 0.35966933402467088365384029202687088 4`10.};
          indToPtsAssocC[331] = {2.02500000000000355271367880050092935 56`10.,
              15.6500000000000003553`10., -0.71973070327931676359867196879349648952`10.};
          indToPtsAssocC[332] = {1.57500000000000177635683940025046467 78`10.,
              15.6500000000000003553`10., -0.64033066597532917185731093923095613718`10.};
          indToPtsAssocC[165] = {1.57500000000000177635683940025046467 78`10.,
              0, 0.35966933402467088365384029202687088 4`10.};
          indToPtsAssocC[169] = {1.57500000000000177635683940025046467 78`10.,
              0, -0.64033066597532917185731093923095613718`10.};
          indToPtsAssocC[251] = {2.02500000000000355271367880050092935 56`10.,
              0, 0.28026929672068318089017679994867648929`10.};
          indToPtsAssocC[254] = {2.02500000000000355271367880050092935 56`10.,
              0, -0.71973070327931676359867196879349648952`10.};
```

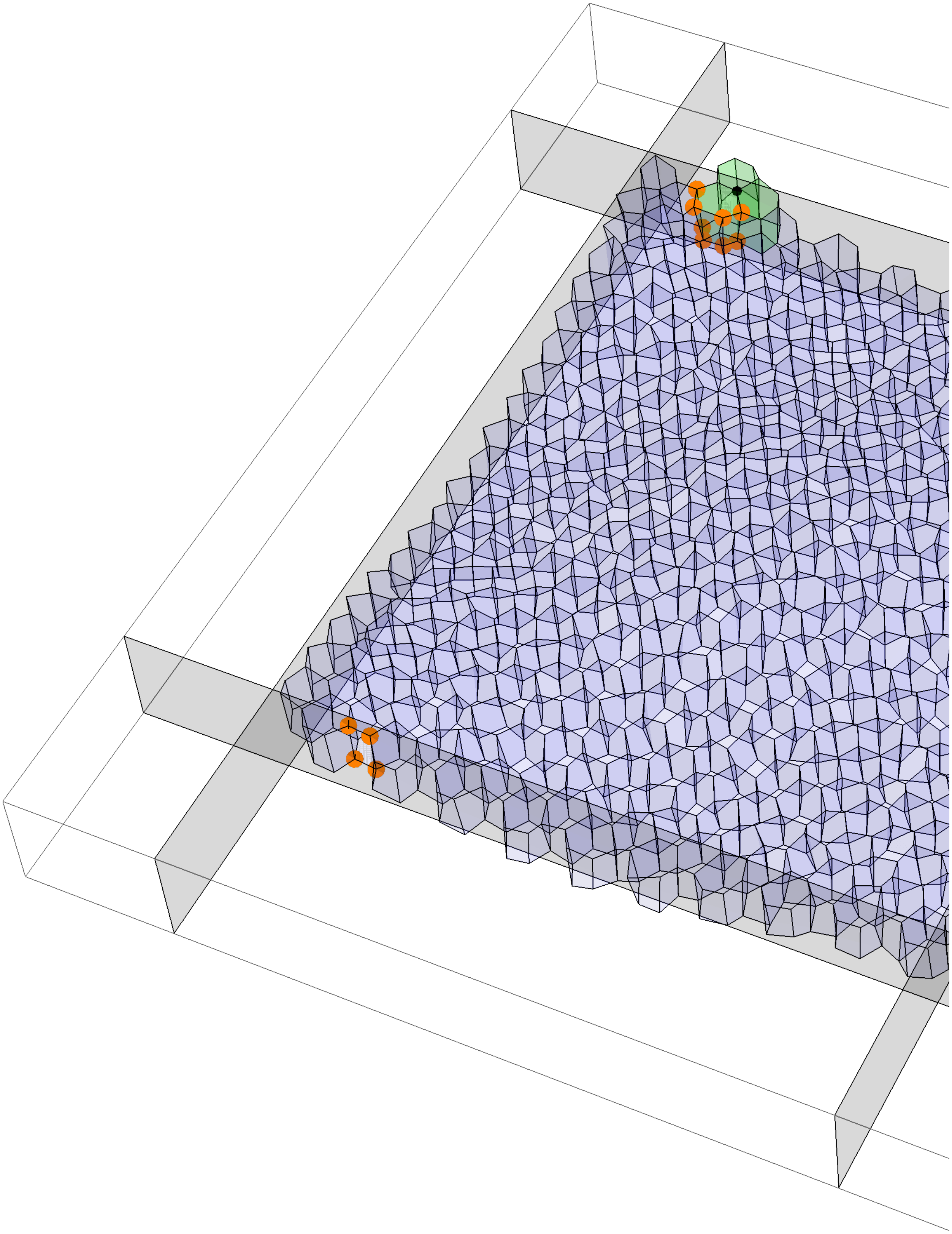
```
In[131]:= wrappedMatC = AssociationThread[
              Keys[cellVertexGrouping] → Map[Lookup[indToPtsAssocC, #] /. periodicRules &,
                Lookup[cellVertexGrouping, Keys[cellVertexGrouping]], {2}]];
```

```
In[132]:= faceListCoordsC = Map[Lookup[indToPtsAssocC, #] &, cellVertexGrouping, {2}];

In[133]:= ptsToIndAssocC = AssociationMap[Reverse, indToPtsAssocC];

In[134]:= boundarycellsC = outerCellsFn[faceListCoordsC, vertexToCell, ptsToIndAssocC]

Out[134]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 40, 41, 60, 61,
           80, 81, 100, 101, 120, 121, 140, 141, 160, 161, 180, 181, 200, 201, 220, 221, 240,
           241, 260, 261, 280, 281, 300, 301, 320, 321, 340, 341, 360, 361, 380, 381, 382, 383,
           384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400}

In[•]:= Graphics3D[Polyhedron /@ Map[Lookup[indToPtsAssoc, #] &,
           Lookup[cellVertexGrouping, boundarycells], {2}], ImageSize → Large]

Out[•]=
```

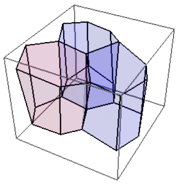


```
In[135]:= wrappedMatCSel = KeyTake[wrappedMatC, boundarycellsC];
```

```
In[∘]:= Block[{xx, yy, zz, edgelen, edgesel, keys, dset, id = 328, point},
         point = indToPtsAssocC[id];
         {xx, yy, zz} = getLocalTopology[ptsToIndAssocC, indToPtsAssocC, vertexToCell,
            cellVertexGrouping, wrappedMatCSel, faceListCoordsC][indToPtsAssocC[id]];
         keys = Keys@xx;
         Print@Graphics3D[{{Opacity[0.05], Blue, Polyhedron /@ Values@faceListCoordsC},
            Black, PointSize[0.007], Point@point, Green, Point[point /. periodicRules],
            PointSize[0.012], Orange, Map[Point, wrappedMatC[60], {2}],
            Opacity[0.15], Green, Polyhedron /@ Values[xx],
            Black, InfinitePlane[{{0, 0, 0}, {0, xLim[[2]], 0}, {0, xLim[[2]], 1}}],
            InfinitePlane[
             {{0, yLim[[2]], 0}, {xLim[[2]], yLim[[2]], 0}, {xLim[[2]], yLim[[2]], 1}}],
            InfinitePlane[{{xLim[[2]], yLim[[2]], 0}, {xLim[[2]], yLim[[2]], 1},
              {xLim[[2]], yLim[[1]], 1}}],
            InfinitePlane[{{xLim[[2]], yLim[[1]], 0}, {xLim[[2]], yLim[[1]], 1},
              {0, yLim[[1]], 1}}]},
           ImageSize → 1024];
         <|{"vertex" → id,
           "vertices in topology" → Length@DeleteDuplicates@Flatten[Values@xx, 2],
           "cell indices" → keys, "translated cell indices" → yy, (*"transvec"→zz,*)
           "localtopology" →
            Graphics3D[{Opacity[0.1], RandomColor[], Polyhedron /@ Lookup[xx, keys],
              Blue, If[yy ≠ {}, Polyhedron /@ Lookup[xx, yy]]}, ImageSize → Tiny]}|>
        ]
```



```
Out[∘]= ⟨|vertex → 328, vertices in topology → 26, cell indices → {60, 41, 61},

         translated cell indices → {41, 61}, localtopology →
```



```
         |⟩
```

```
In[∘]:= Keys[getLocalTopology[ptsToIndAssocC, indToPtsAssocC, vertexToCell, cellVertexGrouping,
                wrappedMatCSel, faceListCoordsC][indToPtsAssocC[#]] // First] & /@
          Range[Max@ptsToIndAssocC] // Counts@*Map[Length] // AbsoluteTiming

Out[∘]= {1.00874, ⟨|3 → 1760|⟩}
```