

(\*code modified from Eduardo Sandoval/Jian Liu\*)

Dump

# Mains()

```
In[ ]:= Quiet@Remove["Global`*"]; Quiet@ClearAll["Global`*"];
```

```
In[ ]:= Clear@interpmembraneProfile;
interpmembraneProfile[ $\rho$ _, h_,  $\eta$ _,  $\Delta s$ _, nsteps_, interpolationnumber_] :=
Block[{index1, index2,  $\rho$ c, hc,  $\rho$ new, hnew, interpolationpts, interpolationFn,
interpolation,  $\rho$ interp, hinterp, interpolationFn2, interpolation2,  $\rho$ newf, hnewf},

If[True,
(*to extrapolate we first take whatever our extrapolation number is defined as
interpolation number because when we consider the total vesicle/cell,
technically we are just interpolating as we are filling in between known points),
and find the minimum index for either end of the
contour that corresponds to this value *)
index1 = First@Flatten@Position[#, Min@#] &&
Abs[ $\rho$ [[IntegerPart[ $\eta$  * nsteps] ;; nsteps + 1]] - interpolationnumber];
index2 = First@Flatten@Position[#, Min@#] &&
Abs[ $\rho$ [[1 ;; IntegerPart[ $\eta$  * nsteps]]] - interpolationnumber];
(*we use the index to construct a new
vector and thus solution that reflects the rest of the solution
about the y axis and generates a vector to fill in the rest*)
 $\rho$ c =  $\rho$ [[index2 ;; index1 +  $\eta$  * nsteps]];
hc = h[[index2 ;; index1 +  $\eta$  * nsteps]];

(*for first interpolation we use newradius and newheight*)
 $\rho$ new = ConstantArray[0, 2 Length[ $\rho$ c]];
 $\rho$ new[[1 ;; Length@ $\rho$ c]] =  $\rho$ c;
 $\rho$ new[[Length[ $\rho$ c] + 1 ;;]] = -Reverse[ $\rho$ c];

hnew = ConstantArray[0, 2 Length[hc]];
hnew[[1 ;; Length@hc]] = hc;
hnew[[Length[hc] + 1 ;;]] = Reverse[hc];

interpolationpts = Range[-interpolationnumber, interpolationnumber,  $\Delta s$ ];

interpolationFn =
Interpolation[Thread[{ $\rho$ new[[Length[ $\rho$ new] / 2 - interpolationnumber ;;
Length[ $\rho$ new] / 2 + interpolationnumber]],
hnew[[Length[ $\rho$ new] / 2 - interpolationnumber ;; Length[ $\rho$ new] / 2 +
interpolationnumber]]], Method -> "Spline", InterpolationOrder -> 2];
```

```

interpolation = Table[interpolationFn[x], {x, interpolationpts}];
(*for second interpolation we use interpradius and interpheight*)
pinterp = ConstantArray[0, 2 Length@ρc];
pinterp[[1 ;; Length@ρc]] = Reverse@ρc;
pinterp[[Length[ρc] + 1 ;;]] = -ρc;
hinterp = ConstantArray[0, 2 Length@hc];
hinterp[[1 ;; Length@hc]] = Reverse@hc;
hinterp[[Length[hc] + 1 ;;]] = hc;

interpolationFn2 =
  Interpolation[Thread[{pinterp[[Length[ρnew] / 2 - interpolationnumber
    ;; Length[ρnew] / 2 + interpolationnumber]],
    hinterp[[Length[ρnew] / 2 - interpolationnumber ;; Length[ρnew] / 2 +
      interpolationnumber]]}], Method → "Spline", InterpolationOrder → 2];
interpolation2 = Table[interpolationFn2[x], {x, interpolationpts}];

(*final vectors with interpolations incorporated*)
ρnewf = ConstantArray[0, 2 Length[ρc] + 2 Length[interpolationpts]];
ρnewf[[1 ;; Length[interpolationpts]]] = interpolationpts;
ρnewf[[Length[interpolationpts] + 1 ;; Length[ρc] + Length[interpolationpts]]] = ρc;
ρnewf[[Length[ρc] + Length[interpolationpts] + 1
  ;; Length[ρc] + 2 Length[interpolationpts]]] = -interpolationpts;
ρnewf[[Length[ρc] + 2 Length[interpolationpts] + 1 ;;
  2 Length[ρc] + 2 Length[interpolationpts]]] = -Reverse[ρc];

hnewf = ConstantArray[0, 2 Length[ρc] + 2 Length[interpolationpts]];
hnewf[[1 ;; Length[interpolationpts]]] = interpolation2;
hnewf[[Length[interpolationpts] + 1 ;; Length[ρc] + Length@interpolationpts]] = hc;
hnewf[[Length[ρc] + Length[interpolationpts] + 1
  ;; Length[ρc] + 2 Length[interpolationpts]]] = interpolation;
hnewf[[Length[ρc] + 2 Length[interpolationpts] + 1 ;;
  2 Length[ρc] + 2 Length[interpolationpts]]] = Reverse[hc];

{ρnewf, hnewf}
]
];

```

In[ ]:=

```

Clear@membraneShapeFn;
membraneShapeFn[C_, nsteps_, η_, δθ_, ξθ_, ρθ_, σ_, κ_, P_, extrap_ : True | False,
  interpolationnumber_] := Block[{θ, δ, ξ, ρ, h, Δs, Cvector, H, ρiter, hiter},
  Δs = C / nsteps;
  H = η  $\left( \frac{C}{\Delta s} \right)$ ;
  Cvector = Subdivide[C, nsteps];
  θ = δ = ξ = ρ = h = ConstantArray[0, Length@Cvector];

```

```

ρ[[1]] = ρ0;
ρiter = ρ0;
h[[1]] = 0;
hiter = 0.;
θ[[1]] = π / 2;
δ[[1]] = δ0;
ξ[[1]] = ξ0;

```

$$\varrho[[1]] = N \left[ -\frac{1}{2} \delta[[1]]^3 - 2 \frac{\cos[\theta[[1]]]}{\rho[[1]]} \xi[[1]] + \frac{3}{2} \frac{\sin[\theta[[1]]]}{\rho[[1]]} \delta[[1]]^2 + \frac{3 \cos[\theta[[1]]]^2 - 1}{2 \rho[[1]]^2} \delta[[1]] - \frac{\cos[\theta[[1]]]^2 + 1}{2 \rho[[1]]^3} \sin[\theta[[1]]] + \frac{\sigma[[1]]}{\kappa[[1]]} \left( \delta[[1]] + \frac{\sin[\theta[[1]]]}{\rho[[1]]} \right) + \frac{P}{\kappa[[1]]} \right];$$

```
Do[
```

```
Which[j == 2,
```

```
(*we use a central finite difference method, which results in a modified step
for the second timestep of the simulation. we assume θ[[i-2]]=θ[[i-1]]*)
```

```
θ[[j]] = θ[[j - 1]] + 2 δ[[j - 1]] Δs;
```

```
δ[[j]] = δ[[j - 1]] + 2 ξ[[j - 1]] Δs;
```

```
ξ[[j]] = ξ[[j - 1]] + 2 ρ[[j - 1]] Δs;
```

```
(*we update the radius and height for each step,
this is estimated by using trigonometry and assuming that our
Δs is small enough that this error will not be far off*)
```

```
ρiter += Δs * Cos[θ[[j]]] // N;
```

```
ρ[[j]] = ρiter;
```

```
hiter += Δs * Sin[θ[[j]]] // N;
```

```
h[[j]] = hiter;
```

```
(*again we have the shape equation*)
```

$$\varrho[[j]] = N \left[ -\frac{1}{2} \delta[[j]]^3 - 2 \frac{\cos[\theta[[j]]]}{\rho[[j]]} \xi[[j]] + \frac{3}{2} \frac{\sin[\theta[[j]]]}{\rho[[j]]} \delta[[j]]^2 + \frac{3 \cos[\theta[[j]]]^2 - 1}{2 \rho[[j]]^2} \delta[[j]] - \frac{\cos[\theta[[j]]]^2 + 1}{2 \rho[[j]]^3} \sin[\theta[[j]]] + \frac{\sigma[[1]]}{\kappa[[1]]} \left( \delta[[j]] + \frac{\sin[\theta[[j]]]}{\rho[[j]]} \right) + \frac{P}{\kappa[[1]]} \right];$$

```
,
```

```
j ≤ IntegerPart[nsteps / 2] + 1,
```

```
Which[j ≤ IntegerPart[ℋ] + 1,
```

```
θ[[j]] = θ[[j - 2]] + 2 * δ[[j - 1]] * Δs;
```

```
(*this if loop is purely for debugging
and seeing where a simulation breaks to adjust guesses*)
```

```

If[ $\theta[j] > 2\pi \ || \ \theta[j] < -2\pi$ ,
  (*this part should not run*)
  Print@j;
   $\delta[j] = \delta[j - 2] + 2 \zeta[j - 1] \Delta s$ ;
   $\zeta[j] = \zeta[j - 2] + 2 \varrho[j - 1] \Delta s$ ;
   $\rho_{iter} += N \Delta s * \text{Cos}[\theta[j]]$ ;
   $\rho[j] = \rho_{iter}$ ;
   $h_{iter} += N \Delta s * \text{Sin}[\theta[j]]$ ;
   $h[j] = h_{iter}$ ;

  
$$\varrho[j] = N \left[ -\frac{1}{2} \delta[j]^3 - 2 \frac{\text{Cos}[\theta[j]]}{\rho[j]} \zeta[j] + \frac{3}{2} \frac{\text{Sin}[\theta[j]]}{\rho[j]} \delta[j]^2 + \frac{3 \text{Cos}[\theta[j]]^2 - 1}{2 \rho[j]^2} \right.$$


$$\left. \delta[j] - \frac{\text{Cos}[\theta[j]]^2 + 1}{2 \rho[j]^3} \text{Sin}[\theta[j]] + \frac{\sigma[1]}{\kappa[1]} \left( \delta[j] + \frac{\text{Sin}[\theta[j]]}{\rho[j]} \right) + \frac{p}{\kappa[1]} \right];$$


  Continue[],

   $\delta[j] = \delta[j - 2] + 2 \zeta[j - 1] \Delta s$ ;
   $\zeta[j] = \zeta[j - 2] + 2 \varrho[j - 1] \Delta s$ ;
   $\rho_{iter} += N \Delta s * \text{Cos}[\theta[j]]$ ;
   $\rho[j] = \rho_{iter}$ ;
   $h_{iter} += N \Delta s * \text{Sin}[\theta[j]]$ ;
   $h[j] = h_{iter}$ ;

  
$$\varrho[j] = N \left[ -\frac{1}{2} \delta[j]^3 - 2 \frac{\text{Cos}[\theta[j]]}{\rho[j]} \zeta[j] + \frac{3}{2} \frac{\text{Sin}[\theta[j]]}{\rho[j]} \delta[j]^2 + \frac{3 \text{Cos}[\theta[j]]^2 - 1}{2 \rho[j]^2} \right.$$


$$\left. \delta[j] - \frac{\text{Cos}[\theta[j]]^2 + 1}{2 \rho[j]^3} \text{Sin}[\theta[j]] + \frac{\sigma[1]}{\kappa[1]} \left( \delta[j] + \frac{\text{Sin}[\theta[j]]}{\rho[j]} \right) + \frac{p}{\kappa[1]} \right];$$


],
j == IntegerPart[ $\mathcal{H}$ ] + 2,
(*this part should not run for our case*)
(*we have reached our phase boundary
or transition. Relevant formulae can be found in Jian Liu's
paper: endocytic vesicle scission by lipid phase boundary forces*)
 $\theta[j] = \theta[j - 2] + 2 \delta[j - 1] * \Delta s$ ;
 $\rho_{iter} += N \Delta s * \text{Cos}[\theta[j]]$ ;
 $\rho[j] = \rho_{iter}$ ;
 $h_{iter} += N \Delta s * \text{Sin}[\theta[j]]$ ;
 $h[j] = h_{iter}$ ;
phase1 $\delta = \delta[j - 2] - 2 \zeta[j - 1] \Delta s$ ;
phase1 $\zeta = \zeta[j - 2] - 2 \varrho[j - 1] \Delta s$ ;
phase2 $\delta = \frac{\kappa[1]}{\kappa[2]} \text{phase1}\delta - \frac{\text{Sin}[\theta[j]] (\kappa[2] - \kappa[1])}{\kappa[2] * \rho[j]}$ ;
 $\delta[j] = \text{phase2}\delta$ ;

```

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phase2ξ =  $\frac{\kappa[1]}{\kappa[2]} \left( \text{phase1}\xi - \frac{\text{Sin}[\theta[j]] \text{Cos}[\theta[j]]}{\rho[j]^2} + \text{Cos}[\theta[j]] \frac{\text{phase1}\delta}{\rho[j]} \right) -$ 
 $\frac{\text{Cos}[\theta[j]] \delta[j]}{\rho[j]} + \frac{\text{Sin}[\theta[j]] \text{Cos}[\theta[j]]}{\rho[j]^2};$ 
ξ[j] = phase2ξ;
e[j] = N[ $-\frac{1}{2} \delta[j]^3 - 2 \frac{\text{Cos}[\theta[j]]}{\rho[j]} \xi[j] + \frac{3}{2} \frac{\text{Sin}[\theta[j]]}{\rho[j]} \delta[j]^2 + \frac{3 \text{Cos}[\theta[j]]^2 - 1}{2 \rho[j]^2} \delta[j] -$ 
 $\frac{\text{Cos}[\theta[j]]^2 + 1}{2 \rho[j]^3} \text{Sin}[\theta[j]] + \frac{\sigma[2]}{\kappa[2]} \left( \delta[j] + \frac{\text{Sin}[\theta[j]]}{\rho[j]} \right) + \frac{p}{\kappa[2]}]$ ;
,
True
,
,
θ[j] = θ[j - 2] + 2 δ[j - 1] Δs;
δ[j] = δ[j - 2] + 2 ξ[j - 1] Δs;
ξ[j] = ξ[j - 2] + 2 e[j - 1] Δs;
ρiter += N@Δs * Cos[θ[j]];
ρ[j] = ρiter;
hiter += N@Δs * Sin[θ[j]];
h[j] = hiter;
e[j] = N[ $-\frac{1}{2} \delta[j]^3 - 2 \frac{\text{Cos}[\theta[j]]}{\rho[j]} \xi[j] + \frac{3}{2} \frac{\text{Sin}[\theta[j]]}{\rho[j]} \delta[j]^2 + \frac{3 \text{Cos}[\theta[j]]^2 - 1}{2 \rho[j]^2} \delta[j] -$ 
 $\frac{\text{Cos}[\theta[j]]^2 + 1}{2 \rho[j]^3} \text{Sin}[\theta[j]] + \frac{\sigma[2]}{\kappa[2]} \left( \delta[j] + \frac{\text{Sin}[\theta[j]]}{\rho[j]} \right) + \frac{p}{\kappa[2]}]$ ;
],
j == IntegerPart[ $\frac{\text{nsteps}}{2}$ ] + 2,

(*when we have reached halfway through the simulation,
we have to back calculate the remaining half,
since we began at half of the contour. We start by initializing a
new index to make the computation easier, since now we are back
calculating and will subtract the index by one each step *)
i = IntegerPart[nsteps / 2];
(*we then take simulation results so far and
define them as the second half of the solution i.e. we have a
1000 length vector and got to 500 and now must backcalculate,
we redefined the last 500 of the vector to our current result *)
θ[i + 1 ;;] = θ[1 ;; j - 1];
ξ[i + 1 ;;] = ξ[1 ;; j - 1];
δ[i + 1 ;;] = δ[1 ;; j - 1];
e[i + 1 ;;] = e[1 ;; j - 1];
(*the bottom four lines may not be needed and are only used to placate
paranoia that previously defined values are not refound and maintain
inaccuracies *)

```

```

 $\theta[1 ;; i] = 0;$ 
 $\varrho[1 ;; i] = 0;$ 
 $\xi[1 ;; i] = 0;$ 
 $\delta[1 ;; i] = 0;$ 

 $h[i + 1 ;; j] = h[1 ;; j - 1];$ 
 $\rho[i + 1 ;; j] = \rho[1 ;; j - 1];$ 
 $h[1 ;; i] = 0;$ 
 $\rho[1 ;; i] = 0;$ 
 $\theta[i] = \theta[i + 2] - 2 \delta[i + 1] \Delta s;$ 
 $\rho_{iter} = \rho \theta;$ 
 $\rho_{iter} -= N \Delta s * \cos[\theta[i]];$ 
 $\rho[i] = \rho_{iter};$ 
 $h_{iter} = 0;$ 
 $h_{iter} -= N \Delta s * \sin[\theta[i]];$ 
 $h[i] = h_{iter};$ 

If[i == IntegerPart[ $\mathcal{H}$ ] + 2,
  (*this part should not run for our case*)
  phase1 $\delta$  =  $\delta[i + 2] - 2 \xi[i + 1] \Delta s;$ 
  phase1 $\xi$  =  $\xi[i + 2] - 2 \varrho[i + 1] \Delta s;$ 
  phase2 $\delta$  =  $\frac{\kappa[1]}{\kappa[2]} \text{phase1}\delta - \frac{\sin[\theta[i]] (\kappa[2] - \kappa[1])}{\kappa[2] \times \rho[i]};$ 
   $\delta[i] = \text{phase2}\delta;$ 
  phase2 $\xi$  =  $\frac{\kappa[1]}{\kappa[2]} \left( \text{phase1}\xi - \frac{\sin[\theta[i]] \cos[\theta[i]]}{\rho[i]^2} + \cos[\theta[i]] \frac{\text{phase1}\delta}{\rho[i]} \right) -$ 
 $\frac{\cos[\theta[i]] \delta[i]}{\rho[i]} + \frac{\sin[\theta[i]] \cos[\theta[i]]}{\rho[i]^2};$ 
   $\xi[i] = \text{phase2}\xi;$ 
  (*note that since we are in the second phase, the membrane shape is now using
  the second measurements of membrane tension and binding modulus*)
   $\varrho[i] = N \left[ -\frac{1}{2} \delta[i]^3 - 2 \frac{\cos[\theta[i]]}{\rho[i]} \xi[i] + \frac{3}{2} \frac{\sin[\theta[i]]}{\rho[i]} \delta[i]^2 + \frac{3 \cos[\theta[i]]^2 - 1}{2 \rho[i]^2} \delta[i] - \right.$ 
 $\left. \frac{\cos[\theta[i]]^2 + 1}{2 \rho[i]^3} \sin[\theta[i]] + \frac{\sigma[2]}{\kappa[2]} \left( \delta[i] + \frac{\sin[\theta[i]]}{\rho[i]} \right) + \frac{p}{\kappa[2]} \right];$ 
  ,

   $\delta[i] = \delta[i + 2] - 2 \xi[i + 1] \Delta s;$ 
   $\xi[i] = \xi[i + 2] - 2 \varrho[i + 1] \Delta s;$ 
   $\varrho[i] = N \left[ -\frac{1}{2} \delta[i]^3 - 2 \frac{\cos[\theta[i]]}{\rho[i]} \xi[i] + \frac{3}{2} \frac{\sin[\theta[i]]}{\rho[i]} \delta[i]^2 + \frac{3 \cos[\theta[i]]^2 - 1}{2 \rho[i]^2} \delta[i] - \right.$ 

```

```

        
$$\frac{\cos[\theta[i]]^2 + 1}{2\rho[i]^3} \sin[\theta[i]] + \frac{\sigma[2]}{\kappa[2]} \left( \delta[i] + \frac{\sin[\theta[i]]}{\rho[i]} \right) + \frac{p}{\kappa[2]} \Bigg];$$

    ],

    True,

    i = i - 1;
     $\theta[i] = \theta[i + 2] - 2 \delta[i + 1] \Delta s;$ 
    If[ $\theta[i] > 2\pi$  ||  $\theta[i] < -2\pi$ ,
      Print@j;
      Break[],

       $\rho_{iter} = N \Delta s * \cos[\theta[i]];$ 
       $h_{iter} = N \Delta s * \sin[\theta[i]];$ 
       $\delta[i] = \delta[i + 2] - 2 \xi[i + 1] \Delta s;$ 
       $\xi[i] = \xi[i + 2] - 2 \varrho[i + 1] \Delta s;$ 
       $\rho[i] = \rho_{iter};$ 
       $h[i] = h_{iter};$ 
       $\varrho[i] = N \Bigg[ -\frac{1}{2} \delta[i]^3 - 2 \frac{\cos[\theta[i]]}{\rho[i]} \xi[i] + \frac{3}{2} \frac{\sin[\theta[i]]}{\rho[i]} * \delta[i]^2 + \frac{3 \cos[\theta[i]]^2 - 1}{2 \rho[i]^2}$ 
        
$$\delta[i] - \frac{\cos[\theta[i]]^2 + 1}{2\rho[i]^3} \sin[\theta[i]] + \frac{\sigma[2]}{\kappa[2]} \left( \delta[i] + \frac{\sin[\theta[i]]}{\rho[i]} \right) + \frac{p}{\kappa[2]} \Bigg];$$

    ];
  ];
  , {j, 2, Length[Cvector]}
];
{ $\rho$ , h};
If[extrap, interpmembraneProfile[ $\rho$ , h,  $\eta$ ,  $\Delta s$ , nsteps, interpolationnumber]]
];

```

```

In[ ]:= data = Import["C:\\Users\\hashmial\\Downloads\\nuclei
    curvature and dP fit estimates\\dump\\shape.xlsx", "Data"][[1]];

```

```

In[ ]:= {headings, data} = {First@data, Rest@data};

```

```

In[ ]:= initialslopeList = data[[All, 1]];
initialcurvatureList = data[[All, 2]];
sigmalist = data[[All, 3]];
Plist = data[[All, 4]];
baselinesigma = 0.1;
C = 1000  $\pi$ ;
baselineP = -2 baselinesigma / (C /  $\pi$ );
initialradiiList = data[[All, 5]];
extrapolationNumbers = data[[All, 6]];

```

```

In[ ]:= Clear@membraneShapePhasePlane;
membraneShapePhasePlane[] := Block[{ },
  maxradius = minradius = maxheight = minheight = 0;
  C = 1000  $\pi$ ;
  nsteps = 2000;

   $\eta$  = 1 / 2;
   $\kappa$  = {16000, 16000};
   $\sigma$  = {1 / 10, 1 / 10};
  P = -2  $\sigma$ [[1]] / (C /  $\pi$ );
  extrapolation = True;
  interpolationNumber = 10;

   $\rho_0$  = C /  $\pi$ ;
   $\delta_0$  = 1 / (C /  $\pi$ );
   $\xi_0$  = 0;
  spacing = 0.2;
  magicnum = 3000;
  (* rest of the code comes here*)
  Table[

     $\sigma$  = {baselinesigma, sigmalist[[1]]};
     $\delta_0$  = initialslopeList[[1]];
     $\xi_0$  = initialcurvatureList[[1]];
     $\rho_0$  = initialradiiList[[1]];
    P = Plist[[1]];
    interpolationNumber = extrapolationNumbers[[1]];
    {radius, height} =
      membraneShapeFn[C, nsteps,  $\eta$ ,  $\delta_0$ ,  $\xi_0$ ,  $\rho_0$ ,  $\sigma$ ,  $\kappa$ , P, True, interpolationNumber];

    (*changeinrad= $\frac{\text{sigmalist}[[1]]-\text{baselinesigma}}{\text{baselinesigma spacing}}$  *magicnum;

    changeinheight= $\frac{P-\text{baselineP}}{\text{baselineP spacing}}$  *magicnum;

```



```

height+=changeinheight;

radius+=changeinrad;*)

If[extrapolation, {sigmalist[[1]] / baselinesigma, P / baselineP,
  (*ListLinePlot[Thread@{radius,height},AspectRatio→{1,1},Axes→False,ImageSize→
    Tiny,MaxPlotPoints→200,InterpolationOrder→1,PlotStyle→RandomColor[]],*)
  Graphics[{FaceForm[Directive[Opacity[0.2], RandomColor[]]],
    EdgeForm[Directive[Black]], FilledCurve[BSplineCurve[
      Thread[{radius, height}], SplineClosed → True]]], ImageSize → Tiny],
  Graphics[{FaceForm[Directive[RGBColor[0.255, 0.333, 0.902, 0.2]]],
    EdgeForm[Directive[Black]], FilledCurve[BSplineCurve[
      Thread[{radius, height}], SplineClosed → True]]], ImageSize → Tiny]
  }]
, {1, Length[Plist]}
]

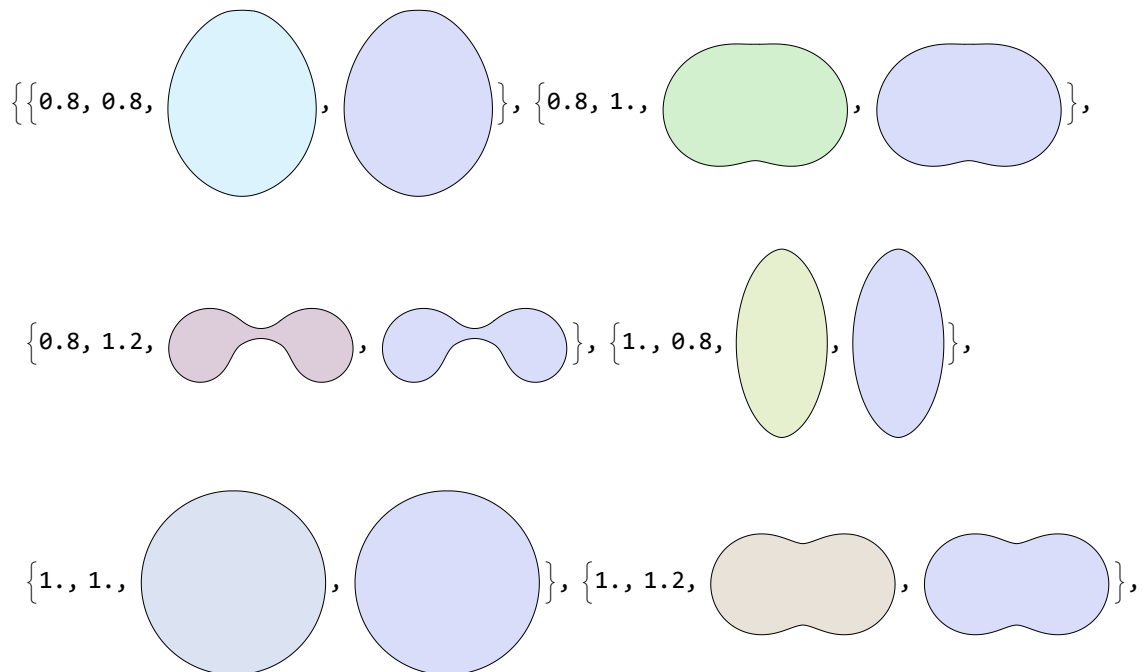
];

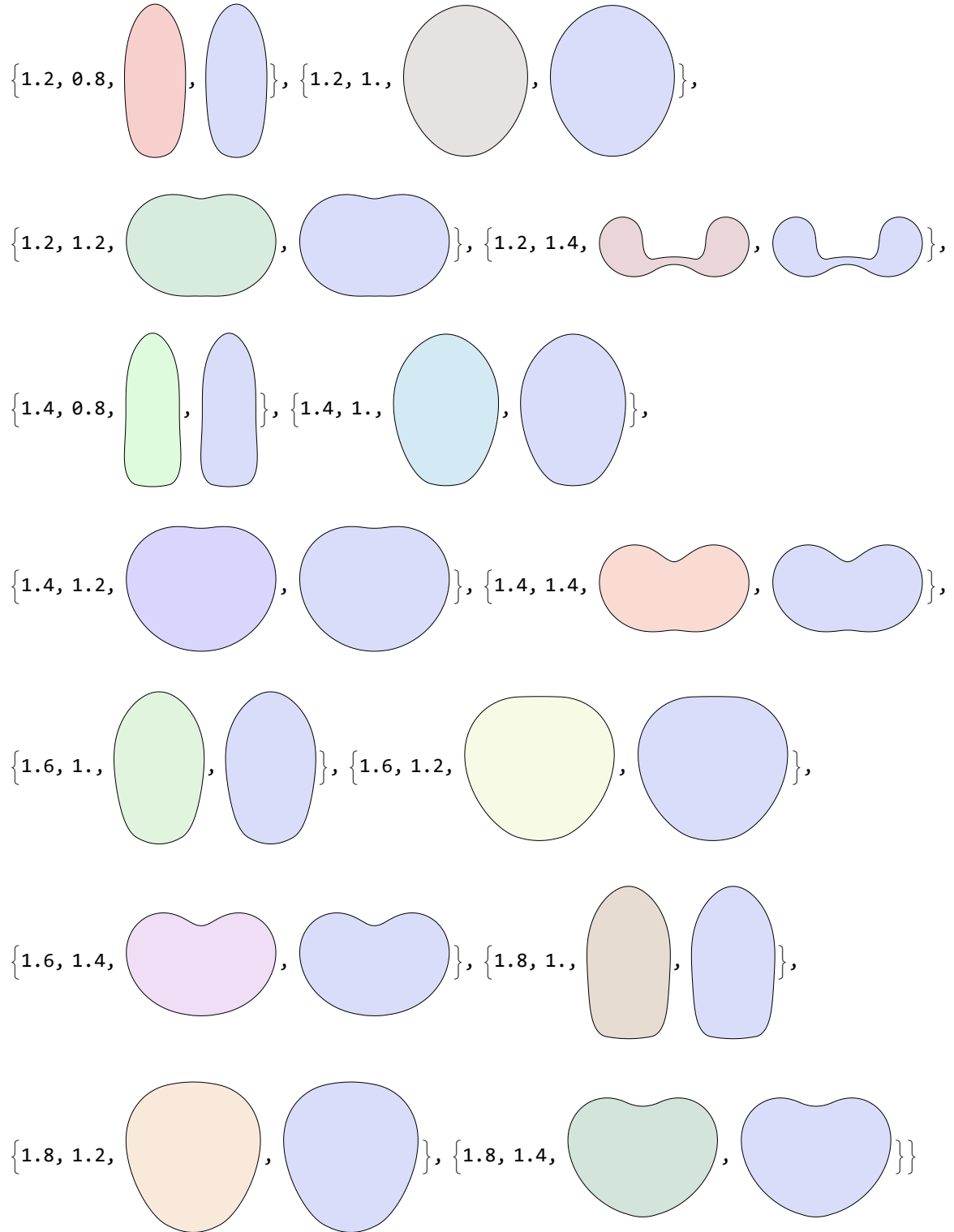
```

In[ ]:= res = membraneShapePhasePlane[];

In[ ]:= res1 = SortBy[res, First]

Out[ ]:=



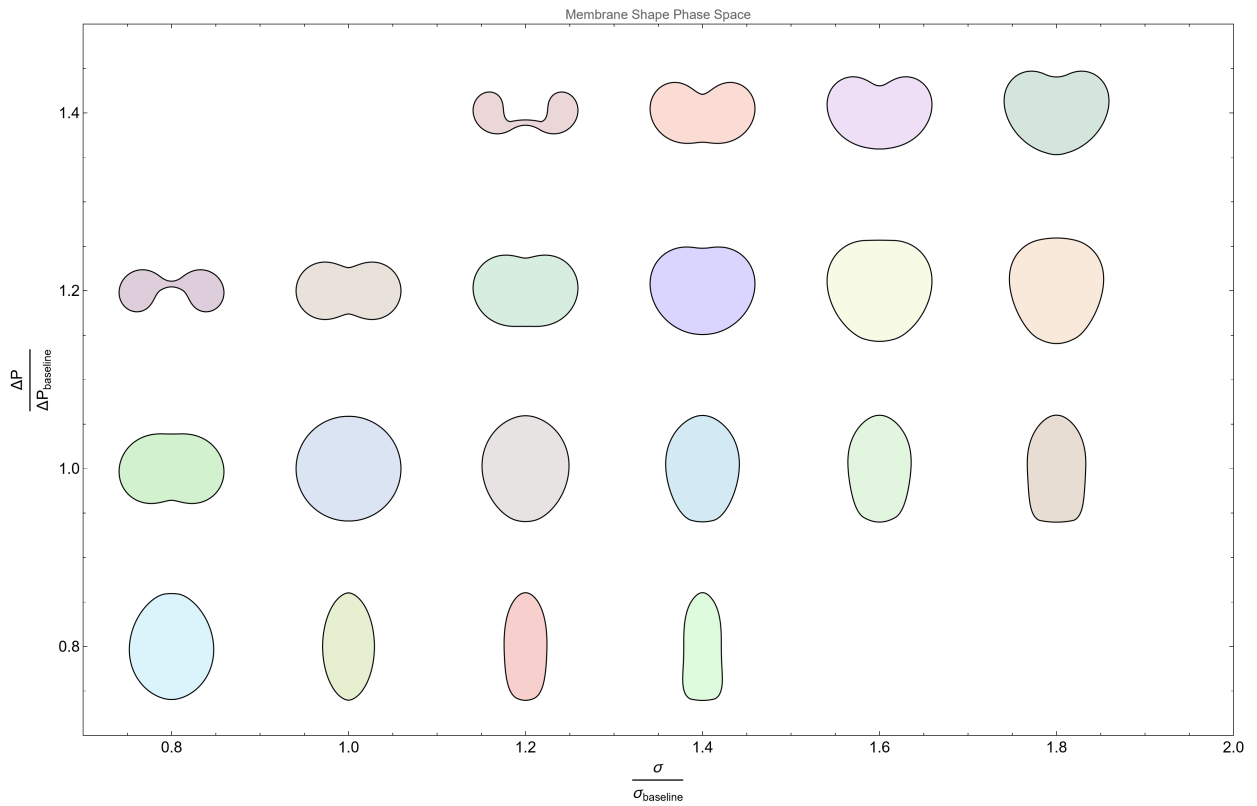


```

In[ ]:= Rasterize[
  ListPlot[Thread[{res[[All, 1]], res[[All, 2]]}], PlotStyle -> None,
  Epilog -> (Inset[#, {#[[1]], #[[2]]} & /@ res), ImageSize -> Full,
  PlotRange -> {{0.7, 2}, {0.7, 1.5}}, PlotLabel -> "Membrane Shape Phase Space",
  Frame -> True, FrameStyle -> Directive[Black, 14], FrameLabel -> {
    " $\frac{\sigma}{\sigma_{\text{baseline}}}$ ", " $\frac{\Delta P}{\Delta P_{\text{baseline}}}$ "}]]

```

Out[ ]=



```

In[ ]:= Rasterize[
  ListPlot[Thread[{res[[All, 1]], res[[All, 2]]}], PlotStyle -> None,
  Epilog -> (Inset[#[[-1]], {#[[1]], #[[2]]} & /@ res), ImageSize -> Full,
  PlotRange -> {{0.7, 2}, {0.7, 1.5}}, PlotLabel -> "Membrane Shape Phase Space",
  Frame -> True, FrameStyle -> Directive[Black, 14],
  FrameLabel -> {
    " $\frac{\sigma}{\sigma_{\text{baseline}}}$ ", " $\frac{\Delta P}{\Delta P_{\text{baseline}}}$ "
  }, FrameTicksStyle -> Blue]]

```

Out[ ]:=

