

Class Exercises: Probabilistic Analysis & Quicksort

Análisis y Diseño de Algoritmos

May 16th, 2024

Ejercicio 1. Let X be a random variable that represents the number of heads in two tosses of a fair coin. What is the value of $E[X^2]$? What is the value of $E[X]^2$?

Ejercicio 2. Let X be a random variable that represents the sum of the results in the roll of n dice. What is the value of $E[X]$?

Ejercicio 3. Consider the problem of hiring n people (HIRE-ASSISTANT). Let X_i be the random variable that is 1 if the i -th candidate is hired and 0 if not. Find X_1, X_2, X_3 when $n = 3$ and the candidates are $\{1, 2, 3\}$. Find X_1, X_2, X_3, X_4 when $n = 4$ and the candidates are $\{1, 2, 3, 4\}$.

Ejercicio 4. In the pseudocode of HIRE-ASSISTANT, assuming the candidates are presented in a uniformly random manner, what is the probability that exactly one candidate is hired? What is the probability that n candidates are hired? What is the probability that 2 candidates are hired?

Ejercicio 5. Consider the following algorithm that determines the largest and smallest element of a vector $v[1 \dots n]$ with distinct positive numbers.

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LARGESTSMALLEST( $v, n$ )
1:  $largest = v[1]$ 
2:  $smallest = v[1]$ 
3: for  $i = 2$  to  $n$ 
4:   if  $v[i] > largest$ 
5:      $largest = v[i]$ 
6:   else
7:     if  $v[i] < smallest$ 
8:        $smallest = v[i]$ 
9: return  $largest, smallest$ 
```

Assume the input of the algorithm is a permutation of 1 to n chosen uniformly from all permutations of 1 to n . What is the expected number of comparisons executed in line 7 of the algorithm? What is the expected number of assignments performed in line 8 of the algorithm?

Ejercicio 6. Illustrate the PARTITION operation on the array $A = [13, 19, 9, 5, 12, 8, 7, 4, 21, 2, 6, 11]$.

Ejercicio 7. Write a function that receives a vector with n letters A and B and, through swaps, moves all the A to the beginning of the vector. Your function should have an execution time of $O(n)$.

Ejercicio 8. What is the execution time of QUICKSORT when all the elements of the array A have the same value?

Ejercicio 9. Prove that $\max_{0 \leq q \leq n-1} \{q^2 + (n - q - 1)^2\} = (n - 1)^2$