

Probabilistic Analysis & Quicksort

CS3026 – Analysis & Design of Algorithms

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Probabilistic Analysis

HIRE-ASSISTANT(n)

```
1  best = 0          // candidate 0 is a least-qualified dummy candidate
2  for  $i = 1$  to  $n$ 
3      interview candidate  $i$ 
4      if candidate  $i$  is better than candidate best
5          best =  $i$ 
6      hire candidate  $i$ 
```

Figure 1: Cormen, Introduction to Algorithms

3 Quicksort

Algorithm

```
QUICKSORT( $A, p, r$ )
```

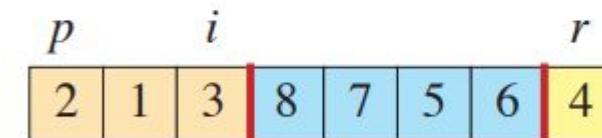
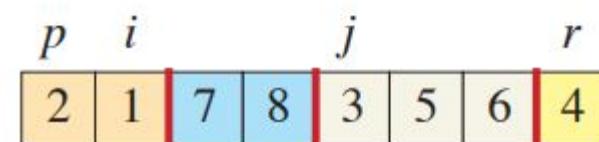
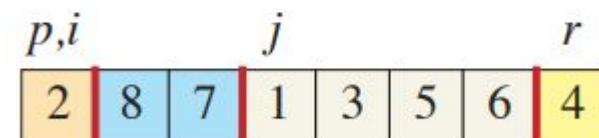
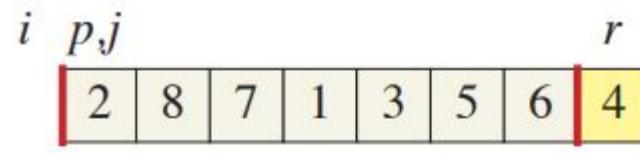
```
1  if  $p < r$ 
2      // Partition the subarray around the pivot, which ends up in  $A[q]$ .
3       $q = \text{PARTITION}(A, p, r)$ 
4      QUICKSORT( $A, p, q - 1$ ) // recursively sort the low side
5      QUICKSORT( $A, q + 1, r$ ) // recursively sort the high side
```

Partition

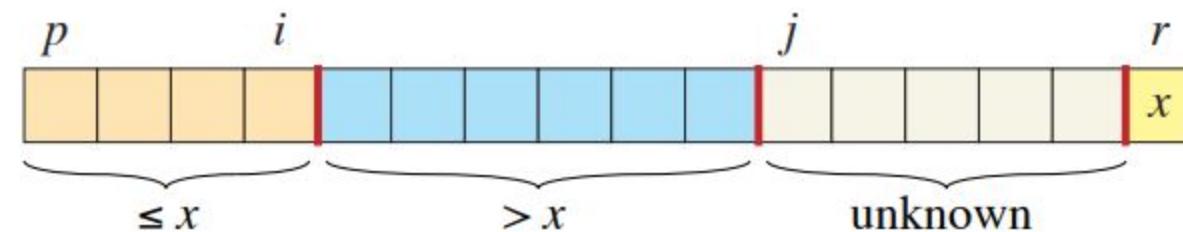
PARTITION(A, p, r)

```
1   $x = A[r]$                                 // the pivot
2   $i = p - 1$                             // highest index into the low side
3  for  $j = p$  to  $r - 1$           // process each element other than the pivot
4      if  $A[j] \leq x$                 // does this element belong on the low side?
5           $i = i + 1$                   // index of a new slot in the low side
6          exchange  $A[i]$  with  $A[j]$  // put this element there
7  exchange  $A[i + 1]$  with  $A[r]$  // pivot goes just to the right of the low side
8  return  $i + 1$                   // new index of the pivot
```

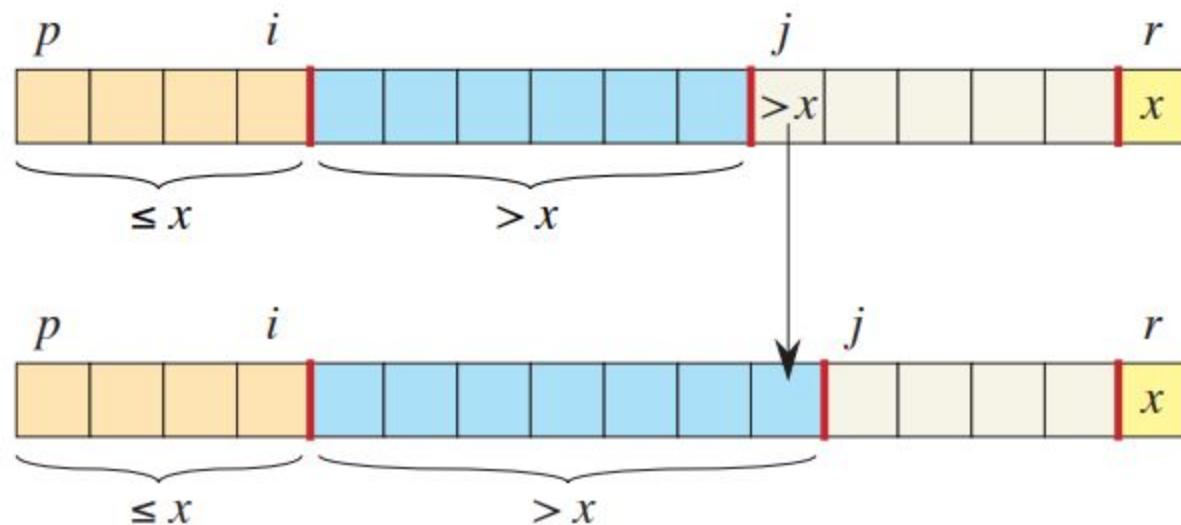
Visualization



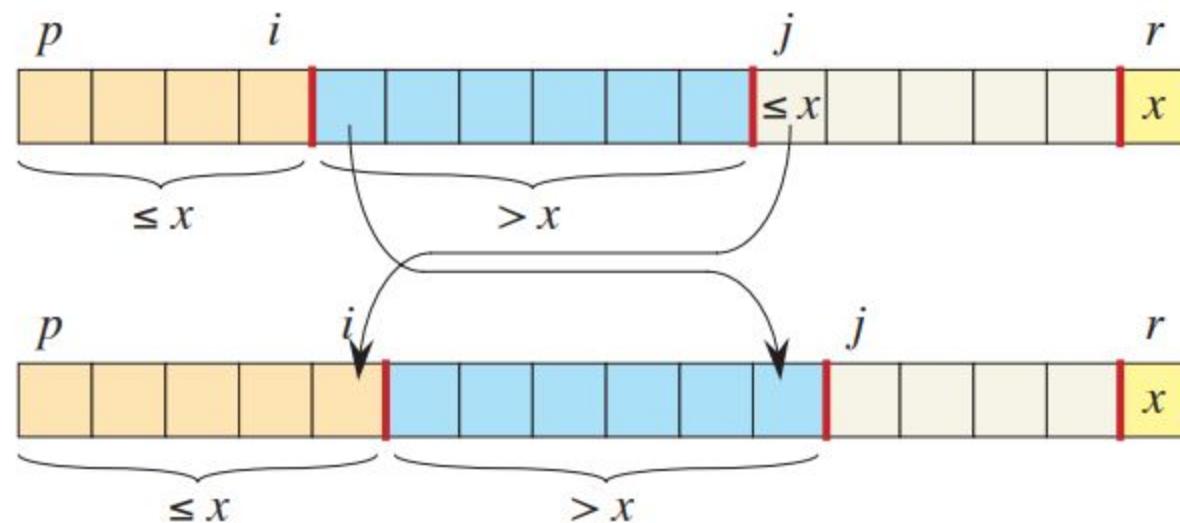
Invariant



$A[j] > x$



$A[j] \leq x$



General Formula

-

$$T(n) = \max \{T(q) + T(n - 1 - q) : 0 \leq q \leq n - 1\} + \Theta(n)$$

Visualization

$$T(n) \geq \min_{0 \leq q \leq n-1} \{ T(q) + T(n-1-q) \} + \Omega(n).$$

Thank you!

Let's keep finding mistakes!