Running times

ADA

24 de marzo de 2024

Ejercicio 1. Show the following equations using the definitions.

(a) $n^2 - 10n + 2 = O(n^2)$

(b) $\lceil n/3 \rceil = O(n)$

(c) $\lg n = O(\log_{10} n)$

(d) $n = O(2^n)$

(e) $\lg n$ is not $\Omega(n)$

(f) $n/100 \neq O(1)$

(g) $n^2/2 \neq O(n)$

(h) $\frac{1}{3}(n+1)(n-2) - 5 = \Theta(n^2)$.

(i) $n \lg n - \lceil 2n/3 \rceil - \lg n + 4 = \Omega(2n \lg n)$

(j) $\lg n! = \Omega(n \lg n)$

(k) $n \lg n - \lceil 2n/3 \rceil - \lg n + 4 = \Omega(2n \lg n)$

(l) $\lg n! = \Omega(n \lg n)$

Ejercicio 2. Prove or disprove

(a) $\lg \sqrt{n} = O(\lg n)$

(b) $f(n) = O(g(n))$ and $g(n) = O(h(n))$ imply $f(n) = O(h(n))$

(c) $f(n) = O(g(n))$ and $g(n) = \Theta(h(n))$ imply $f(n) = \Theta(h(n))$

(d) $f(n) = O(g(n))$ then $2^{f(n)} = O(2^{g(n)})$

(e) $o(g(n)) \cap \omega(g(n)) = \emptyset$

(f) $\max\{f(n), g(n)\} = \Theta(f(n) + g(n))$, for any nonnegative functions $f(n)$ and $g(n)$.

(*g*) $(n+a)^b = \Theta(n^b)$, where $a, b \in \mathbb{R}$ y $b > 0$.

(*h*) $\sqrt{n} = O(\lg^2 n)$.

(*i*) $\sum_{k=1}^{n} k^{99} = \Theta(n^{100})$

(*j*) Assume that $\lg(g(n)) \geq 1$ and $f(n) \geq 1$ for each positive integer n sufficiently large. In this case, if $f(n) = O(g(n))$ then $\lg(f(n)) = O(\lg(g(n)))$.

(*k*) $\sum_{k=1}^{n} k^{99} = \Theta(n^{100})$