Heap

Analysis and Design of Algorithms

January 1, 2026

Exercise 1. Show that a heap with n elements has height $\lfloor \lg n \rfloor$.

Exercise 2. Show by induction that there are at most $\lceil n/2^{h+1} \rceil$ nodes of height h in a heap with n nodes.

Exercise 3. Let $A[1..n]$ be a heap, let i be an index in the heap. What are the elements that appear in the subheap rooted at i? Express your answer as a mathematical expression in the most compact form possible.

Exercise 4. Illustrate the operation MAX-HEAPIFY starting at node 2 and node 3 in the array $[27, 3, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0]$.

Exercise 5. Illustrate the operation BUILD-MAX-HEAP on the array $[5, 3, 17, 10, 84, 19, 6, 22, 9]$. You must show all steps involved.

Exercise 6. Illustrate the HEAPSORT operation on the array $[5, 13, 2, 25, 7, 17, 20, 8, 4]$. You must show all steps involved.

Exercise 7. Illustrate the HEAPSORT operation on the array $[21, 3, 15, 25, 17, 12, 10, 4, 8]$. You must show all steps involved.

Exercise 8. Illustrate the operation MAX-HEAP-INSERT$(A, 10)$ on the heap
$[15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2]$.

Exercise 9. The operation HEAP-DELETE deletes the item at node i from heap A. Give an implementation of HEAP-DELETE that runs in $O(\lg n)$ for a max-heap with n elements.

Exercise 10. Write in pseudocode each of the following procedures, which implement a priority queue with a min-heap: HEAP-MINIMUM, HEAP-EXTRACT-MIN, HEAP-DECREASE-KEY, and HEAP-INSERT.

Exercise 11. Given an array $A[1..n]$ of distinct numbers, indicate an $O(n \lg k)$ algorithm that finds the k-th smallest element. For example, if $A = [3, 1, 2, 7, 5]$ and $k = 4$, your algorithm should return 5. **You must use a priority queue** with MAXHEAP.